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A CONTROL SYSTEM DESIGN APPROACH FOR FLEXIBLE SPACECRAFT

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Abstract

A control system design approach for flexible spacecraft is presented. The control system design is carried out in two steps. The first step consists of determining the "ideal" control system in terms of a desirable dynamic performance. The second step consists of designing a control system using a limited number of actuators that possess a dynamic performance that is close to the ideal dynamic performance. The effects of using a limited number of actuators is that the actual closed-loop eigenvalues differ from the ideal closed-loop eigenvalues. A method is presented to approximate the actual closed-loop eigenvalues so that the calculation of the actual closed-loop eigenvalues can be avoided. Depending on the application, it also may be desirable to apply the control forces as impulses. The effect of digitizing the control to produce the appropriate impulses is also examined.

Introduction

A great deal of work has been dedicated to the development of structural control theories. Although the developments are extensive, the designer often finds it difficult to apply many of these theories to "real structural control problems." Indeed, it is of primary concern to bridge the gap between engineering design and the structural control theories.

The focus of the research into structural control theories is diverse. Many of the researchers are concerned with practical implementation problems and toward that end, promote decentralized control (Refs. 1 and 2). Others promote a centralized modal control approach and toward that end, point out that a control theory should not destroy certain characteristics which are natural to a structure (Refs. 3-5). Still others, in search for a global optimum, are concerned with distributed controls (Refs. 6 and 7). Much attention is also given to describing the robustness of the control theories in the presence of modelling errors, particularly in view of the fact that it is difficult to characterize structural stiffness in mathematical models (Refs. 8-10).

All of these concerns support the objective to uniformly dampen the motion of a spacecraft. As it turns out, a uniform damping control is a robust, decentralized, natural control with near globally optimal performance (Ref. 11). Thus, a uniform damping control answers the concerns raised in the previously cited references.

In this paper, an engineering design approach to structural control is described. The design of a uniform damping control system is carried out in two independent steps. The first step consists of identifying the solution which leads to the ideal dynamic performance. Toward that end, one recognizes that the state of a spacecraft is distributed over its domain, implying that the ideal dynamic performance will require distributed actuation and sensing devices. On the other hand, it is recognized that the use of these distributed devices is, for the most part, impractical. The second step consists of constructing a control system of minimal cost which exhibits dynamic performance that is as close as possible to the ideal. Therefore, the second step consists of implementing the uniform damping control obtained in the first step using discrete actuation and discrete sensing devices. As it turns out, ideal performances can be obtained with a relatively small number of actuators.

II. Mathematical Description

The equations of motion of a flexible structure can be expressed in the form

$$M\ddot{\underset{\sim}{x}}(t) + K\underset{\sim}{x}(t) = \underset{\sim}{F}(t) \tag{1}$$

where $\underset{\sim}{x}(t)$ is an n-dimensional vector of nodal displacements and slopes and $\underset{\sim}{F}(t)$ are forces and moments at the corresponding nodes. M and K denote n by n mass and stiffness matrices, respectively, and overdots represent differentiations with respect to time. The mass and stiffness matrices are obtained using the finite element method. Common computer programs capable of generating the mass and stiffness matrices include NASTRAN and SAP.

Associated with the equations of motion, one commonly defines the eigenvalue problem

$$\lambda M\underset{\sim}{\phi} = K\underset{\sim}{\phi} \tag{2}$$

The solution of this problem is known as the eigensolution which consists of the eigenvector $\underset{\sim}{\phi}$ and the associated eigenvalue λ. There exist n eigensolutions, i.e. n eigenvectors $\underset{\sim}{\phi}_r$ (r = 1, 2, ..., n) and n associated eigenvalues λ_r (r = 1, 2, ..., n). Structural dynamicists commonly refer to the eigenvectors as natural modes of vibration. The associated eigenvalues are related to the natural frequencies ω_r by $\lambda_r = \omega_r^2$ ($r = 1, 2, \ldots, n$). As a general rule of thumb, the computed eigensolution with higher associated natural frequencies are inexact. Indeed, modelling error will significantly effect these quantities. Only the eigensolutions with lower associated natural frequencies can be computed accurately. However, more often than not, we are only concerned with the lower modes, so this presents no difficulty. NASTRAN and SAP are two typical computer programs capable of computing the eigensolution (Ref. 12).

The natural modes can be normalized so that

$$\underset{\sim}{\phi}_r^T M \underset{\sim}{\phi}_s = \delta_{rs} \tag{3}$$

where $\delta_{rs} = 0$ for $r \neq s$ and $\delta_{rr} = 1$. We express the displacement vector $\underset{\sim}{X}(t)$ as a linear combination of the lowest m modes, written

$$\underset{\sim}{X}(t) = \underset{\sim}{\phi}_1 u_1(t) + \underset{\sim}{\phi}_2(t) u_2(t) + \ldots + \underset{\sim}{\phi}_m u_m(t) \tag{4}$$

where $m \ll n$, and $u_r(t)$ ($r = 1, 2, \ldots, m$) are modal displacements which express the degree to which the modes participate in the system response. Generally, the higher modes do not contribute significantly in the response so they are not included in Equation (4). The modal displacements are governed by the scalar equations,

$$\ddot{u}_r(t) + \omega_r^2 u_r(t) = f_r(t), \quad (r = 1, 2, \ldots, m) \tag{5}$$

where the modal forces $f_r(t)$ are related to the nodal force $\underset{\sim}{F}(t)$ by

$$f_r(t) = \underset{\sim}{\phi}_r^T \underset{\sim}{F}(t), \quad (r = 1, 2, \ldots, m) \tag{6}$$

We have assumed here that the modes are normalized, i.e. that Equation (3) is satisfied. It remains to compute the modal displacements in Equation (6). Toward this end, we first distinguish between rigid-body modes for which $\omega_r = 0$ and flexible-body modes for which $\omega_r \neq 0$.

(A) Rigid-body Modal Responses ($\omega_r = 0$)

We rewrite Equation (5) in the state space by introducing the change of variables $\underset{\sim}{u}_r(t) = [\dot{u}_r(t)\ u_r(t)]^T$ and obtain the modal equations

$$\dot{\underset{\sim}{u}}_r(t) = A\underset{\sim}{u}_r(t) + \underset{\sim}{B}f_r(t) \tag{7}$$

where

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad \underset{\sim}{B} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \tag{8}$$

The solution to Equation (8) can be converted into a difference equation. Letting T denote the time step, and letting $\dot{u}_r(k)$ and $u_r(k)$ denote the modal velocity and modal dispacement at time kT, (k = 0, 1, 2, ...) we obtain the difference equations

$$\dot{u}_r(k+1) = \dot{u}_r(k) + Tf_r(k) \tag{9a}$$

$$u_r(k+1) = \dot{u}_r(k)T + u_r(k) + \frac{T^2}{2}f_r(k) \tag{9b}$$

Equation (9) is used to compute the response of a rigid-body mode.

(B) Flexible-body Modal Responses ($\omega_r \neq 0$)

Equation (5) describes the motion of an undamped oscillator. However, structures experience small degrees of structural damping. We can introduce some damping into the mathematical model at the exponential rate α_r by replacing Equation (5) with

$$\ddot{u}_r(t) + 2\alpha_r\dot{u}_r(t) + (\alpha_r^2 + \omega_r^2)u_r(t) = f_r(t) \tag{10}$$

The natural frequency in Equation (10) is identical to that in Equation (5). We rewrite Equation (10) by introducing the change of complex variables $u_r(t) = \mathrm{Re}\{w_r(t)\}$, $\dot{u}_r(t) = \mathrm{Re}\{\lambda_r w_r(t)\}$ where $\lambda_r = -\alpha_r + i\omega_r$, and we obtain the complex modal state equations

$$\dot{w}_r(t) = \lambda_r w_r(t) + f_r(t)/(i\omega_r) \tag{11}$$

Letting T denote the time step, the response to Equation (11) is given by the difference equation

$$w_r(k+1) = \Phi_r w_r(k) + \Gamma_r f_r(k) \quad (r = 1, 2, \ldots, m) \qquad (12)$$

where

$$\Phi_r = e^{\lambda_r T}, \Gamma_r = (\Phi_r - 1)/(i\lambda_r \omega_r) \qquad (13)$$

Equation (12) is used in order to compute the response of a flexible-body mode. For these purposes, it is desirable to take a time step smaller than one tenth of the smallest flexible body period of oscillation.

III. Control System Design

The control system design is carried out in two steps. In the first step, one constructs the "ideal" control system with the best dynamic performance that nature will allow. Such a system requires distributed forces which are certainly impractical for most applications. The second step consists of designing a control system of minimal cost and greatest simplicity and one which imitates the ideal control system. Perhaps the simplest way to carry out the second step is to consider various designs and to compare the dynamic performances of these designs with the dynamic performance of the ideal control system.

Step 1: The ideal control system.

For vibration suppression, pointing, and shape control, the ideal control system is one which dampens all the modes of vibration at a single exponential rate α (Ref. 11). The linear feedback control law is

$$\underset{\sim}{F}(t) = -2\alpha M\dot{\underset{\sim}{x}}(t) - \alpha^2 M\underset{\sim}{x}(t) \qquad (14)$$

Substituting Equation (4) into Equation (14) while considering the orthonormality conditions, Equation (3), we obtain the expressions for the modal control forces

$$f_r(t) = -2\alpha\dot{u}_r(t) - \alpha^2 u_r(t), \quad (r = 1, 2, \ldots, m) \qquad (15)$$

We observe from Equation (15) that only the rth modal displacement and the rth modal velocity control the rth modal force. Such a control is referred to as natural because the modal coordinates do not couple the equations of motion (Refs. 7 and 11). Substituting Equation (15) into Equation (5), we obtain the closed-loop modal equations

$$u_r(t) + 2\alpha\dot{u}_r(t) + (\alpha^2 + \omega_r{}^2)\, u_r(t) = 0 \quad (r = 1, 2, \ldots, m) \qquad (16)$$

The corresponding closed-loop eigenvalues are given by

$$\lambda_{1,2} = 1/2[-2\alpha \pm \sqrt{(2\alpha)^2 - 4(\alpha^2 + \omega_r{}^2)}] = -\alpha \pm i\omega_r \qquad (17)$$

From Equation (17), the closed-loop modes all decay at the same exponential rate α and the closed-loop frequencies of oscillation are identical to the natural frequencies. Also, observe that the control law, Equation (14), is independent of the spacecraft stiffness. As a general rule of thumb, when a control system is designed to dampen modes in a more non-uniform manner, the control law will tend to depend more on the structural stiffness. Therefore, in the interest of designing a robust control system and one which does not depend explicitly on the fidelity of the mathematical model of stiffness, we uniformly dampen the motion.

The objective to uniformly dampen the motion can also be arrived at from other points of view. For example, let us assume that we wish to drive the motion of a given point on the structure to equilibrium at the exponential rate α, i.e. we wish that a given point be exponentially stable. Then, it can be shown that this point will be exponentially stable at the exponential decay rate α only if all of the natural modes of vibration are exponentially stable at the rates α_r not less than α. Also, note that any effort to dampen a given mode at an exponential rate α_r strictly greater than α will require unnecessary fuel. Therefore, the most effective way to drive the motion of any point to equilibrium at the exponential decay rate α is by damping the motion of the natural modes uniformly at the exponential decay rate α (Ref. 11).

Finally, we observe that the uniform damping control law, Equation (14), is decentralized. Because the mass matrix is diagonal, if we write

$$M = \mathrm{diag}(m_1, m_2, \ldots, m_n), \qquad (18)$$

then Equation (14) becomes

$$F_r(t) = -2\alpha m_r \dot{x}_r(t) - \alpha^2 m_r x_r(t), \quad (r = 1, 2, \ldots, n) \qquad (19)$$

Clearly, Equation (19) represents a set of independent control laws, which suggests that uniform damping is relatively easy to implement. As a matter of theoretical interest, uniform damping control represents a close approximation to globally optimal control (Ref. 11).

In view of the considerations presented in the previous paragraphs, the objective to uniformly dampen the motion has been chosen, and for the purpose of design, it will be viewed herein as an ideal.

Step 2: Implementation of the ideal control system.

It is usually impractical to consider a large number of control forces as in Equation (19). Therefore, we arrive at the second step and design a control system that performs as closely as possible to the ideal control system. The control law obtained in the second step can be given by

$$\underset{\sim}{F}(t) = -C\dot{\underset{\sim}{x}}(t) - D\underset{\sim}{x}(t) \qquad (20)$$

where C and D are usually sparce matrices because in most applications only a relatively small number of control forces are required. It is of immediate concern to describe the degradation in performance due to implementing the controls with a limited number of control forces. As it turns out, the degradation in performance can be marginal. Substituting Equation (20) into Equation (6) and considering Equations (3) and (4), we obtain the modal equations

$$\ddot{u}_r(t) + \omega_r^2 u_r(t) = -\sum_{s=1}^{m} (\underset{\sim}{\phi}_r^T C \underset{\sim}{\phi}_s \dot{u}_s(t) + \underset{\sim}{\phi}_r^T D \underset{\sim}{\phi}_s u_s(t)), \quad (r = 1, 2, \ldots, m) \qquad (21)$$

Equation (21) can be rewritten in the form

$$\ddot{u}_r(t) + 2\alpha\dot{u}_r(t) + (\alpha^2 + \omega_r^2)u_r(t) = -\sum_{s=1}^{m}[(\phi_r^T C\phi_s - 2\alpha\delta_{rs})\dot{u}_s(t) + (\phi_r^T D\phi_s - \alpha^2\delta_{rs})u_s(t)], \quad (r = 1, 2, \ldots, m) \tag{22}$$

The flexible-body modes and the rigid-body modes in Equation (22) can we rewritten in the state space by introducing the complex change of variables

$$u_r(t) = \text{Re}\,\{w_r(t)\}, \quad \dot{u}_r(t) = \text{Re}\,\{\lambda_r w_r(t)\}, \quad (r = 1, 2, \ldots, m) \tag{23}$$

where $\lambda_r = -\alpha + i\omega_r$ are the system eigenvalues that would be obtained using the ideal control system. We obtain the complex modal state equations

$$\dot{w}_r(t) = \lambda_r w_r(t) + 1/2\sum_{s=1}^{m}(g_{rs}w_s(t) + \overline{g_{rs}w_s}(t)) \tag{24}$$

where

$$g_{rs} = (\alpha^2\delta_{rs} - \phi_r^T D\phi_s)/(i\omega_r) + (2\alpha\delta_{rs} - \phi_r^T C\phi_s)\lambda_s/(i\omega_r), \quad (r,s = 1, 2, \ldots, m) \tag{25}$$

The eigenvalues of the controlled spacecraft lie in the circles with centers C_r and associated radii R_r, given by

$$C_r = \lambda_r + g_{rr}/2, \quad R_r = \sum_{\substack{s=1 \\ s\neq r}}^{m} \left| g_{rs} \right| \tag{26}$$

Note that the centers C_r are also first-order approximations of the eigenvalues associated with the ideal control system. Equation (26) can be used in order to compare the performance of the control system design with the performance of the ideal control system.

IV. Digitization of the Controls

In the previous section, distributed controls were discretized in space leading to the implementation of the controls using a limited number of control forces. The controls acted continuously in time. The controls can also be discretized in time leading to digital controls. In the process, the dynamic performance of the controls are expected to change depending on the level of digitization. The question arises, at what level of digitization will the dynamic performance of the spacecraft vary significantly from the dynamic performance of the spacecraft with an ideal control system. Consider the continuous controls acting at the rth node with the associated control law

$$F_r(t) = -2\alpha m_r \dot{x}_r(t) - \alpha^2 m_r x_r(t) \tag{27}$$

Here, m_r refers to the mass of the region within which the control force $F_r(t)$ acts. Over a small time increment T, we apply an impulse

$$I_r(t) = \int_t^{t+T} F_r(\tau)d\tau = F_r(t)T \tag{28}$$

so that

$$I_r(t) = -2\alpha T m_r \dot{x}_r(t) - \alpha^2 T m_r x_r(t) \tag{29}$$

Instead of applying continuously acting controls as suggested by Equation (27), let us apply an impulse every k seconds. Then, we replace the continuous control law, Equation (27), with the digital control law

$$I_r(t) = -2\alpha K T m_r \dot{x}_r(t) - \alpha^2 K T m_r x_r(t) \tag{30}$$

where the impulse $I_r(t)$ is applied every KT seconds. The particular effects of implementing Equation (30) rather than Equation (27) are described in the numerical example.

V. Uniform Damping of a Simply Supported Beam

As an illustrative example, we control a simply supported beam of length a = 10.0 units with unit mass per unit length and unit stiffness density. For this simple example, the equations of motion admit closed-form expressions. The normalized eigenfunctions and natural frequencies are

$$\phi_r(x) = (2/a)^{1/2} \sin\left(\frac{r\pi x}{a}\right) \quad \omega_r = \left(\frac{r\pi}{a}\right)^2, \quad r = 1, 2, \ldots, m \tag{31}$$

For the sake of this example, we assume that the lowest m = 10 modes of vibration contribute significantly to the overall system response and that the contribution of the remaining modes to the motion is negligible. The beam is given an initial unit step input at $x = 4.0$ for 2.0 seconds. We design for a uniform exponential decay rate of $\alpha = 1.0$ and we assume that 1 percent structural damping is present in the beam.

As a first step, the ideal control system is designed. The free response is shown in Figure 1 and the ideal control system response is shown in Figure 2. The ideal closed-loop eigenvalues are given in Table 1. Next we consider implementing the control system using a discrete number of control forces. In order to approximate the ideal control system, we locate control forces along the beam at the points P_r, $(r = 1, 2, \ldots, s; s = 4, 5)$ (See Table 2). The associated control laws are given by

$$F_r(t) = -2\alpha m_r \dot{x}_r(t) - \alpha^2 m_r x_r(t), \quad m_r = a/s, \quad (r = 1, 2, \ldots, s) \tag{32}$$

where $x_r(t)$ is the displacement at P_r. Here, again, m_r represents the mass in the region of the rth control force. The responses of the beam with the discrete controls are shown in Figures 3 and 4. The corresponding fuels consumed by the controls are shown in Figures 5 and 6. Also, the corresponding first-order approximations of the closed-loop eigenvalues are given in Tables 3 and 4.

Next we digitize the control law Equation (30). The responses of the beam using digitized discrete controls are shown in Figures 7 and 8. The corresponding fuels consumed by the controls are shown in Figures 9 and 10. A computer program listing is given in Appendix A.

VI. Conclusions

A control system design approach for flexible spacecraft has been presented. The control system design is carried out in two steps. The first step consists of determining an "ideal" uniform exponential rate at which we desire the spacecraft motion to dampen. Next, we construct a control with dynamic performance that is close to the "ideal" using a limited number of actuators. It is also shown that the controls can be digitized when it is desirable to create forces using impulses.

The control system design approach is demonstrated with a simple numerical example in which it is shown that close to ideal dynamic performances can be obtained with a relatively small number of actuators. Also, the effects of digitizing the controls on the dynamic performance is illustrated.

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Ideal Closed-Loop Eigenvalues

$$\lambda_r = -\alpha + i\omega_r$$

r	$-\alpha + i\omega_r$
1	-1.0 + i0.098
2	-1.0 + i0.394
3	-1.0 + i0.888
4	-1.0 + i1.579
5	-1.0 + i2.467
6	-1.0 + i3.553
7	-1.0 + i4.836
8	-1.0 + i6.316
9	-1.0 + i7.994
10	-1.0 + i9.869

Table 1

Locations P_r of the Control Forces

	P_1	P_2	P_3	P_4	P_5
Five Forces	1.0	3.0	5.0	7.0	9.0
Four Forces	2.0	4.0	6.0	8.0	--

Table 2

First-Order Approximation of the

Closed-Loop Eigenvalues Using Five Control Forces

r	$\lambda_r = -\alpha_r + i\omega_r$
1	-1.0 + i0.098
2	-1.0 + i0.394
3	-1.0 + i0.888
4	-1.0 + i1.579
5	-2.0 + i2.264
6	-1.0 + i3.553
7	-1.0 + i4.836
8	-1.0 + i6.316
9	-1.0 + i7.994
10	0.0 + i9.920

Table 3

First Order Approximation of the

Closed Loop Eigenvalues Using Four Control Forces

r	$\lambda_r = -\alpha_r + i\omega_r$
1	-1.25 + i0.078
2	-1.25 + i0.747
3	-1.25 + i1.167
4	-1.25 + i1.500
5	-1.25 + i2.760
6	0.00 + i3.517
7	-1.25 + i4.810
8	-1.25 + i6.296
9	-1.25 + i7.978
10	0.00 + i9.920

Table 4

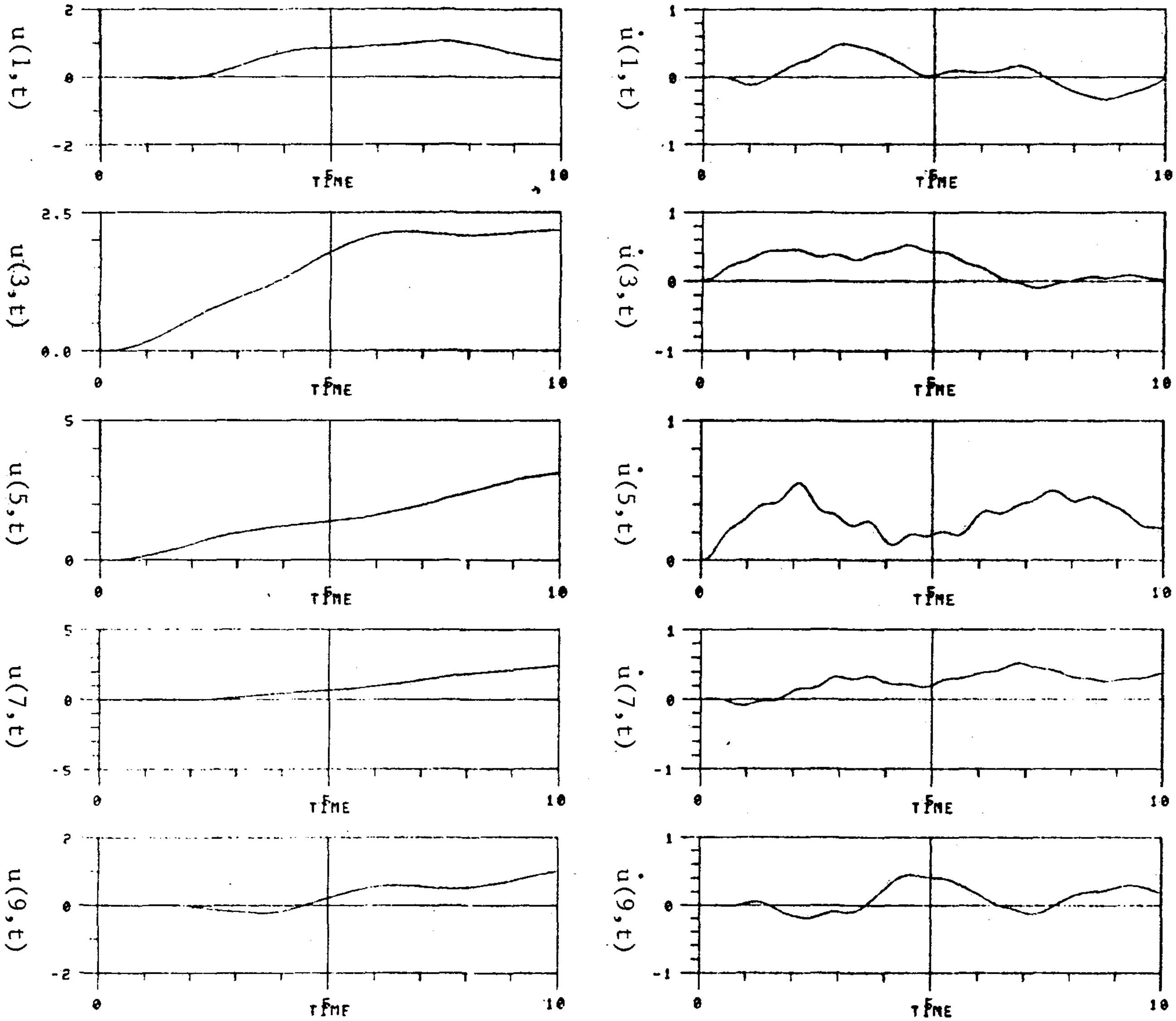

Figure 1. Free Response.

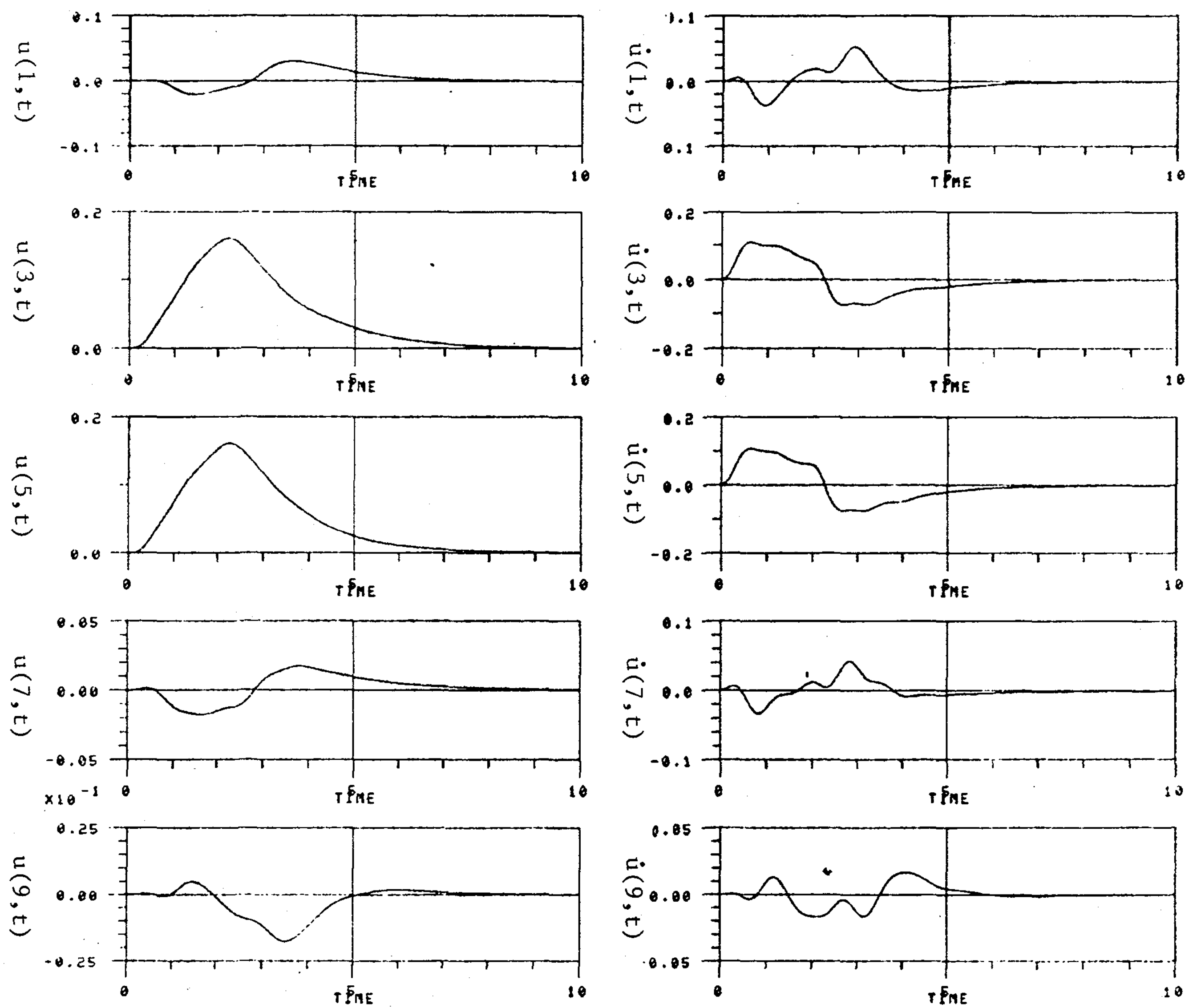

Figure 2. Controlled Response-Distributed Forces, Continuous in Time.

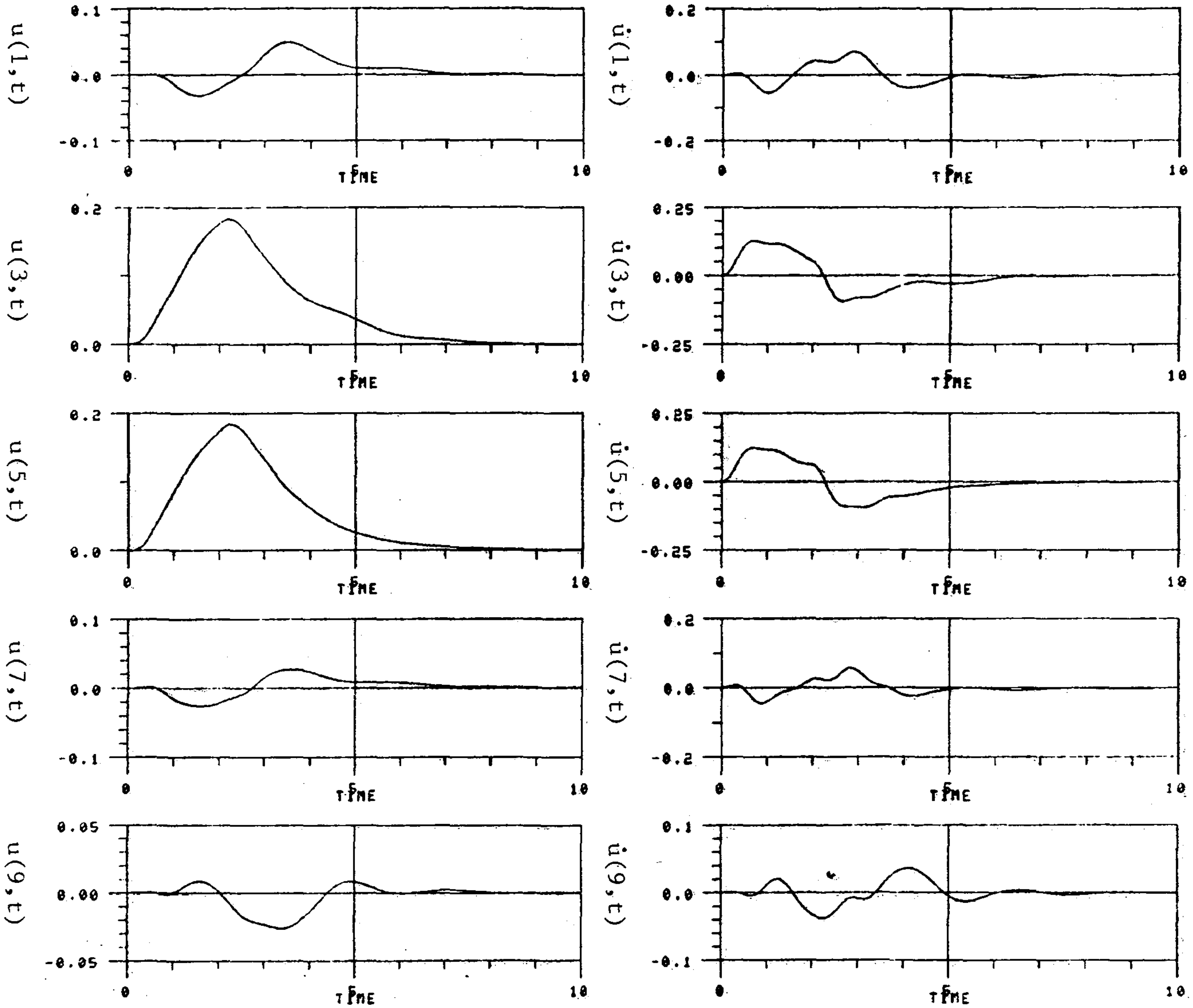

Figure 3. Controlled Response-Five Control Forces, Continuous in Time.

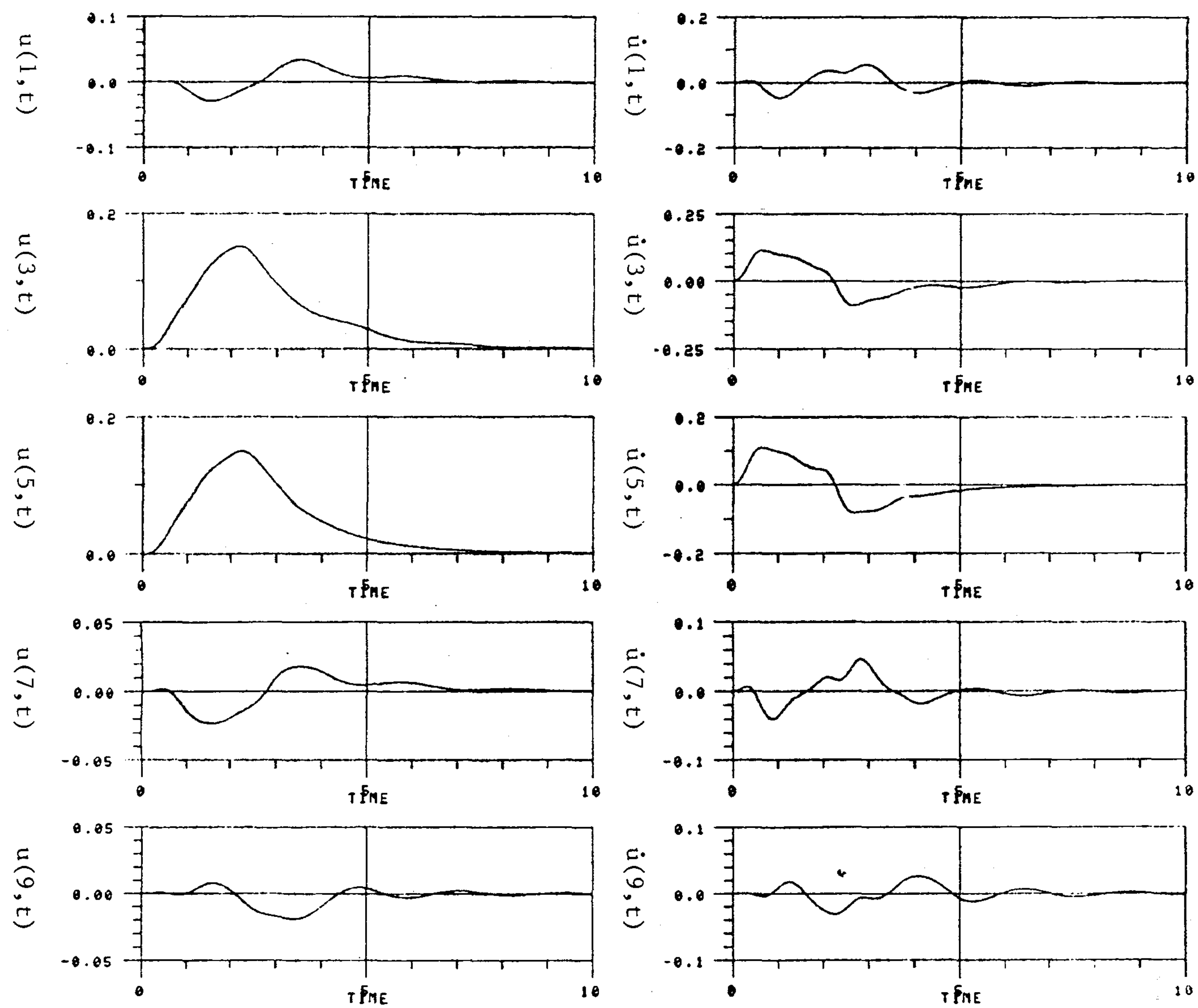

Figure 4. Controlled Response-Four Control Forces, Continuous in Time.

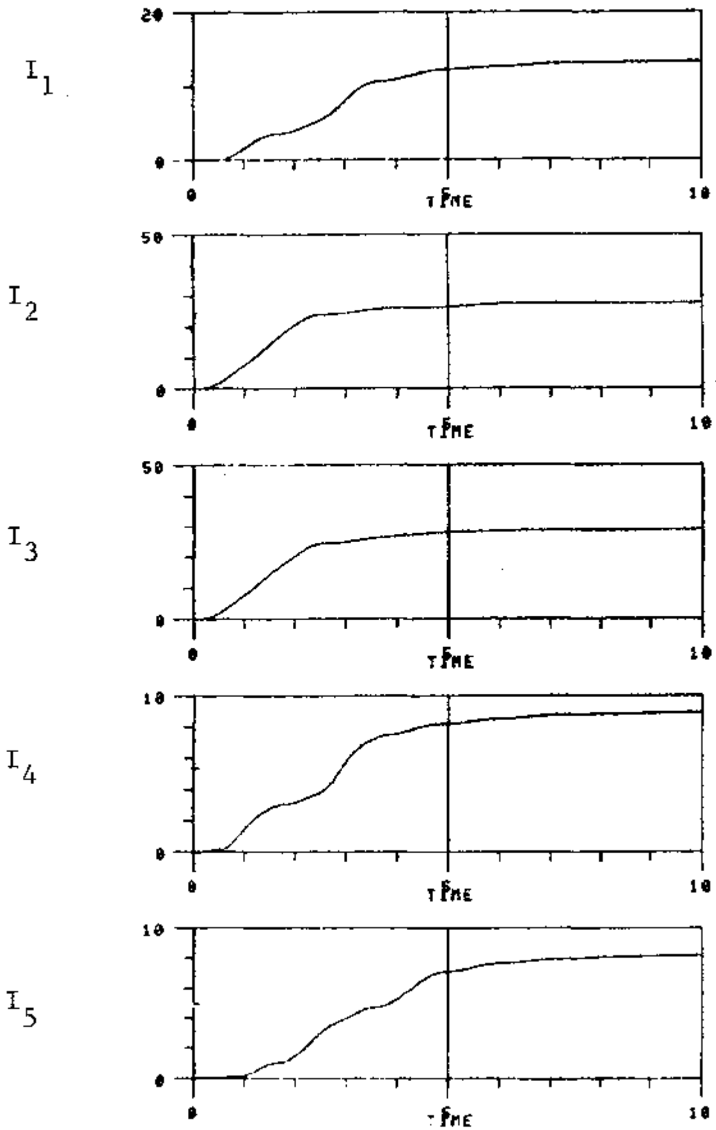

Figure 5. Fuel-Five Control Forces, Continuous in Time.

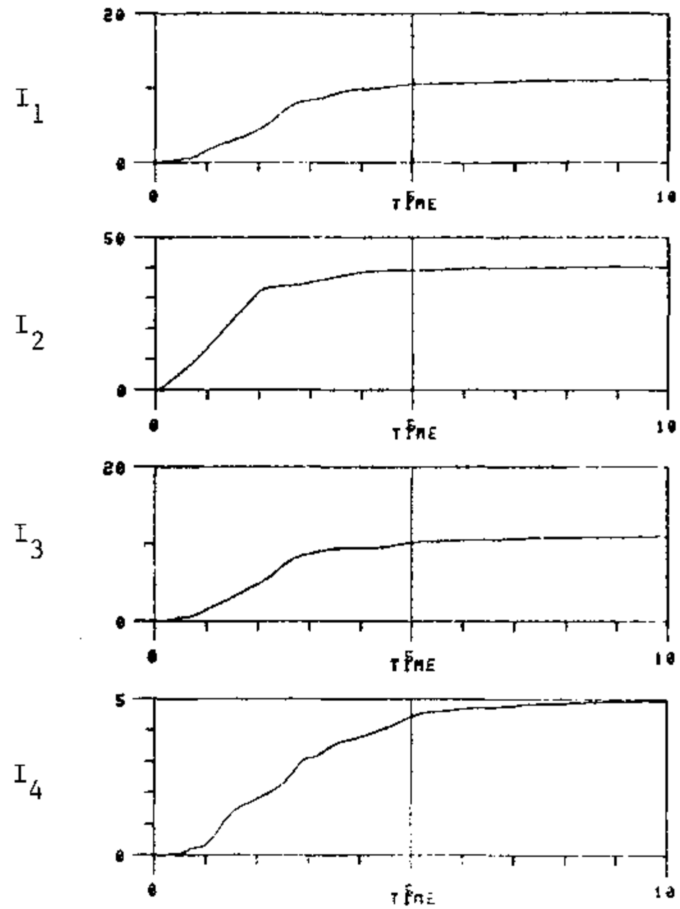

Figure 6. Fuel-Four Control Forces, Continuous in Time.

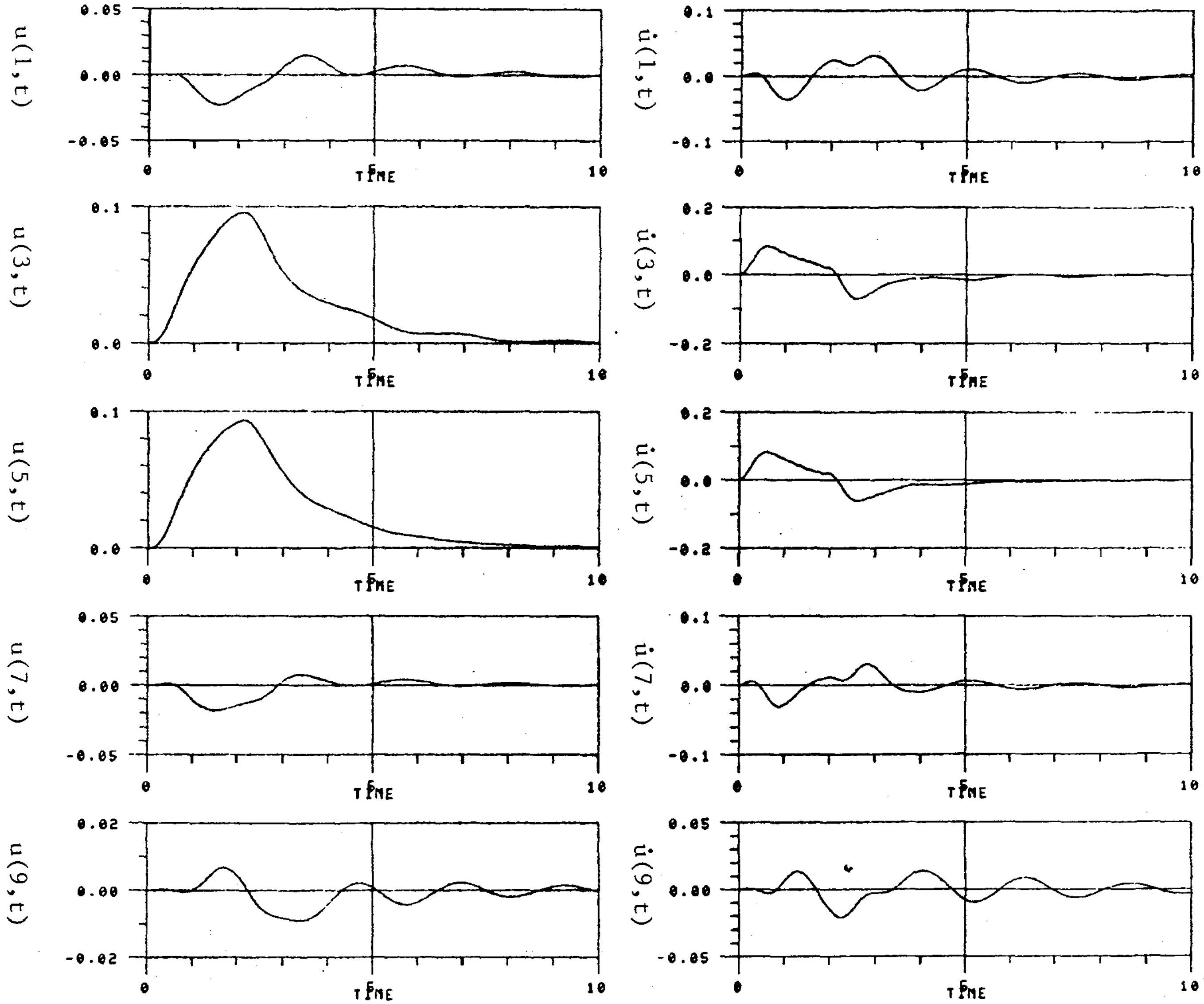

Figure 7. Controlled Response-Four Control Forces, Impulses Every 0.2 Seconds.

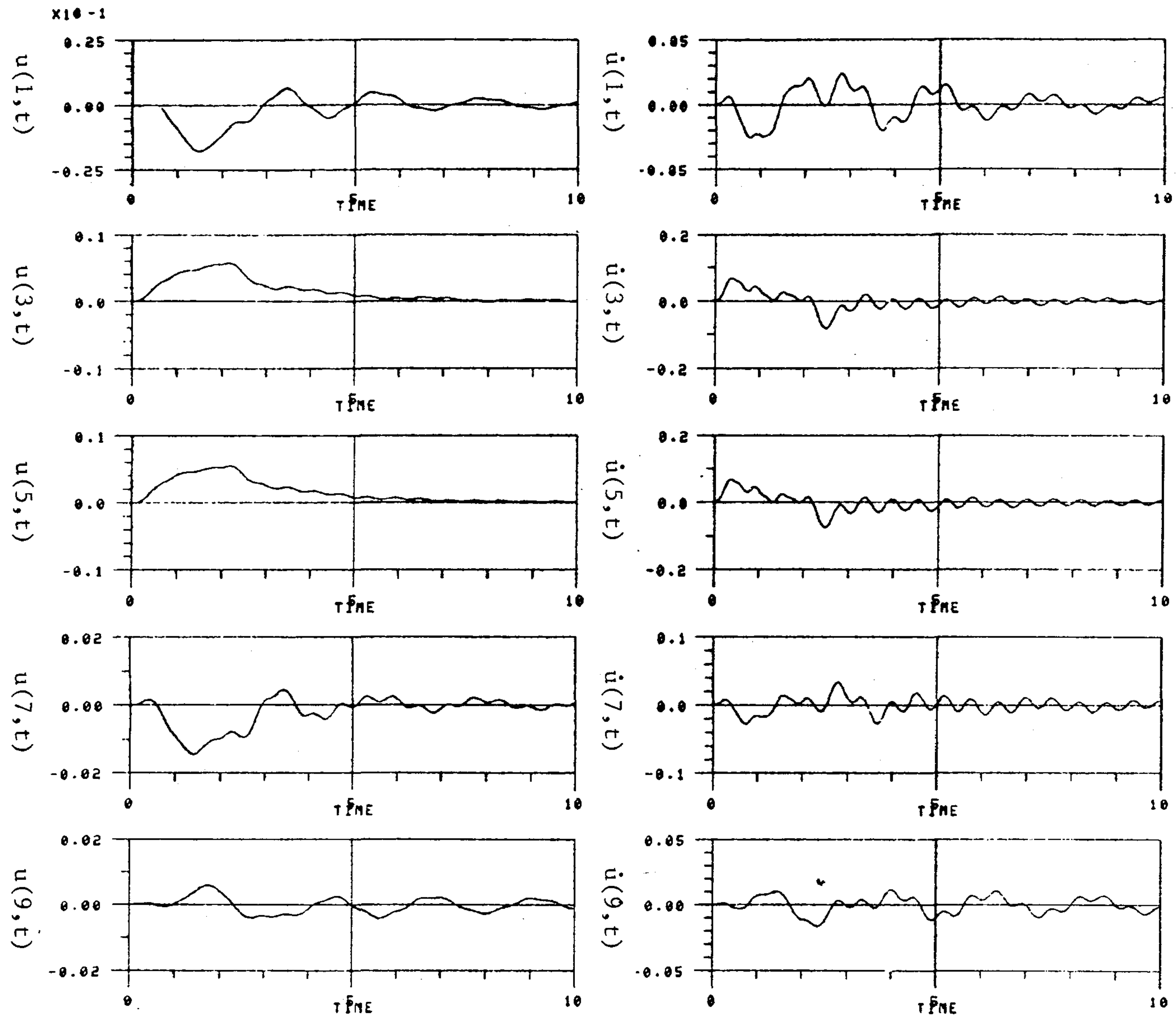

Figure 8. Controlled Response-Four Control Forces, Impulses Every 0.3 Seconds.

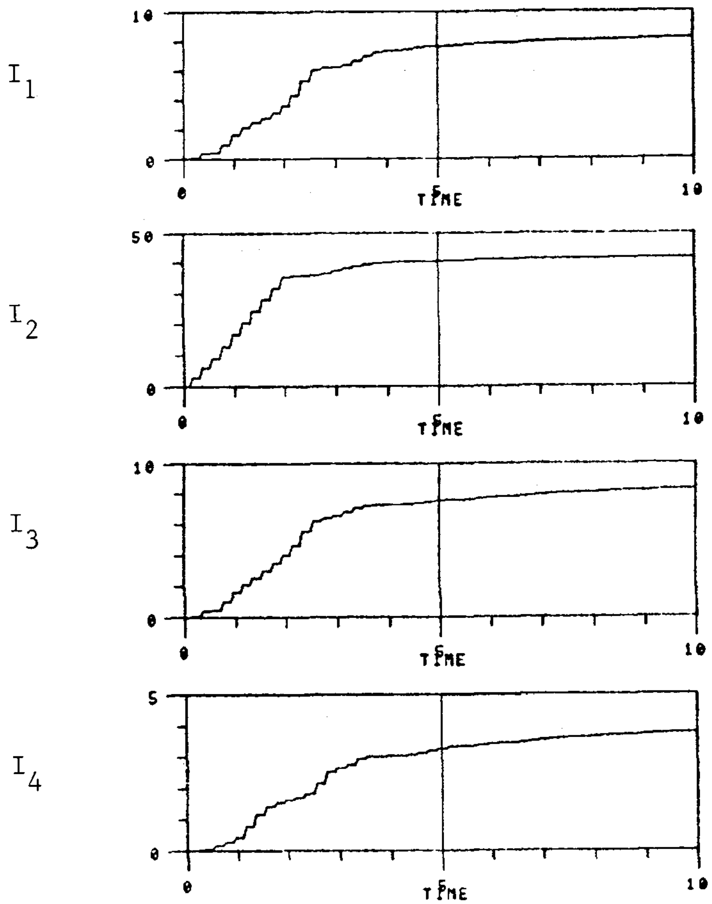

Figure 9. Fuel-Four Control Forces, Impulses Every 0.2 Seconds.

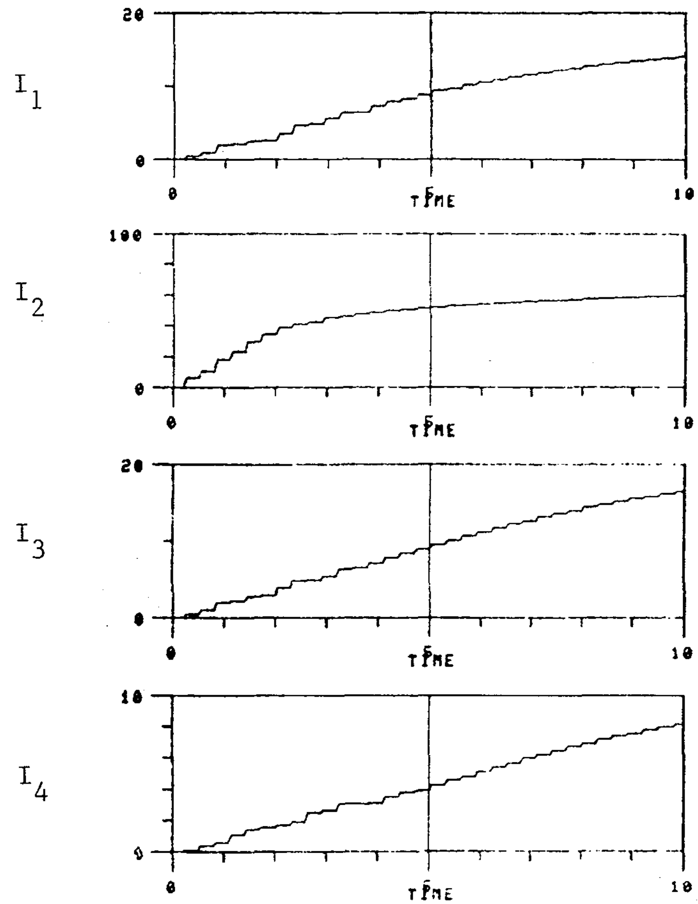

Figure 10. Fuel-Four Control Forces, Impulses Every 0.3 Seconds.

```
.NULL.
CCCC   CCCCCCCCCCCCCCCCCCCCCCC
CCCC   CCCCCCCCCCCCCCCCCCCCCCC
CCCC
CCCC   DATA GENERATION PROGRAM
CCCC
CCCC     THE STRUCTURAL SYSTEM PARAMETERS
CCCC     ARE DEFINED INCLUDING THE NATURAL
CCCC     FREQUENCIES AND THE NATURAL MODES.
CCCC
CCCC   CCCCCCCCCCCCCCCCCCCCCCC
CCCC   CCCCCCCCCCCCCCCCCCCCCCC
CCCC
       REAL*8 VEC(25)
       COMPLEX*16 LAMDA
       OPEN(UNIT=11,FILE='DAT',STATUS='UNKNOWN')
       M=10
       N=9
       TSTEP=0.05
       NSTEP=200
       WRITE(11,*)M
       WRITE(11,*)N
       WRITE(11,*)TSTEP
       WRITE(11,*)NSTEP
       PI=ACOS(-1.)
       AA=10.
       ZETA=0.01
       SQ2OA=SQRT(2./AA)
       DO 1  I=1,M
       OMEGA=(I*PI/AA)**2
```

```
      OMEGA=(I*PI/AA)**2
      ALFA=2.*ZETA*OMEGA
      LAMDA=(0.,1.)*OMEGA-ALFA
      WRITE(11,*)LAMDA
 1    CONTINUE
      DO 3 I=1,N
      DO 2  J=1,M
      N1=N+1
      VEC(J)=SQ2OA*SIN(J*PI*(I-0.0)/N1)
 2    CONTINUE
      WRITE(11,100)(VEC(J),J=1,M)
 3    CONTINUE
 100  FORMAT(2X,5E15.6)
      CLOSE(11)
      STOP
      END
BOTTOM
```

```
.NULL.
CCCC
CCCC   CCCCCCCCCCCCCCCCCCCCCC
CCCC   CCCCCCCCCCCCCCCCCCCCCC
CCCC
CCCC   EXTERNAL DISTURBANCE PROGRAM
CCCC
CCCC     THE EXTERNAL FORCES NOT INCLUDING CONTROL
CCCC     FORCES ARE DEFINED.
CCCC
CCCC   CCCCCCCCCCCCCCCCCCCCCC
CCCC   CCCCCCCCCCCCCCCCCCCCCC
CCCC
       OPEN(UNIT=13,FILE='FORCES',STATUS='UNKNOWN')
       NP=1
       IFOR1=4
       WRITE(13,*)NP
       WRITE(13,*)IFOR1
       F1=1
       F4=0
       DO 1 K=1,40
 1     WRITE(13,*)F1
       DO 2 K=21,200
 2     WRITE(13,*)F4
       CLOSE(13)
       STOP
       END
BOTTOM
```

```
.NULL.
CCCC
CCCC    CCCCCCCCCCCCCCCCCCCCCCCC
CCCC    CCCCCCCCCCCCCCCCCCCCCCCC
CCCC
CCCC    THE CONTROL PARAMETERS ARE DEFINED
CCCC
CCCC    CCCCCCCCCCCCCCCCCCCCCCCC
CCCC    CCCCCCCCCCCCCCCCCCCCCCCC
CCCC
        REAL*8 XMASS(9)
        OPEN(UNIT=11,FILE='CONTROL',STATUS='UNKNOWN')
        OPEN(UNIT=12,FILE='DAT',STATUS='UNKNOWN')
        ALFA=1.0
        READ(12,*)MMM
        READ(12,*)N
        DO 10 J=1,N
        XMASS(J)=10./N
 10     CONTINUE
        KTIME=1
        WRITE(11,*)(XMASS(I),I=1,N)
        WRITE(11,*)KTIME
        WRITE(11,*)ALFA
        CLOSE(11)
        CLOSE(12)
        STOP
        END
BOTTOM
```

```
.NULL.
        SUBROUTINE LAW(FOR,X,XDOT,I,FORT,XMASS,KTIME,ALFA)
CCCC
CCCC    CCCCCCCCCCCCCCCCCCCCCCCC
CCCC    CCCCCCCCCCCCCCCCCCCCCCCC
CCCC
CCCC    SUBROUTINE LAW
CCCC
CCCC      THE CONTROL LAW IS DEFINED.
CCCC
CCCC    CCCCCCCCCCCCCCCCCCCCCCCC
CCCC    CCCCCCCCCCCCCCCCCCCCCCCC
CCCC
        REAL*8 X(9),XDOT(9),FOR(9),XMASS(9),FORT(9)
        DO 1 K=1,9
        IF((I/KTIME)*KTIME.NE.I)GOTO1
        FORK=-ALFA*XMASS(K)*KTIME*(2.*XDOT(K)+ALFA*X(K))
        FOR(K)=FOR(K)+FORK
        FORT(K)=FORT(K)+ABS(FORK)
 1      CONTINUE
        RETURN
        END
BOTTOM
```

```
.NULL.
      SUBROUTINE RESP(VEC,VAL,X,XDOT,T,FOR,M,N,U,UDOT,W)
CCCC
CCCC  CCCCCCCCCCCCCCCCCCCC
CCCC  CCCCCCCCCCCCCCCCCCCC
CCCC
CCCC  SUBROUTINE RESP
CCCC
CCCC    THE SYSTEM RESPONSE IS UPDATED FOR EACH
CCCC    TIME STEP. THE COMPUTATION DISTIGUISHES
CCCC    BETWEEN RIGID-BODY MOTION AND FLEXIBLE-BODY
CCCC    MOTION.
CCCC
CCCC  CCCCCCCCCCCCCCCCCCCC
CCCC  CCCCCCCCCCCCCCCCCCCC
CCCC
      REAL*8 VEC(9,25),X(9),XDOT(9),FOR(9),U(25),UDOT(25)
      COMPLEX*16 VAL(25),W(25),PSI,GAMA,OMI
      DO 3 J=1,M
      F=0
      DO 1 K=1,N
 1    F=F+VEC(K,J)*FOR(K)
      IF(CDABS(VAL(J)).LT.1.D-6)GOTO 2
      PSI=CDEXP(VAL(J)*T)
      OM=(0.,-1.)*VAL(J)
      OMI=(0.,1.)*OM
      GAMA=(PSI-1)/VAL(J)/OMI
      W(J)=PSI*W(J)+GAMA*F
      U(J)=W(J)
      UDOT(J)=VAL(J)*W(J)
```

```
      UDOT(J)=VAL(J)*W(J)
      GOTO 3
 2    U(J)=U(J)+T*UDOT(J)+T**2/2.*F
      UDOT(J)=UDOT(J)+T*F
 3    CONTINUE
      DO 4 K=1,N
      X(K)=0
      XDOT(K)=0
      DO 4 J=1,M
      X(K)=X(K)+VEC(K,J)*U(J)
      XDOT(K)=XDOT(K)+VEC(K,J)*UDOT(J)
 4    CONTINUE
      RETURN
      END
BOTTOM
```

```
.NULL.
CCCC
CCCC    CCCCCCCCCCCCCCCCCC
CCCC    CCCCCCCCCCCCCCCCCC
CCCC
CCCC    SYSTEM RESPONSE PROGRAM
CCCC
CCCC      THE RESPONSE OF THE CONTROLLED SYSTEM IS
CCCC      COMPUTED AT VARIOUS POINTS.
CCCC
CCCC    CCCCCCCCCCCCCCCCCC
CCCC    CCCCCCCCCCCCCCCCCC
CCCC
        REAL*8 VEC(9,25),X(9),XDOT(9),FOR(9),U(25),UDOT(25),FORT(9)
        REAL*8 XMASS(9)
        COMPLEX*16 VAL(25),W(25)
        INTEGER IFOR(9)
        OPEN(UNIT=11,FILE='DAT',STATUS='UNKNOWN')
        OPEN(UNIT=13,FILE='FORCES',STATUS='UNKNOWN')
        OPEN(UNIT=14,FILE='OUT1',STATUS='UNKNOWN')
        OPEN(UNIT=15,FILE='OUT2',STATUS='UNKNOWN')
        OPEN(UNIT=16,FILE='OUT3',STATUS='UNKNOWN')
        OPEN(UNIT=17,FILE='OUT4',STATUS='UNKNOWN')
        OPEN(UNIT=18,FILE='OUT5',STATUS='UNKNOWN')
        OPEN(UNIT=6,FILE='FOR1',STATUS='UNKNOWN')
        OPEN(UNIT=7,FILE='FOR2',STATUS='UNKNOWN')
        OPEN(UNIT=8,FILE='FOR3',STATUS='UNKNOWN')
        OPEN(UNIT=9,FILE='FOR4',STATUS='UNKNOWN')
        OPEN(UNIT=10,FILE='FOR5',STATUS='UNKNOWN')
        OPEN(UNIT=19,FILE='CONTROL',STATUS='UNKNOWN')
```

```
      OPEN(UNIT=19,FILE='CONTROL',STATUS='UNKNOWN')
      READ(11,*)M
      READ(11,*)N
      READ(11,*)T
      READ(11,*)L
      READ(11,*)(VAL(I),I=1,M)
      READ(11,*)((VEC(I,J),J=1,M),I=1,N)
      READ(19,*)(XMASS(I),I=1,N)
      READ(19,*)KTIME
      READ(19,*)ALFA
      WRITE(14,*)L,L,L
      WRITE(15,*)L,L,L
      WRITE(16,*)L,L,L
      WRITE(17,*)L,L,L
      WRITE(18,*)L,L,L
      WRITE(6,*)L,L,L
      WRITE(7,*)L,L,L
      WRITE(8,*)L,L,L
      WRITE(9,*)L,L,L
      WRITE(10,*)L,L,L
      TM=0
      DO 1 I=1,M
      U(I)=0
      UDOT(I)=0
      W(I)=0
1     CONTINUE
      DO 2 K=1,N
      FORT(K)=0
      IFOR(K)=0
      X(K)=0
```

```
        X(K)=0
        XDOT(K)=0
2       CONTINUE
        READ(13,*)NP
        READ(13,*)(IFOR(K),K=1,NP)
        DO 4 I=1,L
        DO 3 K=1,N
        FOR(K)=0
3       CONTINUE
        READ(13,*)(FOR(IFOR(K)),K=1,NP)
        CALL LAW(FOR,X,XDOT,I,FORT,XMASS,KTIME,ALFA)
        CALL RESP(VEC,VAL,X,XDOT,T,FOR,M,N,U,UDOT,W)
        WRITE(14,100)TM,X(1),XDOT(1)
        WRITE(15,100)TM,X(3),XDOT(3)
        WRITE(16,100)TM,X(5),XDOT(5)
        WRITE(17,100)TM,X(7),XDOT(7)
        WRITE(18,100)TM,X(9),XDOT(9)
        WRITE(6,100)TM,FORT(1),FORT(2)
        WRITE(7,100)TM,FORT(3),FORT(4)
        WRITE(8,100)TM,FORT(5),FORT(6)
        WRITE(9,100)TM,FORT(7),FORT(8)
        WRITE(10,100)TM,FORT(9),FORT(9)
        TM=T+TM
4       CONTINUE
100     FORMAT(F6.3,2E22.13)
        CLOSE(11)
        CLOSE(13)
        CLOSE(14)
        CLOSE(15)
        CLOSE(16)
```

```
      CLOSE(16)
      CLOSE(17)
      CLOSE(18)
      CLOSE(6)
      CLOSE(7)
      CLOSE(8)
      CLOSE(9)
      CLOSE(10)
      STOP
      END
BOTTOM
```

```
.NULL.
CCCC   CCCCCCCCCCCCCCCC
CCCC   CCCCCCCCCCCCCCCC
CCCC
CCCC   CONTROL ROBUSTNESS PROGRAM
CCCC
CCCC     IDEALLY, A DESIRABLE DYNAMIC PERFORMANCE
CCCC     REQUIRES DISTRIBUTED SENSING AND ACTUATION
CCCC     WHICH IS FOR THE MOST PART IMPRACTICAL.
CCCC     THEREFORE, ONE RESORTS TO FINITE-DIMENSIONAL
CCCC     SENSING AND ACTUATION.THIS PROCESS OF GOING FROM
CCCC     DISTRIBUTED TO DISCRETE IS CALLED CONTROL
CCCC     DISCRETIZATION.THIS PROGRAM LOOKS AT THE EFFECTS
CCCC     OF CONTROL DISCRETIZATION ON THE DYNAMIC PERFORMANCE.
CCCC     TOWARD THIS END, WE LOOK AT:
CCCC     1) CHANGES IN THE NEIGHBOURHOODS OF THE CLOSED-LOOP
CCCC        EIGENVALUES.
CCCC     2) FIRST-ORDER PERTUBATIONS OF THE CLOSED-LOOP
CCCC        EIGENVALUES.
CCCC
CCCC
CCCC   CCCCCCCCCCCCCCCC
CCCC   CCCCCCCCCCCCCCCC
CCCC
       REAL*8 VEC(25,25),C(25,25),D(25,25),XMASS(25),RAD(25)
       COMPLEX*16 VAL(25),CEN(25),LAM(25),GRS,GJS,GJI,OM(25)
       OPEN(UNIT=11,FILE='DAT',STATUS='UNKNOWN')
       OPEN(UNIT=12,FILE='CIRCLE',STATUS='UNKNOWN')
       OPEN(UNIT=13,FILE='CONTROL',STATUS='UNKNOWN')
       READ(11,*)M
```

```
      READ(11,*)M
      READ(11,*)N
      READ(11,*)T
      READ(11,*)L
      READ(11,*)(VAL(I),I=1,M)
      READ(11,*)((VEC(I,J),J=1,M),I=1,N)
      READ(13,*)(XMASS(I),I=1,N)
      READ(13,*)KTIME
      READ(13,*)ALFA
      DO 1 I=1,N
      DO 1 J=1,N
      C(I,J)=0
      D(I,J)=0
      IF(I.EQ.J)C(I,I)=2.*ALFA*XMASS(I)
      IF(I.EQ.J)D(I,I)=ALFA**2*XMASS(I)
1     CONTINUE
      DO 2 I=1,M
      OMM=(0.,-1.)*VAL(I)
      OM(I)=(0.,1.)*OMM
      CEN(I)=-ALFA+OM(I)
      LAM(I)=CEN(I)
2     CONTINUE
      DO 7 IR=1,M
      RAD(IR)=0
      DO 6 IS=1,M
      GRS=0
      DO 5 J=1,N
      GJS=0
      DO 4 I=1,N
      GJI=-(C(J,I)*LAM(IS)+D(J,I))/OM(IR)
```

```
         GJI=-(C(J,I)*LAM(IS)+D(J,I))/OM(IR)
         GJS=GJS+GJI*VEC(I,IS)
 4       CONTINUE
         GRS=GRS+VEC(J,IR)*GJS
 5       CONTINUE
         IF(IR.EQ.IS)GRS=GRS+(2.*LAM(IR)*ALFA+ALFA**2)/OM(IR)
         IF(IR.EQ.IS)CEN(IR)=CEN(IR)+GRS*0.5
         IF(IR.NE.IS)RAD(IR)=RAD(IR)+CDABS(GRS)
 6       CONTINUE
 7       CONTINUE
         WRITE(12,100)(LAM(I),I=1,M)
 100     FORMAT(2X,'IDEAL EIGENVALUES'//,25(2X,2E15.5/))
         WRITE(12,150)(XMASS(I),I=1,N)
 150     FORMAT(2X,'REGIONAL MASSES'//,25(E15.5/))
         WRITE(12,200)
         WRITE(12,300)(CEN(I),RAD(I),I=1,M)
 200     FORMAT(2X,'NEIGHBOURHOODS OF THE CLOSED-LOOP'
     1    ,' EIGENVALUES'/,/2X,4X,'CENTERS(FIRST-ORDER APPROX)',5X
     1    ,2X,'RADII'/)
 300     FORMAT(2X,2E15.5,5X,E15.5)
         CLOSE(11)
         CLOSE(12)
         STOP
         END
BOTTOM
```

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15. Supplementary Notes

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16. Abstract

A control system design approach for flexible spacecraft is presented. The control system design is carried out in two steps. The first step consists of determining the "ideal" control system in terms of a desirable dynamic performance. The second step consists of designing a control system using a limited number of actuators that possess a dynamic performance that is close to the ideal dynamic performance. The effects of using a limited number of actuators is that the actual closed-loop eigenvalues differ from the ideal closed-loop eigenvalues. A method is presented to approximate the actual closed-loop eigenvalues so that the calculation of the actual closed-loop eigenvalues can be avoided. Depending on the application, it also may be desirable to apply the control forces as impulses. The effect of digitizing the control to produce the appropriate impulses is also examined.

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