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Semiempirical Method of Determining Flow Coefficients for Pitot Rake Mass Flow Rate Measurements

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SEMIEMPIRICAL METHOD OF DETERMINING FLOW COEFFICIENTS

FOR PITOT RAKE MASS FLOW RATE MEASUREMENTS

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SUMMARY

Pitot rakes are often used to measure mass flow rate in circular or annular ducts. Often the rakes are area weighted and a simple summation is used to determine the average velocity. Errors in flow rate measurement are inherent in this technique because of the discretization of the velocity profile. The error decreases as the number of tubes on the rakes increases, and resolution of the velocity profile improves. A study was conducted to determine the error in measuring mass flow rate with pitot rakes in an annulus. The ideal flow rate was determined by using a unique semiempirical analysis for fully developed, turbulent flow. The velocity profile obtained from this analysis was imposed on the pitot rake, and an area-weighted summation was used to determine the flow rate that the rake would indicate. Results in terms of flow coefficient, or the ratio of ideal to indicated flow rate, ranged from 0.903 for one probe placed at a radius dividing two equal areas to 0.984 for a 10-probe area-weighted rake. Flow coefficients were not a strong function of annulus hub-to-tip radius ratio for rakes with three or more probes.

INTRODUCTION

The measurement of airflow rate is of primary interest in the testing and development of aircraft propulsion systems. Several methods of measuring flow rate are available including choked-exit devices, venturis, and bellmouths. These methods generally require the installation of additional hardware for the specific purpose of measuring flow rate. Where installation of such hardware is impractical, other means of measuring flow rate must be devised. For inlet testing there is usually an array of total pressure rakes at the diffuser exit station to measure total pressure recovery and distortion. Such rakes are generally area weighted and may also be used to measure flow rate. Each ring (probe in each rake at a common radius) in the array is assigned an area of the circular or annular duct. If the rakes are area weighted, these are equal areas. This area combined with the measured ring total pressure, a representative static pressure, and the total temperature can be used to calculate a mass flow rate for that ring. Subsequent summation of all ring flow rates results in an estimate of the total mass flow rate. Because of the discrete nature of the measurements significant error exists in the indicated value of flow rate, especially in high velocity gradient regions such as boundary layers. Obviously the larger the number of probes on the rakes, the finer the resolution of the velocity profile and the smaller the error. To account for this discrepancy between the measured and ideal flow rates, experimentally determined flow coefficients applicable to a particular rake geometry and annulus hub-to-tip radius ratio have normally been used (ref. 1). However,

since the flow coefficient varies with the number of probes on the rakes and the hub-to-tip radius ratio of the annulus, a semiempirical procedure for determining an applicable flow coefficient would be desirable in many cases.

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This investigation was conducted to determine the error in the rake mass flow rate measurement for turbulent flow in an annulus due to the discretization of the velocity profile and the variation of this error with the number of probes on an area-weighted rake. Also investigated was the effect of annulus hub-to-tip radius ratio on the error.

A semiempirical method for determining the velocity profile in fully developed, turbulent flow in an annulus is presented. The integral of this velocity profile is compared with summation of velocities as measured by an area-weighted rake subjected to the same velocity profile. Results are presented as flow coefficients, or the ratio of the integrated profile to the summed, for area-weighted rakes of up to 10 probes and a range of annulus hubto-tip radius ratios.

SYMBOLS

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Aj	area	assigned	το ρι	rope	٦,	m-	

An integration constant for region n

Bn constant for region n

CF flow coefficient

C_n constant for region n

D_n constant for region n

En constant for region n

F(n) function of region n

F(x) function of variable x

I number of probes on rake

 K_n dimensionless distance from origin to inner radius of region n, r/R_t

 K'_n dimensionless distance from hub surface to inner radius of region n, (r - R_h)/(R_t - R_h)

m_n ratio of turbulent to laminar viscosity in region n

N number of regions in semiempirical method

p static pressure gradient in flow direction, Pa/m

r distance from origin in radial direction, m

2

R _h	annulus hub radius, m
Rt	annulus tip radius, m
u	dimensionless distance from origin in radial direction, r/R_t
v	velocity in z-direction, m/sec
v _{max}	maximum velocity in z-direction, m/sec
v _n	velocity in z-direction at interface of regions n and $n - 1$, m/sec
μ(2)	coefficient of laminar viscosity, Pa sec/m ²
μ(t)	coefficient of turbulent viscosity, Pa sec/m ²
τ <mark>(ջ)</mark> τz	laminar transport of z-momentum in r-direction, Pa/m^2
τ <mark>(t)</mark> rz	turbulent transport of z-momentum in r-direction, Pa/m^2
Subscript	S:
h	hub surface
1	individual probe on rake

N outermost	region	in sem	iempirica	method
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individual region in semiempirical method n

radial direction r

t tip surface

axial direction Z

Superscripts:

4:

- laminar (2)
- turbulent (t)

APPROACH

Derivation of Semiempirical Velocity Profiles

To develop the turbulent velocity profiles, a semiempirical approach is used under the following assumptions:

(1) Two-dimensional (axisymmetric) flow(2) Steady, fully developed flow

(3) Incompressible flow

(4) Linear pressure gradient in flow direction

Figure 1 depicts the flow situation and the coordinate system. The time-smoothed z-component of the momentum equation under the preceding assumptions becomes

$$p = \frac{1}{r} \frac{d}{dr} \left[r \left(\tau_{rz}^{(l)} + \tau_{rz}^{(t)} \right) \right]$$
(1)

Before equation (1) can be integrated to yield velocity profiles, an additional equation is needed to relate the laminar and turbulent flux of z-momentum in the radial direction to the radial velocity gradient. To develop this relation, the annular cross section is divided into an arbitrary number of annular regions N. Within each region it is assumed that the negative of the radial velocity gradient is proportional to the sum of the laminar and turbulent momentum fluxes. Further the constant of proportionality in a particular region n is the sum of the laminar viscosity and a turbulent viscosity that varies from region to region. In equation form

$$\tau_{rz}^{(l)} + t_{rz}^{(t)} = -(\mu^{(l)} + \mu_{n}^{(t)})\frac{dv}{dr}$$
(2)

Expressed in terms of the ratio of turbulent to laminar viscosity, which is constant in a particular region n, equation (2) becomes

$$\tau_{rz}^{(\ell)} + \tau_{rz}^{(t)} = -\mu^{(\ell)}(1 + m_n) \frac{dv}{dr}$$
(3)

where

$$m_{n} = \frac{\mu(t)}{\mu(\ell)}$$
(4)

Combining equations (1) and (3), there results for a region n

$$p = -\frac{1}{r} \frac{d}{dr} \left[r \mu^{(\ell)} (1 + m_n) \frac{dv}{dr} \right]$$
(5)

Integrating over the radial direction yields after rearrangement

$$\frac{dv}{dr} = -\frac{1}{\mu(\ell)(1+m_n)} \left(\frac{pr}{2} + \frac{A_n}{r}\right)$$
(6)

where $A_{\rm I}$ is the integration constant for region n. Defining $K_{\rm I}$ as the ratio of the inner boundary radius of region n to the annulus tip radius $R_{\rm t}$ and defining $V_{\rm I}$ as the velocity at the inner boundary radius of region n,

equation (6) can be integrated from the inner boundary of region n to an arbitrary radius within the region:

$$\int_{V_{n}}^{V} dv = -\frac{1}{\mu^{(\ell)}(1 + m_{n})} \int_{K_{n}R_{t}}^{r} \left(\frac{pr}{2} + \frac{A_{n}}{r}\right) dr$$
(7)

Finally for any region n

$$v(r) = V_{n} - \frac{1}{\mu^{(\ell)}(1 + m_{n})} \left\{ \frac{pR_{t}^{2}}{4} \left[\left(\frac{r}{R_{t}} \right)^{2} - K_{n}^{2} \right] + A_{n} \ln \left(\frac{r}{K_{n}R_{t}} \right) \right\}$$
(8)

Note that K_1 is equal to the hub-to-tip radius ratio and K_{N+1} is equal to 1. At the interface of two adjacent regions the solutions are "spliced" together by equating the velocity gradients. Writing equation (6) for regions n and n + 1 at a common radius K_{n+1} and equating the two resulting expressions yields

$$\frac{1}{1 + m_n} \left(\frac{pK_{n+1}R_t}{2} + \frac{A_n}{K_{n+1}R_t} \right) = \frac{1}{1 + m_{n+1}} \left(\frac{pK_{n+1}R_t}{2} + \frac{A_{n+1}}{K_{n+1}R_t} \right)$$
(9)

Solving for A_{n+1} results in a recursion relation

$$A_{n+1} = \left[\frac{1 + m_{n+1}}{1 + m_n} \left(\frac{pK_{n+1}R_t}{2} + \frac{A_n}{K_{n+1}R_t}\right) - \frac{pK_{n+1}R_t}{2}\right] K_{n+1}R_t$$
(10)

Velocities at the interfaces between regions can be computed by evaluating equation (8) at $r = K_{n+1}R_t$:

$$V_{n+1} = v(K_{n+1}R_t) = V_n - \frac{1}{\mu^{(2)}(1 + m_n)} \left[\frac{pR_t^2}{4} \left(K_{n+1}^2 - K_n^2 \right) + A_n \ln \left(\frac{K_{n+1}}{K_n} \right) \right]$$
(11)

Applying the no-slip boundary condition at the tip radius R, equation (11) becomes

$$0 = V_{N} - \frac{1}{\mu^{(2)}(1 + m_{N})} \left[\frac{pR_{t}^{2}}{4} \left(K_{N+1}^{2} - K_{N}^{2} \right) + A_{N} \ln \left(\frac{K_{N+1}}{K_{N}} \right) \right]$$
(12)

Now $\,V_{\rm N}\,$ can be successively replaced by using equation (11) until an equation of the form

$$V_{1} = \sum_{n=1}^{N} F(n)$$

$$F(n) = \frac{1}{\mu^{(2)}(1 + m_{n})} \left[\frac{pR_{t}^{2}}{4} \left(K_{n+1}^{2} - K_{n}^{2} \right) + A_{n} \ln \left(\frac{K_{n+1}}{K_{n}} \right) \right]$$
(13)

results. Since $V_{\rm l}$ is the velocity at the inner boundary of region l, it is the velocity at the hub surface and is zero. Hence

$$0 = \sum_{n=1}^{N} F(n)$$
 (14)

Equation (14) along with the N - 1 equations obtained from the recursion relation for A_n (eq. (10)) can be arranged into the following form:

$$C_1A_1 + C_2A_2 + \dots + C_{N-1}A_{N-1} + C_NA_N = B$$
 (14)

$$E_1 A_1 - A_2 = -D_1$$
 (10)

•

$$E_2 A_2 - A_3 = -D_2$$
 (10)

 $E_{N-1}A_{N-1} - A_{N} = -D_{N-1}$ (10)

where

$$B = -\frac{pR_{t}^{2}}{4}\sum_{n=1}^{N}\frac{K_{n+1}^{2} - K_{n}^{2}}{1 + m_{n}}$$
(15)

$$C_{n} = \frac{1}{1 + m_{n}} \ln\left(\frac{K_{n+1}}{K_{n}}\right)$$
 (16)

$$D_{n} = \frac{p(K_{n+1}R_{t})^{2}}{2} \left(\frac{1 + m_{n+1}}{1 + m_{n}} - 1 \right)$$
(17)

$$E_{n} = \frac{1 + m_{n+1}}{1 + m_{n}}$$
(18)

Cramer's rule can be used to solve for the integration constant in region 1 (A₁), after which repeated application of equation (10) yields all other A_n's. The velocity profile for the entire annulus is now computed by using equation (8), beginning at the hub surface (where $V_1 = 0$) and progressing to the outer radius of region 1. At the outer radius of region 1 a solution for V_2 results, and equation (8) is applied again for region 2. In a similar manner the solution progresses across the annulus to the tip surface, where the no-slip boundary condition has previously been satisfied in equation (14).

Values of turbulent-to-laminar viscosity ratio $m_{\rm R}$ and region boundaries, as well as the number of regions used N in this investigation are depicted in figure 2. These constants were found to best fit available experimental velocity profile data over a wide range of hub-to-tip radius ratios with only the region 5 viscosity ratio m_5 varying with hub-to-tip radius ratio. In figure 3 nondimensional velocity profiles obtained by this procedure show good agreement with the experimental data of reference 2.

Calculation of Flow Coefficients

For the purpose of this analysis flow coefficient is defined as the ideal flow rate divided by the measured or indicated flow rate. The ideal flow rate is determined by using the semiempirical velocity profile. The indicated flow rate is determined by using an area-weighted summation of velocity values taken from the semiempirical profile at the rake probe locations. Under the assumption of incompressible flow the flow coefficient reduces to the following:

$$C_{F} = \frac{2 \int_{K_{1}}^{1} v(u)u \, du}{(1 - K_{1}^{2}) \sum_{i=1}^{I} A_{i}v_{i}} \qquad \left(u = \frac{r}{R_{t}}\right) \qquad (19)$$

where v_1 is the velocity at probe location i, A_1 is the annular area fraction associated with that probe, and I is the number of probes on the rake. If the rake is area weighted, all A_1 are equal. Solutions to equation (19) were obtained by using the Fortran computer program listed in the appendix. The program evaluates the numerator by applying a sixth-order numerical integration technique known as Weddle's method to equation (8). Inputs to the program are annulus hub-to-tip ratio and the number of probes on the rake. A subroutine automatically locates the probes at area-weighted radii although provisions exist to input other rake geometries through an input data set. Turbulent-to-laminar viscosity ratios, region boundaries, and the number of regions used in conjunction with equation (8) are input through a separate input data set. All results presented were obtained by using the values in figure 2.

RESULTS

Figure 4 depicts the differences between the ideal velocity profile and the discretized profile that is summed in obtaining the indicated flow rate. For all probes compensating errors occur. However, probes near the hub and tip surfaces will clearly overpredict the flow rate because of the no-slip condition at these surfaces. The discretized profiles more closely approximate the ideal profile as the number of probes increases. Figure 5 presents the flow coefficients obtained with the Fortran program for area-weighted rakes with one to 10 probes over a range of annulus hub-to-tip radius ratios.

CONCLUSIONS

An investigation was conducted to determine the error in the rake mass flow rate measurement for turbulent flow in an annulus due to the discretization of the velocity profile and the variation of this error with the number of probes on an area-weighted rake. The following conclusions were drawn:

1. The semiempirical method presented for determining fully developed, turbulent velocity profiles in an annulus agreed adequately with experimental data.

2. Flow coefficients ranged from 0.903 for one probe placed at a radius dividing two equal areas to 0.984 for a 10-probe area-weighted rake.

3. Flow coefficients were not a strong function of annulus hub-to-tip radius ratio for rakes having three or more probes.

APPENDIX - COMPUTER PROGRAM LISTING

00100	C	
0300	C	UUNUI.SUUKCE.IAPVA
0400	C C	THIS PROGRAM COMPUTES SEMI-EMPIRICAL VELOCITY PROFILES FOR FULLY-DEVELOPED TURBULENT FLOW IN AN ANNULUS. THESE PROFILES ARE COMPARED TO DISCRETIZED PROFILES AS INDICATED
0600	C	BY A PITOT RAKE SUBJECTED TO THE SEMI-EMPIRICAL FLOW FIELD. FLOW COEFFICIENTS ARE
0800	č	FLOWRATE IS OBTAINED BY A NUMERICAL INTEGRATION OF THE SEMI-EMPIRICAL PROFILE. THE
1000	C	INDICATED FLOWRATE IS OBTAINED BY AN AREA-WEIGHTED SUM OF VELOCITIES OBTAINED FROM The semi-empirical velocity profile at the rake probe locations. Results are output
1100	C C	GRAPHICALLY.
1300	•	DOUBLE PRECISION CAPPA(10), VISCO(10)
1500		DOUBLE PRECISION VISCUL, PRESS, RADIUS DOUBLE PRECISION B(10), C(10), D1, D2, D3, D(10), E(10), BSUM, ATERM, ANUM, ADEN, A(10)
1600	с	DOUBLE PRECISION V(2000),VINTER(10),RATIO,VREF,H(10),HSUB(10),VSUM,SUBSUM(10),P1,P2,P3,RV,COEFF(7)
1800		DIMENSION NINT(10),VEL(2000),RAD(2000)
2000		DIMENSION PFLOW(2)
2200	с	DATA COEFF/1.,5.,1.,6.,1.,5.,1./
2300		DIMENSION XVARS(10), YVARS(10)
2500	ç	
2700	Схихии	INPUT REGION GEOMETRY AND FLOW VARIABLES
2800	C	READ(5,1000) NREG
3000		READ(5,1500) ALPHA
3200		READ(5,1100) CAPPA(I), NINT(I), VISCO(I)
3400	100	CAPPA(I)=ALPHA+(I-ALPHA)*CAPPA(I) CONTINUE
3500	C	READ(5.1200) VISCOL
3700		READ(5,1200) PRESS
3900	c	
4100	с	CAPPA(NREG+1)=1.
4200	Сккхки	COMPUTE INTEGRATION CONSTANT FOR REGION ONE
4400	Ċ	DO 200 N=1.NRFG
4600	c	
4800		C(N)=(DLOG(CAPPA(N+1)/CAPPA(N)))/(1+VISCO(N))
5000		D2=PRE55*(CAPPA(N+1)**2)*(RADIUS**2)/2
5200		D(N)=D2*(D1-1) F(N)=D1
5300 5400	~	
	200	CONTINUE
15500	200 C	CONTINUE
5500 5600 5700	200 C C C C X X X X X X	CONTINUE <solve a(1)="" a(n)'s<="" and="" compute="" for="" matrix="" nxn="" remaining="" td=""></solve>
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155000 155000 155000 155000 159000 160000 162000 162000 164000 164000 164000 164000 166000 166000 166000 172000 1770000 1770000 1770000 1770000 1770000 1770000 1770000 1770000 17700000000	C 200 C 220 C 220 C 240 C 260 C 260 C 300	CONTINUE SOLVE NXN MATRIX FOR A(1) AND COMPUTE REMAINING A(N)'S BSUM=0 DO 220 I=1,HREG BSUM=BSUM=BSUM=BS(1) CONTINUE ATERM=D(1) ANUM=ANUM-C(1)*ATERM ATERM=ATERMAE(1)+D(1) CONTINUE ATERM=1 ADEH=0 DO 260 I=1,HREG ADEH=ADEH+C(1)*ATERM ATERM=1 ATERM=1 ATERM=C(1) CONTINUE A(1)=ANUM/ADEN DO 300 H=2,NREG A(N)=D(N-1)+E(N-1)*A(N-1) CONTINUE
	C 200 C 220 C 220 C 240 C 260 C 260 C 300 C 300 C 240	CONTINUE SOLVE NXN MATRIX FOR A(1) AND COMPUTE REMAINING A(N)'S BSUM=0 DO 220 I=1,NREG BSUM=BSUM=BSUM=BSUM=B(1) CONTINUE ATERM=D(1) ANUM=ANUM-C(I) ANUM=ANUM-C(I) ATERM=ATERMAE(I)+D(I) CONTINUE ATERM=1 ADEN=0 DO 260 I=1,NREG ADEN=ADEN+CC(I) ATERM=ATERMAE(I) CONTINUE A(1)=ANUM/ADEN DO 300 N=2,NREG A(N)=D(N-1)+E(N-1)*A(N-1) CONTINUE A(U)=ANUM/ADEN DO 300 N=2,NREG A(N)=D(N-1)+E(N-1)*A(N-1) CONTINUE
155000 155000 155000 155000 159000 1661000 1662000 1662000 1662000 1664000 1665000 1665000 1665000 1665000 1665000 1665000 1771000 17750000 17750000 17750000000000	с 200 С 220 с 220 с 240 с 260 с 300 с хинин)	CONTINUE KSOLVE HXH MATRIX FOR A(1) AND COMPUTE REMAINING A(N)'S BSUM=0 D0 220 I=1,NREG BSUM=5(1) CONTINUE ATERM=D(1) ANUM=SSUM D0 240 I=2,NREG ANUM=ANUM-C(1)*ATERM ATERM=ATERM*AC(1)*D(1) CONTINUE ATERM=1 ADEN=0 D0 260 I=1,NREG ADEN=0 D0 260 I=1,NREG ADEN=0 D0 260 I=1,NREG ADEN=0 D0 260 I=1,NREG ADEN=0 D0 260 I=1,NREG ACDEN=0 D0 260 I=1,NREG ACDEN=0 D0 260 I=1,NREG ACDEN=0 D0 260 I=1,NREG ACDEN=0 D0 260 I=1,NREG ACDEN=0 D0 300 N=2,NREG A(1)=ANUM/ADEN D0 300 N=2,NRE0 A(1)=ANUM/ADEN D0 300 N=2,NRE0 A(1)=CN-1)+E(N-1)*A(N-1) CONTINUE CONTINUE
15500 15500 15500 15500 15500 15500 15500 15500 15500 15500 15500 15500 15500 15500 15500 166000 166000 1667000 1667000 1667000 1667000 177000 1775000 1775000 1775000 1775000 1775000 1775000 1775000 1775000 1775000 1775000 1775000 1775000 1775000 1830000 1830000 1840000 1850000 1850000 18870000 18870000 18870000 18870000 18870000 18870000 18870000 1880000 1880000	с 200 С 220 с 220 с 240 с 260 с 300 с схижи	CONTINUE CONTINUE ASOLVE MXN MATRIX FOR A(1) AND COMPUTE REMAINING A(N)'S BSUM=0 D0 220 1=1,NREG BSUM=5(1) CONTINUE ATERM=0(1) ANUM=SSUM D0 240 1=2,NREG ANUM=ANUM-C(1)*ATERM ATERM=1ERM*E(1)+D(1) CONTINUE ATERM=1 ADEN=0 D0 260 1=1,NREG ADEN=0 D0 260 1=1,NREG ADEN=0 D0 260 1=1,NREG ADEN=0 D0 260 1=1,NREG ATERM=ATERM*E(1)+D(1) CONTINUE A(1)=ANUM-ADEN D0 300 H=2,NREG A(1)=ANUM-ADEN D0 300 H=2,NREG A(1)=2,NREG A(1)=2,NREG A(1)=2,NREG A(1)=1)*E(N-1)*A(N-1) CONTINUE (NUMERICALLY INTEGRATE VELOCITY PROFILE FROM CAPPA(1) TO 1
15500 15500 155700 155700 155700 155700 155700 155700 155700 155700 155700 155700 155700 155700 155700 161000 162000 164000 164000 164000 164000 164000 164000 164000 164000 164000 174000 177000 177000 177000 177000 177000 177000 177000 177000 177000 177000 177000 183000 184000 184000 184000 184000 184000 184000 184000 184000 184000 184000 184000 </th <th>C 200 C 220 C 220 C 240 C 260 C 300 C 260 C 300 C 260 C 200 C 200</th> <th>CONTINUE (SOLVE NXN MATRIX FOR A(1) AND COMPUTE REMAINING A(N)'S BSUM=0 DD 220 1=1,HREG BSUM=SUM-B(1) CONTINUE ATERM=D(1) ATERM=D(1) ANUM=SUM DD 240 1=2,REG ANUM=ASUM DD 240 1=2,REG ANUM=ASUM CONTINUE ATERM=ATERMAE(1)+D(1) CONTINUE ATERM=ATERMAE(1)+D(1) CONTINUE ATERM=ATERMAE(1) CONTINUE A(1)=ANUM/ADEN DO 300 N=2,REG A(N)=D(N-1)+2(N-1)+A(N-1) CONTINUE (NUMERICALLY INTEGRATE VELOCITY PROFILE FROM CAPPA(1) TO 1 VSUM=0. VSUM=0</th>	C 200 C 220 C 220 C 240 C 260 C 300 C 260 C 300 C 260 C 200 C 200	CONTINUE (SOLVE NXN MATRIX FOR A(1) AND COMPUTE REMAINING A(N)'S BSUM=0 DD 220 1=1,HREG BSUM=SUM-B(1) CONTINUE ATERM=D(1) ATERM=D(1) ANUM=SUM DD 240 1=2,REG ANUM=ASUM DD 240 1=2,REG ANUM=ASUM CONTINUE ATERM=ATERMAE(1)+D(1) CONTINUE ATERM=ATERMAE(1)+D(1) CONTINUE ATERM=ATERMAE(1) CONTINUE A(1)=ANUM/ADEN DO 300 N=2,REG A(N)=D(N-1)+2(N-1)+A(N-1) CONTINUE (NUMERICALLY INTEGRATE VELOCITY PROFILE FROM CAPPA(1) TO 1 VSUM=0. VSUM=0
	$\begin{array}{c} 0.2 \\ 0.0 \\$	10200 CHRHH 10300 C 10400 C 10500 C 10500 C 10800 C 1000 C 1100 C 1100 C 1100 C 1100 C 1100 C 1100 C 1100 C 1100 C 1200 C 1200 C 1200 C 1200 C 1200 C 1400

SUBSUM(N)=0. H(N)=(CAPPA(N+1)-CAPPA(N))/NIHT(N) HSUB(N)=H(N)/6. RATIO=CAPPA(N) 0009400 0009500 0009600 0009700 0009800 C 0009900 0010000 0010100 C NSTOP=NINT(N) DO 500 NSUB=1,NSTOP 0010100 C 0010200 0010300 C 0010400 C 0010500 C 0010600 C 0010700 0010800 JSTOP=JSTART+6 K=1 DO 400 J=JSTART, JSTOP P1=1./(VISCOL*(1+VISCO(N))) P2=PRESS*RADIUS**2*(RATIO**2-CAPPA(N)**2)/4 0010900 0011000 0011100 0011200 C P3=A(H)*DLOG(RATIO/CAPPA(H)) V(J)=VINTER(H)-P1*(P2+P3) RV=V(J)×RATIO 0011200 C 0011300 0011400 0011500 0011700 C 0011800 0011900 C 0012000 0012200 C 0012200 C 0012400 C 0012500 0012600 C RAD(J)=RATIO SUBSUM(N)=SUBSUM(N)+COEFF(K)×RV K=K+1 RATIO=RATIO+HSUB(N) 400 CONTINUE RATID=CAPPA(N)+NSUB*H(N) JSTART=J **500 CONTINUE** SUBSUM(N)=.3×HSUB(N)×SUBSUM(N) VSUM=VSUM+SUBSUM(N) VINTER(N+1)=V(J) U12700 VINTER(N+1)-V(J) 012800 C 012900 600 CONTINUE 013000 C 013000 C 013200 CHHXHHCOMPUTE POINT OF MAXIMUM VELOCITY (VREF), AND VAVG 013200 C 013400 N=(NREG+1)/2 013500 C 013500 C 013500 C 013600 P1=1./(VISCOL*(1+VISCO(N))) 013700 P2=PRESS*RADIUSHM2+(RATIOHX-CAPPA(N)HM2)/4 014000 P3=A(H)+D106(RATIO/CAPPA(H)) 01400 VREF=VINTER(N)-P1*(P2+P3) 014400 VREF=VINTER(N)-P1*(P2+P3) 014400 VAVG=2HVSUM/(VREF*(1-CAPPA(1)HM2)) 014600 C 014600 C 014600 D0 650 I=1,JSTOP 014900 VEL(I)=V(I)/VREF 015000 650 CONTINUE 015100 C VADEC(J)=0 0012800 C CLOT THEORETICAL 00 D0 650 I=1, JSTOP 014900 VEL(I)=V(I)/VREF 0015000 650 CONTINUE 0015100 C 0015200 XVARS(1)=9 0015300 XVARS(2)=6. 0015400 XVARS(3)=0. 0015500 XVARS(5)=1 0015700 XVARS(5)=1 0015900 001690 XVARS(1)=9 XVARS(2)=6. XVARS(3)=0. XVARS(5)=1. XVARS(5)=1. XVARS(6)=5. XVARS(6)=6. XVARS(8)=6. XVARS(8)=6. 0015900 0016100 0016200 C CALL XAXIS(1.,1.,XVARS) 0016300 0016400 0016500 YVARS(1)=9 YVARS(2)=8. YVARS(2)=8. YVARS(3)=90. YVARS(4)=CAPPA(1) 0016500 0016600 0016700 0016800 0016900 0017000 0017100 YVARS(5)=1. YVARS(5)=5. YVARS(6)=5. YVARS(7)=1. YVARS(8)=2. YVARS(9)=0. CALL YAXIS(1.,1.,YVARS) 0017200 0017300 C 0017400 0017500 C CALL CORNER(1) 0017500 0017700 0017700 0017800 0017900 0018000 0018100 IVARS(1)=2 IVARS(2)=JSTART CALL GPLOT(VEL,RAD, IVARS) CALL AVRAD(VEL,RAD, JSTART, VAVG, RADH, RADT)
 0018100
 CALL AVRADUCEL, RAD, START, VAUDARADD, RADD, R TSUM=0. READ(6,1300) NTUBE 0018800 0018900 C

```
DO 700 I=1,NTUBE
Read(6,1400) TLOC(I),AREA(I)
700 Continue
 0019000
0019100 DU /00 1-1,000

0019100 READ(6,1400) TLOC(I),AREA(I)

0019200 700 CONTINUE

0019300 C

0019400 C

0019500 CXXXXXCOMPUTE POINTS OF DISCONTINUITY

0019600 C

4TOT=1-CAPPA(1)XX2
 0019700
0019800
                                                  ATOT=1-CAPPA(1)**2
                                                 DISC(1)=CAPPA(1)
 0019900 C
0020000
                                                 ISTOP=NTUBE+1
 0020100
                                                 D0 750 I=2,ISTOP
DISC(I)=59RT((AREA(I-1)*ATOT)+DISC(I-1)**2)
0020200 DISC(1)=SQRT((AREA(I-1)*ATOT)+DISC(I-1)**2)
0020300 750 CONTINUE
0020400 C
0020500 C
0020500 C
0020700 C*****COMPUTE VELOCITY AT EACH TUBE LOCATION AND DO WEIGHTED SUM
0020800 C
0020900 DO 800 J=1,NTUBE
 0021000 C
0021100
                                  DO 850 N=1,NREG
IF(CAPPA(N+1).GE.TLOC(J))GO TO 875
850 CONTINUE
875 CONTINUE
 0021100
0021200
0021300
0021400
0021500 C
0021600
                                                  RATIO=TLOC(J)
 0021700 C
                                                 P1=1./(VISCOL*(1+VISCO(N)))
P2=PRESS*RADIUS**2*(RATIO**2-CAPPA(N)**2)/4
P3=A(N)*DLOG(RATIO/CAPPA(N))
V(J)=(VINTER(N)-P1*(P2+P3))/VREF
TSUM=TSUM+AREA(J)*V(J)
 0021800
 0022000
0022100
0022200

        0022200
        TSUM=TSUM+AREA(J)*V(J)

        0022300
        C

        0022400
        800

        0022500
        C

        0022600
        C

        0022600
        C

        0022600
        C

        0022800
        CFLOW=VAVG/TSUM

        0023100
        C

        0023200
        CHLOW=VAVG/TSUM

        0023100
        C

        0023300
        C

        0023300
        C

        0023400
        NPOINT=NTUBE*2

        0023500
        C

        0023400
        D0 900

        0023500
        C

0023500 C
0023600
0023700
0023900
0024000
0024100 C
0024200
0024200
0024400
0024400
0024500
                                  DO 900 I=1,NTUBE
II=I×2
PVEL(II)=V(I)
PVEL(II-1)=V(I)
900 CONTINUE
                                                 PLOC(1)=CAPPA(1)
D0 950 I=1,NTUBE
II=I*2
PLOC(II)=DISC(I+1)
PLOC(II+1)=DISC(I+1)
CONTINUE
 0024600
                                   950 CONTINUE
 0024700
0024800 C
0025000 C
0025100
0025200 C
0025300 C
                                                 IVARS(2)=NPOINT
                                                 CALL GPLOT(PVEL, PLOC, IVARS)
 0025400
                                                 DO 975 I=1,NTUBE
TVEL(I)=V(I)
0025500
0025500
0025700 C
0025900
002600
0026100
0026400 C
0026500
0026400 C
0026500
0026500
0026700 C
0026900
0027000
0027000 C
                                   975 CONTINUE
                                                 IVARS(1)=6
IVARS(2)=HTUBE
IVARS(3)=3
IVARS(4)=62
                                                 IVAR5(6)=20
                                                 CALL GPLOT(TVEL,TLOC,IVARS)
                                                 CALL NUMBER(4,CFLOW,6,4,PFLOW)
CALL CHARS(6,PFLOW,0,4.8,.5,15)
CALL CHARS(12,'FLOW COEFF =',0,3.,.5,15)
CALL CHARS(12, FLOW COEFF =

0027100 C

0027200 PRINT1500, CFLOW

0027300 CALL DISPLA(1)

0027400 C

0027600 C

0027600 C

0027600 C

0027700 C

0027800 CHARS(12)

0028000 1000 FORMAT(12)

0028000 1000 FORMAT(F10.5)

0028200 1200 FORMAT(F10.5)

0028500 1500 FORMAT(F10.5)

0028500 1500 FORMAT(F10.5)

0028600 C

0028700 END
  0028700
                                                  END
```

11

REFERENCES

- Sanders, Bobby W.: Dynamic Response of a Mach 2.5 Axisymmetric Inlet and Turbojet Engine with a Poppet-Valve-Controlled Inlet-Stability Bypass System when Subjected to Internal and External Airflow Transients. NASA TP-1531, 1980.
- 2. Brighton, J.A.; and Jones, J.B.: Fully Developed Turbulent Flow in Annuli. J. Basic Eng., vol. 86, no. 4, Dec. 1964, pp. 835-844.



Figure 1. - Annulus nomenciature and coordinate system.



Figure 2. - Annular region geometry and turbulent-to-laminar viscosity ratios used in semiempirical scheme.













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16. Abstract						
Flow coefficients applica ments are presented for f lent velocity profile is radius ratio and integrat The calculated velocities rate as indicated by the the particular rake geome rate by the rake-indicate one probe placed at a rad area-weighted rake. Flow hub-to-tip radius ratio f method used to generate t	ble to area-weig ully developed, generated semiem ed numerically t at each probe l rake is obtained try is subsequen ed flow rate. Fl ius dividing two coefficients we or rakes with th the turbulent velo	hted pitot rake turbulent flow pirically for a o determine the ocation are then . The flow coef tly obtained by ow coefficients equal areas to re not a strong ree or more prof ocity profiles	mass flow rat in an annulus. given annulus ideal mass fl n summed, and fficient to be dividing the ranged from O 0.984 for a 1 function of a bes. The semi is described i	e measure- A turbu- hub-to-tip ow rate. the flow used with ideal flow .903 for 0-probe nnulus empirical n detail.		
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