

JPL PUBLICATION 85-100

Direct Model Reference Adaptive Control of a Flexible Robotic Manipulator

Deirdre R. Meldrum

(NASA-CR-176659) DIRECT MODEL REFERENCE
ADAPTIVE CONTROL OF A FLEXIBLE ROBOTIC
MANIPULATOR (Jet Propulsion Lab.)

HC A05/MF A01

90 P
CSCI 05H

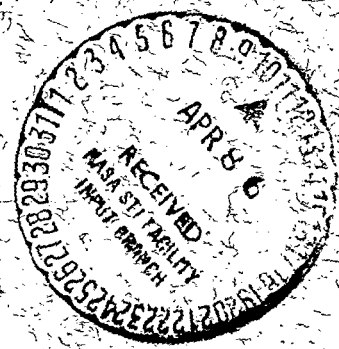
N86-22113

G3/54 Unclas
05856

December 15, 1985



National Aeronautics and
Space Administration
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California



JPL PUBLICATION 85-100

Direct Model Reference Adaptive Control of a Flexible Robotic Manipulator

Deirdre R. Meldrum

December 15, 1985

NASA

National Aeronautics and
Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

The research described in this publication was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not constitute or imply its endorsement by the United States Government or the Jet Propulsion Laboratory, California Institute of Technology.

ABSTRACT

Quick, precise control of a flexible manipulator in a space environment is essential for future Space Station repair and satellite servicing. Numerous control algorithms have proven successful in controlling rigid manipulators with colocated sensors and actuators; however, few have been tested on a flexible manipulator with noncolocated sensors and actuators. In this thesis, a model reference adaptive control (MRAC) scheme based on command generator tracker theory is designed for a flexible manipulator. Quicker, more precise tracking results are expected over nonadaptive control laws for this MRAC approach.

Equations of motion in modal coordinates are derived for a single-link, flexible manipulator with an actuator at the pinned-end and a sensor at the free end. An MRAC is designed with the objective of controlling the torquing actuator so that the tip position follows a trajectory that is prescribed by the reference model. An appealing feature of this direct MRAC law is that it allows the reference model to have fewer states than the plant itself. Direct adaptive control also adjusts the controller parameters directly with knowledge of only the plant output and input signals. No a priori knowledge of the plant is necessary.

Simulations are performed to test both nonadaptive and adaptive model reference control on the flexible manipulator model. Although nonadaptive control gives satisfactory tracking results, the adaptive control does *not* due to the inability of the noncolocated system to satisfy a necessary positive realness condition. When the sensor and actuator are nearly colocated excellent tracking results are achieved.

TABLE OF CONTENTS

CHAPTER		Page
1	INTRODUCTION	1
	1.1 Motivation	1
	1.2 Literature Review	3
	1.2.1 Control of Flexible Structures	3
	1.2.2 Adaptive Control	6
	1.3 Outline	9
	1.4 Summary of Results	9
2	MODEL FORMULATION FOR A ONE LINK FLEXIBLE MANIPULATOR	10
	2.1 Introduction	10
	2.2 Model Description	10
	2.3 Derivation of the Equations of Motion	10
	2.4 Solving the Equations of Motion	12
	2.5 Characteristics of the Model	22
3	MODEL REFERENCE ADAPTIVE CONTROL	26
	3.1 Introduction	26
	3.2 Command Generator Tracker Theory	26
	3.3 Nonadaptive Model Reference Control	30
	3.4 Positive Realness	31
	3.5 Model Reference Adaptive Control	32

PRECEDING PAGE BLANK NOT FILMED

TABLE OF CONTENTS (Continued)

CHAPTER		Page
4	SIMULATION RESULTS AND DISCUSSION	38
	4.1 Introduction	38
	4.2 Plant and Reference Model Parameters	39
	4.3 Case 1: Nonadaptive, Noncolocated, 6th Order System	42
	4.4 Case 2: Adaptive, Noncolocated, 6th Order System	46
	4.5 Case 3: Adaptive, Two Sensor/One Actuator, 6th Order System	48
	4.6 Case 4: Adaptive, Colocated, 6th Order System with Reaction Wheel	52
	4.7 Case 5: Adaptive, Noncolocated, 6th Order System with Reaction Wheel	56
5	CONCLUSION	60
	5.1 Results	60
	5.2 Future Research Recommendations	62
	REFERENCES	64
	APPENDIX A	74
	APPENDIX B	80

LIST OF TABLES

Table	Page
4.I. Model Parameters	39
4.II. Natural Frequencies	40

LIST OF FIGURES

Figure	Page
2.1 Flexible Manipulator Geometry	11
2.2 Pinned-Free Rigid Body Mode Shape, $\phi_0(x)$	23
2.3 Pinned-Free Flexible Mode Shape #1, $\phi_1(x)$	23
2.4 Pinned-Free Flexible Mode Shape #2, $\phi_2(x)$	24
2.5 Pinned-Free Flexible Mode Shape #3, $\phi_3(x)$	24
2.6 Pinned-Free Flexible Mode Shape #4, $\phi_4(x)$	25
2.7 Pinned-Free Flexible Mode Shape #5, $\phi_5(x)$	25
3.1 "Modified" Model Reference Adaptive Control Block Diagram	35
4.1 Case 1: Plant Output, YP, and Model Output, YM	44
4.2 Case 1: Torque Control, UP	45
4.3 Case 1: Position of Manipulator Tip, PPOS, and Model $\sin 5t$, MPOS	45
4.4 Case 1: Velocity of Manipulator Tip, PVEL, and Model $2.5 \cos 5t$, MPOS	46
4.5 Case 3: Position of Manipulator Tip, YP(1), and Model $\sin 5t$, YM(1)	50
4.6 Case 3: Position of Manipulator 1 m from Pin, YP(2), and Model $.5 \cos 5t$, YM(2)	50
4.7 Case 3: Torque Control, UP	51
4.8 Case 3: Output Error $EY=YP-YM$	51

LIST OF FIGURES (Continued)

Figure		Page
4.9	Case 4: Manipulator Output, YP, and Model $\sin 5t + .25 \cos 5t$, YM	53
4.10	Case 4: Torque Control, UP	54
4.11	Case 4: Position of Manipulator Tip, PPOS, and Model $\sin 5t$, MPOS	54
4.12	Case 4: Velocity of Manipulator Tip, PVEL, and Model $.25 \cos 5t$, MVEL	55
4.13	Case 4: Output Error $EY=YP-YM$	55
4.14	Case 5: Manipulator Output, YP, and Model $\sin 5t + .25 \cos 5t$, YM	57
4.15	Case 5: Torque Control, UP	57
4.16	Case 5: Position of Manipulator Tip, PPOS, and Model $\sin 5t$, MPOS	58
4.17	Case 5: Velocity of Manipulator Tip, PVEL, and Model $.25 \cos 5t$, MVEL	58
4.18	Case 5: Output Error $EY=YP-YM$	59

CHAPTER 1

INTRODUCTION

1.1 Motivation

The function of a robotic manipulator is to follow a commanded trajectory into a workspace and then to perform a given task with its end effector. Conventional manipulators of today control the position of the end effector by commanding an appropriate set of joint-angle values which are derived through a real-time kinematic transformation. Large, massive manipulators that are very rigid must be used for this method to ensure that the end effector will move to the desired location. Since the position sensor is colocated with the actuator, stable servo control exists and the control system is easy to design.

In space, flexible robotic arms capable of quick, precise tracking of trajectories and performance of tasks are necessary for in-orbit assembly of the Space Station and autonomous satellite retrieval and repair. The arms are flexible because they are made as lightweight as possible to keep the energy consumption and costs of shipping and operating in space to a minimum. Control of the tip position of a flexible manipulator requires a control scheme and an actuator/sensor setup that is unlike that used for the conventional rigid manipulator. If the sensor for the manipulator is placed at the tip then the position is determined directly without the use of real-time kinematic transformations.

The objective in this study is to test a Model Reference Adaptive Control (MRAC) scheme on a flexible manipulator with a torquing actuator at the pinned-end of the arm and a sensor at the free-end. When the actuator and sensor are *not* located at the same place on the manipulator, the system is referred to as being

“noncolocated”. The function of MRAC is to command the tip of the manipulator to track a prescribed reference signal with speed and accuracy.

Stable control of a noncolocated flexible manipulator like the one used in this project is a difficult control problem that has only recently been addressed. Schmitz [SCH-1] has experimentally shown that with a noncolocated manipulator system it is possible to control the end of a flexible arm using feedback and experimental identification. The tracking results he obtains are satisfactory but quicker, more precise tracking results are expected with adaptive control.

A direct MRAC scheme is chosen for this problem since the plant for the flexible manipulator is a complex, distributed-parameter system whose parameters are uncertain and may change with time. Direct adaptive control adjusts the controller parameters directly with knowledge of only the plant output and input signals whereas indirect adaptive control requires adaptive identification of the plant parameters. The direct MRAC algorithm that is used to control the flexible manipulator is developed in detail in [WEN-1]. An especially appealing feature of this MRAC law is that it allows the reference model to have fewer states than the plant itself. Hence, more states may be used to represent the plant more accurately without the cost of implementing a large order reference model.

Although MRAC has not yet successfully controlled a system that has noncolocated sensors and actuators, it has been applied in a number of cases where the sensors and actuators are colocated. In particular, the Power Tower configuration of the future space station has been modelled and successfully controlled in simulations with this control scheme [IH-1]. A historical review of the control of flexible structures (including manipulators) and the MRAC law follows.

1.2 Literature Review

In a literature review on adaptive control of a remote manipulator arm it is appropriate to discuss first the research on control of flexible structures. Next the history of adaptive control is presented.

1.2.1 Control of Flexible Structures

Stability is a crucial topic that must be addressed in the control of flexible structures. Since very low frequency modes and spillover effects from the use of truncated modal models may easily lead to instability problems it is important to be able to verify the stability of a control system design. This issue was made abundantly clear when the first spacecraft placed in orbit by the United States, Explorer I, went unstable due to energy dissipation of its elastic whip antennas [PIL-1].

Early work related to control system design for elastic spacecraft was performed in [LIK-1], [LIK-2], [FLE-1], [FLE-2], where the focus was on developing useful dynamic simulation tools. More recently, control of elastic structures for space applications has included communications satellites with long flexible solar panels, the Space Shuttle Remote Manipulator System (SRMS), and the future Space Station [GRA-1], [GUP-1], [LYO-1]. In [NGU-1], [RAV-1] the independent joint servo control of the SRMS is described.

Currently a "design challenge" called SCOLE (Spacecraft Control Laboratory Experiment) is being developed at NASA Langley Research Center [BA-1], [TAY-1]. The challenge consists of both a mathematical problem and an experimental test specimen for which control laws are to be developed to slew and stabilize the radio frequency axis of a flexible off-set antenna attached to the space shuttle

by a flexible beam. The dynamics are described by a distributed parameter free-free beam equation with rigid bodies attached at each end. Hence, the mathematical formulation involves various techniques for the solution of partial differential equations (PDE) with delta-functions on the boundary. Many of the issues that must be addressed in SCOLE are similar to those for the flexible remote manipulator.

At the Jet Propulsion Laboratory (JPL) a project called Space Power-100 KW (SP-100) has been modelled very much like the SCOLE project—a free-free beam with rigid bodies attached at each end [SPA-1]. Free and forced vibration studies have been made and a proportional-derivative (PD) controller has been designed. Future work will implement higher order compensators, full-state feedback, and multi-actuator/sensor controllers.

The spacecraft projects described above demonstrate that research in the control of flexible space structures has many issues in common with the problem of active control of an elastic manipulator. Numerous control schemes have been developed for manipulators. Book [BOO-1], [BOO-2], designed a PD joint-angle feedback controller for a two-link, planar manipulator. The French Atomic Energy Commission (C.E.A.) [LIE-1] developed a dynamic model of a six degree-of-freedom, lightweight MA-23 manipulator and implemented servo control of a single joint of the manipulator with a full-state feedback law.

In order to test many of the controllers that have been designed, several experimental beams and manipulators have been built. A one-meter-long experimental arm with colocation was built by Truckenbrodt [TRU-1]. Various control laws were tested such as output feedback and state-feedback using a reduced-order estimator. In [USO-1] an optimal full-state feedback regulator based on a quadratic perfor-

mance index was designed for a two-link elastic arm. Other experimental arms that have been built include the JPL beam [SC-1], [SC-2], the Lockheed Toysat beam [BRE-1], [BRE-2], and the NASA Langley beam [MON-1]. JPL built a pinned-free beam with colocated sensors and actuators and demonstrated active shape control, active dynamic control, and state estimation.

All of the examples listed above use *only* collocation of the sensors and actuators. Noncolocated systems often lead to non-minimum phase systems; that is, systems with right half plane zeros. This is an undesirable feature that creates a difficult control problem. Horowitz and Sidi [HOR-1] address non-minimum phase systems but not for the case of a system having lightly damped elastic modes. In [EDM-1], problems associated with control system design for elastic systems having noncollocation are identified and optimal control designs using output feedback and full-state feedback are evaluated in terms of robustness and performance.

A number of successful designs have been demonstrated for noncolocated systems. The Galileo spacecraft [CHO-1], and the Orbiting Solar Observatory-8 (OSO-8), [YOC-1], [SLA-1], implement control designs for noncolocated systems. In [BAU-1], [CAN-2], the feasibility of actively controlling the elastic vibration modes of a lightly damped mechanical system has been demonstrated with noncolocated position control. The tests were performed on the Lockheed flexible offset-feed antenna and the Stanford four-disk system.

At Stanford, Schmitz [SCH-1] has recently completed end-point position feedback experiments on a very flexible beam with actuation at the pinned-end and sensing at the other. He implemented a direct discrete Linear Quadratic Gaussian (LQG) design and also a reduced-order LQG compensator design. Both controllers yielded a four-fold improvement in bandwidth over what is typically achieved with

joint-angle feedback.

1.2.2 Adaptive Control

Adaptive control was initiated in the late 1950's when it was found that high system performance under varying conditions is difficult to achieve with constant linear feedback [KAL-1]. Significant strides have since been made using primarily two approaches called MRAC and self-tuning regulator (STR). Given a deterministic system, MRAC drives the difference between the plant output and the reference model output to zero asymptotically. An STR divides the problem of controlling a stochastic system into a controller and an estimation scheme [EGA-1], [AST-2]. A survey of the main results for adaptive control may be found in [AST-1], [LAN-3]. In this research a deterministic system is to be controlled by an MRAC; hence, the ensuing discussion will focus primarily on the history of direct adaptive control.

Direct adaptive control was first designed in 1961 using the index minimization method [WHI-1]. Improvements on the design rule were made in [DON-1], [WIN-2] but by 1966 still none of them were globally stable. In 1966, Butchart and Shackcloth [BUT-1] first suggested the use of a quadratic Lyapunov function which was immediately applied to MRAC [PAR-1]. Other adaptive algorithms employing the direct Lyapunov stability approach were developed by Monopoli for single-input, single-output (SISO) systems [MO-1], [MO-2]. For multi-input, multi-output (MIMO) systems satisfying Erzberger's perfect model following conditions [ERZ-1], MRAC algorithms were also developed in [GIL-1], [POR-1], [WIN-1].

Aside from the direct Lyapunov method, two other approaches for stability analysis have been applied to MRAC systems: Popov's Hyperstability Theorem [POP-1], [POP-2], and the Kalman-Yakubovich Lemma [MO-4]. Landau was

the first to apply Popov's hyperstability criterion to MRAC design of continuous systems [POP-1], [POP-2]. The same technique was used by Landau [LAN-1], [LAN-2], and Bethoux [BET-1], to treat discrete-time MRAC problems.

The Kalman-Yakubovich Lemma has been used for stability analysis in [ION-1], [MO-3], [MOR-1], [MOR-2], [NAR-1], [NAR-2], [NAR-3], and [SUZ-1]. In [MO-3], Monopoli uses the lemma in conjunction with an augmented error signal to eliminate pure differentiators when the reference model is not positive real. Narendra, Valavani, and Morse [MOR-1], [MOR-2], [NAR-1], [NAR-2], [NAR-3], design globally stable, asymptotic output tracking algorithms but under the assumption that the relative degree of the plant transfer function is known. Similar techniques were developed for discrete SISO systems by Narendra [NAR-3], Ionescu [ION-1], and Suzuki [SUZ-1]. A projection theorem was used by Goodwin [GOO-1] to obtain a class of globally convergent adaptive algorithms for the multi-variable discrete case provided that certain a priori knowledge of the plant is available.

The adaptive controllers mentioned so far all require many assumptions on the unknown plant and the size of the reference model in order to ensure stability. In 1979, the Command Generator Tracker (CGT) theory was developed by Broussard [BRO-1], for the model following problem with known parameters. This theory has since led to some major developments in MRAC design.

Using the CGT law and a direct Lyapunov stability approach, Sobel, Kaufman, and Mabus, [SOB-1], [SOB-2], [SOB-3], designed a direct MRAC algorithm that forces the error between the outputs of the plant and model to approach zero. Although the algorithm requires the same number of outputs and control inputs and compliance with the condition of strict positive realness of the closed-loop transfer function matrix, the reference model need not be the same size as the plant. In

addition, no a priori knowledge of the plant is necessary. Bar-Kana [BAR-2] relaxed the condition of strict positive realness to simply positive realness.

Positive realness [AND-1], [CHE-1], [KAL-2], [LAN-3], [LI-1], [LJU-1], [MO-4], [NAR-4], [POP-1], [POP-2], is a strong condition that is very difficult to satisfy for many systems. Wen and Balas [WEN-1], [WEN-2], [WEN-3], have relaxed the condition even further by designing a "modified" MRAC scheme that requires only the condition of "almost" positive realness. The algorithm has also been generalized to infinite dimensional systems. Studies on the effects of unmodelled dynamics and modal truncations on the stability of systems have been addressed in [BAL-2], [IOA-1], [IOA-2], [IOA-3], [JOH-1], [ORT-1].

MRAC has been successfully applied to systems with colocated sensors and actuators. In [IH-1] and [WAN-1] the planar model of the Space Station is controlled with MRAC in simulation studies. At JPL a tuned feedback controller for an elastic spacecraft, Galileo, has been designed [KOP-1], [MAC-1]. The success of the design remains to be seen until the spacecraft is launched in 1986.

Direct MRAC is very attractive since it eliminates the need for a priori knowledge of the system to be controlled. Successful application of this scheme to a flexible manipulator with noncollocation is highly desirable especially when changes are encountered in the reference trajectory and the tip mass. In this research a study is made on applying direct MRAC to a flexible remote manipulator with noncollocation. MRAC using various reference model tracking objectives is applied to the flexible manipulator for several sensor configurations. The prominent issue that is addressed is for what manipulator configuration and tracking objectives can the almost positive realness condition be satisfied. This condition is extremely difficult to achieve for a nonminimum phase system.

1.3 Outline

In order to satisfactorily analyze the properties of a flexible remote manipulator with noncolocation and boundary control it is important to exactly model the system. A detailed derivation of the model using a distributed parameter approach is shown in Chapter 2 and features peculiar to the flexible manipulator are pointed out. Chapter 3 presents MRAC algorithms using the CGT approach. Chapter 4 is devoted to the application of the controllers derived in Chapter 3 to the manipulator model described in Chapter 2. Various sensor configurations and reference model objectives are tried and the results are discussed. A summary and recommendations for future research are presented in Chapter 5.

1.4 Summary of Results

The results of this study include:

1. Nonadaptive model reference control of the flexible manipulator with noncolocation is possible and gives satisfactory results.
2. MRAC of the manipulator with actuation at the pinned-end and sensing at the free-end is *not* possible due to the positive realness condition.
3. MRAC with two sensors and one actuator results in a bounded error but not proper tracking by the manipulator tip.
4. MRAC with a reaction wheel and a sensor located at the tip of the manipulator give excellent tracking results.

CHAPTER 2

MODEL FORMULATION FOR A ONE LINK FLEXIBLE MANIPULATOR

2.1 Introduction

The main goal in this chapter is to acquire an understanding of the general structure and fundamental characteristics of flexible manipulator dynamics. First an exact infinite dimensional representation to evaluate the system analytically is derived. Then a reduced-order model is obtained by choosing a finite number of system modes which can be used to test the adaptive control algorithm in simulation.

2.2 Model Description

The manipulator is represented as a uniform pinned-free beam of length L , moving in the horizontal plane, as shown in Figure 2.1. Properties of the beam are as follows: E is the Young's modulus of elasticity, I is the second moment of area, A , of the beam cross-section, and ρ is the density per unit volume of the beam. At the pinned-end an external torque T may be applied to create an angle $\theta(t)$ with respect to the beam's neutral axis. A horizontal displacement of any point along the beam's neutral axis at a distance x from the pinned end is given by $u(t, x)$.

2.3 Derivation of the Equations of Motion

In order to obtain a partial differential equation (PDE) for the model shown in Figure 2.1, apply Hamilton's principle [KAN-1]:

$$\delta \int_{t_1}^{t_2} (K - V) dt = 0 \quad (2.1)$$

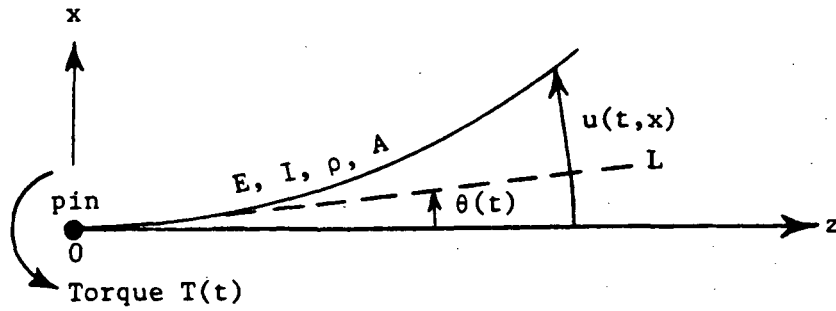


Fig. 2.1 Flexible Manipulator Geometry

where δ is the Kronecker delta, t_1 and t_2 are two arbitrary times ($t_1 < t_2$), K is the kinetic energy, and V is the potential energy.

The kinetic energy of the beam is

$$2K = \rho A \int_0^L \left(\frac{\partial u}{\partial t} \right)^2 dx \quad (2.2)$$

Neglecting the effects of shear displacement for this model, the strain potential energy is expressed as follows:

$$2V_e = EI \int_0^L \left(\frac{\partial^2 u}{\partial x^2} \right)^2 dx \quad (2.3)$$

The external torque T contributes a potential energy of:

$$V_a = -T\theta \quad (2.4)$$

Energy dissipation of the system will be added later as a damping term in the modal state space formulation.

Applying the Hamiltonian of (2.1), where $V_a + V_e$ is the total potential energy V , the following fourth order homogeneous PDE is obtained:

$$EI \frac{\partial^4 u}{\partial x^4} + \rho A \frac{\partial^2 u}{\partial t^2} = 0 \quad (2.5)$$

with the non-homogeneous boundary conditions:

$$\begin{aligned}
 u(t, 0) &= 0 & (2.6) \\
 EI \frac{\partial^2 u}{\partial x^2} \Big|_{x=0} &= T(t) \\
 EI \frac{\partial^2 u}{\partial x^2} \Big|_{x=L} &= 0 \\
 EI \frac{\partial^3 u}{\partial x^3} \Big|_{x=L} &= 0
 \end{aligned}$$

The dynamic equations describing the motion of the flexible manipulator are given by the fourth order PDE (2.5) with its four boundary conditions (2.6). Next these equations shall be solved in a manner that will result in a modal state space form.

2.4 Solving the Equations of Motion

To solve the equations of motion (2.5), (2.6), apply a method proposed by Meirovitch [MEI-1] whereby a homogeneous PDE with non-homogeneous boundary conditions is transformed into a non-homogeneous PDE with homogeneous boundary conditions.

Assume a solution of (2.5) in the form:

$$u(t, x) = v(t, x) + h(x)T(t) \quad (2.7)$$

This gives boundary conditions for $v(t, x)$ as

$$v(t, 0) = -h(0)T(t) \quad (2.8)$$

$$EI \frac{\partial^2 v(t, x)}{\partial x^2} \Big|_{x=0} = T(t) - EI \frac{d^2 h(x)}{dx^2} \Big|_{x=0} T(t)$$

$$EI \frac{\partial^2 v(t, x)}{\partial x^2} \Big|_{x=L} = -EI \frac{d^2 h(x)}{dx^2} \Big|_{x=L} T(t)$$

$$EI \frac{\partial^3 v(t, x)}{\partial x^3} \Big|_{x=L} = -EI \frac{d^3 h(x)}{dx^3} \Big|_{x=L} T(t)$$

To render the boundary conditions for the variable $v(t, x)$ homogeneous, $h(x)$ must satisfy the following equations:

$$h(0) = 0$$

$$EI \frac{d^2 h(x)}{dx^2} \Big|_{x=0} = 1 \quad (2.9)$$

$$EI \frac{d^2 h(x)}{dx^2} \Big|_{x=L} = 0$$

$$EI \frac{d^3 h(x)}{dx^3} \Big|_{x=L} = 0$$

Equation (2.9) may be written as:

$$\frac{d^2 h(x)}{dx^2} = \frac{1}{EI} \left(\frac{x^2}{L^2} - \frac{2x}{L} + 1 \right) \quad (2.10)$$

In view of (2.9), (2.10) has the solution

$$h(x) = \frac{1}{EI} \left(\frac{x^4}{12L^2} - \frac{x^3}{3L} + \frac{x^2}{2} \right) \quad (2.11)$$

where $h(x)$ is zero at $x = 0$.

The transformed problem consists of the nonhomogeneous PDE:

$$EI \frac{\partial^4 v(t, x)}{\partial x^4} + \rho A \frac{\partial^2 v(t, x)}{\partial t^2} = -EI \frac{d^4 h(x)}{dx^4} T(t) - \rho A h(x) \frac{d^2 T(t)}{dt^2} \quad (2.12)$$

and the homogeneous boundary conditions:

$$v(t, 0) = 0 \quad (2.13)$$

$$EI \frac{\partial^2 v(t, x)}{\partial x^2} \Big|_{x=0} = 0$$

$$EI \frac{\partial^2 v(t, x)}{\partial x^2} \Big|_{x=L} = 0$$

$$EI \frac{\partial^3 v(t, x)}{\partial x^3} \Big|_{x=L} = 0$$

In order to solve (2.12), (2.13) by using modal expansion, first use modal analysis to obtain a solution to the eigenvalue problem for the flexible modes that consists of the differential equation:

$$EI \frac{d^4 \phi_n(x)}{dx^4} - \omega_n^2 \rho A \phi_n(x) = 0, \quad n = 1, \dots, \infty \quad (2.14)$$

with the boundary conditions:

$$\phi_n(0) = 0 \quad (2.15)$$

$$EI \frac{d^2 \phi_n(x)}{dx^2} \Big|_{x=0} = 0 \quad (2.16)$$

$$EI \frac{d^2 \phi_n(x)}{dx^2} \Big|_{x=L} = 0 \quad (2.17)$$

$$EI \frac{d^3 \phi_n(x)}{dx^3} \Big|_{x=L} = 0 \quad (2.18)$$

where ϕ_n represents the infinite set of natural, orthogonal mode shapes of the system with their associated natural frequency, ω_n .

The general solution of equation (2.14) is:

$$\phi_n(x) = C_1 \sin k_n x + C_2 \sinh k_n x + C_3 \cos k_n x + C_4 \cosh k_n x \quad (2.19)$$

When dealing with end conditions it is useful to write (2.19) in the following equivalent form:

$$\begin{aligned} \phi_n(x) = & A(\cos k_n x + \cosh k_n x) + B(\cos k_n x - \cosh k_n x) + \\ & C(\sin k_n x + \sinh k_n x) + D(\sin k_n x - \sinh k_n x) \end{aligned} \quad (2.20)$$

From (2.15) and (2.16) obtain $A = 0$ and $B = 0$, respectively. The remaining linear system obtained from (2.17) and (2.18) is:

$$\begin{bmatrix} -\sin k_n L + \sinh k_n L & -\sin k_n L - \sinh k_n L \\ -\cos k_n L + \cosh k_n L & -\cos k_n L - \cosh k_n L \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (2.21)$$

Solving for the coefficients two relationships are obtained:

$$\frac{C}{D} = \frac{\sin k_n L + \sinh k_n L}{-\sin k_n L + \sinh k_n L} \quad (2.22)$$

$$\frac{C}{D} = \frac{\cos k_n L + \cosh k_n L}{-\cos k_n L + \cosh k_n L}$$

Therefore, the frequency equation is given by

$$\tan k_n L = \tanh k_n L \quad (2.23)$$

where the eigenvalues obtained by solving (2.23) are approximately:

$$k_n \approx \frac{(n + \frac{1}{4})\pi}{L}, \quad n = 1, 2, 3, \dots, \infty \quad (2.24)$$

From (2.14), (2.20), (2.24), the infinite set of natural, orthogonal modes $\phi_n^*(x)$ and the associated natural frequencies ω_n are:

$$\phi_n(x) = [(1 + \sigma_n) \sin k_n x + (\sigma_n - 1) \sinh k_n x], \quad n = 1, 2, 3, \dots, \infty \quad (2.25)$$

$$\text{where } \sigma_n = \frac{C}{D}$$

$$\omega_n = \left(\frac{k_n}{L}\right)^2 \sqrt{\frac{EI}{\rho A}}, \quad n = 1, 2, 3, \dots, \infty \quad (2.26)$$

Normalize the eigenfunctions (2.25) so that the orthogonal modes satisfy the relation:

$$\rho A \int_0^L \phi_n(x) \phi_m(x) dx = \delta_{mn}; \quad m, n = 1, 2, 3, \dots, \infty \quad (2.27)$$

After solving (2.27) for the normalization constant the flexible normalized eigenfunctions become [BIS-1]:

$$\phi_n(x) = \frac{1}{\sqrt{\rho AL}} [(1 + \sigma_n) \sin kx + (\sigma_n - 1) \sinh kx] \quad (2.28)$$

Equations (2.26), (2.28) are the solution to the eigenvalue problem for the flexible modes. A pinned-free beam has one rigid body mode, rotation about the hinge, that must also be taken into account. Assuming no gravitational force field, a rigid body displacement added to the motion does not affect the potential energy; hence, for a rigid body mode, the eigenvalue equation becomes [MEI-1]:

$$EI \frac{d^4 \phi_0(x)}{dx^4} = 0 \quad (2.29)$$

which has the general solution:

$$\phi_0(x) = D_1 + D_2 x + D_3 x^2 + D_4 x^3 \quad (2.30)$$

To satisfy the homogeneous boundary conditions:

$$\phi_0(0) = 0 \quad (2.31)$$

$$EI \frac{d^2 \phi_0(x)}{dx^2} \Big|_{x=0} = 0$$

$$EI \frac{d^2 \phi_0(x)}{dx^2} \Big|_{x=L} = 0$$

$$EI \frac{d^3 \phi_0(x)}{dx^3} \Big|_{x=L} = 0$$

the general solution (2.30) becomes:

$$\phi_0(x) = D_2 x \quad (2.32)$$

Upon normalizing (2.32), the rigid body eigenfunction may be written as:

$$\phi_0(x) = x\sqrt{\frac{3}{L^3}} \quad (2.33)$$

with the corresponding natural frequency:

$$\omega_0 = 0 \quad (2.34)$$

Now that the natural eigenfunctions of the system have been obtained, use modal expansion to assume a solution of (2.12) in the form:

$$v(t, x) = \sum_{n=0}^{\infty} \phi_n(x) \eta_n(t) \quad (2.35)$$

where $\eta_n(t)$ are time-dependent generalized coordinates. Introducing (2.35) into (2.12) the following is obtained:

$$\begin{aligned} \sum_{n=0}^{\infty} \left[\eta_n(t) \frac{d^4 \phi_n(x)}{dx^4} + \frac{d^2 \eta_n(t)}{dt^2} \frac{\rho A}{EI} \phi_n(x) \right] \\ = - \frac{d^4 h(x)}{dx^4} T(t) - \frac{\rho A}{EI} h(x) \frac{d^2 T(t)}{dt^2} \end{aligned} \quad (2.36)$$

Since $\phi_n(x)$ and ω_n satisfy (2.14) and (2.29), (2.36) reduces to:

$$\sum_{n=0}^{\infty} \left[\frac{d^2 \eta_n(t)}{dt^2} + \omega_n^2 \eta_n(t) \right] \frac{\rho A}{EI} \phi_n(x) = - \frac{d^4 h(x)}{dx^4} T(t) - \frac{\rho A}{EI} h(x) \frac{d^2 T(t)}{dt^2} \quad (2.37)$$

Equation (2.37) contains all the generalized coordinates $\eta_n(t)$ so, in effect, it is a coupled equation. To uncouple it multiply both sides of the equation by $\phi_m(x)$ and integrate with respect to x over its domain. If this is done and in addition the notation

$$H_n = \int_0^L \phi_n(x) h(x) dx \quad (2.38)$$

$$H_n^* = \frac{EI}{\rho A} \int_0^L \phi_n(x) \frac{d^4 h(x)}{dx^4} dx \quad (2.39)$$

is introduced, an infinite set of uncoupled ordinary differential equations is obtained:

$$\frac{d^2 \eta_n(t)}{dt^2} + \omega_n^2 \eta_n(t) = -H_n^* T(t) - H_n \frac{d^2 T(t)}{dt^2} \quad (2.40)$$

Equation (2.40) is simply the modal representation of the transformed non-homogeneous PDE (2.12).

In order to implement an adaptive control scheme on this system, a state space representation is desired in terms of control torque $T(t)$ only — not $T(t)$ and $\frac{d^2 T(t)}{dt^2}$ as in (2.40). This can be done by performing the following transformation. Assuming zero initial conditions, take the Laplace transform of (2.40):

$$\mathcal{L}[(2.40)] \equiv s^2 \hat{\eta}_n(s) + \omega_n^2 \hat{\eta}_n(s) = -H_n^* \hat{T}(s) - s^2 H_n \hat{T}(s) \quad (2.41)$$

Solve for $\hat{T}(s)$ to obtain:

$$\hat{T}(s) = \frac{(s^2 + \omega_n^2) \hat{\eta}_n(s)}{-H_n^* - H_n s^2} \quad (2.42)$$

Define $\hat{z}_n(s)$ as

$$\hat{z}_n(s) = \frac{\hat{\eta}_n(s)}{-H_n^* - H_n s^2} \quad (2.43)$$

then

$$T(t) = \frac{d^2 z_n(t)}{dt^2} + \omega_n^2 z_n(t) \quad (2.44)$$

$$\eta_n(t) = -H_n \frac{d^2 z_n(t)}{dt^2} - H_n^* z_n(t) \quad (2.45)$$

Solve for $\frac{d^2 z_n(t)}{dt^2}$ in (2.44) and substitute into (2.45) to obtain:

$$\eta_n(t) = (H_n \omega_n^2 - H_n^*) z_n(t) - H_n T(t) \quad (2.46)$$

A solution to the original PDE (2.5) may now be written in terms of the generalized coordinates $\eta_n(t)$ (2.46) and the eigenfunctions of the system $\phi_n(x)$ (2.28), (2.33). In (2.7), a solution of (2.5) was assumed in the form

$$u(t, x) = v(t, x) + h(x)T(t) \quad (2.47)$$

From (2.7), (2.11), and (2.46) it follows that

$$\begin{aligned} u(t, x) &= \sum_{n=0}^{\infty} \eta_n(t) \phi_n(x) + h(x)T(t) \\ &= \sum_{n=0}^{\infty} (H_n \omega_n^2 - H_n^*) z_n(t) \phi_n(x) - \sum_{n=0}^{\infty} H_n \phi_n(x) T(t) + h(x)T(t) \end{aligned} \quad (2.48)$$

The expansion of $h(x)$ is $\sum_{n=0}^{\infty} H_n \phi_n(x)$; therefore,

$$- \sum_{n=0}^{\infty} H_n \phi_n(x) T(t) + h(x)T(t) = 0 \quad (2.49)$$

leaving

$$u(t, x) = \sum_{n=0}^{\infty} (H_n \omega_n^2 - H_n^*) z_n(t) \phi_n(x) \quad (2.50)$$

For the expansion (2.48), both strong and weak solutions u give uniform and pointwise convergence for u and u' . If u'' is continuous, then the expansion $\sum_{n=0}^{\infty} \eta_n(t) \phi_n(x) + h(x)T(t)$ also converges uniformly and pointwise [STR-1]. Although the difference between the weak and strong formulation is mentioned, it is important to note that both formulations ultimately result in the same dynamic equations.

Since (2.50) is a solution of $u(t, x)$ in terms of $z_n(t)$, equation (2.44) is an exact representation of $u(t, x)$. In a flexible structure such as the manipulator, structural damping of about .5 percent is inherent. This may be heuristically represented by adding the damping term $2\zeta\omega_n \frac{dz_n(t)}{dt}$ to (2.44) as

$$T(t) = \frac{d^2 z_n(t)}{dt^2} + 2\zeta\omega_n \frac{dz_n(t)}{dt} + \omega_n^2 z_n(t) \quad (2.51)$$

The term H_n^* may be expanded with the eigenfunction $\phi_n(x)$ as

$$\begin{aligned} H_n^* &= \left\langle \frac{d^4 h(x)}{dx^4}, \phi_n(x) \right\rangle \\ &= \frac{d^3 h(x)}{dx^3} \phi_n(x) \Big|_0^L - \frac{d^2 h(x)}{dx^2} \frac{d\phi_n(x)}{dx} \Big|_0^L + \frac{dh(x)}{dx} \frac{d^2 \phi_n(x)}{dx^2} \Big|_0^L \\ &\quad - h(x) \frac{d^3 \phi_n(x)}{dx^3} \Big|_0^L + \left\langle h(x), \frac{d^4 \phi_n(x)}{dx^4} \right\rangle \end{aligned} \quad (2.57)$$

From the boundary conditions for $h(x)$, (2.9), obtain

$$\begin{aligned} H_n^* &= \frac{d\phi_n(0)}{dx} + \omega_n^2 \langle h(x), \phi_n(x) \rangle \\ &= \frac{d\phi_n(0)}{dx} + \omega_n^2 H_n \end{aligned} \quad (2.58)$$

Hence, the output $y(t)$ may be expressed as

$$y(t) = -\frac{d\phi_n(0)}{dx} \phi_n(L) z_n(t) \quad (2.59)$$

In state-space form,

$$y(t) = Cz(t) + DT(t) \quad (2.60)$$

with

$$C = \begin{bmatrix} -\frac{d\phi_0(0)}{dx} \phi_0(L) & -\alpha \frac{d\phi_0(L)}{dx} \phi_0(L) & -\frac{d\phi_1(0)}{dx} \phi_1(L) & -\alpha \frac{d\phi_1(L)}{dx} \phi_1(L) \\ \dots & -\frac{d\phi_\infty(0)}{dx} \phi_\infty(L) & -\alpha \frac{d\phi_\infty(L)}{dx} \phi_\infty(L) \end{bmatrix} \quad (2.61)$$

$$D = 0$$

where α is the weighting factor of the position versus rate measurement.

An equivalent realization for the B and C matrices given in (2.55) and (2.61)

is:

$$B = \begin{bmatrix} 0 \\ \frac{d\phi_0(0)}{dx} \\ 0 \\ \frac{d\phi_1(0)}{dx} \\ \vdots \\ 0 \\ \frac{d\phi_\infty(0)}{dx} \end{bmatrix} \quad (2.62)$$

$$C = [\phi_0(L) \quad \alpha\phi_0(L) \quad \phi_1(L) \quad \alpha\phi_1(L) \quad \cdots \quad \phi_\infty(L) \quad \alpha\phi_\infty(L)] \quad (2.63)$$

For the simulation studies, the flexible manipulator system matrices will be represented as in (2.54), (2.62), and (2.63).

2.5 Characteristics of the Model

It is important to recognize a few fundamental characteristics of the flexible manipulator model that have just been derived. Look at the mode shapes of the pinned-free beam in Figures 2.2 - 2.7 as given by equations (2.28) and (2.33). Two important observations can be made:

1. The slopes of the mode shapes at the pinned end $\left. \frac{d\phi_n(x)}{dx} \right|_{x=0}$ become larger for increasing modal frequencies.
2. The signs of the modal deflection $\phi_n(L)$ alternate from one mode to the next due to the nonminimum phase property of the system.

These characteristics will be shown to have a substantial effect on the system's performance under adaptive control.

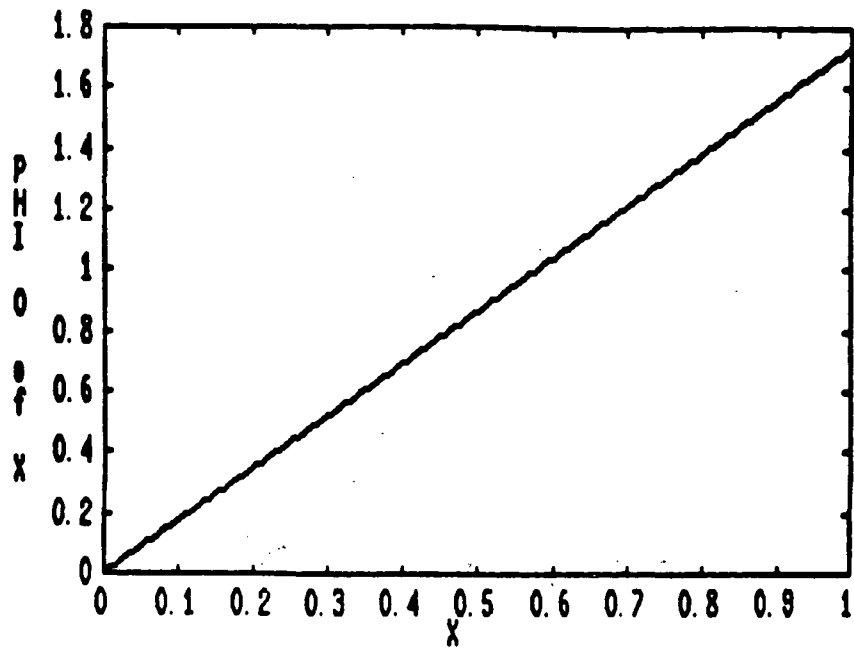


Fig. 2.2 Pinned-Free Rigid Body Mode Shape, $\phi_0(x)$

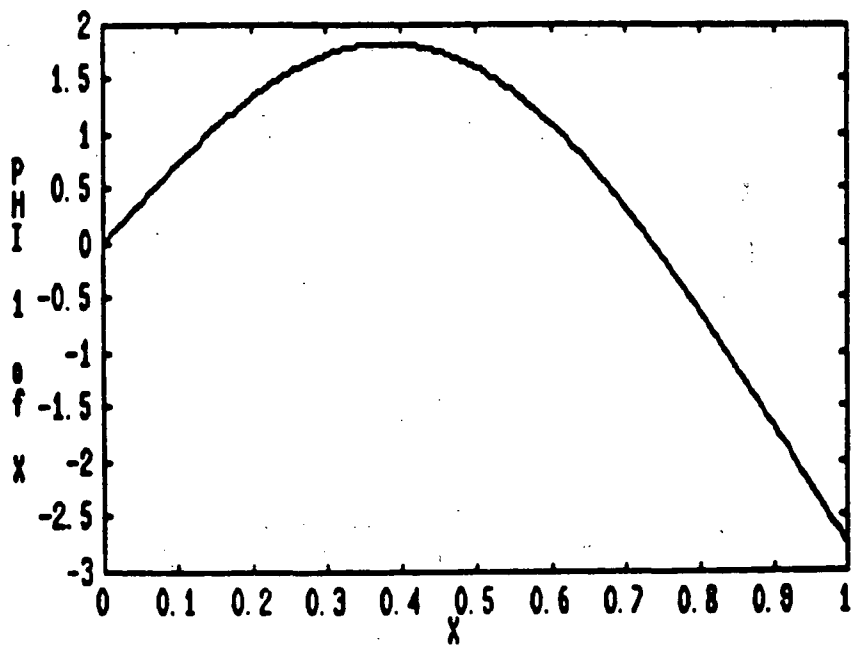


Fig. 2.3 Pinned-Free Flexible Mode Shape #1, $\phi_1(x)$

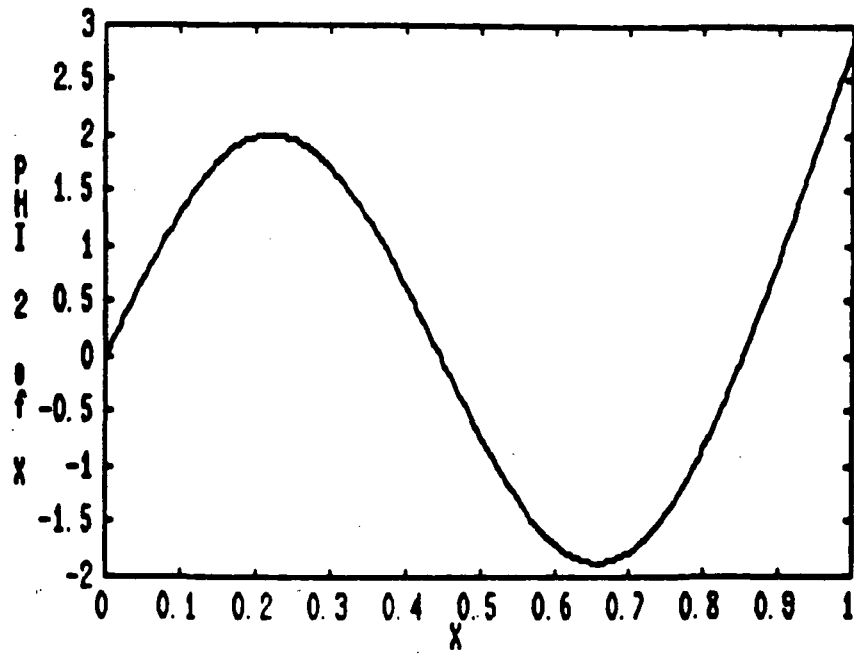


Fig. 2.4 Pinned-Free Flexible Mode Shape #2, $\phi_2(x)$

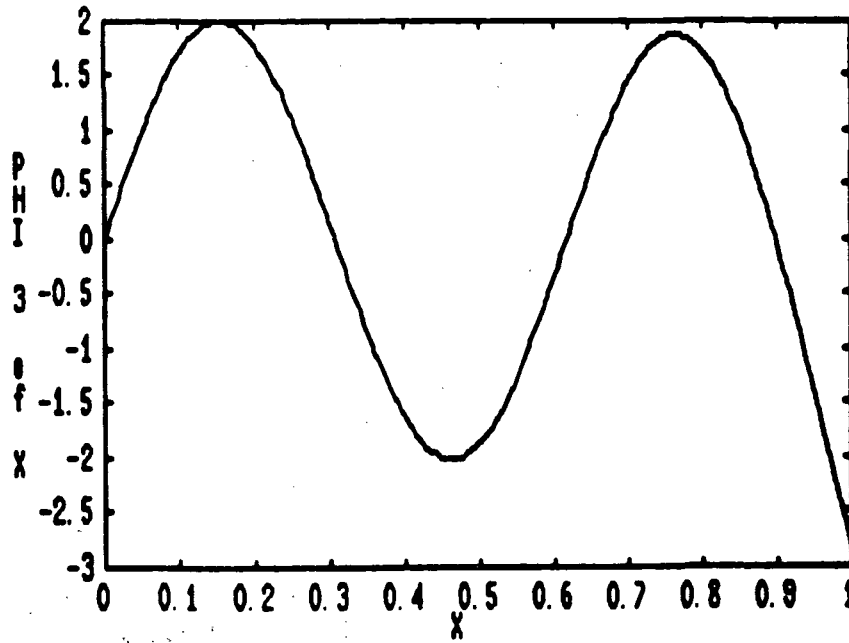


Fig. 2.5 Pinned-Free Flexible Mode Shape #3, $\phi_3(x)$

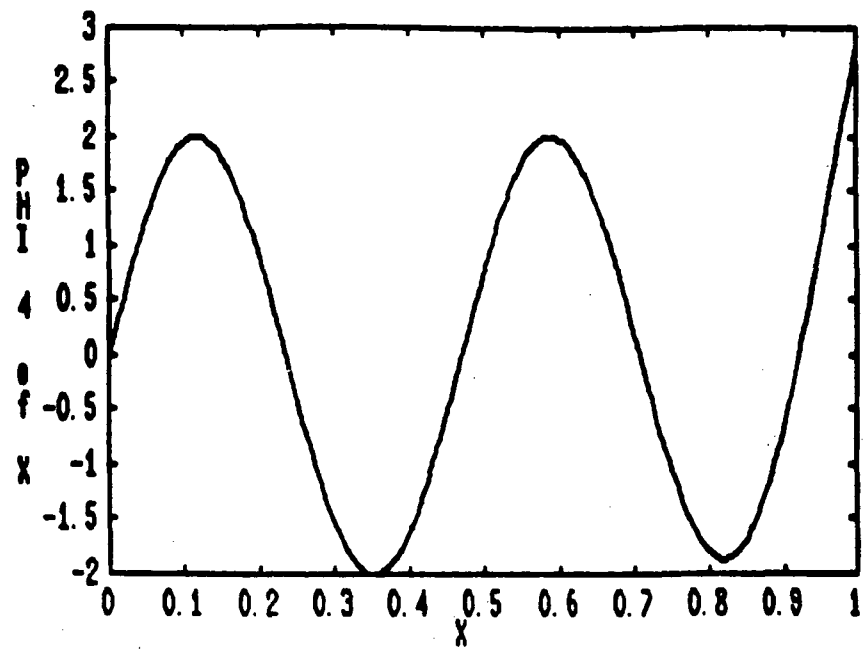


Fig. 2.6 Pinned-Free Flexible Mode Shape #4, $\phi_4(x)$

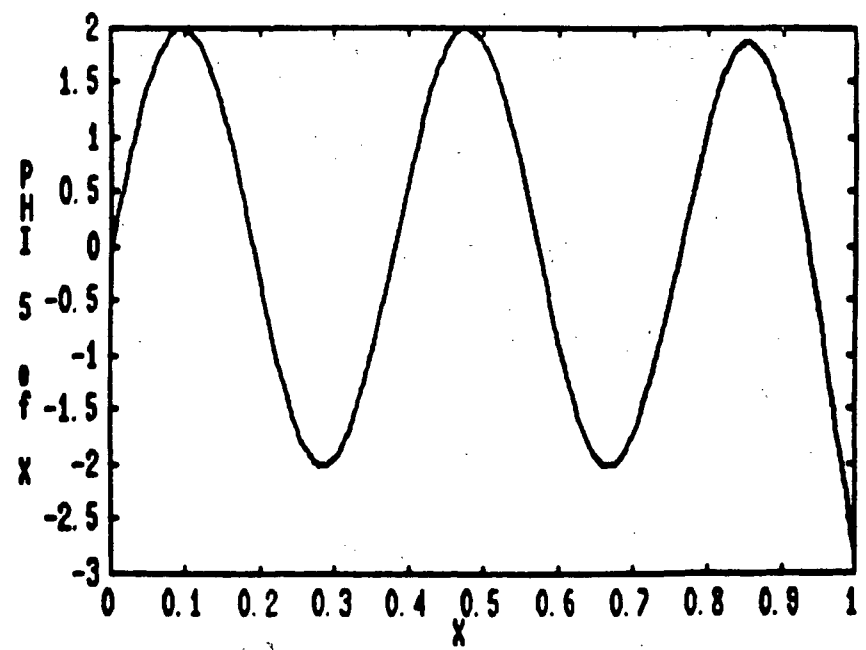


Fig. 2.7 Pinned-Free Flexible Mode Shape #5, $\phi_5(x)$

CHAPTER 3

MODEL REFERENCE ADAPTIVE CONTROL

3.1 Introduction

In this chapter direct model reference controllers are developed for the flexible manipulator. The Command Generator Tracker (CGT) theory, the basis of the controllers, is introduced first. A *nonadaptive* model reference controller is then designed using CGT theory and explicit knowledge of the system plant. Positive realness, a condition that must hold for MRAC of a system, is defined before finally developing adaptive control and "modified" adaptive control schemes.

3.2 Command Generator Tracker Theory

The plant under consideration was developed in Chapter 2 as

$$\frac{dz(t)}{dt} = Az(t) + BT(t)$$

$$y(t) = Cz(t) + DT(t)$$

Taking a finite-dimensional plant, it is desired to find a finite dimensional controller so that the output $y(t)$ tracks a desirable output trajectory $y_m(t)$. This output trajectory is generated by a finite dimensional reference model

$$\frac{dz_m(t)}{dt} = A_m z_m(t) + B_m T_m(t) \quad (3.1)$$

$$y_m(t) = C_m z_m(t); \quad z_m(0) = z_{m0}, \quad t \geq 0$$

with the only requirement that the model output is of the same dimension as the plant output. The order of the model may be much smaller than the order of

the plant which makes this method very attractive as far as implementation is concerned.

Since the dimensions of z and z_m are not the same, the error signal must be created via the CGT approach as developed in [BAL-3], [WEN-1]. We assume there exists an "ideal" intermediate system:

$$\frac{dz^*(t)}{dt} = Az^*(t) + BT^*(t) \quad (3.2)$$

$$y^*(t) = Cz^*(t); \quad z^*(0) = z_0^*, \quad t \geq 0$$

Provided that the state space dimension of the reference model is not bigger than the dimension of a completely controllable and completely observable subsystem of the plant under piecewise constant T_m , the following exact model matching CGT condition is usually satisfied [WEN-1]:

$$\begin{bmatrix} z^*(t) \\ T^*(t) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} z_m(t) \\ T_m(t) \end{bmatrix} \quad (3.3)$$

$$y^*(t) = y_m(t), \quad \forall t \geq 0 \quad (3.4)$$

where S_{11} , S_{12} , S_{21} , S_{22} are bounded linear operators. This condition is not always easily verified. A sufficient but not necessary condition for CGT has been proposed in [BAL-3] and is described below.

Assume that $z^*(t)$ and $T^*(t)$ are linearly related to the model state vector $z_m(t)$ and command vector $T_m(t)$ as in (3.3):

$$z^*(t) = S_{11}z_m(t) + S_{12}T_m(t) \quad (3.5)$$

$$T^*(t) = S_{21}z_m(t) + S_{22}T_m(t) \quad (3.6)$$

Since $T_m(t)$ is constant, differentiation of (3.5) with respect to t leads to:

$$\begin{aligned}
 \frac{dz^*(t)}{dt} &= S_{11} \frac{dz_m^*(t)}{dt} \\
 &= S_{11} A_m z_m(t) + S_{11} B_m T_m(t) \\
 &= (A S_{11} + B S_{21}) z_m(t) + (A S_{12} + B S_{22}) T_m(t) \\
 &= A (S_{11} z_m(t) + S_{12} T_m(t)) + B (S_{21} z_m(t) + S_{22} T_m(t)) \\
 &= A z^*(t) + B T^*(t)
 \end{aligned} \tag{3.7}$$

and

$$z^*(0) = S_{11} z_m(0) + S_{12} T_m(t)$$

$$\begin{aligned}
 y^*(t) &= C z^*(t) \\
 &= C S_{11} z_m(t) + C S_{12} T_m(t) \\
 &= C_m z_m(t) \\
 &= y_m(t)
 \end{aligned} \tag{3.8}$$

Equations (3.7) and (3.8) may be rewritten as

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} S_{11} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_m & B_m \\ C_m & 0 \end{bmatrix} \tag{3.9}$$

If (3.9) is satisfied then the CGT condition (3.3) holds. In order to verify (3.9), the following manipulation is made: Let

$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = \begin{bmatrix} A^{-1}(I - B(CA^{-1}B)^{-1}CA^{-1}) & A^{-1}B(CA^{-1}B)^{-1} \\ (CA^{-1}B)^{-1}CA^{-1} & -(CA^{-1}B)^{-1} \end{bmatrix} \tag{3.10}$$

Then (3.9) can be written as

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \Omega_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} A_m & B_m \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ C_m & 0 \end{bmatrix} \tag{3.11}$$

Equation (3.11) is of the form $S = F(S)$; hence, if a fixed point of F exists, then (3.3) is satisfied.

However, it has been demonstrated that in some cases the CGT condition (3.3) is satisfied even though it is impossible to satisfy the sufficient condition (3.9). A heuristic justification for this is as follows [WEN-1]: A stabilizable, detectable system can be decomposed into

$$\frac{dq(t)}{dt} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} q(t) + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} T(t) \quad (3.12)$$

$$y(t) = [C_1 \quad C_2] q(t)$$

where $q(t) = P^{-1}z(t)$ is a linear coordinate transform, (A_1, B_1, C_1) is a finite-dimensional minimal system and A_2 generates an exponentially stable system.

Assuming that the dimension of A_1 is the same as that of A_m , A_m has distinct eigenvalues, and $T_m(t)$ is a constant input, choose $T(t) = Gq_1(t) + v$. The constant gain G is chosen such that the eigenvalues of $(A_1 + B_1G)$ are exactly the same as the eigenvalues of A_m . Another coordinate change gives

$$\frac{dx}{dt} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} x(t) + \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} v \quad (3.13)$$

$$y(t) = [\gamma_1 \quad \gamma_2] x(t)$$

where the eigenvalues of Λ_1 and $(A_1 + B_1G)$ are the same and likewise the eigenvalues of Λ_2 and A_2 are the same. If $x_2(0)$ is chosen to be $-\Lambda_2^{-1}\beta_2v$ then, x_2 will be $-\Lambda_2^{-1}\beta_2v$ for all $t \geq 0$. Y is thus a linear combination of $e^{-\lambda_i t}$ and a constant where λ_i represents the eigenvalues of A_m . With proper choice of $T(t)$ and $x_1(0)$, the CGT condition can be satisfied provided that y_m does not contain any modes that do not appear in y . In [WEN-1] additional strategies are developed for the

cases when y_m does contain modes not present in y and when the reference model is chosen higher than the largest controllable-observable subsystem of the plant. For an example of how to verify the CGT condition using the technique described in (3.12)-(3.13) see Appendix A.

3.3 Nonadaptive Model Reference Control

Before developing the direct adaptive control law which uses no a priori knowledge of the plant parameters, the nonadaptive control law is investigated. Since the dimensionality mismatch between the plant and the reference model has been accounted for by the intermediate system introduced in (3.2), the following error system can now be created:

$$e(t) = z(t) - z^*(t)$$

$$\frac{de(t)}{dt} = Ae(t) + BT(t) - BT^*(t); \quad e(0) = z_0 - z_0^*, \quad t \geq 0 \quad (3.14)$$

$$e_y(t) = y(t) - y^*(t) = Ce(t)$$

The objective of model reference control based on the error system (3.14) is to find a bounded control signal that drives $e_y(t)$ to zero asymptotically and keeps $e(t)$ uniformly bounded. A derivation of the control law follows.

Assume (A, B, C) are known in (2.52), (2.60), and it is possible to find a static output feedback gain G such that $(A + BGC)$ is strictly stable. Find S_{21} and S_{22} in (3.3) by solving for the CGT condition. With these parameters the model following nonadaptive control law may be constructed:

$$T(t) = G(y(t) - y_m(t)) + S_{21}z_m(t) + S_{22}T_m(t) \quad (3.15)$$

Substituting (3.15) into (3.14) and applying the CGT conditions (3.3) and (3.4), the following error equation is obtained:

$$\begin{aligned} \frac{de(t)}{dt} &= Ae(t) + BG(y(t) - y_m(t)) + B(S_{21}z_m(t) + S_{22}T_m(t) - T^*(t)) \\ &= (A + BGC)e(t) \end{aligned} \quad (3.16)$$

Since $(A + BGC)$ is strictly stable by assumption, $e(t) \rightarrow 0$ as $t \rightarrow \infty$ and the control objective is satisfied.

Explicit knowledge of the plant is required to actually compute G , S_{21} , and S_{22} in (3.15). Since the plant is generally not known, an adaptive control law may be derived based on (3.15) after the concept of positive realness is defined. In the nonadaptive control law (3.15) it is sufficient for $(A + BGC)$ to be just output stabilizable; but, in adaptive control the positive realness condition must also be satisfied.

3.4 Positive Realness

The system (A, B, C, D) , $A \in R^{n \times n}$, $B \in R^{n \times m}$, $C \in R^{m \times n}$, $D \in R^{m \times m}$ is "strictly positive real" if there exists $P \in R^{n \times n}$, positive definite, $W \in R^{n \times m}$, $Q \in R^{m \times n}$ and $\epsilon > 0$ sufficiently small such that [AND-1]:

$$A^T P + PA = -(\epsilon I + Q^T Q) \quad (3.17)$$

$$B^T P = C + W^T Q$$

$$W^T W = \frac{(D + D^T)}{2}$$

If (3.17) is replaced by

$$A^T P + PA = -Q^T Q$$

then (A, B, C, D) is "positive real."

This definition is equivalent to the frequency condition of the Kalman-Yakubovich Lemma [LI-1]:

$$\operatorname{Re}[W^T(C(j\omega I - A)^{-1}B + D)W] \geq 0, \quad \forall W \in C^m, \quad \forall \omega \in R \quad (3.18)$$

For the adaptive control law presented in the next section, it is required that the *closed-loop* system be positive real. Hence, in (3.18) A is replaced by $A_c = A + BGC$. To actually verify positive realness of a system, (3.18) says that the Nyquist plot of $(A_c, B, C, 0)$ must be only in the closed right half plane. A necessary condition that must also be satisfied for positive realness is that

$$CB > 0 \quad (3.19)$$

3.5 Model Reference Adaptive Control

The objective of model reference *adaptive* control is to find a bounded control signal that drives $e_y(t)$ as in (3.14) to zero asymptotically and keeps $e(t)$ uniformly bounded. This objective is similar to that of nonadaptive control but now *no a priori* knowledge of the plant parameters is required.

Assuming that $(A + BGC)$ is stable and $(A_c, B, C, 0)$ is *strictly* positive real, an adaptive version of (3.15) may be constructed [SOB-2]:

$$T(t) = G(t)(y(t) - y_m(t)) + S_{21}(t)z_m(t) + S_{22}(t)T_m(t) \quad (3.20)$$

where $G(t)$, $S_{21}(t)$, $S_{22}(t)$ are the adaptive estimates of G , S_{21} , S_{22} respectively. Define

$$\Delta G(t) = G(t) - G \quad (3.21)$$

$$L(t) = S(t) - S$$

$$w(t) = [z_m(t)^T \quad T_m(t)^T]^T$$

$$S(t) = [S_{21}(t) \quad S_{22}(t)]$$

$$S = [S_{21} \quad S_{22}]$$

From (3.14) and (3.20) the following closed-loop dynamic equation is obtained:

$$\frac{de(t)}{dt} = (A + BGC)e(t) + B\Delta G(t)e_y(t) + BL(t)w(t) \quad (3.22)$$

$$e_y(t) = Ce(t)$$

Applying Lyapunov's Direct Method to select an adaptive strategy that will stabilize (3.22) yields [WEN-1]:

$$\frac{dG(t)}{dt} = \frac{d\Delta G(t)}{dt} = -\Gamma_1 e_y(t)e_y^T(t) \quad (3.23)$$

$$\frac{dS(t)}{dt} = \frac{dL(t)}{dt} = -\Gamma_2 e_y(t)w^T(t)$$

where Γ_1, Γ_2 are constant positive definite matrices of dimension $m \times m$. LaSalle's Theorem [LAS-1] is then used to verify that $e \rightarrow 0$ and $e_y \rightarrow 0$ as $t \rightarrow \infty$. Since $G(t), S_{21}(t), S_{22}(t)$ are uniformly bounded the control objective is satisfied.

A modification of the adaptive algorithm (3.23) is made by Wen [WEN-1] by adding an extra term to generalize the algorithm to infinite dimensions and to add robustness:

$$\frac{dG(t)}{dt} = \frac{d\Delta G(t)}{dt} = -\gamma_1 G(t) - \Gamma_1 e_y(t)e_y^T(t) \quad (3.24)$$

$$\frac{dS(t)}{dt} = \frac{dL(t)}{dt} = -\gamma_2 S(t) - \Gamma_2 e_y(t)w^T(t)$$

However, this modification weakens the global asymptotic stability to Lagrange stability. The convergence rate is globally exponential with rate λ but strict positive realness of $(A_c, B, C, 0)$ is still a requirement.

In [BAR-1] the control law (3.20) is modified in order to relax the condition on $(A_c, B, C, 0)$ from being strictly positive real to positive real. The control (3.20) is now:

$$T(t) = G(t)(y(t) - y_m(t)) + S_{21}(t)z_m(t) + S_{22}(t)T_m(t) + K(y(t) - y_m(t)) \quad (3.25)$$

where K is a positive definite *constant* matrix. If the positive realness and CGT conditions are satisfied then asymptotic output stabilizability is achieved.

Adaptive control laws (3.20), (3.23), (3.24), and (3.25) are well suited for systems that are static output feedback stabilizable and positive real. However, these restrictions rule out the possibility of adaptively controlling many interesting systems that are "almost" feedback positive real. "Almost" positive real means that there exists an output feedback gain G that is $-d$ -stabilizing with $d \geq 0$ for $(A, B, C, 0)$ where $-d$ -stabilizing means the Nyquist plot is in the left half plane a very small distance d from the imaginary axis. This distance " d " is called the "positive realness index" (PRI). In [WEN-1] a "modified" adaptive controller is derived to allow systems of this type to be successfully controlled (see Figure 3.1):

$$T(t) = G(t)e_y(t) + S(t)w(t) - he_y(t) \quad (3.26)$$

where

$$\frac{dG(t)}{dt} = -\gamma_1 G(t) - \Gamma_1 e_y(t) e_y^T(t) - \xi \Gamma_1 G(t) e_y(t) e_y^T(t) \quad (3.27)$$

$$\frac{dS(t)}{dt} = -\gamma_2 S(t) - \Gamma_2 e_y(t) w^T(t)$$

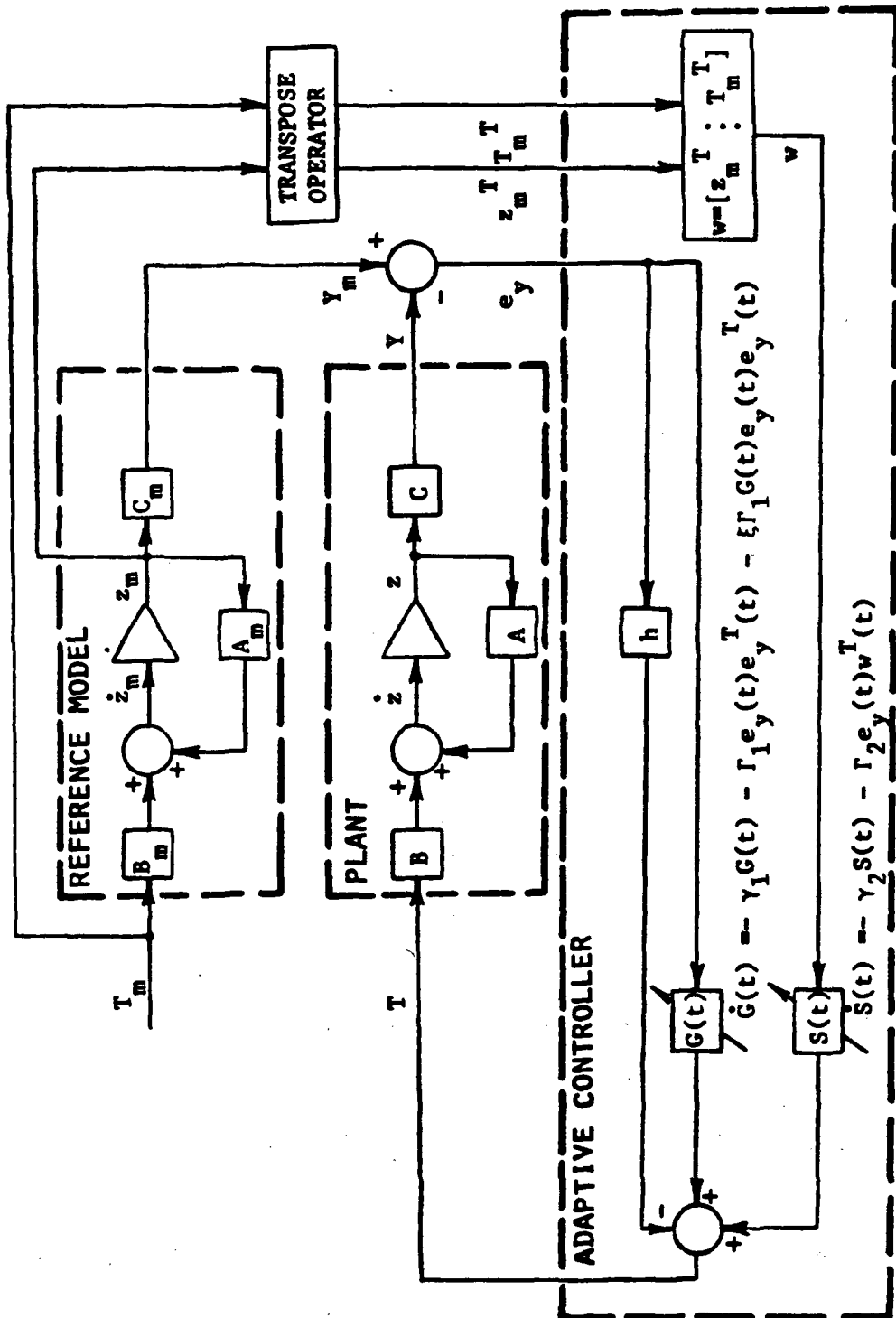


Figure 3.1 "Modified" Model Reference Adaptive Control Block Diagram

The adaptive control laws (3.26), (3.27) are based on the assumption that the following statements are true:

1. There exists $G \in R^{m \times m}$ that is $-d$ -stabilizing, $d > 0$.
2. CGT condition (3.3) holds.
3. The reference model is stable.

When these conditions are satisfied a robust controller with Lagrange stability is obtained. Parameters γ_1 , γ_2 , ξ , and h are constants which must be chosen to satisfy the following conditions: Given PRI sufficiently small,

$$\begin{aligned}
 h &= g & (3.28) \\
 \xi &= \frac{1}{g} \\
 dg &\leq 0.25 \left(\frac{c-1}{c} \right) \\
 \gamma_2 &> cd \|\Gamma_2\| M_w^2 \\
 c &> 1
 \end{aligned}$$

where g is the upperbound on the feedback gain G such that $(A + BGC)$ is exponentially stable and M_w is the uniform upperbound of w . Positive definite parameters Γ_1 , Γ_2 affect the ultimate bound of the norm ϵ . The larger Γ_1 , Γ_2 are chosen, the faster the rate of convergence will be between the outputs y and y_m .

For experimentation purposes in this project, an additional MRAC controller is derived based on (3.23) and (3.26) for a system with *two* sensors and one control actuator. The output of the sensor that satisfies both the positive realness and CGT conditions goes to zero asymptotically. For the sensor that satisfies only the CGT condition, the output stays bounded. The derivation is presented in Appendix B and results in the following MRAC law:

$$T(t) = G(t)(y(t) - y_m(t)) + S_{21}(t)z_m(t) + S_{22}(t)T_m(t) - h(y_2(t) - y_{m_2}(t)) \quad (3.29)$$

$$\frac{dG_1(t)}{dt} = \frac{d\Delta G_1(t)}{dt} = -\Gamma_1 e_{y_2}(t) e_{y_1}^T(t)$$

$$\frac{dG_2(t)}{dt} = \frac{d\Delta G_2(t)}{dt} = -\Gamma_2 e_{y_2}(t) e_{y_2}^T(t)$$

$$\frac{dS(t)}{dt} = \frac{dL(t)}{dt} = -\Gamma_2 e_{y_2}(t) w^T(t)$$

In the simulation results that follow the control algorithms presented in this chapter are implemented. The choice of the controller used in each case depends on which positive realness condition is satisfied by the system.

CHAPTER 4

SIMULATION RESULTS AND DISCUSSION

4.1 Introduction

A state-space representation for a flexible beam with actuation at the pinned-end and sensing at the free-end was formulated in Chapter 2. In this chapter, five different simulation cases of model reference control with the flexible manipulator are presented. Using a fourth order model of the system with a desired reference model, nonadaptive (case 1) and adaptive (case 2) model reference controllers are designed and simulated. In case 2 the MRAC scheme is not capable of controlling the manipulator. Until recently *all* adaptive control laws have been designed on the basis that the system being controlled is minimum phase. In [MOR-3] an adaptive control law has been stated for a first order, non-minimum phase system that has a relative degree of one in the transfer function. Although this algorithm may be a step in the right direction, the result is a highly complex, nonlinear controller.

Due to the inability to adaptively control the noncolocated, nonminimum phase manipulator system that does *not* satisfy positive realness, two additional model configurations are constructed and tested with MRAC. Case three has an actuator at the pinned-end controlling two sensors that are located respectively at the tip and one meter from the pin. The next two cases have a torque producing reaction wheel located close to the sensor at the tip so that positive realness is satisfied. Case 4 has the reaction wheel perfectly colocated with the sensor and case 5 has the reaction wheel located one meter from the sensor.

After stating the parameters chosen for the manipulator model, simulation results are presented and discussed for each case described above. All simulations

are performed on an IBM-PC-AT computer with PC-MATLAB [PCM-1], a software package designed by the Mathworks, Inc.

4.2 Plant and Reference Model Parameters

In order to simulate the manipulator model and implement the control algorithms, the parameters as shown in Table 4.I were chosen. These values are consistent with experimental data that has been obtained from a pinned-free, flexible beam experiment presented in [SCH-1].

TABLE 4. I
Model Parameters

Parameter	Value
L=arm length	4.0 m
A=x-sectional area	0.0016 m ²
E=modulus of elasticity	7.311 × 10 ¹⁰ N - m ²
I=moment of inertia	1.0 × 10 ⁻⁸ m ⁴
ρ =density/unit volume	2699.0 kg/m ³
ζ =structural damping	0.5%
α = weighting factor of position vs. rate measurement	.5 (case 1) 0.05 (cases 2, 3, 4, 5)

Now that E , I , ρ , A , and L have been chosen, the natural frequencies of the manipulator system may be calculated from (2.26) (shown below for convenience):

$$\omega_n = \left(\frac{k_n}{L} \right)^2 \sqrt{\frac{EI}{\rho A}}, \quad n = 0, 1, 2, \dots, \infty$$

In Table 4.II the eigenvalues, k_n , and natural frequencies ω_n in rad/sec and f_n in Hz are listed.

TABLE 4. II
Natural Frequencies

mode # (n)	k_n	ω_n (rad/sec)	f_n (Hz)
0	0	0	0
1	3.927	12.542	1.996
2	7.069	40.637	6.467
3	10.210	84.773	13.492
4	13.352	144.976	23.074
5	16.493	221.210	35.207

Using the parameter values in Table 4.I and Table 4.II the sixth order state space model matrices may be obtained based on equations (2.52)-(2.55), (2.60), (2.62), and (2.63):

$$\frac{dz(t)}{dt} = Az(t) + BT(t) \quad (4.1)$$

$$y(t) = Cz(t)$$

where

$$z(t) = \begin{bmatrix} z_0(t) \\ \frac{dz_0(t)}{dt} \\ z_1(t) \\ \frac{dz_1(t)}{dt} \\ z_2(t) \\ \frac{dz_2(t)}{dt} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & & & & 0 \\ 0 & 0 & & & & \\ & & 0 & 1 & & \\ & & -157.3 & -.1254 & & \\ & & & & \ddots & \\ 0 & & & & & 0 & 1 \\ & & & & & -1651.4 & -.4064 \end{bmatrix} \quad (4.2)$$

$$B = [0 \quad 36.6543 \quad 0 \quad 157.1943 \quad 0 \quad 299.9147]^T \quad (4.3)$$

$$C = [.866 \quad \alpha.866 \quad -1.3759 \quad \alpha(-1.3759) \quad 1.4165 \quad \alpha 1.4165] \quad (4.4)$$

This model is used in cases 1 and 2. In case 3 the same system is used but with an additional row in C to represent the sensor located at one meter from the pin, $\phi(1)$:

$$C = \begin{bmatrix} .866 & \alpha.866 & -1.3759 & \alpha(-1.3759) & 1.4165 & \alpha 1.4165 \\ .2165 & \alpha.2165 & .7778 & \alpha.7778 & .9854 & \alpha.9854 \end{bmatrix} \quad (4.5)$$

Cases 4 and 5 have the actuator placed at the tip and one meter from the tip, respectively. Thus, the systems they use are (4.1) and (4.4) with the B matrices:

$$\text{case 4 : } B = [0 \quad 36.6543 \quad 0 \quad -228.7695 \quad 0 \quad 423.6336]^T \quad (4.6)$$

$$\text{case 5 : } B = [0 \quad 36.6543 \quad 0 \quad -201.5686 \quad 0 \quad 202.6958]^T \quad (4.7)$$

Although the ultimate interest is in the *position* of the manipulator tip, velocity must also be included in the output matrix C in order to obtain output feedback stabilizability. This is a requirement for both the nonadaptive and adaptive controllers. By choosing parameter α , the weighting factor of position versus rate measurement (Table 4.I), very small, the *majority* of the torque controller is devoted to satisfying the position requirement. Parameter ζ , the percent of structural damping present in the manipulator, has been chosen to represent the amount of damping typically found in flexible space structures.

To implement model reference control on the manipulator model created with the parameters in Tables 4.I and 4.II, an appropriate reference model must also be constructed. For all five simulation cases, the desired trajectory that the manipulator tip is to track is $\sin 5t$. Hence, the reference model is chosen as follows:

$$\frac{dz_m(t)}{dt} = A_m z_m(t) \quad (4.8)$$

where

$$\begin{aligned} z_m(t) &= \begin{bmatrix} z_{m_1}(t) \\ \frac{dz_{m_1}(t)}{dt} \end{bmatrix}, & z_{m_0} &= \begin{bmatrix} 0 \\ 5 \end{bmatrix} \\ A_m &= \begin{bmatrix} 0 & 1 \\ -25 & 0 \end{bmatrix}, & u_m &= 0 \\ y_m(t) &= C_m z_m(t) \end{aligned} \quad (4.9)$$

where

$$C_m = [1 \quad \alpha]$$

The output y_m is equal to $\sin 5t + 5\alpha \cos 5t$ where $5\alpha \cos 5t$ is included to control the velocity component in the plant output $y(t)$.

For case 3 there are *two* outputs due to the two sensors. A requirement on the reference model is that it must have the same number of outputs as plant outputs. If the manipulator tip is commanded to track $\sin 5t$ it is reasonable for the sensor at one meter from the pinned-end to track a motion with a smaller magnitude and a phase shift of 90° . In this simulation the second sensor is commanded to track a trajectory of $.5 \cos 5t$. This may be represented by combining (4.8) with the following for case 3:

$$y_m(t) = C_m z_m(t) \quad (4.10)$$

where

$$C_m = \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

4.3 Case 1: Nonadaptive, Noncolocated, 6th Order System

Before attempting model reference *adaptive* control, design and test the non-adaptive controller (3.15) presented in Section 3.3 for the sixth order plant (4.1), (4.2), (4.3), (4.4):

$$T(t) = G(y(t) - y_m(t)) + S_{21}z_m(t) + S_{22}T_m(t)$$

Gain G is chosen so that output feedback stabilizability of the plant is obtained. Matrices S_{21} and S_{22} are determined by satisfying the CGT condition described in Section 3.2. A detailed derivation of S_{21} and S_{22} (see Appendix A) results in the following nonadaptive model reference controller:

$$T(t) = -.001 (y(t) - y_m(t)) + [-.3791 \quad -.0003] z_m(t) \quad (4.11)$$

Since the CGT and output feedback stabilizability conditions are satisfied it is expected that the controller will perform well in simulation. Figure 4.3 shows that the tip position of the manipulator tracks the desired trajectory of $\sin 5t$ with a small error of .1 within 3.5 seconds. The implications of these excellent results is that a higher order system can be successfully controlled with nonadaptive model reference control as long as CGT and output feedback stabilizability conditions are satisfied.

Although successful control has been demonstrated for this case, the issue of robustness is still of concern. In (4.11) the static output feedback gain, G , is chosen very small, -.001, in order to achieve output feedback stabilizability. This results in a small stability margin and hence little robustness. The tracking results for this case are comparable to those shown in [SCH-1] for a similar manipulator where dynamic feedback is used to achieve a very robust system. It is highly probable that dynamic output feedback used with nonadaptive model reference control would result in a more robust system for this case. However, a limiting factor will be the computation of the CGT matching conditions.

There are several drawbacks to nonadaptive control that should be mentioned. The most significant drawback is that in nearly all control situations, exact knowledge of the plant is *not* available. To design this nonadaptive controller,

exact knowledge of the plant is necessary. If it is desired that the reference signal be tracked well in less than 3.5 seconds, nonadaptive control has no means of tuning the gain parameters for a quicker response in this case. Any disturbance or change in the reference model would require a change in the S matrix which is not possible with this nonadaptive controller. In addition, as can be seen by looking at Appendix A, the calculations required to determine the S gains are *very tedious* — especially when a large order plant is used.

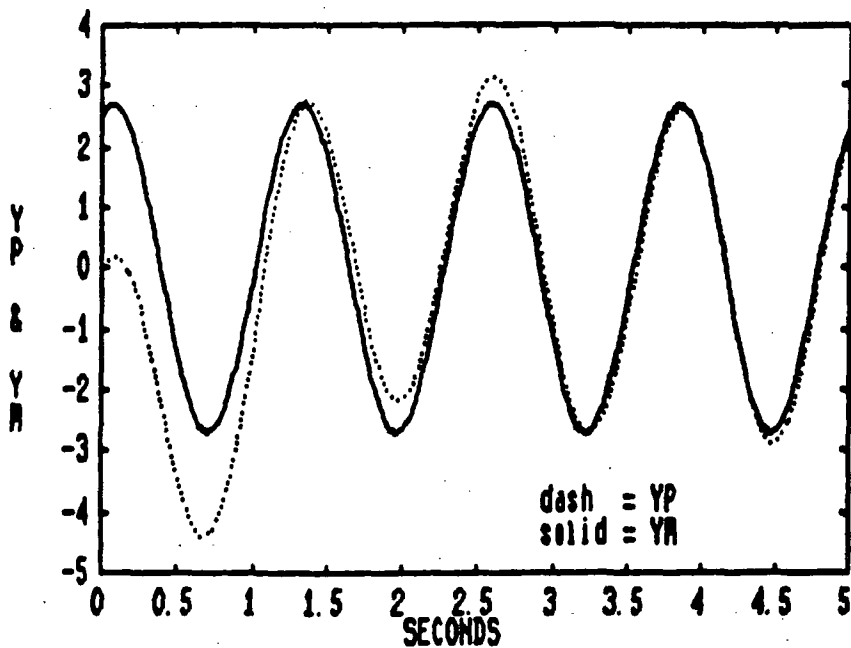


Fig. 4.1 Case 1: Plant Output, YP, and Model Output, YM

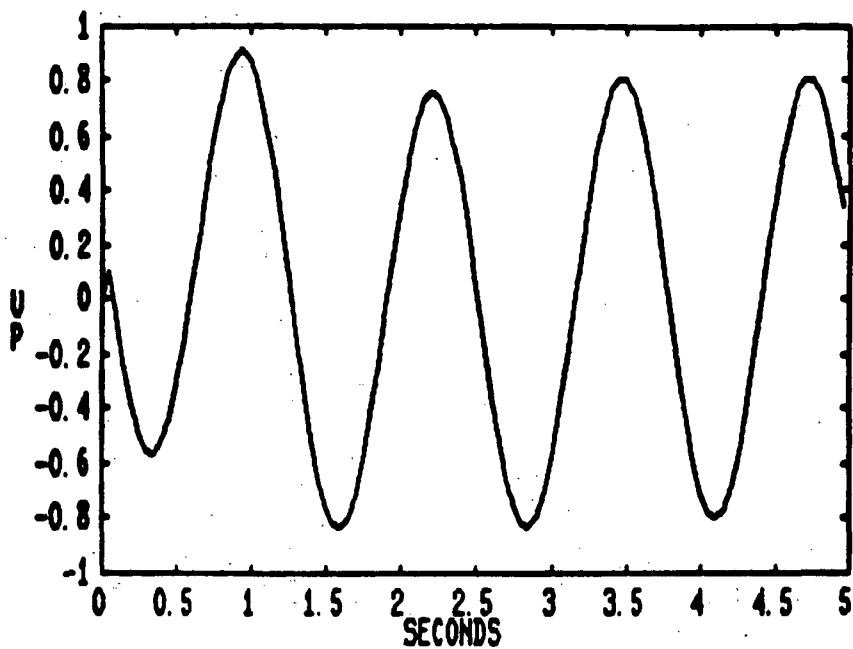


Fig. 4.2 Case 1: Torque Control, UP

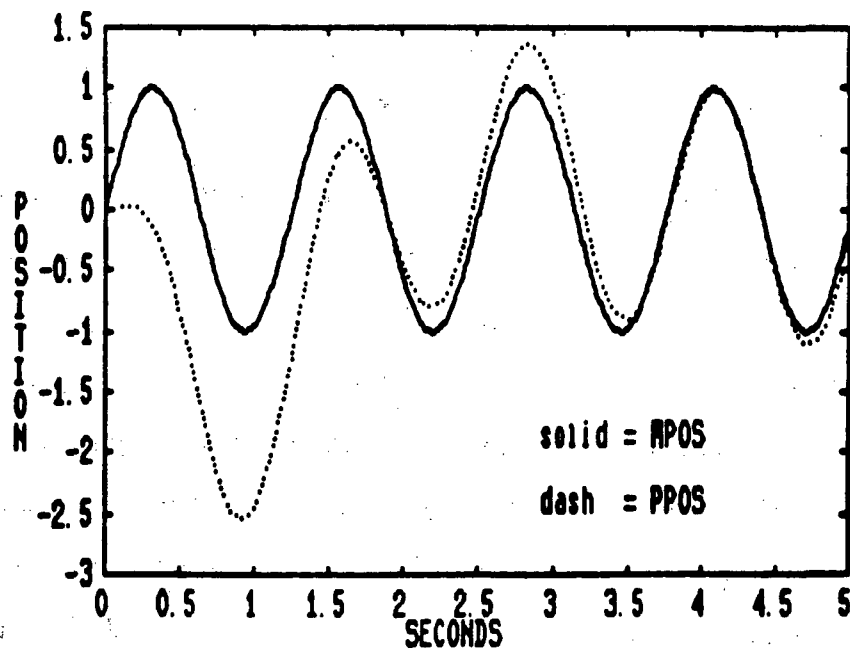


Fig. 4.3 Case 1: Position of Manipulator Tip, PPOS, and Model $\sin 5t$, MPOS

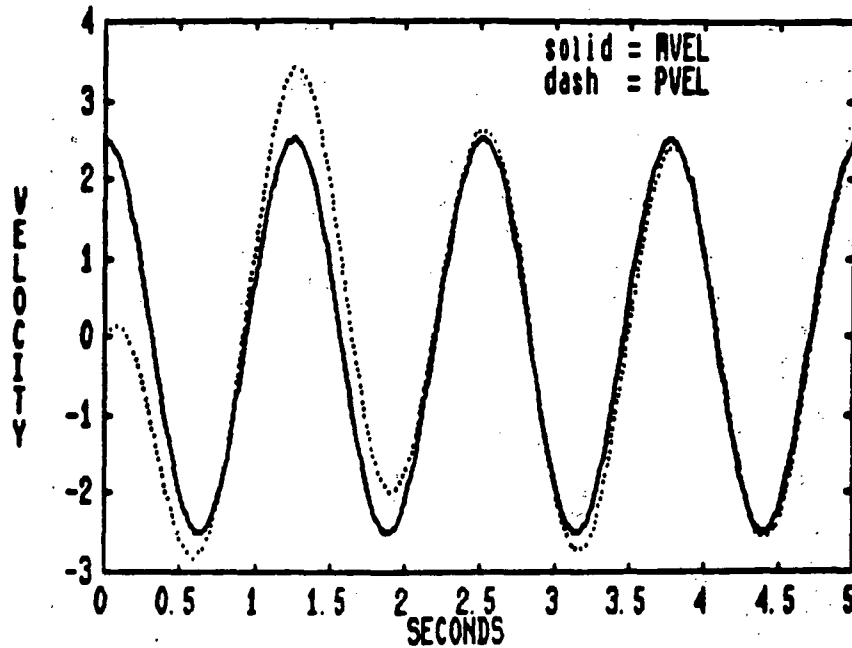


Fig. 4.4 Case 1: Velocity of Manipulator Tip, PVEL, and Model $2.5 \cos 5t$, MPOS

4.4 Case 2: Adaptive, Noncolocated, 6th Order System

This case is a test of the primary objective in our study: to adaptively control the tracking of a flexible manipulator tip that has a noncolocated actuator located at the opposite end of the arm. Successful MRAC is highly desirable for this flexible manipulator because quicker tracking of the reference signal and robustness to disturbances and changes in the reference model would be possible. As noted in Section 3.5 there are *three* major requirements that must be satisfied before MRAC will work. Not only must the plant satisfy the CGT condition and be output feedback stabilizable, but the quadruplet $(A + BGC, B, C, 0)$ must be at least "almost" positive real.

It is possible for the plant (4.1), (4.2), (4.3), (4.4) to satisfy the CGT condition

and output feedback stabilizability as shown in the previous section. The question that remains is can “almost” positive realness be obtained at the *same time*? A great deal of time has been spent trying to simultaneously satisfy these conditions for this system. The necessary condition $CB > 0$ is satisfied and “almost” positive realness of $(A + BGC, B, C, 0)$ is also satisfied but *not* for an output feedback gain G that stabilizes the system. By looking at the B and C plant matrices (4.3), (4.4), a key point can be made:

$$B = [0 \quad 36.6543 \quad 0 \quad 157.1943 \quad 0 \quad 299.9147]^T$$

$$C = [.866 \quad \alpha.866 \quad -1.3759 \quad \alpha(-1.3759) \quad 1.4165 \quad \alpha.4165]$$

The pinned-free flexible beam with boundary control and end-point sensing has an eigenfunction that is composed of sines and hyperbolic sines (2.28). As n goes from $1 \rightarrow \infty$, the C parameters, $\phi_n(L)$, alternate in sign like a sine series. Negative output feedback therefore has an opposing effect on the system that causes some poles to cross over the imaginary axis into the right half plane causing instability. Only a very small G will stabilize the system by placing the poles very close to the imaginary axis in the left half plane. Unfortunately, for G 's of this small magnitude, “almost” positive realness of the system is impossible.

Although it is not possible to satisfy the three conditions required for MRAC, controllers (3.25) and (3.26) were designed and implemented in simulation. Many combinations of controller parameters were tested and as might be expected, the system went unstable.

The results obtained for this case, albeit not surprising, are disappointing. It is conceivable that if the manipulator were stiffer than the one used in this project, then MRAC of simply the rigid body mode would be possible. Perhaps an upper

bound on the flexibility possible for successful MRAC of a noncolocated system like this one can be found in future research.

In the next three sections different sensor-actuator configurations for the flexible manipulator are tested with MRAC in an attempt to successfully attain our tracking objective.

4.5 Case 3: Adaptive, Two Sensor/One Actuator, 6th Order System

The motivation for this case is that the addition of one sensor to the manipulator system of Section 4.4 may allow MRAC to be successful. A sensor located at one meter from the actuator at the pinned-end of the arm results in the model (4.1), (4.2), (4.3), (4.5). Define C_1 and C_2 to be the first and second rows of C in (4.5), respectively, where each row of C represents a different sensor. The system $(A, B, C_1, 0)$ with the sensor at the tip is called "system 1" and the system $(A, B, C_2, 0)$ with the sensor near the pin is "system 2." The objective for this setup is for the plant to track the signals produced by the reference model (4.8) and (4.10). System 1 is to follow $\sin 5t$ and system 2 is to follow $.5 \cos 5t$.

In Appendix B an adaptive control law similar to (3.23) and (3.26) is derived that allows for two sensors and one actuator. The algorithm is derived keeping in mind that both systems satisfy the CGT condition but only system 2 is simultaneously output feedback stabilizable *and* positive real. Stated below is the control law derived in Appendix B:

$$T(t) = G(t)e_y(t) + S(t)w(t) - h(y_2(t) - y_{m_2}(t)) \quad (4.12)$$

where

$$G(t) = [G_1(t) \quad G_2(t)]$$

$$e_y(t) = \begin{bmatrix} y_1(t) - y_{m_1}(t) \\ y_2(t) - y_{m_2}(t) \end{bmatrix} \quad (4.13)$$

$$\frac{dG_1(t)}{dt} = \frac{d\Delta G_1(t)}{dt} = -\Gamma_1 e_{y_2}(t) e_{y_1}^T(t)$$

$$\frac{dG_2(t)}{dt} = \frac{d\Delta G_2(t)}{dt} = -\Gamma_1 e_{y_2}(t) e_{y_2}^T(t)$$

$$\frac{dS(t)}{dt} = \frac{dL(t)}{dt} = -\Gamma_2 e_{y_2}(t) w^T(t)$$

Since system 1 is not positive real but system 2 is, the algorithm only guarantees that $\|e_{y_1}\|$ is bounded and $\|e_{y_2}\| \rightarrow 0$. Simulation results of control law (4.12) with plant (4.1), (4.2), (4.3), (4.5) and reference model (4.8), (4.10), are presented in figures 4.5 - 4.8. The controller parameters were chosen as $\Gamma_1 = \Gamma_2 = 100$ and $h = 20$. Comparing the plots on figure 4.5, it can be seen that the tip position tracking is at least bounded but it still does *not* track the desired trajectory of $\sin 5t$. The position of the manipulator at sensor 2 tracks the trajectory of $.5 \cos 5t$ extremely well (Figure 4.6). This is because system 2 satisfies the positive realness condition.

Again, as in Section 4.4, the limiting factor in achieving the desired tracking with the manipulator tip, is the inability to satisfy the positive realness condition. Various controller parameters and reference models were again implemented in simulation but without much success. If it were somehow possible to slow down the convergence of e_{y_2} to zero (see Figure 4.8), it may be possible to achieve better tracking results with system 1. The reason for this is made clear by looking at the control law (4.13). When e_{y_2} reaches zero, no more adaptation takes place in the controller since $\frac{dG_1(t)}{dt}$, $\frac{dG_2(t)}{dt}$, and $\frac{dS(t)}{dt}$ equal zero.

The last two sections propose different sensor-actuator configurations that satisfy the positive-realness condition.

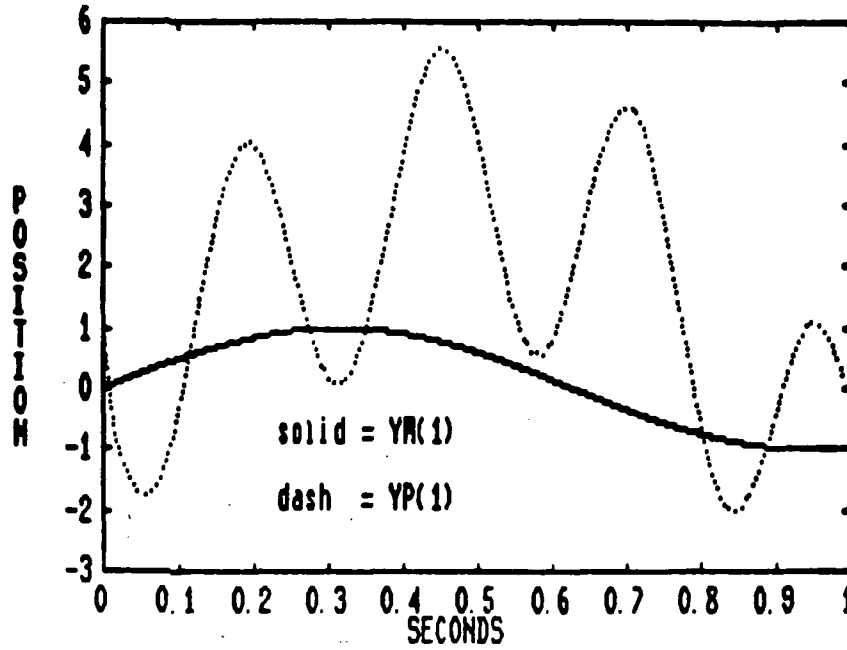


Fig. 4.5 Case 3: Position of Manipulator Tip, $YP(1)$, and Model $\sin 5t$, $YM(1)$

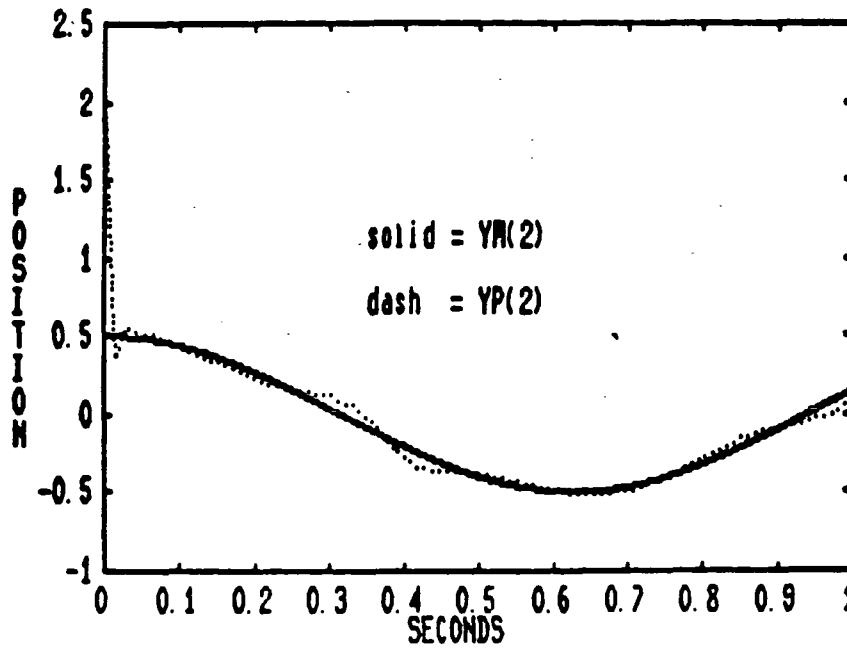


Fig. 4.6 Case 3: Position of Manipulator 1 m from Pin, $YP(2)$ and Model $.5 \cos 5t$, $YM(2)$

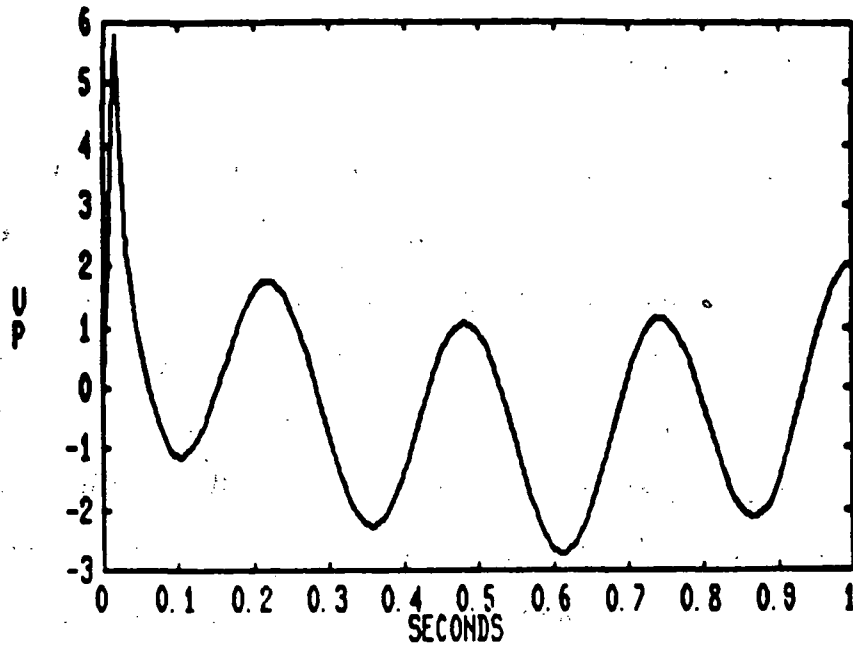


Fig. 4.7 Case 3: Torque Control, UP

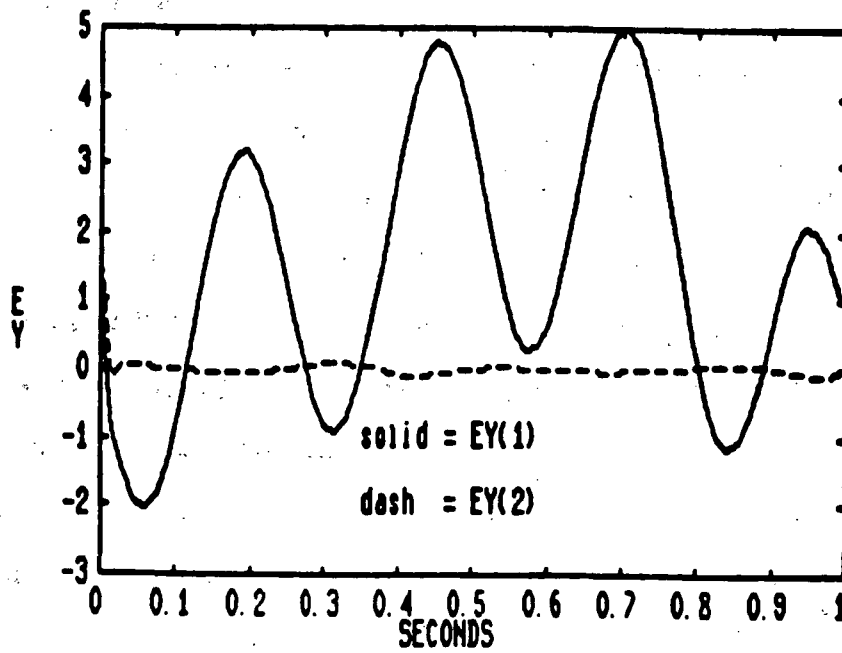


Fig. 4.8 Case 3: Output Error $EY=YP-YM$

4.6 Case 4: Adaptive, Colocated, 6th Order System with Reaction Wheel

As demonstrated in the previous cases, MRAC of the flexible manipulator is not possible when positive realness is not satisfied. If the actuator is moved close to the sensor at the tip of the beam then positive realness is finally satisfied. At first the hesitation to this approach is the practicality of locating a torquing actuator at the *free* end of a flexible manipulator. As Sir Isaac Newton said, "To every action there is always opposed an equal reaction;..." [NEW-1]. When the actuator is located at the pinned-end, actuating torque is produced by a reaction against the basebody inertia. A torque at the free end can be produced by a reaction against a rotating inertia. This requires a reaction wheel type actuator similar to those used extensively for spacecraft attitude control. In space applications of a flexible manipulator the reaction wheel actuator may ultimately prove to be better than the pinned-end actuator as far as disturbances to the spacecraft go. The actuator at the pinned-end may impart a large, undesirable moment to the spacecraft whereas the reaction wheel may produce only a small linear force.

For this case a reaction wheel is placed at the tip with the sensor-system (4.1), (4.2), (4.4), (4.6). Again, the objective is to control the tip position so that it follows the trajectory produced by the reference model (4.8), (4.9). After verifying that this system simultaneously satisfies the CGT condition, "almost" positive realness and output feedback stabilizability, the following control parameters were chosen in accordance with (3.28): $\Gamma_1 = \Gamma_2 = 100$, $\gamma_1 = 2$, $\gamma_2 = 20$, $h = 20$, and $\xi = 0.05$. The resulting "modified" adaptive controller (3.26) is:

$$T(t) = G(t)e_y(t) + S(t)w(t) - 20e_y(t) \quad (4.14)$$

where

$$\frac{dG(t)}{dt} = -2G(t) - 100e_y(t)e_y^T(t) - (.05)(100)G(t)e_y(t)e_y^T(t)$$

$$\frac{dS(t)}{dt} = -20S(t) - 100e_y(t)w^T(t)$$

This adaptive controller was implemented with the manipulator model (4.1), (4.2), (4.4), (4.6) to obtain the results shown in figures 4.9 - 4.13. At last, quick, precise tracking of the reference model trajectory $\sin 5t$ is obtained by the tip of the flexible manipulator! By comparing figures 4.11 - 4.13, it can be seen that the majority of the error, $e_y(t)$, is due to the velocity component of the plant output.

By colocating the actuator and sensor it has been shown that MRAC of the flexible manipulator is now possible. An important question still needs to be answered before a claim is made that this setup is a viable solution. How far can the reaction wheel be located from the sensor while maintaining satisfactory tracking results? Perfect colocation is difficult to achieve and in some cases may not be desirable. This question is addressed in the next section.

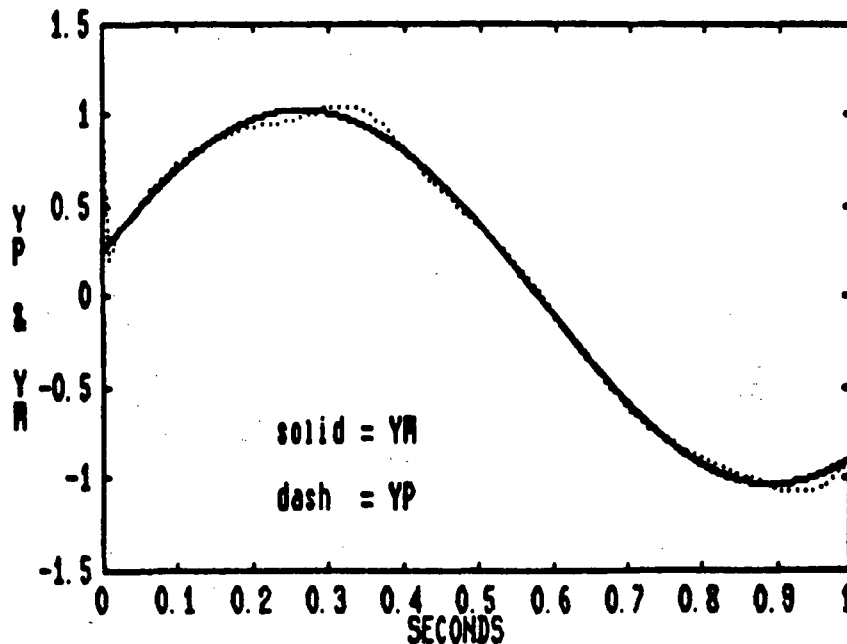


Fig. 4.9 Case 4: Manipulator Output, YP, and Model $\sin 5t + .25 \cos 5t$, YM

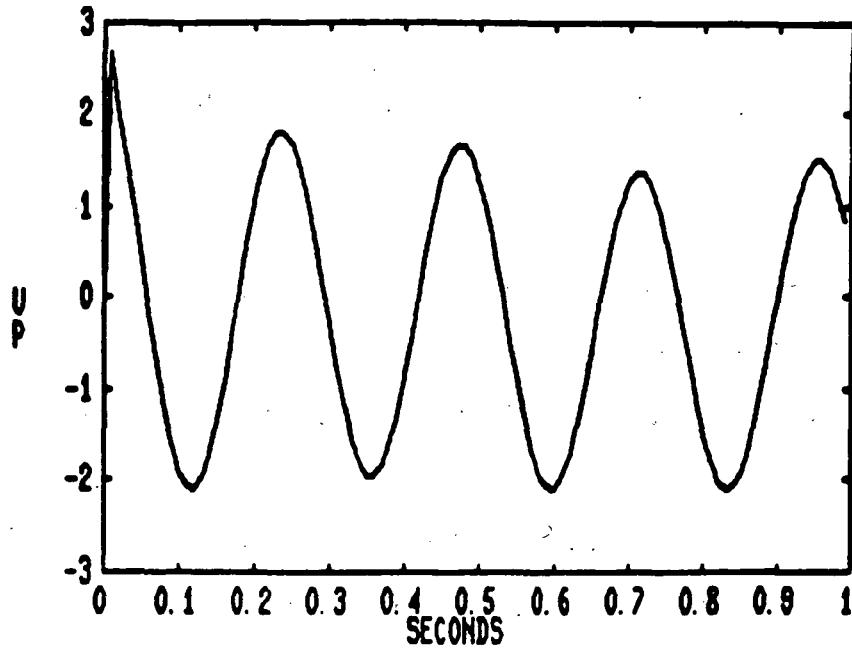


Fig. 4.10 Case 4: Torque Control, UP

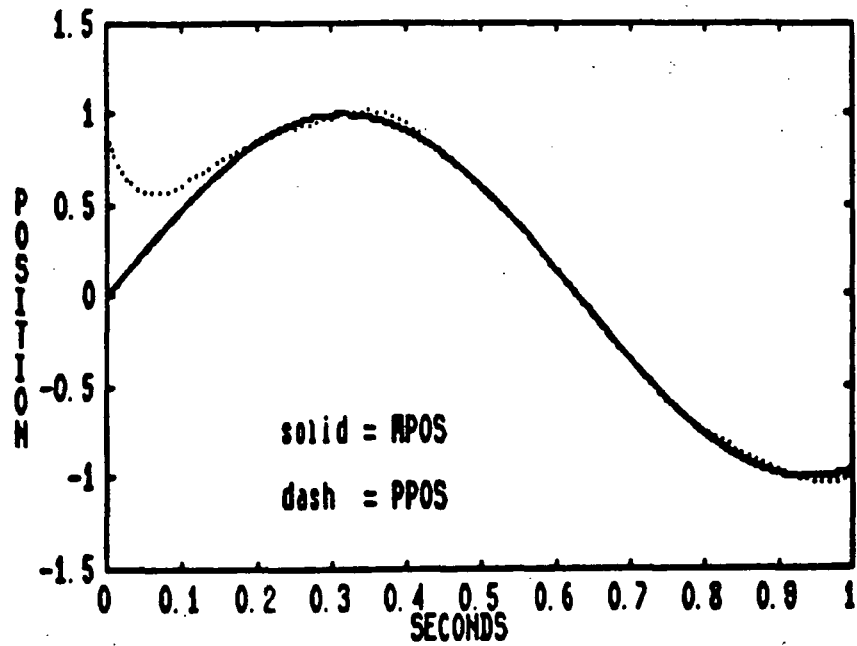


Fig. 4.11 Case 4: Position of Manipulator Tip, PPOS, and Model $\sin 5t$, MPOS

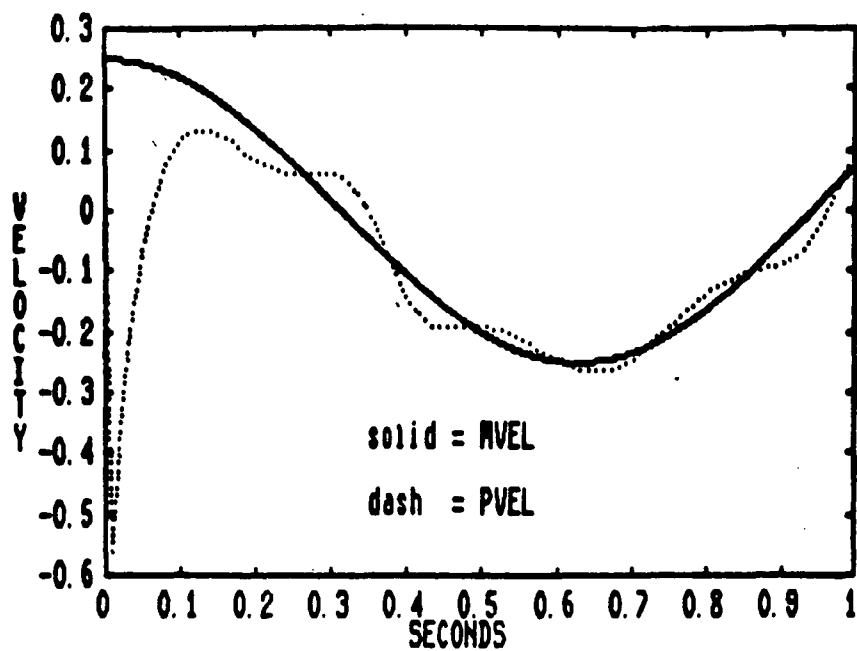


Fig. 4.12 Case 4: Velocity of Manipulator Tip, PVEL, and Model $.25 \cos 5t$, MVEL

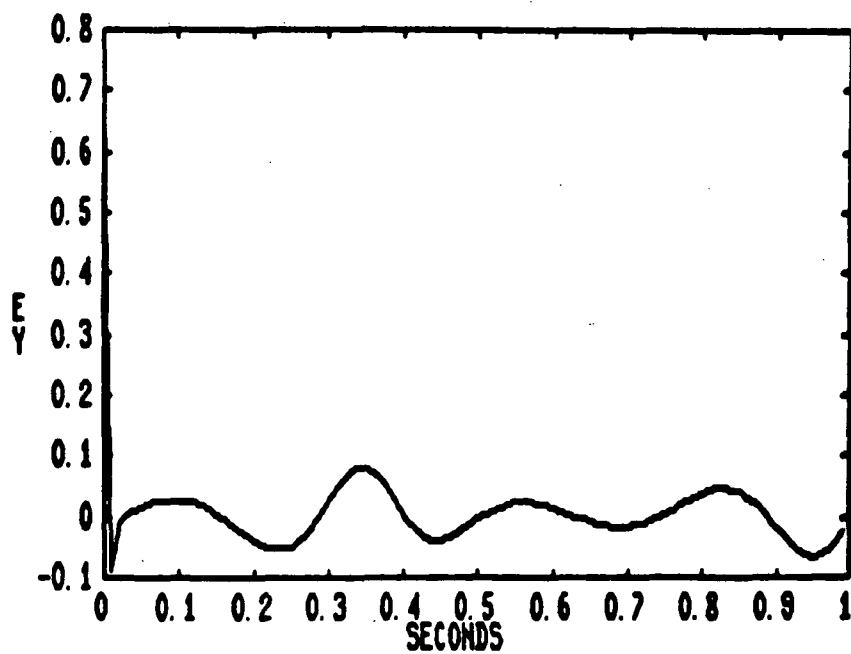


Fig. 4.13 Case 4: Output Error $EY = YP - YM$

4.7 Case 5: Adaptive, Noncolocated, 6th Order System with Reaction Wheel

In the previous section it was demonstrated that excellent tracking results can be obtained for MRAC of a flexible manipulator with a *colocated* sensor and reaction wheel. However, perfect colocation is difficult to achieve and in some cases may not be desirable. For instance, if the manipulator has an end effector attached to the tip, a reaction wheel may obstruct its performance. To test MRAC (4.14) on the manipulator when the reaction wheel is moved in from the tip use model (4.1), (4.2), (4.4), (4.7). With the reaction wheel placed one meter away from the tip and using the same parameter settings and objective as in Section 4.6, the simulation results shown in figures 4.14 - 4.18 are obtained. Satisfactory tracking of the tip position is again achieved but with a higher error than that shown in Figure 4.11 for the colocated system.

Other simulations were run to determine that the maximum distance the actuator may be moved from the sensor for this model is ≈ 1.5 meters. This is dependent on the parameters that have been chosen for this case. The more the actuator and sensor are separated, the higher the position error becomes due to the positive realness condition. As d increases, c increases also to satisfy the conditions stated in (3.28); therefore, more error is incurred.

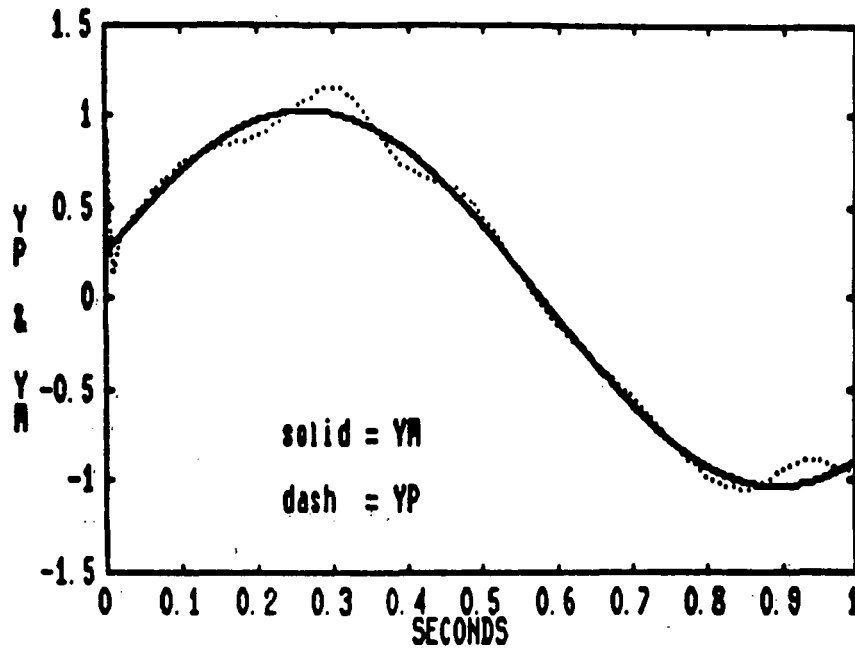


Fig. 4.14 Case 5: Manipulator Output, YP, and Model $\sin 5t + .25 \cos 5t$, YM

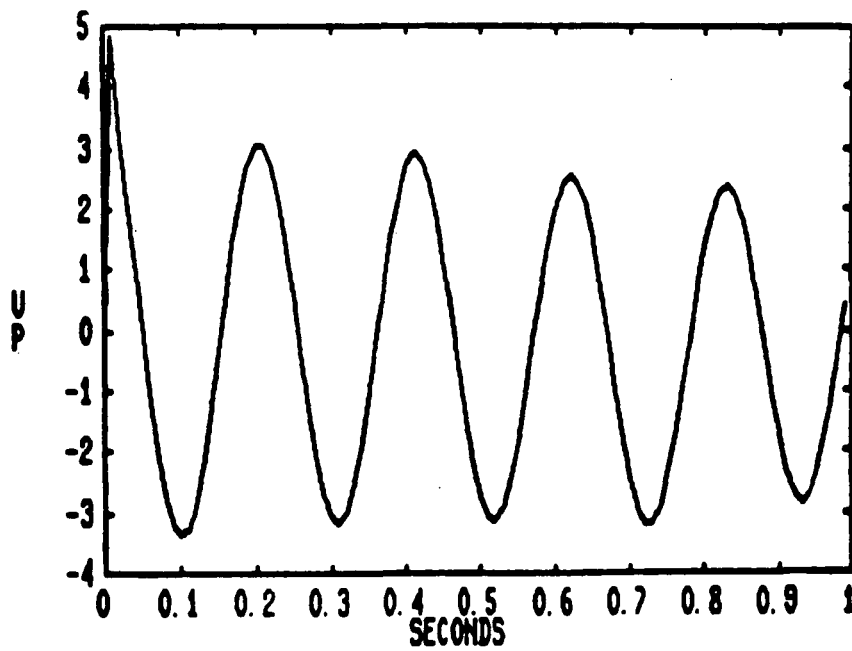


Fig. 4.15 Case 5: Torque Control, UP

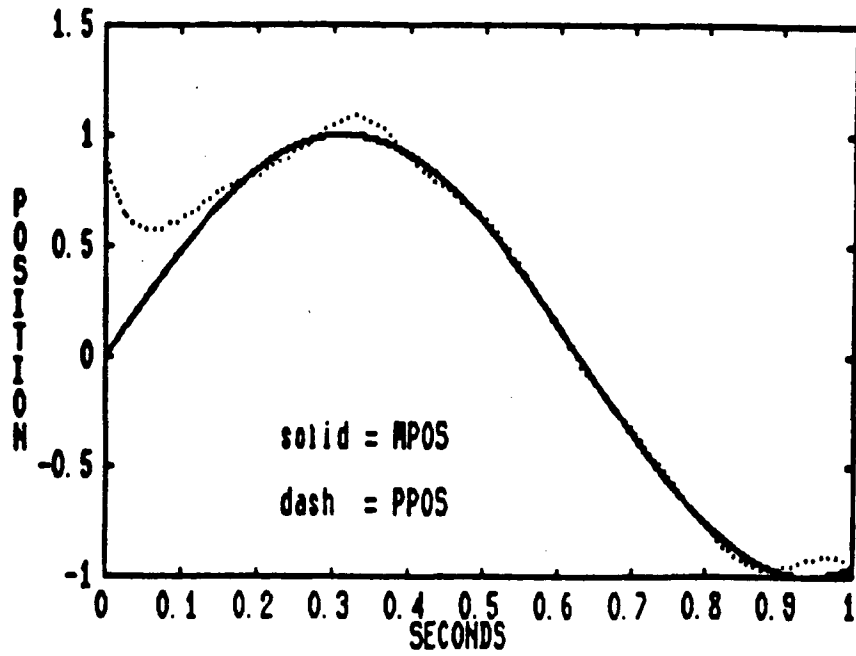


Fig. 4.16 Case 5: Position of Manipulator Tip, PPOS, and Model $\sin 5t$, MPOS

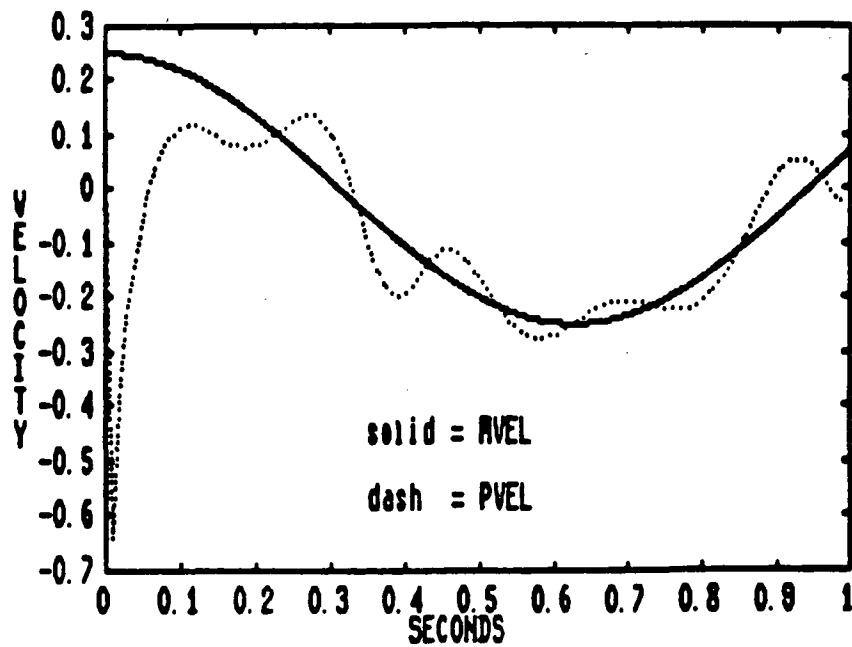


Fig. 4.17 Case 5: Velocity of Manipulator Tip, PVEL, and Model $.25 \cos 5t$, MVEL

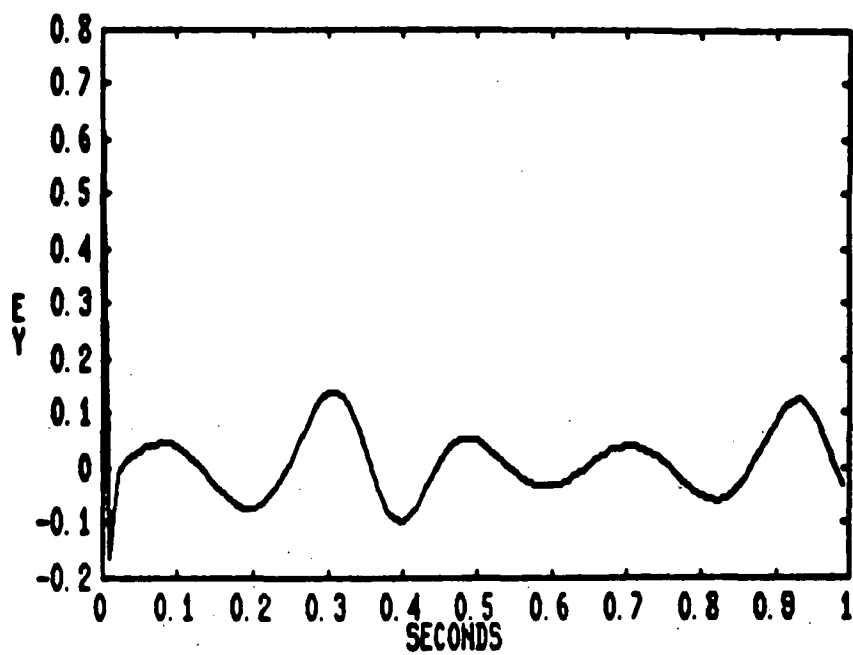


Fig. 4.18 Case 5: Output Error $EY=YP-YM$

CHAPTER 5

CONCLUSION

5.1 Results

As stated in Section 1.1, the objective in this study was to invoke MRAC as the control scheme by which the tip of a flexible manipulator with a torquing actuator at the pinned-end and a sensor at the free-end would track a prescribed trajectory. Through analysis and computer simulations the following results were obtained.

1. An exact modal representation of the pinned-free flexible manipulator with boundary control at the free-end was derived.
2. A reduced-order state space model was obtained by selecting a finite number of the system modes. This model was used for simulation studies.
3. The nonadaptive and adaptive model reference control laws were stated. These laws were implemented with the state-space model in simulations.
4. A nonadaptive control law was derived for a sixth order model of the flexible manipulator. In simulation satisfactory results were acquired.
5. The MRAC control laws stated in Section 3.5 were unsuccessful in controlling the same sixth order model of the flexible manipulator used in 4. Due to the inherent nonminimum phase properties of a noncolocated pinned-free beam, it is impossible to satisfy "almost" positive realness of $(A + BGC, B, C, 0)$ for an output feedback gain that will stabilize the system.
6. In an attempt to achieve stability and proper tracking by the system, an additional sensor located one meter from the pin was added to the model. After modifying the MRAC law to include two sensors, simulation tests were run. The tip position error was at least bounded for this case but proper

tracking by the tip was still not possible.

7. Since MRAC is definitely not possible when positive realness is not satisfied, a torque producing reaction wheel colocated with the sensor at the tip was proposed. This setup satisfies positive realness and excellent tracking results are obtained in simulation.
8. To demonstrate the robustness of the MRAC when perturbations of the actuator placement are present, the reaction wheel was moved away from the sensor at the tip. Simulation results verify that satisfactory tracking of the tip is possible as long as positive realness is satisfied. However, the position error of the tip increases as the actuator is moved further away from the sensor.

Based on these results several important conclusions can be drawn. If the system can be configured such that "almost" positive realness is satisfied, then MRAC is a powerful algorithm. Without knowledge of the plant and with a reference model that may be of a smaller order than the plant, a robust controller may be designed to give quick, precise tracking results. For the flexible manipulator with a reaction wheel type actuator excellent tracking by the tip was demonstrated.

By producing torque with a reaction wheel located at the tip of the flexible manipulator many dynamic problems that may occur when the actuator is placed at the pinned-end are resolved. In [SCH-1] it has been shown that when the actuator and sensor are noncolocated that the speed of response to commands is ultimately limited by the inherent wave-propagation delay for the beam. By colocating the reaction wheel with the sensor this problem does not need to be coped with. Another advantage of using the reaction wheel is that only a small linear force as opposed to a large torque is imparted to the basebody that the manipulator is pinned to.

For a flexible manipulator with an actuator at the pinned-end and a sensor at

the free-end, MRAC is not advised as a suitable control law unless the algorithm is modified such that the positive realness condition is omitted. Although the nonadaptive control scheme which requires *exact* knowledge of the plant performed well in simulation for this setup, it also is not recommended. Not only are the control gains obtained by performing tedious calculations, but the controller is not robust when disturbances are imparted to the plant or if the reference model changes.

5.2 Future Research Recommendations

The work presented in this thesis merely demonstrates the advantages and disadvantages of the MRAC laws as applied to a flexible manipulator. Many interesting issues remain as future research topics. A few are listed below:

1. The "almost" positive realness condition is a very restricting condition that precludes MRAC of many interesting systems. Is a modification to the MRAC law possible such that positive realness can be omitted entirely?
2. Simulation studies have been made that include only the first three modes of the flexible manipulator system. A useful study would be to test MRAC on an experimental manipulator with a reaction wheel.
3. MRAC is designed to adjust gain parameters *on-line* in the event a disturbance or change to the system occurs. Successful demonstration of the ability of MRAC to handle a change in the tip mass would be extremely valuable for robotic applications requiring the retrieval of a payload.
4. MRAC may be possible if the controller parameters are chosen for the two sensor/one actuator system (case 3) such that the convergence of e_{y_2} slows down to allow more adaptation for the control of e_{y_1} .
5. It may be possible to add more sensors to the setup in case 3 such that the

nonminimum phase system becomes minimum phase. If this is true then the chances of MRAC working increase significantly. In [SCH-1] regulation results were improved by adding a hub-rate sensor *and* a strain gauge.

6. In [WEN-1] a bound is determined for the magnitude of α , the weighting factor of position versus rate measurement, that will give an output feedback positive real system. For the model derived in this study α seems to behave in an inverse manner from the condition Wen states. This issue should be investigated.

REFERENCES

- [AND-1] Anderson, B.D.O., S. Vongpanitlerd, *Network Analysis and Synthesis: A Modern Systems Theory Approach*. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1973.
- [AND-2] Anderson, B.D.O., "A System Theory Criterion for Positive Real Matrices," *Journal SIAM Control*, vol.5, no. 2, 1967.
- [AND-3] Anderson, B.D.O., "A Simplified Viewpoint of Hyperstability," *IEEE Transactions on Automatic Control*, vol.AC-13, pp.292-294, 1968.
- [AST-1] Astrom, K.J., "Theory and Applications of Adaptive Control— A Survey," *Automatica*, vol.19, pp.471-486, 1983.
- [AST-2] Astrom, K.J., B. Wittenmark, "On Self-Tuning Regulators," *Automatica*, vol.9, pp.185-199, 1973.
- [BA-1] Balakrishnan, A.V., "A Mathematical Formulation of the SCOLE Control Problem: Part I," NASA Contractor Report 172581, NASA Langley Research Center, May 1985.
- [BAL-1] Balas, M.J., "Feedback Control of Flexible Systems," *IEEE Transactions on Automatic Control*, vol.AC-23, no. 4, August 1978.
- [BAL-2] Balas, M.J., "Some Critical Issues in Stable Finite-Dimensional Adaptive Control of Linear Distributed Parameter Systems," in *Proceedings 4th Yale Conference on Adaptive Control*, Yale, Conn., 1983.
- [BAL-3] Balas, M.J., H. Kaufman, J. Wen, "Stable Direct Adaptive Control of Linear Infinite Dimensional Systems Using a Command Generator Tracker Approach," presented at the Workshop on Identification and Control of Flexible Space Structures, San Diego, CA, June 1984.
- [BAR-1] Bar-Kana, I., "Direct Multivariable Model Reference Adaptive Control with Application to Large Structural Systems," Ph.D. Dissertation, Rensselaer Polytechnic Institute, Troy, NY, 1983.
- [BAR-2] Bar-Kana, I., H. Kaufman, M.J. Balas, "Model Reference Adaptive Control of Large Structural Systems," *Journal Guidance and Control*, vol.6, No. 2, pp.112-118, 1983.
- [BAU-1] Bauldry, R.D., J.A. Breakwell, G.J. Chambers, K.F. Johansen, N.C. Nguyen, and D.B. Schaechter, "A Hardware Demonstration of Control for a Flexible Offset-feed Antenna," *The Journal of the Astronautical Sciences*, vol.31, no. 3, July-September 1983.

REFERENCES (Continued)

- [BET-1] Bethoux, B., B. Courtiol, "A Hyperstable Discrete Model Reference Adaptive Control System," in *Proceedings 3rd IFAC Symposium on Sensitivity, Adaptivity, and Optimality*, Ischia, Italy, June 1973.
- [BIS-1] Bishop, R.E.D., D.C. Johnson, *The Mechanics of Vibration*. Cambridge, MA: At the University Press, 1960.
- [BOO-1] Book, W.J., "Modeling, Design and Control of Flexible Manipulator Arms," Ph.D. Dissertation, M.I.T., Cambridge, MA, April 1974.
- [BOO-2] Book, W.J., O. Maizza-Neto, and D.E. Whitney, "Feedback Control of Two Beam, Two Joint Systems with Distributed Flexibility," *ASME Journal of Dynamic Systems, Measurement and Control*, vol.97, no. 4, December 1975.
- [BRE-1] Breakwell, J.A., "Control of Flexible Spacecraft," Ph.D. Dissertation, Stanford University, May 1980.
- [BRE-2] Breakwell, J.A., G.J. Chambers, "The Toysat Structural Control Experiment," *The Journal of the Astronautical Sciences*, vol.32, no. 3, July-September 1983.
- [BRO-1] Broussard, J., M. O'Brien, "Feedforward Control to Track the Output of a Forced Model," in *17th IEEE Conference on Decision and Control*, Jan. 1979.
- [BUT-1] Butchart, R.L., B. Shackcloth, "Synthesis of Model Reference Adaptive Control Systems by Lyapunov's Second Method," in *Proceedings 1965 IFAC Symposium Adaptive Control*, Teddington, England, 1966.
- [CAN-1] Cannon, R.H., E. Schmitz, "Initial Experiments on the End-Point Control of a One Link Flexible Experimental," *The International Journal of Robotics Research*, vol.3, no. 3, Fall 1984.
- [CAN-2] Cannon, R.H., D.E. Rosenthal, "Experiments in Control of Flexible Structures with Noncolocated Sensors and Actuators," *Journal of Guidance and Control*, vol.7, no. 5, September-October, 1984.
- [CAN-3] Cannon, R.H., E. Schmitz, "Precise Control of Flexible Manipulators," in *Robotics Research*. Brady, M, R. Paul, ED. Cambridge, MA: MIT Press, 1984, chap. 9, pp. 841-861.
- [CHE-1] Chen, C.T., *Linear System Theory and Design*. New York, NY: CBS College Publishing, 1984.

REFERENCES (Continued)

- [CHO-1] Chodas, J.L., G.K. Man, "Design of the Galileo Scan Platform Control," *Journal of Guidance and Control*, vol.7, no. 4, July-August 1984.
- [DON-1] Donalson, D.D., C.T. Leondes, "A Model Reference Parameter Tracking Technique for Adaptive Control Systems," *IEEE Transactions Appl. Ind.*, vol.241-262, Sept. 1963.
- [EDM-1] Edmunds, R.S., "Robust Control System Design Techniques for Large Flexible Space Structures Having Non-Colocated Sensors and Actuators," Ph.D. Dissertation, University of California at Los Angeles, 1982.
- [EGA-1] Egardt, B., "Stability Analysis of Continuous-time Adaptive Control Systems," *SIAM Journal Control and Optimization*, vol.18, no. 5, pp.540-558, 1980.
- [ERZ-1] Erzberger, "On the Use of Algebraic Methods in the Analysis and Design of Model Following Control Systems," NASA TN D-4663, 1963.
- [FLE-1] Fleischer, G.E., P.W. Likins, "Attitude Dynamics Simulation Subroutines for Systems of Hinge-Connected Rigid Bodies," Technical Report 32-1592, Jet Propulsion Laboratory, November 1, 1972.
- [FLE-2] Fleischer, G.E., P.W. Likins, "Attitude Dynamics Simulation Subroutines for Systems of Hinge-Connected Rigid Bodies with Nonrigid Appendages," Technical Report 32-1598, Jet Propulsion Laboratory, August 15, 1975.
- [GIB-1] Gibson, J.S., private communications, March-September 1985.
- [GIL-1] Gilbert, J.W., R.V. Monopoli, "A Modified Lyapunov Design for Model Reference Adaptive Control Systems," presented at the Conference on Circuit and System Theory, October 1969.
- [GOO-1] Goodwin, G.C., P.J. Ramadge, and P.E. Caines, "Discrete Time Multivariable Adaptive Control," *IEEE Transactions on Automatic Control*, vol.AC-25, no. 3, pp.449-456, June 1980.
- [GRA-1] Gran, R., G. Rodriguez, editor, "Control of Flexible Structures: A Systematic Overview of the Problems," in *Proceedings of the Workshop on Applications of Distributed System Theory to the Control of Large Space Structures*, JPL Publication 83-46, NASA, July 1983.

REFERENCES (Continued)

- [GUP-1] Gupta, N.K., M.G. Lyons, J.N. Aubrun, and G. Margulies, "Modeling, Control and System Identification Methods for Flexible Structures," *Spacecraft Pointing and Position Control*, AGARG-AG-260, 1981.
- [HOR-1] Horowitz, I., M. Sidi, "Optimum Synthesis of Non-Minimum Phase Feedback Systems with Plant Uncertainty," *International Journal Control*, vol.27, no. 3, pp.361-386, 1978.
- [IH-1] Ih, C.H.C., "A Direct Model Reference Adaptive Approach to the Control of Space Stations," Ph.D. Dissertation, University of California at Los Angeles, 1985.
- [IOA-1] Ioannou, P.A., P.V. Kokotovic, *Adaptive Systems with Reduced Models*. New York, NY: Springer-Verlag, 1983.
- [IOA-2] Ioannou, P.A., P.V. Kokotovic, "Instability Analysis and Improvement of Robustness of Adaptive Control," *Automatica*, vol.20, no. 5, pp.583-594, 1984.
- [IOA-3] Ioannou, P.A., P.V. Kokotovic, "Robust Redesign of Adaptive Control," *IEEE Transactions on Automatic Control*, vol.AC-29, 1984.
- [ION-1] Ionescu, T., R. Monopoli, "Discrete Model Reference Adaptive Control with an Augmented Error Signal," *Automatica*, vol.13, pp.507-517, 1977.
- [JAC-1] Jacobsen, L.S., R.S. Ayre, *Engineering Vibrations*. New York, NY: McGraw-Hill Book Company, Inc., 1958.
- [JOH-1] Johnson, C.R., Jr., M.J. Balas, "Reduced-order Adaptive Controller Studies," in *Proceedings Joint Automatic Control Conference*, San Francisco, CA, 1980.
- [KAI-1] Kailath, T., *Linear Systems*. Englewood, NJ: Prentice-Hall, 1980.
- [KAL-1] Kalman, R.E., "Design of Self-Optimizing Control Systems," *Transactions ASME*, vol.80, pp.468-478, 1958.
- [KAL-2] Kalman, R.E., "Lyapunov Functions for the Problem of Lur'e in Automatic Control," *Proceedings N.A.S.*, vol.49, pp.201-205, 1963.
- [KAN-1] Kane, T.R., D.A. Levinson, *Dynamics: Theory and Applications*. New York, NY: McGraw-Hill Book Co., 1985.

REFERENCES (Continued)

- [KNU-1] Knuth, D.E., *The T_EXbook*. Reading, MA: Addison-Wesley Publishing Co., 1984.
- [KOP-1] Kopf, E.H., T.K. Brown, E.L. Marsh, "Flexible Stator Control on the Galileo Spacecraft," presented at the AAS/AIAA Astrodynamics Specialist Conference, Provincetown, MA, June 25-27, 1979.
- [KUO-1] Kuo, B.C., *Automatic Control Systems*. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1982.
- [LAN-1] Landau, I.D., "Synthesis of Discrete Model Reference Adaptive Control," *IEEE Transactions on Automatic Control*, vol.AC-16, pp.507-508, 1971.
- [LAN-2] Landau, I.D., H.M. Silveira, "A Stability Theorem with Applications to Adaptive Control System," in *Proceedings 3rd IFAC Symposium on Sensitivity, Adaptivity, and Optimality*, Ischia, Italy, June 1973.
- [LAN-3] Landau, I.D., "A Survey of Model Reference Adaptive Techniques Theory and Applications," *Automatica*, vol.10, pp.353-379, 1974.
- [LAS-1] LaSalle, J.P., "Some Extensions of Lyapunov's Second Method," *IRE Transactions on Circ. Theory*, vol.520-527, December 1960.
- [LIE-1] Liégois, A., E. Dombre, and P. Borrel, "Learning and Control for a Compliant Computer-Controlled Manipulator," *IEEE Transactions on Automatic Control*, vol.AC-25, no. 6, December 1980.
- [LI-1] Likhtarnikov, A., V. Yakubovich, "The Frequency Theorem for Equations of Evolution Type," *Sibirskii Matematicheskii Zhurnal*, vol.17, pp.1069-1085, 1976.
- [LIK-1] Likins, P.W., "Dynamics and Control of Flexible Space Vehicles," Technical Report 32-1329, Rev. 1, Jet Propulsion Laboratory, Pasadena, CA, January 15, 1970.
- [LIK-2] Likins, P.W., "The New Generation of Dynamic Interaction Problems," *The Journal of Astronautical Sciences*, vol.27, no. 2, pp.103-113, April-June, 1979.
- [LIK-3] Likins, P.W., G.E. Fleischer, "Large-Deformation Modal Coordinates for Nonrigid Vehicle Dynamics," Technical Report 32-1565, Jet Propulsion Laboratory, Pasadena, CA, November 1, 1972.

REFERENCES (Continued)

- [LIK-4] Likins, P.W., "Analytical Dynamics and Nonrigid Spacecraft Simulation," Technical Report 32-1593, Jet Propulsion Laboratory, Pasadena, CA, July 15, 1974.
- [LIN-3] Lin, J.G., H.Y.H. Lin, D.R. Hegg, J.L. Johnson, and J.E. Keat, "Actively Controlled Structures Theory: Theory of Design Methods," Interim Technical Report, Vol. I, R-1249, Charles Stark Draper Laboratory, April 1979.
- [LJU-1] Ljung, L., "On Positive Real Transfer Functions and the Convergence of Some Recursive Schemes," *IEEE Transactions on Automatic Control*, vol.AC-22, no. 4, August 1977.
- [LYO-1] Lyons, M.G., and others, "Across Five (Active Control of Large Space Structures) Phase 1-A," Final Report No. RADC-TR-82-21, Lockheed Missiles and Space Company Inc., Palo Alto, California.
- [MAC-1] Macala, G.A., "Tuned Feedback Damping with Application to the Galileo Spacecraft," presented at the AAS/AIAA Astrodynamics Specialist Conference, Lake Tahoe, NE, August 3-5, 1981.
- [MEI-1] Meirovitch, L., *Analytical Methods in Vibration*. New York, NY: The MacMillan Company, 1967.
- [MEL-1] Meldrum, D., M.J. Balas, "Direct Adaptive Control of a Flexible Remote Manipulator Arm," presented at the ASME Annual Winter Conference, Miami Beach, Florida, November 1985.
- [MEL-2] Meldrum, D., M.J. Balas, "Application of Model Reference Adaptive Control to a Flexible Remote Manipulator Arm," Submitted for Publication.
- [MEL-3] Meldrum, D., "End-Point Adaptive Control of a Flexible Manipulator Arm with a Reaction Wheel and Visual Sensor," Submitted for Publication.
- [MO-1] Monopoli, R.V., "Lyapunov's Method for Adaptive Control System Design," *IEEE Transactions on Automatic Control*, vol.AC-12, pp.334-335, August 1967.
- [MO-2] Monopoli, R.V., J.W. Gilbert, and W.D. Thayer, "Model Reference Adaptive Control Based on Lyapunov-Like Techniques," in *Proceedings of the Second IFAC Symposium on System Sensitivity and Adaptivity*, August 1968.

REFERENCES (Continued)

- [MO-3] Monopoli, R.V., "Model Reference Adaptive Control with an Augmented Error Signal," *IEEE Transactions on Automatic Control*, vol.AC-19, no. 5, pp.474-484, 1974.
- [MO-4] Monopoli, R.V., "The Kalman-Yakubovich Lemma in Adaptive Control System Design," *IEEE Transactions on Automatic Control*, vol.AC-18, no. 5, pp.527-529, 1973.
- [MON-1] Montgomery, R.C., G.C. Horner, and S.R. Cole, "Experimental Research on Structural Dynamics and Control," presented at the 3rd VPI and SU/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft, Blacksburg, VA, June 1981.
- [MOR-1] Morse, A.S., A. Feuer, "Adaptive Control of Single-Input Single-Output Linear Systems," *IEEE Transactions on Automatic Control*, vol.AC-23, pp.557-570, August 1978.
- [MOR-2] Morse, A.S., "Global Stability of Parameter Adaptive Control Systems," *IEEE Transactions on Automatic Control*, vol.AC-25, no. 3, pp.433-439, 1980.
- [MOR-3] Morse, A.S., "New Directions in Parameter Adaptive Control," in *Proceedings of the 23rd IEEE Conference on Decision and Control*, Las Vegas, NE, December 12-14, 1984.
- [NAR-1] Narendra, K.S., P. Kudva, "Stable Adaptive Schemes for System Identification and Control - Parts I and II," *IEEE Transactions on Systems, Man, Cybernetics*, vol.SMC-4, no. 6, pp.542-560, 1974.
- [NAR-2] Narendra, K.S., L.S. Valavani, "Stable Adaptive Controller Design - Direct Control," *IEEE Transactions on Automatic Control*, vol.AC-23, no. 4, pp.570-583, 1978.
- [NAR-3] Narendra, K.S., Y.H. Lin, L.S. Valavani, "Stable Adaptive Controller Design - Part II: Proof of Stability," *IEEE Transactions on Automatic Control*, vol.AC-25, no. 3, pp.440-448, June 1980.
- [NAR-4] Narendra, K.S., J.H. Taylor, *Frequency Domain Criteria for Absolute Stability*. New York, NY: Academic Press, Inc., 1973.
- [NEW-1] Newton, I., *Philosophiae Naturalis Principia Mathematica*. Royal Academy of Sciences: London, England, 1686.

REFERENCES (Continued)

- [NGU-1] Nguyen, P.K., R. Ravindran, R. Carr, and D.M. Gossain, "Structural Flexibility of the Shuttle Remote Manipulator System Mechanical Arm," in *Proceedings of the Guidance and Control Conference*, AIAA paper no. 82-1536, August 1982.
- [ORT-1] Ortega, R., "Assessment of Stability Robustness for Adaptive Controllers," *IEEE Transactions on Automatic Control*, vol.AC-28, no. 12, pp.1106-1109, 1983.
- [PAR-1] Parks, P.C., "Lyapunov Redesign of Model Reference Adaptive Control Systems," *IEEE Transactions on Automatic Control*, vol.AC-11, no. 3, pp.362-367, 1966.
- [PCM-1] PC-MATLAB, a Computer-Aided Design Package, *PC-MATLAB User's Guide, Version 1.0*. Portola Valley, CA: The Mathworks, Inc., January 1985.
- [PIL-1] Pilkington, W.C., "Vehicle Motions as Inferred from Radio Signal Strength Records," Publication No. 551, Jet Propulsion Laboratory, Pasadena, CA, September 5, 1958.
- [POP-1] Popov, V.M., "The Solution of a New Stability Problem for Controlled Systems," *Automation and Remote Control*, vol.24, pp.1-23, 1963.
- [POP-2] Popov, V.M., *Hyperstability of Control Systems*. New York, NY: Springer-Verlag, 1973.
- [POR-1] Porter, B., M.L. Tatnall, "Stability Analysis of a Class of Multivariable Model Reference Adaptive Systems Having Time Varying Process Parameters," *International Journal of Control*, vol.11, no. 2, pp.325-332, 1970.
- [RAV-1] Ravindran, R., K.H. Doetsch, "Design Aspects of the Shuttle Remote Manipulator Control," in *Proceedings of the Guidance and Control Conference*, AIAA paper No. 82-1581, August 1982.
- [ROH-1] Rohrs, C.E., L.S. Valavani, M. Athans, G. Stein, "Analytic Verification of Undesirable Properties of Direct Model Reference Adaptive Control Algorithms," in *Proceedings 20th IEEE Conference on Decision and Control*, San Diego, CA, 1981.
- [SC-1] Schaechter, D.B., "Hardware Demonstration of Flexible Beam Control," *Journal of Guidance and Control*, vol.15, no. 1, January-February 1982.

REFERENCES (Continued)

- [SC-2] Schaechter, D.B., D.B. Eldred, "Experimental Demonstration of the Control of Flexible Structures," *Journal of Guidance and Control*, vol.7, no. 5, September-October 1984.
- [SCH-1] Schmitz, E., "Experiments on the End-Point Position Control of a Very Flexible One-Link Manipulator," Ph.D. Dissertation, Stanford University, Stanford, CA, 1985.
- [SLA-1] Slafer, L.I., "On-Orbit Evaluation of the Control System/Structural Mode Interactions on OSO-8," *Journal of Guidance and Control*, vol.3, no. 3, May-June 1980.
- [SOB-1] Sobel, K., H. Kaufman, L. Mabus, "Model Reference Output Adaptive Control Systems without Parameter Identification," presented at the 18th *IEEE Conference on Decision and Control*, Ft. Lauderdale, FL, December 1979.
- [SOB-2] Sobel, K., "Model Reference Adaptive Control for Multi-Input Multi-Output Systems," Ph.D. Dissertation, Rensselaer Polytechnic Institute, Troy, NY, June 1980.
- [SOB-3] Sobel, K., H. Kaufman, L. Mabus, "Implicit Adaptive Control Systems for a Class of Multi-Input Multi-Output Systems," *IEEE Transactions on Aerospace and Electronics Systems*, vol.576-590, September 1982.
- [SPA-1] Spanos, J.T., "SP-100 Finite Element Modelling and Preliminary Control Studies," EM-343-898, Jet Propulsion Laboratory, Pasadena, CA, November 1985.
- [STR-1] Strang, G., G.J. Fix, *An Analysis of the Finite Element Method*. Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1973.
- [SUZ-1] Suzuki, T., S. Takashima, "A Hyperstable Scheme for Discrete Model Reference Adaptive Control Systems," *International Journal of Control*, vol.28, pp.245-252, 1978.
- [TAY-1] Taylor, L., compiler, "," in *Proceedings of the SCOLE Workshop*, NASA Langley Research Center, Hampton, Virginia, December 1984.
- [TIM-1] Timoshenko, S., D.H. Young, W. Weaver, Jr., *Vibration Problems in Engineering*. New York, NY: John Wiley and Sons, 1974.

REFERENCES (Continued)

- [TRU-1] Truckenbrodt, A., "Modelling and Control of Flexible Manipulator Structures," in *Proceedings of 4th CISM-IFTOMM Symposium on Theory and Practice of Robots and Manipulators*, Warsaw, 1981.
- [USO-1] Usoro, P.B., R. Nadira, S.S. Mahil, "Control of Lightweight Flexible Manipulators: a Feasibility Study," in *Proceedings of the American Control Conference*, San Diego, CA, June 1984.
- [WAN-1] Wang, S.J., C.H.C. Ih, Y.H. Lin, E. Mettler, "Space Station Dynamic Modelling, Disturbance Accommodation and Adaptive Control," presented at the Workshop on Identification and Control of Flexible Space Structures, San Diego, CA, June 4-6, 1984.
- [WEN-1] Wen, J., "Direct Adaptive Control in Hilbert Space," Ph.D. Dissertation, Rensselaer Polytechnic Institute, Troy, NY, 1985.
- [WEN-2] Wen, J., M.J. Balas, "Direct Adaptive Control in Hilbert Space," Robotics and Automation Laboratory Report, Rensselaer Polytechnic Institute, Troy, NY, 1985.
- [WEN-3] Wen, J., M.J. Balas, "New Method of Adaptive Regulation and MRAC," Robotics and Automation Laboratory Report, Rensselaer Polytechnic Institute, Troy, NY, 1985.
- [WHI-1] Whitaker, H.P., P.V. Osburn, A. Keezer, "New Developments in the Design of Adaptive Control Systems," Institute of Aeronautical Sciences, Paper 61-39, 1961.
- [WIN-1] Winsor, C.A., R.J. Roy, "Design of Model Reference Adaptive Control Systems by Lyapunov's Second Method," *IEEE Transactions on Automatic Control*, vol.AC-13, no. 2, pp.204, April 1968.
- [WIN-2] Winsor, C.A., "Model Reference Adaptive Design," NASA-CR-98453, Nov. 1968.
- [YOC-1] Yocum, J.F., "Control System in the Presence of Severe Structural Dynamics Interactions," *Journal of Guidance and Control*, vol.1, no. 2, March-April 1978.

APPENDIX A

**EXAMPLE ON HOW TO DERIVE CONTROLLER GAINS, S,
VIA THE CGT CONDITION**

Transform the plant matrices (4.2), (4.3), (4.4) to controller canonical form [Kuo-1]:

$$\text{Given } \frac{dz^*(t)}{dt} = Az^*(t) + BG^*(t)$$

If (A, B) is controllable, \exists a nonsingular transform

$$y^*(t) = Qz^*(t)$$

or

$$z^*(t) = Q^{-1}y^*(t)$$

s.t.,

$$\frac{dy^*(t)}{dt} = A_1y^*(t) + B_1T^*(t)$$

A_1 is determined by defining the following:

$$Q = \begin{bmatrix} Q_1 \\ Q_1A \\ \vdots \\ Q_1A^{n-1} \end{bmatrix}$$

where

$$Q_1 = [0 \ 0 \ \dots \ 1][B \ AB \ \dots \ A^{n-1}B]^{-1}$$

Then

$$A_1 = QAQ^{-1}$$

$$B_1 = QB$$

For this example:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -2.5972e^5 & -271.0091 & -1808.7 & -.5318 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Perform pole-placement design with state feedback. The desired poles are chosen to include the same eigenvalues as in the reference model plant, A_m . The remaining poles may be placed anywhere in the left half plane. For this example the desired poles are chosen as $+5i, -5i, -4, -3, -2, -1$.

To determine the feedback gain matrix, G_1 , the following method is used: Let

$$G_1 = [g_1 \ g_2 \ g_3 \ g_4 \ g_5 \ g_6]$$

Then

$$A_1 - B_1 G_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -g_1 & -g_2 & -2.6e^5 - g_3 & -271. - g_4 & -1808.7 - g_5 & -.53 - g_6 \end{bmatrix}$$

The elements of the G_1 matrix are determined by equating coefficients of the $(A_1 - B_1 G_1)$ characteristic equation, (c.e.), with the desired characteristic equation, (d.c.e.):

$$\begin{aligned} \text{c.e.} &= \lambda^6 + (.5318 + g_6)\lambda^5 + (1808.7 + g_5)\lambda^4 \\ &+ (271.0091 + g_4)\lambda^3 + (2.5972e^5 + g_3)\lambda^2 + g_2\lambda + g_1 \end{aligned}$$

$$\text{d.c.e.} = \lambda^6 + 10\lambda^5 + 60\lambda^4 + 300\lambda^3 + 899\lambda^2 + 1250\lambda + 600$$

After equating coefficients, the G_1 matrix becomes

$$G_1 = [600 \quad 1250 \quad -258821 \quad 28.99 \quad -1748.7 \quad 9.4682]$$

G_1 is the feedback gain matrix to give $A_1 - B_1G_1$ the desired eigenvalues. To give the original system, $A + BG$, the same desired eigenvalues, take the negative of G_1 and multiply by the transformation matrix Q :

$$G = [G_1][Q] = [-6.3027e^{-5} \quad -1.3124e^{-4} \quad -.0696 \quad .0053 \quad 5.8703 \quad -.0343] \quad (\text{A.1})$$

The closed-loop system, $A_c = A + BG$, is now

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -.0023 & -.0048 & -2.5516 & .1953 & 215.1727 & -1.259 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -.0099 & -.0206 & -168.2168 & .7124 & 922.7825 & -5.3991 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -.0189 & -.0394 & -20.8776 & 1.5984 & 109.2232 & -10.7075 \end{bmatrix}$$

where the control $T^*(t)$ is $Gz^*(t)$.

Diagonalize the system, A_c , by the following similarity transformation:

$$\Lambda = P^{-1}A_cP$$

$$\Gamma = P^{-1}B$$

$$\beta = CP$$

where

$$P = \begin{bmatrix} -.0001 - .2i & -.0001 + .2i & -.2458 & 1 & -.3519 & -.478 \\ 1 & 1 & 1 & -.9927 & 1 & 1 \\ .0006 + .1621i & .0006 - .1621i & -.1007 & .0267 & -.0739 & -.0556 \\ -.8103 + .0027i & -.8103 - .0027i & .4095 & -.0265 & .2099 & .1163 \\ .0252i & -.0252i & -.02 & .0049 & -.014 & -.0103 \\ -.1257 & -.1257 & .0813 & -.0048 & .0398 & .0216 \end{bmatrix} \quad (\text{A.2})$$

Then

$$\Lambda = \text{diag} [5i \quad -5i \quad -4 \quad -1 \quad -3 \quad -2]$$

$$\Gamma = \begin{bmatrix} -639.52 + 3651.1i \\ -639.52 - 3651.1i \\ 38853 \\ 58980 \\ 47663 \\ 4028 \end{bmatrix}$$

$$\beta = [.892 - .3608i \quad .892 + .3608i \quad -.0818 \quad .7947 \quad -.1913 \quad -.3153] \quad (\text{A.3})$$

The diagonalized system is

$$\frac{d\xi(t)}{dt} = \Lambda \xi(t) + \Gamma T(t)$$

$$y^*(t) = \beta \xi(t)$$

Solving these equations, $\xi_i(t)$ become:

$$\xi_1(t) = e^{5it} \xi_1(0)$$

$$\xi_2(t) = e^{-5it} \xi_2(0)$$

$$\xi_3(t) = e^{-4t} \xi_3(0)$$

$$\xi_4(t) = e^{-t} \xi_4(0)$$

$$\xi_5(t) = e^{-3t} \xi_5(0)$$

$$\xi_6(t) = e^{-2t} \xi_6(0)$$

The desired output based on the reference model (4.8), (4.9) is:

$$\begin{aligned} y^*(t) &= \sin 5t + 2.5 \cos 5t \\ &= \frac{e^{5it} - e^{-5it}}{2i} + 2.5 \left(\frac{e^{5it} + e^{-5it}}{2} \right) \end{aligned} \quad (\text{A.4})$$

Since the only eigenvalues that must be retained to obtain the desired output are $5i$ and $-5i$, the states ξ_3 , ξ_4 , ξ_5 , and ξ_6 may be set to zero by choosing

$$\xi_3(0) = \xi_4(0) = \xi_5(0) = \xi_6(0) = 0$$

From (A.3)

$$y^*(t) = (.892 - .3608i)\xi_{10}e^{5it} + (.892 + .3608i)\xi_{20}e^{-5it} \quad (\text{A.5})$$

Equate coefficients of (A.5) with (A.4) and solve for ξ_{10} and ξ_{20} to yield:

$$\xi_{10} = 1.386705 + .0036197i$$

$$\xi_{20} = 1.386705 - .0036197i$$

The ideal trajectory is now

$$z^*(t) = P\xi(t)$$

Hence,

$$z_1(t) = (-.0001 - .2i)\xi_1(t) + (-.0001 + .2i)\xi_2(t)$$

$$z_2(t) = \xi_1(t) + \xi_2(t)$$

$$z_3(t) = (.0006 + .1621i)\xi_1(t) + (.0006 - .1621i)\xi_2(t)$$

$$z_4(t) = (-.8103 + .0027i)\xi_1(t) + (-.8103 - .0027i)\xi_2(t)$$

$$z_5(t) = .0252\xi_1(t) - .0252\xi_2(t)$$

$$z_6(t) = -.1257\xi_1(t) - .1257\xi_2(t)$$

Substitute the appropriate values for $\xi_1(t)$ and $\xi_2(t)$ (including the initial conditions) and combine in terms of cosine and sine to obtain:

$$z_1(t) = .554689 \sin 5t - .0013255 \cos 5t$$

$$z_2(t) = -.0072394 \sin 5t + 2.77341 \cos 5t$$

$$z_3(t) = -.449574 \sin 5t + .00049054 \cos 5t$$

$$z_4(t) = -.0016222 \sin 5t - 2.2473136 \cos 5t$$

$$z_5(t) = -.06989 \sin 5t - .0001824 \cos 5t$$

$$z_6(t) = .00091 \sin 5t - .3486176 \cos 5t$$

Control $T^*(t)$ becomes:

$$\begin{aligned} T^*(t) &= -6.3027e^{-5} z_1(t) - 1.3124e^{-4} z_2(t) - .0696 z_3(t) \\ &\quad + .0053 z_4(t) + 5.8703 z_5(t) - .0343 z_6(t) \\ &= -.37905874 \sin 5t - .00142196 \cos 5t \end{aligned}$$

The CGT condition is satisfied by

$$\begin{bmatrix} z_1^*(t) \\ z_2^*(t) \\ z_3^*(t) \\ z_4^*(t) \\ z_5^*(t) \\ z_6^*(t) \\ T^*(t) \end{bmatrix} = \begin{bmatrix} .554689 & -.0002651 & 0 \\ -.0072394 & .554682 & 0 \\ -.449574 & .0000981 & 0 \\ -.0016222 & -.449462 & 0 \\ -.06989 & -.00003648 & 0 \\ .00091 & -.069723 & 0 \\ -.37905874 & -.000284392 & 0 \end{bmatrix} \begin{bmatrix} \sin 5t & 5 \cos 5t & 0 \end{bmatrix}$$

where

$$S_{21} = [-.37905874 \quad -.000284392] \quad (\text{A.6})$$

$$S_{22} = [0] \quad (\text{A.7})$$

APPENDIX B

MRAC ALGORITHM FOR SINGLE-INPUT DOUBLE-OUTPUT SYSTEM

An MRAC algorithm is derived for a case with one actuator and two sensors. System 1, denoted as $(A, B, C_1, 0)$, satisfies the CGT condition and system 2, $(A, B, C_2, 0)$, satisfies both the CGT and positive realness conditions. By substituting a signal control law similar to (3.26)

$$T(t) = G(t)e_y(t) + S(t)w(t) - h(y_2(t) - y_{m_2}(t)) \quad (\text{B.1})$$

into the error equation (3.14)

$$\frac{de(t)}{dt} = Ae(t) + BT(t) - BT^*(t)$$

the following closed-loop dynamic equation is obtained:

$$\begin{aligned} \frac{de(t)}{dt} = & (A + BGC)e(t) + B\Delta G_1(t)e_{y_1}(t) \\ & + B\Delta G_2(t)e_{y_2}(t) - he_{y_2}(t) \end{aligned} \quad (\text{B.2})$$

$$e_y(t) = \begin{bmatrix} y_1(t) - y_{m_1}(t) \\ y_2(t) - y_{m_2}(t) \end{bmatrix} = Ce(t) \quad (\text{B.3})$$

Choose the quadratic Lyapunov function candidate as in [WEN-1]:

$$\begin{aligned} V(e, \Delta G, L) = & e^T P e + \text{tr} [\Delta G_1 \Gamma_1^{-1} \Delta G_1^T] \\ & + \text{tr} [\Delta G_2 \Gamma_2^{-1} \Delta G_2^T] + \text{tr} [L \Gamma_2^{-1} L^T] \end{aligned} \quad (\text{B.4})$$

where $P > 0$ is from (3.17) with A replaced by $(A + BGC) = A_c$. Take the time

derivative along the solution (B.2):

$$\begin{aligned}
\frac{dV(e, \Delta G, L)}{dt} &= e^T (A_c^T P + P A_c) e + 2e^T P B \Delta G_1 e_{y_1} \\
&\quad + 2e^T P B \Delta G_2 e_{y_2} + 2e^T P B L w \\
&\quad + 2 \operatorname{tr} \left[\frac{d\Delta G_1(t)}{dt} \Gamma_1^{-1} \Delta G_1^T \right] + 2 \operatorname{tr} \left[\frac{d\Delta G_2(t)}{dt} \Gamma_1^{-1} \Delta G_2^T \right] \\
&\quad + 2 \operatorname{tr} \left[\frac{dL(t)}{dt} \Gamma_2^{-1} L^T \right] - 2e^T P B h e_{y_2}
\end{aligned} \tag{B.5}$$

Since positive realness is satisfied for the system $(A_c, B, C_2, 0)$ then

$$e^T P B = e_{y_2}$$

For the adaptive law chosen as follows:

$$\frac{dG_1(t)}{dt} = \frac{d\Delta G_1(t)}{dt} = -\Gamma_1 e_{y_2}(t) e_{y_1}^T(t) \tag{B.6}$$

$$\frac{dG_2(t)}{dt} = \frac{d\Delta G_2(t)}{dt} = -\Gamma_2 e_{y_2}(t) e_{y_2}^T(t) \tag{B.7}$$

$$\frac{dS(t)}{dt} = \frac{dL(t)}{dt} = -\Gamma_2 e_{y_2}(t) w^T(t) \tag{B.8}$$

Equation (B.5) then becomes

$$\frac{dV(e, \Delta G, L)}{dt} \leq -\|Qe\|^2 - \epsilon \|e\|^2 - h \|e_{y_2}\|^2 \tag{B.9}$$

If $\epsilon > 0$, strict positive realness is satisfied and $\|e\| \rightarrow 0$. If only positive realness is satisfied, $\epsilon = 0$ and $\|e_{y_1}\|$ is bounded while $\|e_{y_2}\| \rightarrow 0$.

For a case that requires only regulation, no model following, then (B.6) may be used with (3.26).

1. Report No. JPL Publication 85-100	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle Direct Model Reference Adaptive Control of a Flexible Robotic Manipulator		5. Report Date December 15, 1985	6. Performing Organization Code
7. Author(s) Deirdre Meldrum		8. Performing Organization Report No.	
9. Performing Organization Name and Address JET PROPULSION LABORATORY California Institute of Technology 4800 Oak Grove Drive Pasadena, California 91109		10. Work Unit No.	11. Contract or Grant No. NAS7-918
12. Sponsoring Agency Name and Address NATIONAL AERONAUTICS AND SPACE ADMINISTRATION Washington, D.C. 20546		13. Type of Report and Period Covered JPL Publication	
14. Sponsoring Agency Code BK-506-46-11-01-00		15. Supplementary Notes	
<p>16. Abstract</p> <p>Quick, precise control of a flexible manipulator in a space environment is essential for future Space Station repair and satellite servicing. Numerous control algorithms have proven successful in controlling rigid manipulators with colocated sensors and actuators; however, few have been tested on a flexible manipulator with non-colocated sensors and actuators. In this thesis, a model reference adaptive control (MRAC) scheme based on command generator tracker theory is designed for a flexible manipulator. Quicker, more precise tracking results are expected over nonadaptive control laws for this MRAC approach.</p> <p>Equations of motion in modal coordinates are derived for a single-link, flexible manipulator with an actuator at the pinned-end and a sensor at the free end. An MRAC is designed with the objective of controlling the torquing actuator so that the tip position follows a trajectory that is prescribed by the reference model. An appealing feature of this direct MRAC law is that it allows the reference model to have fewer states than the plant itself. Direct adaptive control also adjusts the controller parameters directly with knowledge of only the plant output and input signals. No a priori knowledge of the plant is necessary.</p>			
17. Key Words (Selected by Author(s)) Aircraft Stability and Control Spacecraft Communications, Command, and Tracking Electronics and Electrical Engineering Mechanical Engineering		18. Distribution Statement Unclassified - Unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 89	22. Price