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SEMIEMPIRICAL METHODS FOR COMPUTING TURBULENT FLOWS

I. A. Belov, I. P. Ginzburg

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16. Abstract

The paper discusses two semiempirical theories which provide a basis for determining the turbulent friction and heat exchange near a wall -- (1) the Prandtl-Karman theory and (2) the theory utilizing an equation for the energy of turbulent pulsations. A comparison is made between exact numerical methods and approximate integral methods for computing the turbulent boundary layers in the presence of pressure, blowing, or suction gradients. Using the turbulent flow around a plate as an example, it is shown that, when computing turbulent flows with external turbulence, it is perfereable to construct a turbulence model based on the equation for energy of turbulent pulsations.

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SEMIEMPIRICAL METHODS FOR COMPUTING TURBULENT FLOWS

I. A. Belov, I. P. Ginzburg

Determination of turbulent friction and heat exchange in a wall region is based on two semiempirical theories: the Prandtl-Karman mixing path theory and a theory using equations for turbulent pulsation energy with additional Kolmogorov-Rotta hypotheses. /159*

1. **Prandtl-Karman Theory and Effective Viscosity Hypotheses.** Existing models for Prandtl-Karman's semiempirical theory of turbulence applied to boundary layer problems are distinguished by various assumptions regarding the size of the mixing path and the presence of a laminar sublayer. They underlie hypotheses that, when flow at the boundary layer of an axisymmetric wing or body is in question, turbulent flow may be calculated with Prandtl's equation

$$\tau=\rho l^2\left|\frac{\partial v_x}{\partial y}\right|\frac{\partial v_x}{\partial y}, \tag{1.1}$$

where l is the mixing path.

According to the two-layer system, it is assumed that the boundary layer may be broken down into a laminar sublayer in which laws of viscous friction and heat transfer are valid and a turbulent layer in which Prandtl's equation is valid. The thickness of the laminar sublayer δ_l is determined from the condition of a break in the derivatives of velocity at the boundary of the laminar sublayer.

$$\left.\frac{\partial v_x}{\partial y}\right|_{y=\delta_l-0}=K_1\left.\frac{\partial v_x}{\partial y}\right|_{y=\delta_l+0}, \tag{1.2}$$

*Numbers in the margin indicate pagination in the foreign text.

where K_1 is a certain universal constant. If one assumes that $l = Ky$ near the laminar sublayer, the following formula for calculating δ_1 results:

$$\delta_1 = \frac{K_1}{K} \nu \Big/ \sqrt{\frac{\tau}{\rho}}. \tag{1.3}$$

This model was consolidated for a compressible fluid and a gas mixture. In addition to the two universal constants K and K_1, we must introduce the concept of mixing paths for temperatures and for individual components in the mixture or, respectively, $Pr^T = l/l_T$ and $Sm_i^T = l/l_i$, where l_T and l_i are mixing paths for temperatures and for concentrations of the i-th component in a mixture.

As numerical analysis has shown, in calculating friction and heat transfer the hypothesis $l = Ky$ gives satisfactory results which correlate with experiments. The universality of constants K and K_1 produces the limit relationships derived by Kutateladze and Leont'yev [1]. To determine a velocity profile in a boundary layer which correlates with experiments, one must make more general assumptions about the mixing path, taking into account the fact that it has a constant value near the edge of the boundary layer. Here we might point out various functions $l/\delta = f(y/\delta)$ proposed by several authors. Spalding's standardized formula [6] was used at Leningrad State University (LGU) to calculate l/δ in numerical analyses of a turbulent boundary layer. /160

$$\frac{l}{\delta} = \begin{cases} K\frac{y}{\delta}, & \frac{\delta_1}{\delta} \leqslant \frac{y}{\delta} \leqslant \frac{\lambda}{K}; \\ \lambda, & \frac{\lambda}{K} \leqslant \frac{y}{\delta} \leqslant 1, \end{cases} \tag{1.4}$$

where λ is a certain constant close to 0.1; K = 0.4.

In a three-layer system (the Klauzer-Mellor hypothesis [7-8]) the boundary layer is divided into three regions: a laminar sublayer and the inner and outer regions of the boundary layer. It is presumed that:

$$y \leqslant \delta_l, \quad \tau = \mu \frac{\partial v_x}{\partial y}, \quad \delta_l = \frac{K_1}{K} \frac{\nu}{\sqrt{\tau/\rho}};$$

$$\frac{K_1}{K} \leqslant \frac{\sqrt{\frac{\tau}{\rho}}\, y}{\nu} \leqslant \frac{\beta_0}{K} \frac{U\delta^*}{\nu}, \quad \tau = \rho K^2 y^2 \left| \frac{\partial v_x}{\partial y} \right| \frac{\partial v_x}{\partial y}$$

is the boundary layer interior;

$$\frac{\sqrt{\frac{\tau}{\rho}}\, y}{\nu} \geqslant \frac{\beta_0}{K} \cdot \frac{U\delta^*}{\nu}, \quad \tau = \beta_0 \rho U \delta^* \frac{\partial v_x}{\partial y}$$

is the boundary layer exterior, where δ^* is displacement thickness; β_0 is a certain constant (according to Klauzer, it equals 0.016-0.018; in calculations done at LGU, it was set at 0.02); U is the velocity at the outer edge of the boundary layer.

There are also studies where the boundary layer is not divided into a series of regions, but a so-called effective viscosity is introduced, and it is assumed that $\tau = \mu_{ef} (\partial/\partial y) v_x$, whereby $\mu_{ef} = \mu + \mu_T$, where, according to Prandtl $\mu_T = \rho l^2 (\partial/\partial y) v_x$.

Mixing path l is determined differently for different regions of change in y. To account for the fact that the mixing path near a wall approaches zero at $y^{3/2}$, van Driest's equation [9] is used to calculate it in the boundary layer interior.

$$l = Ky\left[1 - \exp\left(-\frac{y}{A\nu}\sqrt{\frac{\tau_w}{\rho}}\right)\right], \quad \frac{y}{\delta} \leqslant \frac{\lambda}{K}, \qquad (1.5)$$

where A is a certain constant (according to van Driest, A = 26). According to Spalding [6], in the boundary layer exterior

$$\frac{l}{\delta} = \lambda, \quad \lambda = 0{,}1; \quad K = 0{,}4. \qquad (1.6)$$

To account for the effect of blowing and a pressure gradient for an incompressible fluid, Sebesi [10] assumes the following for the boundary layer interior /161

$$l = Ky\left[1 - \exp\left(-\frac{y}{A\nu}\sqrt{\frac{\tau}{\tau_w}}\sqrt{\frac{\tau_w}{\rho}}\right)\right]; \qquad (1.7)$$

whereby τ/τ_w is determined from an approximated solution to the equation

$$\rho v_w \frac{\tau}{\mu} = -\frac{dp}{dx} + \frac{\partial\tau}{\partial y}, \quad v_w = v_y|_{y=0}, \qquad (1.8)$$

from which it follows that

$$\tau = -\frac{dp}{dx}\cdot\frac{\mu}{\rho v_w} + \left(\tau_w + \frac{\mu}{\rho v_w}\cdot\frac{dp}{dx}\right)\exp\left(\frac{\rho v_w}{\mu}y\right). \qquad (1.9)$$

In the case of a compressible fluid, (1.8) gives

$$\tau = \left[\frac{dp}{dx}\int_0^y \exp\left(-\int_0^y \frac{\rho v_w}{\mu}dy\right)dy + \tau_w\right]\exp\left(\int_0^y \frac{\rho v_w}{\mu}dy\right).$$

In the absence of blowing,

$$\tau = \tau_w + \frac{dp}{dx}y \qquad (1.9')$$

and equation (1.7) takes the form

$$l = Ky\left[1 - \exp\left(-\frac{yv_*}{A}\sqrt{1 + \frac{y}{\tau_w}\cdot\frac{dp}{dx}}\right)\right]. \qquad (1.7')$$

With regard for blowing,

$$\sqrt{\frac{\tau}{\tau_w}} \cong \sqrt{\frac{\tau_л}{\tau_w}} = \left[\frac{dp}{dx}\cdot\frac{\mu}{\rho v_w}\left(-1 + \exp\frac{\rho v_w}{\mu}\delta_л\right) + \exp\frac{\rho v_w}{\mu}\delta_l\right]^{1/2}, \qquad (1.10)$$

wherein Sebesi assumes $\delta_l = \frac{K_1}{K}\cdot\frac{\nu}{v^*} = 11{,}8\frac{\nu}{v^*}$, where $v_* = \sqrt{\tau_w/\rho}$. In Sharov and Lapin's work [11], the effect of blowing and a pressure gradient was taken into account on the assumption that

$$\delta_1 = \frac{K_1}{K} \cdot \frac{\nu}{\sqrt{\tau_л/\rho}} = \frac{K_1}{K} \cdot \frac{\nu}{v_*} \sqrt{\frac{\tau_w}{\tau_1}} =$$
$$= \frac{K_1}{K} \cdot \frac{\nu}{v_*} \left[\frac{dp}{dx} \cdot \frac{\mu}{\rho v_w} \left(-1 + \exp \frac{\rho v_w}{\mu} \delta_1 \right) + \exp \frac{\rho v_w}{\mu} \delta_1 \right]^{-\frac{1}{2}}. \qquad (1.11)$$

This equation is solved numerically. Note that studies by Sebesi, Lapin, and Sharov divide the boundary layer into interior and exterior parts. In the interior, where l is calculated according to (1.7) with regard for (1.10); in the exterior, μ_T is determined from Klauzer's equation

Velocity profiles are joined on the basis of the equality of turbulent viscosity and of velocity continuity coefficients and velocity gradients.

To evaluate the possibility of using particular models of the semiempirical theory of turbulence, LGU numerically solved equations for movement of an incompressible fluid in a boundary layer in the presence of blowing and a pressure gradient (certain calculation results have been published in articles by G. V. Kocheryzhenkov, S. K. Matveyev, and V. S. Ivanov [2-5]). These calculations showed that discrepancies between results for one system and for the other in functions for friction coefficients and for $K = \delta^*/\delta^{**}$ from x, where x is the distance along a plate, are within limits of calculating accuracy. Figures 1-3* present certain results of the comparison. Note that numerical analysis is naturally simpler in the two-layer system, where there is no need to formulate clarifying hypotheses regarding the mixing path. /162

Along with numerical solution of equations in partial derivatives, approximated integral methods of calculating friction and heat transfer in the case of a turbulent boundary layer are used. These methods are based on use of integral

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*Figures are taken from [2-4].

relationships from boundary layer theory and an approximated concept of friction stress in the form of polynomials of y/δ and v_x/U such as

$$\sqrt{\tau/\tau_w} = 1 + \frac{A}{2}\eta - \left(1 + \frac{A}{2}\right)\eta^2, \tag{1.12}$$

$$\tau/\tau_w = 1 + B\xi - (1 + B)\xi^2, \tag{1.13}$$

where $\eta = y/\delta$, $\xi = v_x/U$.

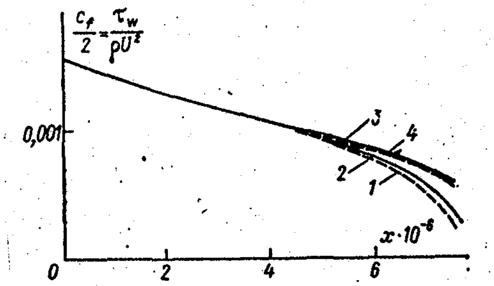

Fig. 1 Change in surface friction factor.
1, 2 - Numerical solution according to Mellor's hypothesis and the two-layer system; 3, 4 - Approximated solution using the same models.

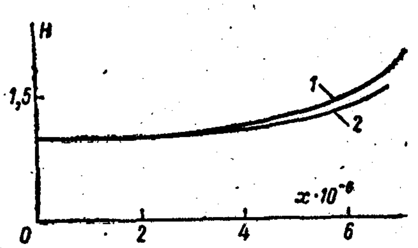

Figure 2. Change in $H = \delta^*/\delta^{**}$.
1 - Numerical solution with Mellor's hypothesis; 2 - Approximated solution using the same model.

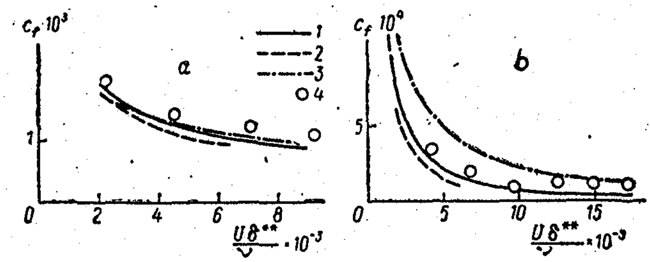

Figure 3. Surface friction factor in the turbulent boundary layer of a plate during blowing:
a - $v_w/U = 0.00386$; b - $v_w/U = 0.0095$; 1 - Two-layer system; 2 - Effective viscosity system; 3 - Sebesi's data; 4 - Simpson, Kays, Moffat experiment.

From conditions taken from motion equations, assuming that they are valid also near a wall, we find that

$$B=\frac{dp}{dx}\cdot\frac{U\mu_w}{\tau_w^2}+\frac{\rho_w v_w U}{\tau_w},$$
$$A=\frac{dp}{dx}\cdot\frac{\delta}{\tau_w}+\frac{\rho v_w \delta}{\mu_w}. \qquad (1.14)$$

Comparison of numerical analyses with approximated solution (if A in (1.12) is calculated from (1.14) when a pressure gradient and blowing are present) showed considerable discrepancy among results. This is because the effect that friction distribution over the surface has on the equation for A is not considered. In studies by G. V. Kocheryzhenkov and S. K. Matveyev [2, 12], A was calculated from the condition $A=\frac{\partial}{\partial\eta}\tau/\tau_w|_{\eta=\eta_f}$. In this case, /163

$$A=\frac{dp}{dx}\cdot\frac{\delta}{\tau_w}+\frac{\delta}{v_*}\cdot\frac{dv_*}{dx}\left(\frac{v_*}{U}\right)^2\beta^2+\frac{v_w\delta}{\nu}\cdot\frac{1}{K_1}\exp\left(K_1-K\frac{U}{v_*}\beta\right), \qquad (1.15)$$

where $\beta=(v_x/U)_{y=\delta_f}$. During calculation, it was assumed that β = 0.6. With regard for A in terms of (1.15), approximated calculations coincided with numerical solutions. The approximated solutions produced on the assumption of the law for τ/τ_w (1.13) and for B (1.14) showed a rather good correlation with the numerical solution.

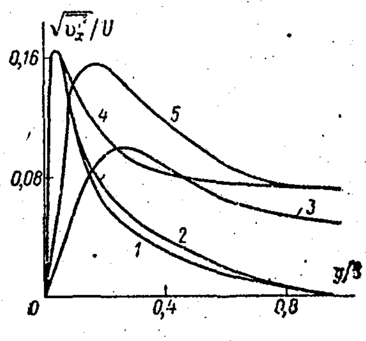

Fig. 4 Profiles of longitudinal root-mean-square velocity pulsation in the boundary layer with external flow turbulence ε_a = 4.6%.
1 - Calculated using van Driest's ratio for mixing flow; 2 - Same as per Spalding; 3-5 - Experiment:
3 - $\sqrt{v'^2_x}/U$ = 0.049;
$Re_x = U_{x/\nu}$ = 0.184 x 10^5;
4 - 0.072; 10^5; 5 - 0.083; 1.53 x 10^5.

In summary, one might note that this comparison of the results of calculations for different systems shows that calculating friction and heat transmission requires numerical computation of boundary layer equations and determination of turbulent friction for the two-layer system. Approximation methods should be used in evaluation, assuming $\tau/\tau_w=f(\xi)$ in accordance with (1.13) and (1.14). To find thermal flows, one can use well-known relationships between heat content and velocity as applied to gas mixtures [13].

2. Semiempirical Theory of Turbulence Involving Use of the Equation for Turbulent Pulsation Energy. In solving the problem of impingement of flows with external turbulence (natural or imposed) upon a barrier, it is essential to account for external flow turbulence when friction and heat exchange with the barrier are being analyzed. It is more convenient to solve this problem using an equation for turbulent pulsation energy, since, in addition to friction and heat exchange, it permits determination of turbulent pulsation energy with regard for external flow turbulence. Note that the hypothesis on mixing flow is unacceptable here, since it gives zero values for pulsations at the outer edge of the boundary layer. An example of a boundary layer with external turbulence is the boundary layer at a plate immersed in a jet flow. The results are given below (fig. 4) [14] for experimental study of such a boundary layer when a plate is immersed in a subsonic jet within the limits of the initial segment of the jet (gradient-free flow) with turbulence intensity at the nozzle exit section

$$\varepsilon_a=\left[\sqrt{\overline{v_x'^2}}\,/\,U\right]_a=4{,}6\%$$

As follows from data in this figure on distribution of longitudinal (along the surface of the barrier) root-mean-square velocity pulsation $\sqrt{\overline{v'^2_x}}$ in the barrier's boundary layer, velocity pulsations in the jet penetrate deep into the boundary layer and create conditions for acceleration of the /164 transition from laminar flow (near the nozzle exit section) to turbulent. Maximum velocity pulsations occur in close proximity to the wall: in the laminar flow area, at a distance of about $y/\delta = 0.2$-0.3, where δ is the thickness of the boundary layer; in the turbulent flow region, at a distance $y/\delta < 0.05$. The figure also presents results of computing velocity pulsation $\sqrt{\overline{v'^2_x}} = l\partial/\partial y(v_x)$, where $v_x/v_* = 5.5+5.75 \lg yv_*/v$; $v_* = \sqrt{\tau_w/\rho}$; l is the mixing path as per van Driest (curve 1) and as per Spalding (curve 2). As these data show, the results of computation correlate well with the experiment only very close to the wall. This fact raises doubts about the existence of a linear relationship between velocity pulsations and the gradient of velocity averaged over time in the turbulent boundary layer's turbulent region.

The semiempirical turbulence theory based on use of energy equations for turbulent pulsations makes it possible to define all flow parameters in greater detail. However, to close the complete system of equations requires we must introduce several additional coefficients which are not always known and not always universal. Let us evaluate these coefficients. The energy equation for turbulent pulsations for an incompressible fluid takes the form

$$\sum_j v_j \frac{\partial E}{\partial x_j} + \sum_j \frac{\partial}{\partial x_j}\left[\frac{1}{2}\overline{v'_j \sum_i v'^2_i} + \frac{1}{\rho}\overline{v'_j p'}\right] + \sum_j \sum_i \overline{v'_j v'_i}\frac{\partial v_i}{\partial x_j} = \nu\left[\frac{\partial^2 E}{\partial x_j \partial x_j} - \overline{\frac{\partial v'_i}{\partial x_j}\cdot\frac{\partial v'_i}{\partial x_j}}\right], \qquad (2.1)$$

where v_j is the constituent in direction x_j for motion velocity averaged over time; v'_j is the pulsation of v_j; $E = \frac{1}{2}\sum_n \overline{v'^2_n}$. It is usually assumed that

$$\overline{\frac{1}{2}v'_j \sum_i v'_i v'_i} + \frac{1}{2}\overline{v'_j p'} = -\frac{\nu_T}{\delta}\frac{\partial E}{\partial x_j};$$

$$\nu\overline{\frac{\partial v'_i}{\partial x_j}\cdot\frac{\partial v'_i}{\partial x_j}} = c_D\frac{E^{3/2}}{l}; \quad \overline{v'_i v'_j} = -\nu_T\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \delta_{ij}\sum_n \frac{\overline{v'_n v'_n}}{3}, \qquad (2.2)$$

where σ is Prandtl's turbulent number for turbulent pulsation energy; $_T$ is turbulent viscosity; c_D is a constant; l is the scale of the turbulent pulsations; δ_{ij} is Kroneker's tensor.

With regard for (2.2), equation (2.1) is rewritten as

$$\sum_j v_j \frac{\partial E}{\partial x_j} = \sum_j \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_T}{\sigma}\right)\frac{\partial E}{\partial x_i}\right] + + \nu_T \sum_j \sum_i \frac{\partial v_i}{\partial x_j}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - c_D \frac{E^{3/2}}{l}. \qquad (2.3)$$

In accordance with Prandtl-Kolmogorov's equation, turbulent viscosity is expressed at $v_T = c_V E^{1/2} l$. One of the most important relationships in (2.3) is that between the kinetic energy of turbulent pulsations and energy dissipation. According to Kolomogorov's hypothesis on local isotropy, c_D in (2.3) can be considered as a universal constant when the subject flow is characterized by very high Reynolds numbers for turbulent pulsations, $Re_T = E^{1/2} l/v$. Given low and moderate Re_T values, the lower the Re_T value, the more c_D deviates from its limit value. Rotta [15] proposed the following approximated function for c_D: /165

$$c_D = \frac{3.93}{Re_T} + 0{,}202. \qquad (2.4)$$

From (2.4) it follows that the limit value for c_D is 0.202 as Re_T approaches infinity. According to Wolfstein's data [16]

$$c_D = \frac{a_D}{1 - \exp(-A_D Re_T)}, \quad A_D = 0{,}263. \qquad (2.5)$$

From data in [17],

$$c_D = Re_T f(R),$$

where

$$f(R)=c^{-2}\left[\frac{1}{2}\alpha^{1/3}\Gamma\left(-\frac{1}{3},\ \frac{\alpha}{R}\right)+R^{2/3}\exp\left(-\frac{\alpha}{R^{s}}\right)\right]^{-2},$$

c = 1.5; x = 4/3; a = 7.5; Γ is a gamma function;

$$R=c_D^{-1/4}l^{5/4}/(\nu^{3/4}E^{3/8}),\quad (0{,}2<R<1000).$$

It follows from the last function that, when Re_T approaches infinity, c_D approaches 0.15 instead of 0.202. At low Re_T values, c_D increases more quickly than is indicated in (2.4).

To find the connection between c_V and c_D in equations for turbulent viscosity and turbulent energy dissipation, let us study the particular case of equation (2.3), which described flow of a fluid in a state of local equilibrium [18]. For this flow, equation (2.3) is rewritten as

$$c_\nu E^{1/2}l\left(\frac{\partial v_x}{\partial y}\right)^2=c_D E^{3/2}/l.$$

But

$$\nu_\tau=-\frac{\overline{v_x'v_y'}}{\frac{\partial v_x}{\partial y}}=c_\nu E^{1/2}l,$$

from which

$$\frac{\tau}{\rho E}=(c_D c_\nu)^{1/2}.$$

According to data in [18]

$$\tau/\rho E\simeq 0{,}32$$

for a uniform transverse flow and for flow in tubes near a wall, but still rather remote from the viscous sublayer (one may approximately state that these flows are in a state of local equilibrium). Consequently, $c_D c_V \simeq 0.1$. Assuming, as did Ng and Spalding [18], $c_D = 0.1$, we obtain $c_V = 1.0$. Taking $c_D = 0.416$, as did Wolfstein [16], we obtain $c_V = 0.24$ (according to Wolfstein's data, $c_V = 0.22$). If c_V does not equal 1, assuming c_v to be constant for the entire range of change in Re_T, Wolfstein recommends the following equation [16]:

$$c_\nu = a_\nu [1 - \exp(-A_\nu \mathrm{Re}_T)], \quad A_\nu = 0{,}016. \tag{2.6}$$

Prandtl's turbulent number incorporated into (2.3) varies within 1.53-2.5 [15, 16, 18]; the nature of the change in scale l in (2.3) must be determined in each specific case. /166

As an example of the use of the turbulent pulsation energy equation, let us calculate the impingement of a flat turbulent jet to a plate set along the normal to the jet flow. This flow is characterized by the existence of a stagnation point on the plate. Directions along the plate (x) and along the normal to it (y) for the area around the stagnation point are equivalent. Therefore, study of this flow requires complete Reynolds equations. These are written in the form of a vortex transfer equation as

$$v_x \frac{\partial \Omega}{\partial x} + v_y \frac{\partial \Omega}{\partial y} = \nu \left[\frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} + \frac{\partial^2}{\partial x \partial y} \left(\overline{v_x'^2 - v_y'^2} \right) + \right.$$
$$\left. + \left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2} \right) \overline{v_x' v_y'} \right], \quad \Omega = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}. \tag{2.7}$$

To equations (2.7) and (2.3), we will add an equation for continuity

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0. \tag{2.8}$$

With regard for the relationship of the components of the Reynolds stress tensor (2.2) and the Prandtl-Kolmogorov equation, the system of equations (2.3), (2.7), and (2.8) is a system of three equations for determining four unknown functions: v_x, v_y, E, and l. Therefore, one more relationship is required to close this system. For an ordinary turbulent boundary layer on the surface of a solid body, this relationship is the familiar mixing path equation $l = Ky$, $K = 0.4$ or any other of the mixing path equations presented above. If there are pulsations at the outer edge of the boundary layer, this approximation may be used as the first approximation.

The velocity profile in the jet remote from the barrier (unperturbed flow) in a direction normal to the barrier is represented as Schlichting's profile. Then, assuming that the velocity constituent normal to the barrier in the area of interaction with the barrier varies linearly with distance from the barrier, we obtain the following directly at the barrier [19] if $0 \leq x \leq r$:

$$U/\sqrt{\gamma\nu} = b + \delta_c\zeta\left(1 - \frac{4}{5}\zeta^{3/2} + \frac{1}{4}\zeta^3\right),$$
$$V/\sqrt{\gamma\nu} = -(1 - \zeta^{3/2})^2 y\sqrt{\gamma/\nu}, \qquad (2.9)$$

where $\zeta = (x-b)/\delta_c$; r is the free jet's boundary; b is the boundary of constant velocity nucleus; δ_c is mixing zone thickness; γ is the velocity gradient at the stagnation point; v is the kinematic viscosity factor. Equation (2.9) is taken as the boundary condition at the outer edge of the viscous mixing of flow and plate $y\sqrt{\gamma/\nu} \to \infty$. Let us formulate boundary conditions for the outer edge of the viscous mixing layer for turbulent pulsation energy. Flow far from the barrier beyond the limits of the boundary layer may be considered with a sufficiently high degree of accuracy to be identical to flow in a jet stream for which [20], by analogy with an axisymmetric jet, we assume:

$$E_c = \frac{1}{2}\left[\overline{v'^2_x} + \overline{v'^2_y} + \overline{v'^2_z}\right]_c, \qquad (2.10)$$

Then /167

$$\sqrt{\overline{v'^2_{xc}}} \simeq \sqrt{\overline{v'^2_{yc}}} \simeq \sqrt{\overline{v'^2_{zc}}} = \varkappa(Y - Y_\infty)\frac{\partial V_c}{\partial x},$$

where

$$\sqrt{\overline{v'^2_{xc}}},\ \sqrt{\overline{v'^2_{yc}}},\ \sqrt{\overline{v'^2_{zc}}}$$

are the velocity constituents of pulsations in a free jet in the direction of the axes of a Cartesian coordinate system x, y, z, respectively; Y is the distance from the nozzle exit section to the barrier; Y_∞ is the distance from the barrier to the cross section in which the effect of the barrier on the flow is negligible (as per [12]), $Y_\infty \sim d_a$, where d_a is the diameter of the nozzle's outlet section; V_c is the free jet constituent normal to the barrier (it is described by Schlichting's profile); x is a constant (as per [20], x = 0.0256).

Equation (2.10) assumes a zero value for pulsation energy at the axis and outer edge of the jet, which corresponds to reality for small distances between nozzle exit section and barrier Y. As Y increases, the equations for E_c suggested in [22] are preferred

$$E_c = \frac{0{,}22\,V_a^2}{(Y - Y_\infty)/d_a}. \tag{2.11}$$

Upon impingement of a plate, pulsation energy changes in the area where the jet interacts with the plate (outside the limits of the flow/plate mixing layer). Therefore, (2.10) or (2.11) may be used only in the first approximation as the boundary condition for pulsation energy at the outer edge of the boundary layer.

Boundary conditions on the plate's surface follow from the equation for fluid adhering to that plate at y = 0:

$$v_x = v_y = E = 0, \quad y = 0. \tag{2.12}$$

We seek the solution for v_x, v_y, E given the well-known law of change in l in the form

$$v_x/\sqrt{\gamma\nu} = r\sqrt{\gamma/\nu}\sum_{m=1}^{\infty}(-1)^{m+1}\zeta^{\frac{3(m-1)}{2}}f'_{m-1},$$

$$v_y/\sqrt{\gamma\nu} = -\sum_{m=1}^{\infty}(-1)^{m+1}\left(\frac{3m-1}{2}\right)\zeta^{\frac{3m-3}{2}}f_{m-1}. \qquad (2.13)$$

Taking

$$E\left(y\sqrt{\gamma/\nu}\to\infty\right)\simeq E_c\sim\left(\frac{\partial V_c}{\partial x}\right)^2\sim F''^2(\zeta),$$

where

$$F'(\zeta)=\left(1-\zeta^{3/2}\right)^2,\quad \zeta=\frac{x-b}{\delta_c},$$

the solution for E takes the form

$$E=\frac{9a^{2}}{r^2}G(\zeta)\left[\sum_{m=1}^{\infty}(-1)^{m+1}\zeta^{\frac{3m-3}{2}}S_{m-1}\right]^2, \qquad (2.14)$$

where f_m, S_m are functions of J subject to definition from equations for vortex transfer and turbulent pulsation energy

$$a=\frac{3}{2}\varkappa(Y-Y_\infty)\frac{2{,}35\,V_a}{\sqrt{(Y-Y_\infty)/d_a}};$$

$$G(\zeta)=\left(\zeta^2-\zeta^{1/2}\right)^2\Big/\left(1-\frac{4}{5}\zeta^{3/2}+\frac{1}{4}\zeta^3\right)^2.$$

The last function is present in the form /168

$$G(\zeta)=1+A_1\zeta^{3/2}+A_2\zeta^3+A_3\zeta^{9/2}+A_4\zeta^6+A_5\zeta^{15/2}+A_6\zeta^9+A_7\zeta^{21/2}+\ldots,$$

where $A_1 = -0.4$; $A_2 = -0.78$; $A_3 = -0.392$; $A_4 = -0.0395$; $A_5 = 0.2231$, $A_6 = 0.3033$; $A_7 = 0.0061$, etc.

After substituting (2.13) and (2.14) into (2.3) and (2.7), equation coefficients with identical degrees of , we obtain a system 2m of ordinary fourth-degree differential equations. Boundary conditions for functions f_m, S_m follow from conditions (2.9)-(2.12) when $y\sqrt{\gamma/v} = 0$ and $y\sqrt{\gamma/v} = \infty$. The system of equations obtained in this way was solved numerically using the Runge-Kutt method on a BESM-6 computer with an accuracy of 10^{-5} by sequential approximation. The number of

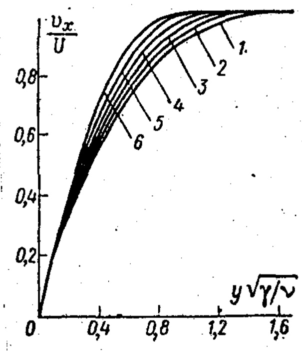

Fig. 5. Profiles of longitudinal average velocity in the boundary layer of a plate when a subsonic flow impinges upon it along the normal: 1 - ζ = x/r = 0.1; 0.2; 2 - 0.3; 3 - 0.4; 4 - 0.5; 5 - 0.6; 0.8; 6 - 0.7.

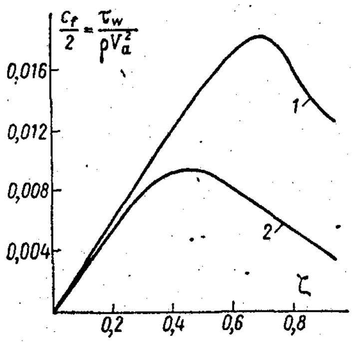

Fig. 6. Distribution of friction over the surface of a barrier when a subsonic jet impinges upon it along the normal: 1 - Current analysis; 2 - data from [109] with x = 0.

approximations is determined from the condition

$$|\theta_m(\infty, t+1) - \theta_m(\infty, t)| < \varepsilon = 10^{-4},$$

where θ_m = f_m or S_m; t, t+1 is the number of approximations; ε is the given accuracy with which the boundary conditions are fulfilled at $y\sqrt{\gamma/v} \to \infty$. Additional boundary conditions required to begin calculation were found with Newton's method to satisfy boundary conditions for f_m, S_m at $y\sqrt{\gamma/v} \to \infty$ with given accuracy ε.

Figures 5 and 6 present the results of calculation with the following raw data: V_a = 20 m/sec; v = $1.5 \cdot 10^{-5}$ m^2/sec; d_a = 10 mm; Y/d_a = 8; $\gamma = V_a/d_a$ [21]. The following approximations were used in calculation: l = Ky, K = 0.4; σ = 1.7; c_D = 0.15; c_v = 0.66. Equation (2.10) was used as the boundary condition for pulsation energy at the outer edge of the boundary layer.

Review of these graphs shows that velocity profiles (fig.

5) in various cross sections along the plate are typical profiles for the turbulent boundary layer when the pressure gradient for the exterior flow is negative. The boundary layer's thickness initially drops as it moves away from the stagnation point, then increases. Maximum thickness corresponds to the cross section with maximum pulsation energy at the outer edge of the boundary layer. The solution shows /169
that exterior jet flow turbulence has an effect on flow in the boundary layer throughout the subject area, including the area around the stagnation point. Friction at the plate (fig. 6) quickly increases due to the negative pressure gradient in the exterior flow, then decreases near the jet boundary due to dissipation effects. One might note the fact that friction at the plate for this type of turbulent flow-plate interaction increases the corresponding figure [19], obtained if the effect of the turbulent jet on flow in the boundary layer around the stagnation point of the plate, is disregarded by a factor of 1.8.

List of symbols used: x, y, x -- axes of the Cartesian coordinate system; l, l_T, l_i -- mixing path, mixing path for temperature, for the i-th component of the mixture; δ, δ_1 -- boundary layer and laminar sublayer thickness; b -- the boundary of the core of constant velocities for a free jet; r -- the free jet's boundary; Y -- distance from the nozzle exit section to the barrier; Y -- distance from the barrier to the cross section at which the effect the barrier has on flow is negligible; v_x, v_y, v_z -- components of velocity toward axes x, y, and z averaged over time; $\sqrt{\overline{v_x'^2}}$, $\sqrt{\overline{v_y'^2}}$, $\sqrt{\overline{v_z'^2}}$ root-mean-square pulsations of velocity components v_x, v_y, v_z; U, V -- velocity components toward x and y on the outer edge of the boundary layer; v_* -- dynamic velocity; ε -- turbulence intensity; γ -- velocity gradient at the stagnation point of a flat barrier set along the normal to the jet; ρ, p -- density, pressure; τ -- friction stress; μ, μ_T, μ_{ef} - coefficient for dynamic viscosity, turbulent viscosity, effective viscosity; E -- turbulent pulsation energy; $\eta = y/\delta$; $\xi = v_x/V$; $\zeta = (x-b)/\delta_c$; $H = \delta^*/\delta^{**}$; K, K_1-- universal constants in (1.3); $\beta = (\xi)_{y=\delta_n}$; β_0 -- constant in Klauzer's equation; A, B -- coefficients in the breakdown of τ/τ_w (1.14); c_D, c_v -- constants in (2.1); x -- constant in (2.10); f_m, S_m -- functions of y defined from (2.7) and (2.3); t -- approximation number; Pr^T, Sm_i^T -- Prandtl's, Schmidt's turbulent number; σ -- Prandtl's turbulent number for turbulent pulsation energy; Re_T -- Reynolds number for turbulent pulsations; δ_{ij} -- Kroneker's tensor.

Subscripts: i, j, n = 1, 2, 3; m = 0, 1, ..., ; a -- parameters at the nozzle's exit section; c -- parameters in a free jet.

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