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SUMMARY

With the advent of monolithic microwave integrated circuit (MMIC) technology, the phased array has become a key component in the design of advanced antenna systems. Array-fed antennas are used extensively in today's multiple beam satellite antennas. In this report, a computer program based on a very efficient numerical technique for calculating the radiated power (Romberg integration), directivity and radiation pattern of a phased array is described. The formulation developed is very general, and takes into account arbitrary element polarization, Eand H-plane element pattern, element location, and complex element excitation. For comparison purposes sample cases have been presented. Excellent agreement has been obtained for all cases. Also included in appendixes A and B are a user guide and a copy of the computer program.

INTRODUCTION

One of the most important radiation characteristics of an antenna system is the directivity. Accurate determination of this parameter is essential for the analysis and design of advanced antenna systems. In general, for most commonly used array elements such as open-ended waveguides, horns, or microstrip patch antennas, an analytical expression of the $(\cos \theta)^q$-type can be used to properly tailor actual element patterns (ref. 1). Experimentally measured element patterns could include mutual coupling, which may be significant in large arrays. Expressions for the directivity and array radiation pattern of a single element and a rectangular array using $(\cos \theta)^q$-type element patterns have been reported (refs. 2 to 11).

It is the purpose of this work to generalize these results and obtain an efficient numerical technique for computing the directivity and the antenna radiation pattern of the generalized array. The generalized array characteristics used in this report includes arbitrary element location, element pattern ($(\cos \theta)^q$-type, other analytically describable functions or experimentally measured), element polarization and element excitation.

ARRAY RADIATION PATTERN

The geometry of the generalized array is shown in figure 1. Given the array geometry and element characteristics, the generalized array problem can be defined as: (1) to determine the power radiated and directivity at a given observation point (this is usually taken in the far-field zone), (2) to determine the co-polarization and cross-polarization component of the electric field (using Ludwig's criterion (ref. 12)). In solving this problem, two sets of coordinate system are used. Figure 2 shows a typical element coordinate system and the reference coordinate system, with the z-axis in the same direction. For an array of M elements located arbitrarily in the reference coordinate system (fig. 1), the m^{th} element radiated field is given by equation A1.1:

$$\vec{E}_m\left(\vec{r}_m'\right) = I_m\left[\frac{e^{-jkr_m'}}{r_m'}\,\hat{\theta}'U_{Em}(\theta')\left(a_m e^{j\psi_m}\cos\varphi' + b_m\sin\varphi'\right) + \hat{\varphi}'U_{Hm}(\theta')\left(-a_m e^{-j\psi_m}\sin\varphi' + b_m\cos\varphi'\right)\right] \tag{A1.1}$$

for $0 \leq \theta' \leq \pi/2$, where

I_m	M^{th} element complex excitation coefficient
U_{Em}, U_{Hm}	M^{th} element E and H plane pattern
a_m, b_m, ψ_m	M^{th} element polarization parameters (see table I)
K	wave number $2\pi/\lambda$
r_m', θ', φ'	spherical coordinates in the element coordinate system

The element pattern U_{Em}, U_{Hm} can be described with an analytical expression ($(\cos\theta)^q$-type or other functions) or with experimentally measured data (discrete). If measured data are used, the pattern may include mutual coupling effects. The polarization parameters in table I are subject to the normalization describe by

$$a_m + b_m = 1 \tag{A1.2}$$

The electric field described by equation (A1.1) is in the element coordinate system. The total electric field due to all M elements is the superposition of the electric field of each element of the array. The total electric field is given by

$$\vec{E}(\vec{r}) = \sum_{m=1}^{M} \vec{E}_m(\vec{r}) \tag{A1.3}$$

where the vector fields $\vec{E}(\vec{r})$ and $\vec{E}_m(\vec{r})$ are defined in the reference coordinate system. A transformation of $\vec{E}_m(\vec{r}_m')$ (eq. (A1.1)) in the element coordinate system into $\vec{E}_m(\vec{r})$ in the reference coordinate system is described next. Figure 3 shows a detailed description of these coordinate systems. The transformation of coordinates for this problem only involves a translation. The transformation procedure is outlined as follows. Knowing the observation coordinates (r,θ,φ) and m^{th} element location (x_m, y_m, z_m), the observation point in the primed coordinate system is found by using:

$$\begin{aligned} x &= r\sin\theta\cos\varphi \\ y &= r\sin\theta\sin\varphi \\ z &= r\cos\theta \end{aligned} \tag{A1.4a}$$

$$\begin{aligned} x' &= x - y_m \\ y' &= y - y_m \\ z' &= z - z_m \end{aligned} \tag{A1.4b}$$

$$r_m' = \sqrt{x'^2 + y'^2 + z'^2}$$

$$\theta' = \cos^{-1} \frac{z'}{r_m'} \qquad \text{(A1.4c)}$$

$$\varphi' = \tan^{-1} \frac{y'}{x'}$$

With equation (A1.4c) computed, all parameters on equation (A1.1) can be calculated. The vector transformation is obtained by using:

$$\begin{bmatrix} E_{Rm} \\ E_{\theta m} \\ E_{\varphi m} \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{bmatrix} \underline{I} \begin{bmatrix} \sin\theta'\cos\varphi' & \cos\theta'\cos\varphi' & -\sin\varphi' \\ \sin\theta'\sin\varphi' & \cos\theta'\sin\varphi' & \cos\varphi' \\ \cos\theta' & -\sin\theta' & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} 0 \\ E_{\theta m}' \\ E_{\varphi m}' \end{bmatrix} \qquad \text{(A1.5)}$$

where $\underline{I}$ is the identity matrix (3 x 3).

Equation (A1.5) transformed $\vec{E}_m(\vec{r}_m')$ into $\vec{E}_m(\vec{r})$ in the reference coordinate system. This process is repeated for each array element. Notice that no constraints have been put into equation (A1.3) regarding the observation distance. This expression (eq. (A1.3)) is valid everywhere except at the location of the source itself. This formulation assumes that each pattern is boresighted in the +z direction. However, the identity matrix $\underline{I}$ in equation (A1.5) could be replaced by a rotation matrix (Euler matrix of transformation) to account for arbitrary pointing.

The array radiation pattern is usually divided into two orthogonal polarizations. Equation (A1.3) may be written:

$$\vec{E}(\vec{r}) = \sum_{m=1}^{M} \vec{E}_m(\vec{r}) = \sum_{m=1}^{M} E_{\theta m}(\vec{r})\hat{\theta} + \sum_{m=1}^{M} E_{\varphi m}(\vec{r})\hat{\varphi} \qquad \text{(A1.5a)}$$

which can be expressed as:

$$\vec{E}(\vec{r}) = E_{\theta}\hat{\theta} + E_{\varphi}\hat{\varphi} \qquad \text{(A1.5b)}$$

The orthogonal components described in equation (A1.5b) are the usual spherical components. Another way of dividing the electric field into two orthogonal polarization is by using Ludwig's definition 3 (ref. 12). The following polarization vectors are introduced:

$$\hat{R} = \hat{\theta}\left(ae^{j\psi} \cos \varphi + b \sin \varphi\right) + \hat{\varphi}\left(-ae^{-j\psi} \sin \varphi + b \cos \varphi\right) \tag{A1.5c}$$

$$\hat{C} = \hat{\theta}\left(ae^{-j\psi} \sin \varphi - b \cos \varphi\right) + \hat{\varphi}\left(ae^{-j\psi} \cos \varphi + b \sin \varphi\right) \tag{A1.5d}$$

The reference polarization and cross polarization expressions of $E(r)$ are:

$$\text{Reference polarization of } \vec{E}\text{:} \quad E_R = \vec{E} \cdot \hat{R}^* \tag{A1.5e}$$

$$\text{Cross polarization of } \vec{E}\text{:} \quad E_C = \vec{E} \cdot \hat{C}^* \tag{A1.5f}$$

With these expressions equation (A1.5a) can be rewritten as:

$$\vec{E}(\vec{r}) = E_R\hat{R} + E_C\hat{C} \tag{A1.5g}$$

The parameters a, b, and ψ can be obtained from table I.

POWER RADIATED

The total power radiated (time-averaged) by the array is given by:

$$P_{rad} = \oiint_S \mathrm{Re}(\vec{E} \times \vec{H}^*) \cdot \vec{ds} \tag{B.1}$$

where

$\nabla \times \vec{E} = -jw\mu_0\vec{H}$ Maxwell equation

ds $\hat{a}_r r^2 \sin \theta \, d\theta \, d\varphi$ differential surface area

S a sphere of radius r

In the far-field of the array (usually taken at $2\Delta^2/\lambda$, Δ is maximum array dimension), the power radiated given in equation (B.1) can be simplified to:

$$P_{rad} = \oiint_S \frac{\vec{E}(\vec{r}) \cdot \vec{E}^*(\vec{r})}{Z_0} r^2 \sin \theta \, d\theta \, d\varphi \tag{B.2}$$

(Z_0 is free space impedance)

Substituting equation (A1.3) into equation (B.2) gives:

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi/2} \left(\sum_{m=1}^{M} \vec{E}_m(\vec{r})\right) \cdot \left(\sum_{n=1}^{M} \vec{E}_n(\vec{r})\right)^* \frac{r^2}{Z_0} \sin \theta \, d\theta \, d\varphi \tag{B.3}$$

In general, the above expression does not have a closed form solution and is evaluated numerically using a Romberg integration algorithm (ref. 13).

Using far-field approximations, equation (A1.1) can be simplified as follows: (This will restrict the observation distance to be only in the far-field of the array.)

$$\vec{E}(\vec{r}) = \sum_{m=1}^{M} \vec{E}_m(\vec{r})\, e^{jk\hat{u} \cdot \vec{\zeta}_m} \tag{B.4}$$

where

$\hat{u}$	unit vector, $\sin\theta \cos\varphi\, \hat{x} + \sin\theta \sin\varphi\, \hat{y} + \cos\theta\, \hat{z}$
$\vec{\zeta}_m$	position vector, $x_m\hat{x} + y_m\hat{y} + z_m\hat{z}$
$e^{jk\hat{u} \cdot \hat{\zeta}_m}$	array factor component

$\vec{E}_m(\vec{r})$ in equation (B.4) is given by:

$$\vec{E}_m(\vec{r}) = I_m \frac{e^{-jkr}}{r}\left[\hat{\theta}' U_{Em}(\theta')\left(a_m e^{i\psi_m} \cos\varphi' + b_m \sin\varphi'\right) + \hat{\varphi}'\, U_{Hm}(\theta')\left(-a_m e^{i\psi_m} \cos\varphi' + b_m \sin\varphi'\right)\right] \tag{B.5}$$

With these far-field approximations all position vectors $\vec{r}_m$ are parallel, making $(\hat{\theta}',\hat{\varphi}')$ equal $(\hat{\theta},\hat{\varphi})$. No further coordinate transformation is required. Substituting equation (B.4) into equation (B.2) produces:

$$P_{rad} = \sum_{m=1}^{M}\sum_{n=1}^{M} I_m I_n^* \left(\frac{1}{Z_o}\int_0^{2\pi}\int_0^{\pi/2} A(\theta,\varphi)\, e^{jk\hat{u} \cdot (\vec{\zeta}_m - \vec{\zeta}_n)} \sin\theta\, d\theta\, d\varphi\right) \tag{B.6}$$

where

$$A(\theta,\varphi) = \Big[\left(U_{Em}U_{En}^* a_m a_n e^{j(\psi_m - \psi_n)} + U_{Hm}U_{Hn}^* b_m b_n\right)\cos^2\varphi + \left(U_{Em}U_{En}^* b_m b_n + U_{Hm}U_{Hn}^* a_m a_n e^{j(\psi_m - \psi_n)}\right)\sin^2\varphi + \left(U_{Em}U_{En}^* - U_{Hm}U_{Hn}^*\right)\left(a_m b_n e^{j\psi_m} + a_m b_n e^{-j\psi_n}\right)\sin\varphi\cos\varphi\Big]$$

By defining

$$R_{mm} = \frac{1}{Z_o}\int_0^{2\pi}\int_0^{\pi/2} A(\theta,\varphi)\, e^{j\hat{u} \cdot (\vec{\zeta}_m - \vec{\zeta}_n)} \sin\theta\, d\theta\, d\varphi \tag{B.7}$$

the equation (B.6) can be expressed in the matrix form:

$$P_{rad} = \sum_{m=1}^{M} \sum_{n=1}^{M} R_{mn} I_m I_n^* \tag{B.8}$$

The coefficient R_{mn} in equation (B.7) is a M x M matrix. The evaluation of R_{mn} is time consuming and it takes the most computer time in the analysis. Reference 10 shows a closed form solution to equation (B.7) for special case of the array element located in the x-y plane having identical polarization parameters.

DIRECTIVITY

The directivity is defined by

$$D(\theta,\varphi) = \frac{4\pi \dfrac{\vec{E}(\vec{r}) \cdot E^*(\vec{r})}{Z_o} r^2}{P_{rad}} \tag{C.1}$$

$D(\theta,\varphi)$ is known as total directivity. Also the reference directivity and the cross directivity can be easily obtained (ref. 14).

Reference directivity:

$$D_R(\theta,\varphi) = \frac{4\pi \dfrac{|\vec{E} \cdot \hat{R}|^2}{Z_o} r^2}{P_{rad}} \tag{C.1a}$$

Cross directivity:

$$D_C(\theta,\varphi) = \frac{4\pi \dfrac{|\vec{E} \cdot \hat{C}|^2}{Z_o} r^2}{P_{rad}} \tag{C.1b}$$

NUMERICAL RESULTS AND DISCUSSIONS

This section presents some numerical results to demonstrate the applications of the computer program. In order to substantiate the accuracy of the generalized array formulation and computer program, detailed comparisons were made with the results presented by King and Wong (ref. 2) and experimental data obtained at NASA Lewis (ref. 15). King and Wong reported on an N x N planar array configuration with symmetrical element patterns of the cos $(\theta)^q$-type. They used direct integration to compute the radiated power. The examples considered were an array of 2 x 2 elements for various element spacings and a 3 x 3 array for which element pattern and frequency were varied. In the NASA Lewis experimental case, a 2 x 2 array of rectangular horns was used. In this case element pattern were measured in the array environment (to account for mutual coupling).

Very good agreement was obtained in all cases. The 2 x 2 array reported by King and Wong assumed a symmetrical E-H-plane patterns with a $q = 3.54$. Figure 4 shows a graphical description of the directivity as a function of element spacing for this array. The 3 x 3 array example used a symmetrical pattern but the performance is described as a function of element pattern, $(\cos(\theta)^q$, varying q) and operating frequency. Table II shows the results from both approaches. In the NASA Lewis experimental case the 2 x 2 array was investigated relative to far-field patterns. These patterns were measured at three different scan angles (boresight, 3° and 5°). The element spacing and frequency were fixed ($S = 2.5\ \lambda$, frequency = 30 GHz). These results are presented in figures 4(a) to (e).

A user guide for the programs developed is presented in appendix A. The implementation of equations (A1.3), (B.8), and (C.1) (antenna pattern, power, and directivity, respectively) with a computer program is given in appendix B. This program (appendix B) can be easily interfaced with available plotting routines for displaying the far-field antenna patterns. The numerical technique used to solve for equation (B.8) is not unique in any sense, but it was found to be faster than just using direct integration. Many other techniques can also be used, and easily implemented in the computer program (appendix B).

CONCLUDING REMARKS

One of the advantages of this generalized array formulation is that is does not break-down for special cases as might occur in approximations using closed forms. Also the formulation developed does not limit the pattern observation to the far-field zone. This can be very useful if the generalized formulation is going to be used with analysis programs for dual reflector configurations. The program developed can be easily modified to be implemented in the analysis of reflectors with phased array feeds.

This computer program is one of the key research tools at the NASA Lewis for analyzing advanced space communication antenna systems. The generalized formulation and computer program provides complete flexibility in analysis of array configurations and in the accurate analysis of experimental data.

APPENDIX A

USER GUIDE

PACAL1

Program Description

Given an angle phi and an array of sources, each with its current magnitude and relative phase, this program calculates the cross polarization, reference polarization, and the far field magnitude for a series of angles theta.

Input (FT05)

X,Y,Z:coordinates of each source (in meters).
AMPL: current amplitude for each source (in amperes).
APHA: relative phase for each source (in radians).
M: total number of sources.
POL: denotes polarization
  1: linear polarization (x-direction)
  2: linear polarization (y-direction)
  3: right-hand circular polarization
  4: left-hand circular polarization
QE,QH: exponent of the cosine function that is used to approximate the element pattern in the analytic form.
INZT: interval between each elevation angle theta.
I: 1+phi, parameter to set for desired cut.

Extraneous Input: (to be changed accordingly)

line 2000 AWAVE=3E8/(frequency)
line 2500 ZETA=(-(total range of angles)+(j-1)*INZT)*PI/180

Output:

(FT08)
  AZETA: value of theta at which the field values are taken (units degrees).
  ERAD: far field magnitude.
(FT07)
  AZETA: (see FT08)
  AREF: reference polarization magnitude
(FT06)
  AZETA: (see FT08)
  ACR: cross polarization magnitude

Using the Program

Create an input file assigned to FT05 and output files assigned to FT06,FT07, and FT08. Running the program will fill FT06,FT07, and FT08 with data that can be used in the program ZPLOT1 to plot the appropriate graphs, while also printing the number of points calculated.

To run:

```
DDEF FT05F001,VS,IN1
DDEF FT06F001,VS,PAOUT3
DDEF FT07F001,VS,PAOUT2
DDEF FT08F001,VS,PAOUT1
PACAL1
```

Example of IN1:

```
 &INPUT    X=.015,-.015,-.015,.015   Y=.015,.015,-.015,-.015
 AMPL=1,1,1,1 APHA=0,0,0,0 M=4,QE=22,QH=16,INZT=.1,I=1 &END
```

PAA1

Program Description

This program takes a series of sources with different amplitudes and phases, and determines the power via two methods.

Input (FT09):

M: number of sources XX,YY,ZZ: x,y and z coordinates of source (meters) A: amplitude of source PHI: relative phase of source.

IPOL: denotex polarity
- 1: X-linear polarized
- 2: Y-linear polarized
- 3: circularly polarized

QE,QH: exponents of cosine functions..

Output:

(FT15) YPOWER: the power using direct integration method. This serves as an input to PADTV.

PADTV

Program Description

This program requires the input file to PAA1 as well as the output file and then calculates the peak directivity.

Input:
- (FT09) Input to PAA1
- (FT15): Output to PAA1

Output (FT06)
- DIR: peak directivity

ZPLOT1

Program Description

This program is designed to plot either the far-field, reference polarization, or cross polarization as calculated in PACAL1.

Input:

(FT06)

X(I),Y(J) (meters)

These values are the output of the previous program PACAL1. If the plot of the far field is desired, the file which was assigned to FT08 in PACAL1 should now be assigned to unit 6. Similarly, if the plot of the reference or cross polarization is desired, the file assigned to FT07 or FT06, respectively, should now be assigned to FT06.

Extraneous Input:

lines 1200,1250

IVARS=NP=total number of points-1

(This can be obtained from the output of PACAL1)

lines 3500,3600

VARS(4)=lower boundary of angles to be plotted.
VARS(5)=upper boundary of angles to be plotted.

Using the Program

After running PACAL1, there will be data in the files assigned to FT08,FT07, and FT06, which in this case will be denoted as A,B,and C. Assign either A,B,orC, to FT06 (after releasing (FT08,FT07, and FT06), depending on whether the far field,reference, or cross polarization plots are desired . Then, run the program with the appropriate plotting device (sidecar 4015) and the plot will appear.

To run:

```
RELEASE FT08
RELEASE FT07
RELEASE FT06
DDEF FT06F001,VS,PAOUT1
ZPLOT1
```

If this routine is executed after that shown in the PACAL1 section, the far field will be plotted.

PROCDEF PAPLOT1

This procdef runs the programs PACAL1 and ZPLOT1 in succession while defining the necessary input and output devices.

As it is listed, file PA114 should contain the input to PACAL1. After assigning the devices FT08,FT07, and FT06 to files PAOUT1, PAOUT2, and PAOUT3, respectively, running PACAL1 will fill the respective file with the far field, reference polarization, and cross polarization magnitudes. After running PACAL1, setting the device FT06 to PAOUT1 will cause ZPLOT1 to plot the far field magnitude.

The Procdef:

```
ERASE PAOUT1 ERASE PAOUT2 ERASE PAOUT3 DDEF
FT08F001,VS,PAOUT1 DDEF FT07F001,VS,PAOUT2 DDEF
FT06F001,VS,PAOUT3 DDEF FT05F001,VS,PA114 PACAL1
RELEASE FT08 RELEASE FT07 RELEASE FT06 RELEASE FT05
GRAPH2D DDEF FT06F001,VS,PAOUT1 ZPLOT1
```

APPENDIX B

PCAL1

```
C*****THIS PROGARM WILL CALCULATE THE TRANVERSE ELECTRIC FIELD*********
C*****AND THE REFERENCE/CROSS POLARIZATION COMPONENTS OF THE FIELD*****



C*****DIMENSION STAMENTS***********************************************

     REAL INZT
      DIMENSION X(100),Y(100),AMPL(100),APHA(100)


C*****INPUT DATA: X(I) ;X-COORDINATE OF HORN I,Y(I) ;Y-COORDINATE OF****
C*****HORN I.AMPL(I) ; AMPLITUDE OF I HORN, APHA(I) ; PHASE OF THE I ***
C*****HORN,M ; NUMBER OF HORNS,POL ; 1=X-POL,2=Y-POL,3-RHCP,4=LHCP.****
C*****QH  ; EXPONENT H PLANE, QE ;EXPONENT OG E-PLANE******************


      NAMELIST/INPUT/X,Y,Z,AMPL,APHA,M,POL,QH,QE,INZT,I
      READ(5,INPUT)
      INPH=1
      J=0
      AWAVE=3E8/10E9
      PI=4.*ATAN(1.)
      AK0=2.*PI/AWAVE
C*****SET THE ANGLES FOR THE PLOT**************************************

 310  CONTINUE
      J=J+1
      ZETA=(-88.+(J-1)*INZT)*PI/180.
      APHI=(I-1)*INPH*PI/180.


      AZETA=ZETA*180./PI
      AAPHI=APHI*180./PI
      IF(AZETA.GT.88.)GO TO 11
      IF(AAPHI.GT.180.)GO TO 999

      RAZP=0.
      AIMZP=0.
      DO 20 K=1,M
C*****START THE ARRAY FACTOR SUMATION**********************************
      ANG=APHA(K)+X(K)*SIN(ZETA)*COS(APHI)+Y(K)*SIN(ZETA)*SIN(APHI)
      ANG=AK0*ANG
      RAZP=RAZP+AMPL(K)*COS(ANG)
      AIMZP=AIMZP+AMPL(K)*SIN(ANG)
20    CONTINUE
C*****SET POLARIZATION PARAMETERS**************************************
      IF(POL.EQ.1.)GO TO 500
      IF(POL.EQ.2.)GO TO 550
      IF(POL.EQ.3.)GO TO 600
      IF(POL.EQ.4.) GO TO 650
      GO TO 999
500   A1=1.
      A2=0.
      SI=0.
      GO TO 800
550   A1=0.
      A2=1.
      SI=0.
      GO TO 800
600   A1=1./SQRT(2.)
      A2=1./SQRT(2.)
      SI=PI/2.
      GO TO 800
650   A1=1./SQRT(2.)
      A2=1./SQRT(2.)
      SI=-PI/2.
```

```
800   CONTINUE


C*****START THE ELEMENT PATTERN CALCULATION***************************
      AUE=(COS(ZETA))**QE
      AUH=(COS(ZETA))**QH
      IF(AUE.LT.1E-3)AUE=1E-3
      IF(AUH.LT.1E-3)AUH=1E-3
      REEZL=A1*COS(SI)*COS(APHI)+A2*SIN(APHI)
      REEZL=AUE*REEZL
      AIMEZL=A1*SIN(SI)*COS(APHI)
      AIMEZL=AUE*AIEMZL
      REEPH=A2*COS(APHI)-A1*SIN(APHI)*COS(SI)
      REEPH=AUH*REEPH
      AIMEPH=-1*A1*SIN(APHI)*SIN(SI)
      AIMEPH=AUH*AIMEPH



C****START THE TOTAL FIELD CALCULATION AT ZETA PHI OBSERVATION*********

      AREZE=RAZP*REEZL-AIMZP*AIMEZL
      AIMZE=AIMZP*REEZL+RAZP*AIMEZL
      AREPH=RAZP*REEPH-AIMZP*AIMEPH
      AIMPH=RAZP*AIMEPH+AIMZP*REEPH
      AEZ=AREZE*AREZE+AIMZE*AIMZE
      AEP=AREPH*AREPH+AIMPH*AIMPH
      ERAD=AEZ+AEP

C*****START THE CALCULATION FOR CROSS POL AND THE REFERENCE POL********
      AB1=A1*COS(SI)*COS(APHI)+A2*SIN(APHI)
      AB2=-1.*A1*SIN(SI)*COS(APHI)
      AB3=-1.*A1*COS(SI)*SIN(APHI)+A2*COS(APHI)
      AB4=A1*SIN(SI)*SIN(APHI)
      AB5=A1*COS(SI)*SIN(APHI)-A2*COS(APHI)
      AB6=A1*SIN(SI)*SIN(APHI)
      AB7=A1*COS(SI)*COS(APHI)+A2*SIN(APHI)
      AB8=A1*SIN(SI)*COS(APHI)



C*** START THE CROSS AND REFERENCE  COMPUTATION*************************
      REER=(AREZE*AB1-AIMZE*AB2)+(AREPH*AB3-AIMPH*AB4)
      AIMER=(AREZE*AB2+AIMZE*AB1)+(AREPH*AB4+AIMPH*AB3)
      RECR=(AREZE*AB5-AIMZE*AB6)+(AREPH*AB7-AIMPH*AB8)
      AIMCR=(AREZE*AB6+AIMZE*AB5)+(AREPH*AB8+AIMPH*AB7)
      AREF=REER*REER+AIMER*AIMER
      ACR=RECR*RECR+AIMCR*AIMCR
      WRITE(8,400)AZETA,ERAD
      WRITE(7,400)AZETA,AREF
      WRITE(6,400)AZETA,ACR
 400  FORMAT(5X,E15.5,5X,E15.5)


      GO TO 310
11    WRITE(12,888)J
888   FORMAT(5X,'TOTAL NUMBER OF POINTS',I5)





999   STOP
      END
```

PAA1

```
      DIMENSION XX(100),YY(100)
      DIMENSION AMAT(100,100),YAMAT(100,100)
      DIMENSION GGXH(100),GGXE(100)
      DOUBLE PRECISION BE(100,100),BH(100,100),CE(100,100),CH(100,100)
      DIMENSION CDE (100,100),CDH(100,100),PHI(100),A(100),THETA(100,100)
      DIMENSION AR(100),AI(100)
      DOUBLE PRECISION QE,QH,P(100,100),XK,XXE,XXH
      DOUBLE PRECISION BESE(100,100),BESH(100,100)
      DOUBLE PRECISION ARHO
      REAL LAMDA
      DOUBLE PRECISION YBE(100,100),YBH(100,100),YCE(100,100),YCH(100,100)
      DOUBLE PRECISION ZA(100),ZB
      DOUBLE PRECISION XTHETA,DTHETA
      DOUBLE PRECISION AP1,AP2
      DOUBLE PRECISION AF11,AF12
C
C
C*****THIS ARE THE INPUTS TO POWER CALCULATION************************
C
      NAMELIST/INPUT/M,XX,YY,ZZ,A,PHI,IPOL,QE,QH
      READ (9,INPUT)
      FREQ=30E9
      LAMDA=3.E8/FREQ
      PI=4.*ATAN(1.)
      XK=2*PI/LAMDA




C*****THIS SECTION WILL CALCULATE THE DISTANCE AND ANGLE M,N***********
      DO 40 JA=1,M
      DO 50 IA=1,M
      IF((XX(IA).EQ.XX(JA)).AND.(YY(IA).EQ.YY(JA)))GO TO 55
      AL1=(XX(IA)-XX(JA))*(XX(IA)-XX(JA))
      AL2=(YY(IA)-YY(JA))*(YY(IA)-YY(JA))
      AB1=SQRT(AL1+AL2)
      AA1=(XX(IA)-XX(JA))/AB1
      AA2=(YY(IA)-YY(JA))/AB1

      THETA(IA,JA)=AA1*AA1-AA2*AA2
      P(IA,JA)=AB1
      GO TO 89
55    P(IA,JA)=0.
      THETA(IA,JA)=1.
89    CONTINUE


50    CONTINUE
 40   CONTINUE
C*****THIS SECTION WILL COMPUTE THE BESSEL AND GAMMA FUNCTIONS*********
      XXE=QE+.5
      XXH=QH+.5
      CALL GMMMA(XXE,GXXE,IER)
      CALL GMMMA(XXH,GXXH,IER)
      AF11=XXE+1
      AF12=XXH+1
      CALL GMMMA(AF11,ARE1,IER)
      CALL GMMMA(AF12,ARH1,IER)
      RATE=GXXE/ARE1
      RATH=GXXH/ARH1
      DO 60 JB=1,M
      DO 70 IB=1,M
      ARHO=XK*P(IB,JB)


      BESE(IB,JB)=BESJP(ARHO,XXE)
      BESH(IB,JB)=BESJP(ARHO,XXH)
  70  CONTINUE
```

```
 60   CONTINUE
C


C*****THIS SECTION WILL CALCULATE THE BE(I,J) AND BH(I,J) COEFICIENTS**


      DO 80 JH=1,M
      DO 90 IH=1,M
      DTHETA=PI/200
      YBE(IH,JH)=0
      YBH(IH,JH)=0
      YCE(IH,JH)=0.0
      YCH(IH,JH)=0.0
      IF(IH.EQ.JH)GO TO 91
      DO 800 IL=1,100
      XTHETA=DTHETA*(IL-1)
      ZA(IL)=ABS(COS(XTHETA))
      ZB=XK*P(IH,JH)*SIN(XTHETA)
      IF(COS(XTHETA).LT.1E-10)GO TO 800
      IF (ZB .LT. 1E-10) GO TO 806
      FACT1=ZA(IL)**(2*QE)
      FACT2=ZA(IL)**(2*QH)
      AP1=0.
      AP2=2.
      ABE1=BESJP(ZB,AP1)
      ABE2=BESJP(ZB,AP2)
      YBE(IH,JH)=YBE(IH,JH)+FACT1*ABE1*SIN(XTHETA)*DTHETA
      YBH(IH,JH)=YBH(IH,JH)+FACT2*ABE1*SIN(XTHETA)*DTHETA
      YCE(IH,JH)=YCE(IH,JH)+FACT1*ABE2*SIN(XTHETA)*DTHETA
      YCH(IH,JH)=YCH(IH,JH)+FACT2*ABE2*SIN(XTHETA)*DTHETA
      GO TO 800
 806  YBE(IH,JH)=YBE(IH,JH)+FACT1*SIN(XTHETA)*DTHETA
      YBH(IH,JH)=YBH(IH,JH)+FACT2*SIN(XTHETA)*DTHETA
 800  CONTINUE
      IF (IH .EQ. JH) GO TO 91
      BB1=(2**(QE-.5))*GXXE/((XK*P(IH,JH))**XXE)
      BB2=(2**(QH-.5))*GXXH/((XK*P(IH,JH))**XXH)
      BE(IH,JH)=BB1*BESE(IH,JH)
      BH(IH,JH)=BB2*BESH(IH,JH)
      GO TO 90
91    BE(IH,JH)=.5/XXE
      BH(IH,JH)=.5/XXH
      YBE(IH,JH)=.5*RATE
      YBH(IH,JH)=.5*RATH


  90  CONTINUE
 80   CONTINUE
C

C*****THIS SECTION WILL CALCULATE THE CH(I,J) AND CE(I,J) COEFICIENTS**
C
      DO 100 JC=1,M
      DO 110 IC=1,M
      CE(IC,JC)=0
      CH(IC,JC)=0
      DO 120 KK=1,12
      K=KK-1
      FACT=1
      DO 130 L=1,KK
 130  FACT=FACT*L
      XE=FLOAT(K)+QE+1.5
      XH=FLOAT(K)+QH+1.5
      CALL GMMMA(XE,GXE,IER)
      CALL GMMMA(XH,GXH,IER)
      IF (P(IC,JC) .EQ. 0) GO TO 225
      CC1=GXXE/((4**K)*FACT*GXE)
      CC2=(XK*P(IC,JC))**(2*K)
      CDE(IC,JC)=CC1*CC2
      AK=FLOAT(K)
      IF (AMOD(AK,2.) .NE. 0) CDE(IC,JC)=-CDE(IC,JC)
      GO TO 226
```

```
 225  CDE(IC,JC)=0
 226  CE(IC,JC)=CE(IC,JC)+.5*CDE(IC,JC)
      IF (P(IC,JC) .EQ. 0) GO TO 235
      DD1=GXXH/((4**K)*FACT*GXH)
      DD2=(XK*P(IC,JC))**(2*K)
      CDH(IC,JC)=DD1*DD2
      IF (AMOD(AK,2.) .NE. 0) CDH(IC,JC)=-CDH(IC,JC)
      GO TO 236
 235  CDH(IC,JC)=0
 236  CH(IC,JC)=CH(IC,JC)+.5*CDH(IC,JC)
 120  CONTINUE
      CH(IC,JC)=CH(IC,JC)-BH(IC,JC)
      CE(IC,JC)=CE(IC,JC)-BE(IC,JC)
 110  CONTINUE
 100  CONTINUE
C
C
C CALCULATING THE POWER
C
      ZREAL=0.
      ZIMAG=0.
      YZREAL=0
      YZIMAG=0
      DO 310 ID=1,M
      AR(ID)=A(ID)*COS(PHI(ID))
 310  AI(ID)=A(ID)*SIN(PHI(ID))
      IF(IPOL.EQ.1)GO TO 150
      IF(IPOL.EQ.2)GO TO 151
      GO TO 152
150   DO 320 J=1,M
      DO 330 I=1,M
      AMAT(I,J)=BE(I,J)+BH(I,J)+THETA(I,J)*(CH(I,J)-CE(I,J))
      YAMAT(I,J)=YBE(I,J)+YBH(I,J)+THETA(I,J)*(YCH(I,J)-YCE(I,J))
 330  CONTINUE
 320  CONTINUE
      GO TO 160
151   DO 220  J=1,M
      DO 230  I=1,M
      AMAT(I,J)=BE(I,J)+BH(I,J)+THETA(I,J)*(CE(I,J)-CH(I,J))
      YAMAT(I,J)=YBE(I,J)+YBH(I,J)+THETA(I,J)*(YCE(I,J)-YCH(I,J))
230   CONTINUE
220   CONTINUE
      GO TO 160
152   DO 420  J=1,M
      DO 430  I=1,M
      AMAT(I,J)=BE(I,J)+BH(I,J)
      YAMAT(I,J)=YBE(I,J)+YBH(I,J)
  430 CONTINUE
420   CONTINUE
160   CONTINUE
      DO 520 J=1,M
      DO 530 I=1,M
      PREAL=(AR(I)*AR(J)+AI(I)*AI(J))*AMAT(I,J)
      YPREAL=(AR(I)*AR(J)+AI(I)*AI(J))*YAMAT(I,J)
      YPIMAG=(AR(J)*AI(I)-AR(I)*AI(J))*YAMAT(I,J)
      PIMAG=(AR(J)*AI(I)-AR(I)*AI(J))*AMAT(I,J)
      ZREAL=ZREAL+PREAL
      ZIMAG=ZIMAG+PIMAG
      YZREAL=YZREAL+YPREAL
      YZIMAG=YZIMAG+YPIMAG
530   CONTINUE
520   CONTINUE
      ZREAL=(1/120.)*ZREAL
      ZIMAG=(1/120.)*ZIMAG
      YZREAL=(1/120.)*YZREAL
      YZIMAG=(1/120.)*YZIMAG
      POWER=SQRT(ZREAL**2+ZIMAG**2)
      YPOWER=SQRT(YZREAL**2+YZIMAG**2)
C
```

```
C  POWER OF SINGLE ELEMENT
      SINGLE=(QE+QH+1)/(60*(2*QE+1)*(2*QH+1))
      RATIO=POWER/SINGLE
      YRATIO=YPOWER/SINGLE


      WRITE(6,500)SINGLE
500   FORMAT(5X,'POWER OF A SINGLE ELEMENT=',1X,E15.5)
      WRITE(6,501)RATIO,YRATIO
501   FORMAT(///5X,'CLOSED FORM RATIO=',1X,E15.5,'DIRECT INT RATIO=',
      1X,1E15.5)
      WRITE(6,502)POWER,YPOWER
502   FORMAT(///5X,'CLOSE FORM POWER=',1X,E15.5,'DIRECT INT POWER=',1X,
      E15.5)
      WRITE (15,600) YPOWER
      WRITE (16,600) POWER
 600  FORMAT (E15.5)
C*****SPECIAL CASES FOR CHECKING RESULTS PREVIOUSLY CALCULATED*********



C*****LARGE SPACING CASE 6 LAMDA OR GREATER****************************

      POWER1=0.
      DO 550 I=1,M
      POWER1=POWER1+A(I)
550   CONTINUE
      POWER1=SINGLE*POWER1
      IF(IPOL.GE.3) GO TO 810
      GO TO 811
810   POWER1=2.*POWER1
811   CONTINUE
      WRITE(6,503)POWER1
503   FORMAT(5X,'LARGE SPACING POWER=',1X,E15.5)


C*****SYMMETRIC PATTERN QH=QE POWER CALCULATION***********************
      RECF=0.
      REDI=0.
      RIMCF=0.
      RIMDI=0.
      DO 560 J=1,M
      DO 561 I=1,M
      CA22=(AR(I)*AR(J)+AI(I)*AI(J))*BE(I,J)
      CA33=(AR(I)*AR(J)+AI(I)*AI(J))*YBE(I,J)
      CA44=(AR(J)*AI(I)-AR(I)*AI(J))*BE(I,J)
      CA55=(AR(J)*AI(I)-AR(I)*AI(J))*YBE(I,J)
      RECF=RECF+CA22
      REDI=REDI+CA33
      RIMCF=RIMCF+CA44
      RIMDI=RIMDI+CA55
561   CONTINUE
560   CONTINUE
      POLPCF=SQRT(RECF**2+RIMCF**2)*(1/60.)
      POLPDI=SQRT(REDI**2+RIMDI**2)*(1/60.)
      IF(IPOL.GE.3)GO TO 710
      GO TO 711
710   POLPCF=2.*POLPCF
      POLPDI=2.*POLPDI
711   CONTINUE
      WRITE(6,510)POLPCF,POLPDI
510   FORMAT(///5X,'POWER CLOSE FORM SYM=',1X,E15.5,'POWER D I SYM=',1X,
      E15.5)
      STOP
      END
```

PADTV

```
      DIMENSION THETA(100),PHI(100),ETHETA(100,100),XX(100)
      DIMENSION YY(100),AMP(100),PHASE(100)
      DIMENSION AR(100,100),AI(100,100),AA(100,100)
      DIMENSION XRADI(100,100),EPHI(100,100)
      DIMENSION UE(100),UH(100),XIL(100,100)
      DIMENSION A(100)
C
C
C READING VARIABLES
C
      NAMELIST/INPUT/M,XX,YY,ZZ,QE,QH,A,PHI,IPOL
      READ (9,INPUT)
      DO 130 I=1,M
      AMP(I)=A(I)
      PHASE(I)=PHI(I)
 130  CONTINUE
      READ (15,131) POWER
 131  FORMAT (E15.5)
C
      PI=4.*ATAN(1.)
      FREQ=29.5E9
      XLAMDA=3.E8/FREQ
      XK=2*PI/XLAMDA
      Z0=377
C
C
C TESTING POLARITY
C
      IF (IPOL .NE. 1) GO TO 102
      C=1
      B=0
      PSI=0
      GO TO 104
C
 102  IF (IPOL .NE. 2) GO TO 103
      C=0
      B=1
      PSI=0
      GO TO 104
C
 103  C=1/SQRT(2.)
      B=1/SQRT(2.)
      IF (IPOL .NE. 3) GO TO 105
      PSI=PI/2
      GO TO 104
 105  PSI=-PI/2
C
C
C SOLVING EQUATIONS
C
 104  DO 100 I=1,100
      DO 101 J=1,100
C
      THETA(I)=(FLOAT(I-1)/99)*PI-PI/2
      PHI(J)=(FLOAT(J-1)/99)*PI-PI/2
C
      UE(I)=(COS(THETA(I)))**QE
      UH(I)=(COS(THETA(I)))**QH
C
      AA1=UE(I)*UE(I)
      AA2=(C*C)*(COS(PHI(J))*COS(PHI(J)))
      AA3=(B*B)*(SIN(PHI(J))*SIN(PHI(J)))
      AA4=2.*C*B*(COS(PSI)*COS(PHI(J))*SIN(PHI(J)))
      ETHETA(I,J)=AA1*(AA2+AA3+AA4)
      BB1=UH(I)*UH(I)
      BB2=C*C*(SIN(PHI(J))*SIN(PHI(J)))
      BB3=(B*B)*(COS(PHI(J))*COS(PHI(J)))
      BB4=AA4
      EPHI(I,J)=BB1*(BB2+BB3-BB4)
```

```

C
      XIL(I,J)=ETHETA(I,J)+EPHI(I,J)
      AR(I,J)=0
      AI(I,J)=0
C
      DO 110 K=1,M
      CC1=XK*(XX(K)*SIN(THETA(I))*COS(PHI(J)))
      CC2=XK*(YY(K)*SIN(THETA(I))*SIN(PHI(J)))
      EEXP=CC1+CC2
      DREAL=AMP(K)*COS(PHASE(K)+EEXP)
      AIMAG=AMP(K)*SIN(PHASE(K)+EEXP)
      AR(I,J)=AR(I,J)+DREAL
      AI(I,J)=AI(I,J)+AIMAG
 110 CONTINUE
C
      AA(I,J)=AI(I,J)*AI(I,J)+AR(I,J)*AR(I,J)
      XRADI(I,J)=AA(I,J)*XIL(I,J)
  101  CONTINUE
 100  CONTINUE
C
C
C SORTING
C
      DO 120 IA=1,100
      DO 121 JA=2,100
      IF (XRADI(IA,1) .GE. XRADI(IA,JA)) GO TO 121
      XA=XRADI(IA,1)
      XRADI(IA,1)=XRADI(IA,JA+1)
      XRADI(IA,JA+1)=XA
 121  CONTINUE
 120  CONTINUE
C
      DO 122 IB=2,100
      IF (XRADI(1,1) .GE. XRADI(IB,1)) GO TO 122
      XB=XRADI(IB,1)
      XRADI(IB,1)=XRADI(1,1)
      XRADI(1,1)=XB
 122  CONTINUE
C
     WRITE(8,973)XRADI(1,1)
973   FORMAT(5X,'THIS IS THE PEAK VALUE',3X,E15.5)
      DIR=4.*PI*XRADI(1,1)/(POWER*ZO)
      DIR =10.*ALOG10(DIR)
      WRITE (6,123) DIR
 123  FORMAT (5X,'DIRECTIVITY=',E15.5)
      STOP
      END
```

ZPLOT1

```
C*****THIS PROGRAM CAN BE USED TO PLOT THE ANTENNA
C*****FAR-FIELD PATTERN.(E-PLANE OR H-PLANE CUTS)
C*****BY R.J. ACOSTA
      DIMENSION X(5000),Y(5000),IVARS(20),VARS(20),RTNARR(2)

      DIMENSION XTITLE(5),YTITLE(5)
       LOGICAL*1 IAXIS
       INTEGER*2 N1
     DATA XTITLE/'ELEV','ATIO','N AN','GLE ','DEG.'/
     DATA YTITLE/'RELA','TIVE',' AMP','LITU','DE  '/
     CALL TITLE(4,20,15,XTITLE)
     CALL TITLE(3,17,15,YTITLE)




      IVARS(1)=2
      IVARS(2)=1760
      NP=1760

      N1=NP





      DO 15 J=1,NP
      READ(6,300)X(J),Y(J)
300   FORMAT(5X,E15.5,5X,E15.5)
15    CONTINUE
      CALL SCLBAK(.FALSE.,N1,Y,RTNARR)
      VMAX=RTNARR(2)



      DO 16 J=1,NP
      Y(J)=Y(J)/VMAX
      IF(Y(J).LT.1E-8)Y(J)=1E-8
      Y(J)=10.*ALOG10(Y(J))
 16   CONTINUE




      VARS(1)=9.
      VARS(2)=8.
      VARS(3)=0.
      VARS(4)=-90.
      VARS(5)=90.
      VARS(6)=6.
      VARS(7)=2.
      VARS(8)=-1.
      VARS(9)=0.
      CALL XAXIS(-1.,-1.,VARS)
      VARS(2)=9.
      VARS(3)=90.
      VARS(5)=0.
      VARS(6)=8.
      VARS(4)=-80.
      CALL YAXIS(-1.,-1.,VARS)
      CALL GPLOT(X,Y,IVARS)
      CALL DISPLA(1)

      READ(9,993)XYZ
993   FORMAT(1A1)
      CALL TERM
     STOP
      END
```

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TABLE I. - POLARIZATION PARAMETERS

Polarization type	a_m	b_m	ψ_m
Linear-X	1	0	0
Linear-Y	0	1	0
RHCP[a]	1/ 2	1/ 2	0.5 π
LHCP[b]	1/ 2	1/ 2	-0.5 π

[a] RHCP (right-hand circular polarized).

[b] LHCP (left-hand circular polarized).

TABLE II. - COMPARISON OF DIRECTIVITY RESULTS WITH THOSE OBTAINED BY KING AND WONG (ref. 2)

Frequency, MHz	S/λ	Element pattern HPBW, deg	Pattern, $\cos^q(\theta)$ $q_E = q_H$	King-Wong measured directivity, dB	King-Wong, dB calculated	NASA Lewis, dB calculated
450	0.687	86.0	1.11	17.1	17.3	17.10
500	.763	92.0	0.96	17.9	18.0	17.83
600	.916	89.4	1.02	18.8	18.5	18.45
700	1.068	94.0	0.91	18.0	17.4	17.69
800	1.220	94.0	0.91	17.4	16.9	17.25

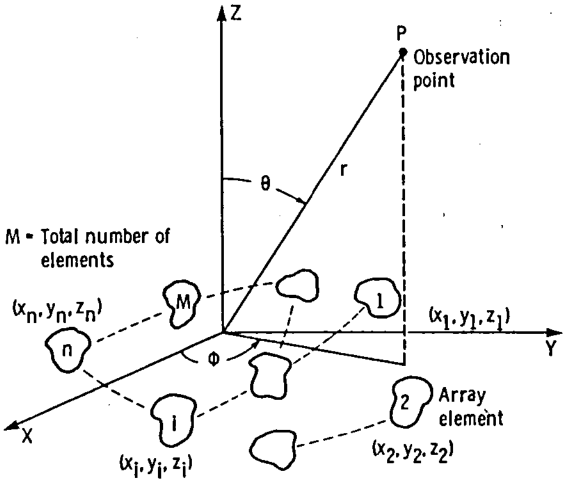


Figure 1. - Geometry of the generalized phased array.

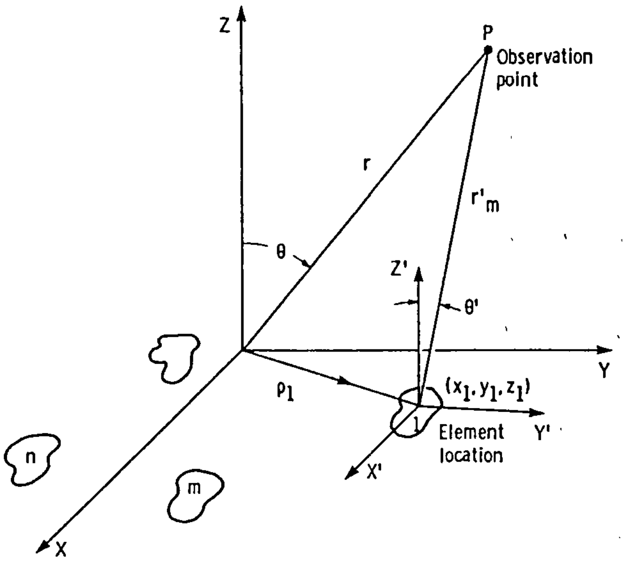


Figure 2. - A typical element coordinate system (X', Y', Z') and the reference coordinate system (X, Y, Z).

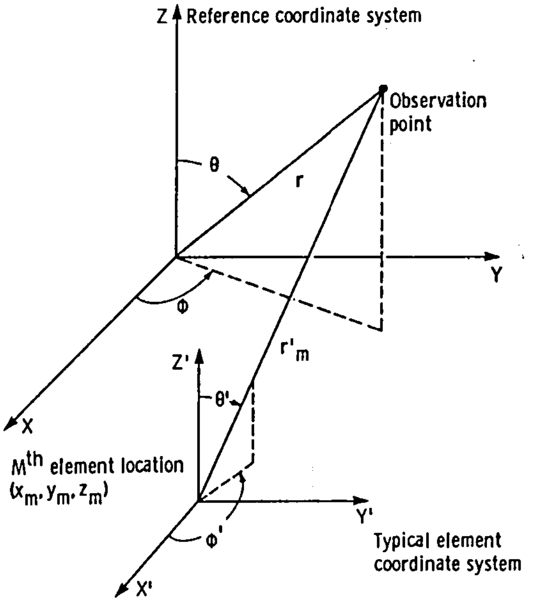


Figure 3. - Geometrical picture of the coordinate transformation between element coordinate system and the reference coordinate system.

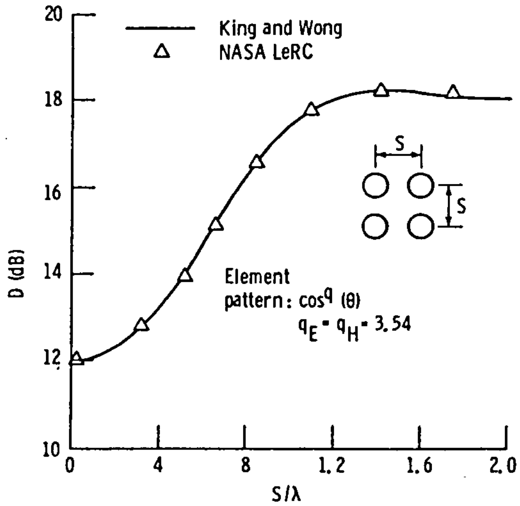
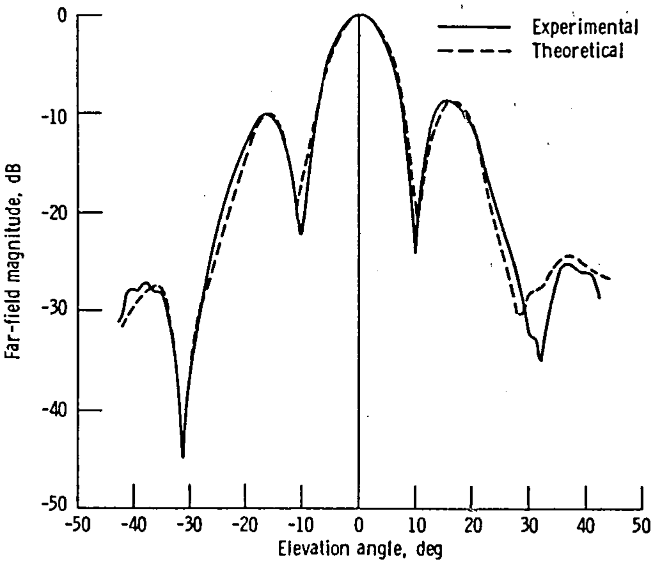
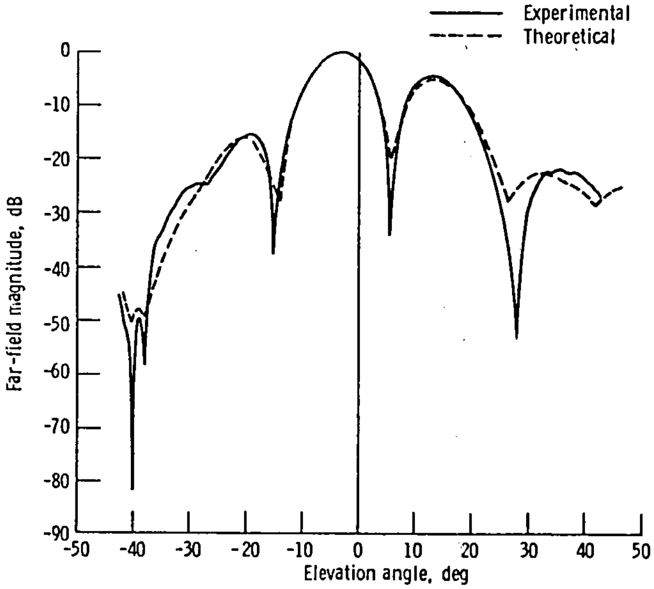


Figure 4. - Compared directivities for a 2x2 array as function of element spacing.



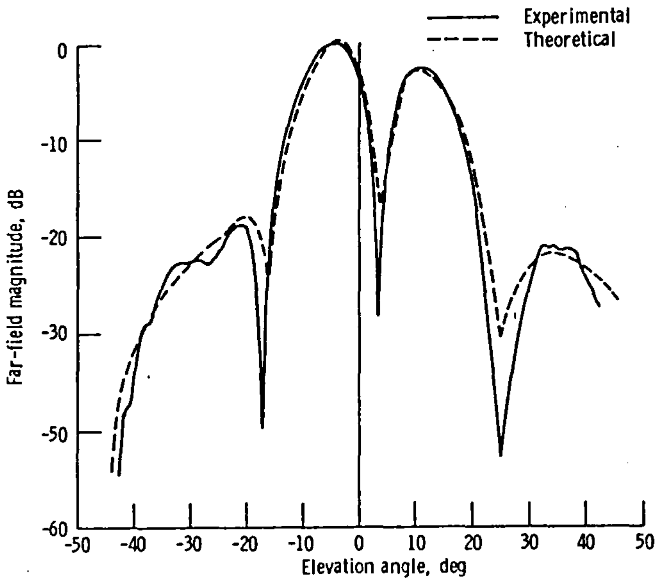
(a) Boresight, reference polarization far-field antenna pattern. (E-plane cut).

Figure 5.



(b) Scan case ($\theta_0 = 3.2^\circ$), reference polarization far-field antenna pattern. (E-plane cut).

Figure 5. - Continued.



(c) Scan case (θ_0 = 5.2°), reference polarization far-field antenna pattern (E-plane cut).

Figure 5. - Concluded.

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16. Abstract

With the advent of monolithic microwave integrated circuit (MMIC) technology, the phased array has become a key component in the design of advanced antenna systems. Array-fed antennas are used extensively in today's multiple beam satellite antennas. In this report, a computer program based on a very efficient numerical technique for calculating the radiated power (Romberg integration), directivity and radiation pattern of a phased array is described. The formulation developed is very general, and takes into account arbitrary element polarization, E- and H-plane element pattern, element location, and complex element excitation. For comparison purposes sample cases have been presented. Excellent agreement has been obtained for all cases. Also included in appendixes A and B are a user guide and a copy of the computer program.

17. Key Words (Suggested by Author(s))
Antenna patterns; Phased array; Numerical analysis; Near-field analysis

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