# Physical Optics Analysis of a Four-Reflector Antenna Part 1 

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#### Abstract

Concern has been raised for the 64-m to 70-m antenna upgrade project that the 70-m system may experience greater S-band beam-pointing perturbations than the $64-\mathrm{m}$ system. The $S$-band perturbations are due to minor (higher order) mode generation, causing subtle cross-polarization fields affecting beam pointing direction, as described herem. For the antennas in their present configuration ( 64 m ), a slight S-band gain degradation of about 0.05 dB can be attributed to these effects. Therefore, a full physical optics analysis was performed for the present-day 64-m system, as described herein The results were compared with past analyses and experimental observations in order to verify the algebra and computer code with the intent of derving a vald analysis method for accurately analyzing the $70-\mathrm{m}$ shaped dual-reflector Cassegramian antenna. The results of the new analysis appear to be in excellent agreement with previous analyses and experimental data, and extension of the analysis methods to the $70-\mathrm{m}$ system will follow in a second article.


## I. Introduction

In support of the DSN $64-\mathrm{m}$ to $70-\mathrm{m}$ antenna upgrade project, several RF performance analyses were made based on a two-reflector, linearly polarized antenna system at X-band and S-band. However, the two-reflector analysis is valid only at X-band. At S-band, the antenna ( $64-\mathrm{m}$ or $70-\mathrm{m}$ ) is in reality a four-reflector system. The present-day DSN 64 -m antennas use conventional parabolodal and hyperboloidal Cassegramian reflectors, with the hyperboloidal subreflector modified in three major ways. First, the vertex is displaced from the main reflector centerline such that a line connects the main reflector focal point, the subreflector vertex, and the primary feedhorn phase center. Second, the subreflector rim is trimmed to describe a cone of $2 \Theta=120^{\circ}$, centered on the main reflector axis of symmetry. Lastly, a special spullover-reducing concal flange is used around the periphery of the offset and asymmetrically trummed hyperboloid. The overall antenna system
employs two additional S-band reflectors near the prumary feeds, one ellipsoidal and the other planar, to make up a fourreflector antenna at S-band. These reflectors, which enable simultaneous S - and X -band use of the overall antenna ( S -band uplink and downlink and X -band downlink), complicate the analyses because of the close proximity to geometric optics focal points (see Fig. 1).

The $70-\mathrm{m}$ Cassegramian subreflector is specially shaped for uniform amplitude illumination of the primary reflector and is slightly asymmetric in order to accommodate offset primary feeds. Two additional reflectors in the primary feed region are also used in the same manner mentioned above for the $64-\mathrm{m}$ antenna.

Nether reflector in the primary feed region can be analyzed using simple geometric optics or usual far-field ap-
proaches. Figure 2 shows the complex physics involved, i.e., the divergence and bunching of the RF energy beam as computed from a near-field analysis. Furthermore, the necessary physical asymmetries involved in these multi-function antenna instruments cause second-order (but important) perturbations in the final aperture beam directions when circular polarization is used. The beam-direction perturbation can be predicted only in a four-reflector antenna analysis that also takes into account near-field and circular polarization effects.

It was necessary to use some approximations in the previous analyses of the present-day system. For example, only a rough analysis of final aperture beam directions was made. Concern exists that the $70-\mathrm{m}$ system, which effectively illuminates the main reflector in a uniform way, may experience greater S-band perturbations or beam squints (i.e., non-coincident right circularly polarized/left circularly polarized beams) than the $64-\mathrm{m}$ system. Accordingly, it was decided to first perform a full analysis of the existing $64-\mathrm{m}$ S-band system, since previous experimental results are avalable for comparison (Ref.1).

If the present analysis effort is verified by comparison with previous experiments, the work will be extended to include the $70-\mathrm{m}$ shaped asymmetric subreflector and shaped symmetric main reflector. For a full analysis, the S-band horn fields will be carried through all four reflectors (resulting in a transmission viewpoint of final antenna system beams) to account fully for all near-field, cross-polarization, and higher order mode generation effects caused by various intentional asymmetries. This appears to be the first time such a complete and rigorous analysis has been performed on such a complex antenna system.

## II. Background

A comprehensive description of the $64-\mathrm{m}$ antenna $\mathrm{S} / \mathrm{X}-$ band reflex feed system was presented by Bathker in 1974 (Ref. 1). An analysis of the observed $64-\mathrm{m}$ antenna S -band beam position offset from antenna boresight was performed by the author in 1977 (Ref. 2). The analysis made use of a diffraction analysis (physical optics) of the feedhorn/ellıpsoid/ dichroic assembly performed by Potter (Ref.3). The analysis by Potter was combined with a geometric-optics analysis of the hyperbolord scattering, and with an aperture field integration computation of the antenna secondary field, to predict the final antenna beam direction. While this early approach worked very well in the $64-\mathrm{m}$ antenna case, the algebra and computer code developed were essentially limited to the hyperbolold/parabolord case, and could not be easily extended to the $70-\mathrm{m}$ dual-shaped reflector system.

The present analysis goes far beyond previous analyses in many ways. The analysis methods can be used for both hyperbolic/parabolic and dual-shaped reflectors. It is a true four-reflector, circular-polarization, near-field physical optics analysis. Figure 1 shows the four-reflector system, consisting of the S-band feedhorn the ellipsoid, the dichroic plate, the asymmetric subreflector, and the symmetric main reflector Figure 3 shows a simplfied block diagram of the analysis approach. Near-field diffraction patterns were computed using standard JPL spherical wave expansion (SWE) techniques deveoped by Ludwig. The far field of each subreflector is first found by use of conventional physical optics integrations. This is followed by a numerical expansion of the far field to obtain the spherical wave mode coefficients. Finally, near-field computations result from using these coefficients (Ref. 4).

## III. Results

The observed RF beam positions of the $64-\mathrm{m}$ antenna are shown in Fig. 4. There are two significant beam position offsets from the antenna boresight, as explained in Ref. 1 The $0.0114^{\circ}$ offset (Fig 4) of the X-band beam is due to the finite thickness of the dichroic plate, which causes a refraction effect as the X -band wave propagates through the dichroic plate. When this was found, the position of the dichroc plate was shimmed by 1 in . The shimming brought the Sand X -band beams together in the direction of center-tocenter line of the S- and X-band feedhorns, as seen in Fig. 4. The S-band right circularly polarized beam has another offset of $0.0086^{\circ}$ normal to this direction. This offset is attributed to the higher order ( $m \neq 1$ ) modes generated at the ellipsoid and is predicted to be observable only for circularly polarized waves. Alternately, this offset can be considered as a depolar1zation effect caused by the asymmetric ellipsoid geometry.

The new numerical results are $90 \%$ in agreement with experimental results observed in Fig. 4 Beam offset of a right circularly polarized wave from the antenna boresight was previously measured at $0.0086^{\circ}$ and computed to be $0.0095^{\circ}$ ( 0.061 beamwidth measured; 0.068 beamwidth computed). (It is expected that the left crrcularly polarized wave offset would be $0.0086^{\circ}$ in the opposite direction from the antenna boresight, although this was not measured.) Beam offset due to a $1-1 \mathrm{n}$. shim in the position of the dichroic was measured at $0.0114^{\circ}$ and computed to be $0.010^{\circ}$ ( 0.081 BW measured, 0.071 BW computed). Table 1 shows the computed beam offset caused by the $1-\mathrm{m}$. shim of the dichroic position. Table 2 shows the right circularly polarized beam offset from antenna boresight. Together, Tables 1 and 2 indicate that the S-band beam position will be offset by $0.010^{\circ}$ and $0.0095^{\circ}$ in the two mutually perpendicular directions.

In addition to good correlation with observed RF beam positions, the present analysis also yields a reflector gann computation that can be compared with previous efficiency computations. A gain of 62.66 dB , corresponding to an efficiency of $77.6 \%$, was computed, including spillover losses, nonuniform illumination and phase, higher order mode ( $m \neq 1$ ), and polarization losses. These losses were previously computed to be (Ref. 1)

$$
\begin{aligned}
\eta_{F S} & =-0.133 \mathrm{~dB} \\
\eta_{R S} & =-0.010 \mathrm{~dB} \\
\eta_{I} & =-0.84 \mathrm{~dB} \\
\eta_{P} & =-0.07 \mathrm{~dB} \\
\eta_{x} & =-0.005 \mathrm{~dB} \\
\eta_{m \neq 1} & =-0.089 \mathrm{~dB}
\end{aligned}
$$

The total losses computed previously (Ref. 1) were -1.147 dB ( $76.8 \%$ ), compared to present computed losses of -1.10 dB (77.6\%). An 1tem-by-item loss comparison is not possible because the present gain computation does not provide a breakdown of individual loss components.

The only small uncertanty of the analysis is that the hyperboloid flange was not included. It is not believed this would make any difference in the beam position prediction. However, efficiency (gain) computation is affected. It is likely that the difference in the two efficiency computations is mostly due to the fact that the subreflector flange was modeled in the earlier analysis. Since future DSN systems will not use such flanges, no effort was expended patching this part of the model into the new software. The difference is less likely due to numerical inaccuracies of either computation.

The new capabilities developed with this work include (1) software for analyzing the arbitrary wave polarization case (linear, cırcular, and elliptıc) at near-field distance (see Appendix A), and (2) a physical optics program for computing symmetric main reflector diffraction when illumınated by asymmetric feed radiation patterns in the form of spherical wave harmonics. These programs enhance physical optics analysis capability and will be useful in other analysis activities. They are applicable to ongoing ground station antenna research and development efforts such as beam waveguide feed systems, holographic data reduction, and verification of GTD analysis software. As stated, this appears to be the first time such a complete and rigorous analysis has been performed on such a complex antenna system. Future work will extend the present results to the planned DSN $70-\mathrm{m}$ dual-shaped reflector system with reflex-dichroic feed.

## Acknowledgment

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## References

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Table 1. Beam offset caused by 1 -in. shim of dichroic position*

| Theta | E Theta |  | E Phı |  | Axial <br> Ratio | Ellupse Tilt Angle | RCP Gain, dB |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Volts | Phase | Volts | Phase |  |  |  |
| 000000 | 148.981325 | -98451 | 148.897799 | 171.534 | 0.005 | 166.91661 | 62.600 |
| 000100 | 149.165831 | -98.443 | 149075554 | 171.540 | 0.006 | 16689899 | 62.610 |
| 0.00200 | 149.331051 | -98435 | 149234343 | 171.547 | 0006 | 166.85301 | 62.620 |
| 0.00300 | 149.476992 | -98426 | 149.374123 | 171.554 | 0007 | 16678256 | 62.628 |
| 0.00400 | 149603628 | -98418 | 149.494875 | 171.561 | 0007 | 16669550 | 62.635 |
| 0.00500 | 149710876 | -98.409 | 149.596622 | 171569 | 0.007 | 16657938 | 62641 |
| 000600 | 149.798769 | -98.400 | 149679274 | 171576 | 0008 | 16645171 | 62646 |
| 0.00700 | 149.867254 | -98.392 | 149742825 | 171584 | 0.008 | 166.30922 | 62.650 |
| 0.00800 | 149916344 | -98.382 | 149787300 | 171.592 | 0.008 | 16614984 | 62.653 |
| 0.00900 | 149.945999 | -98 373 | 149.812653 | 171.600 | 0009 | 165.97935 | 62.654 |
| 0.01000** | 149956234 | -98.363 | 149.818916 | 171.608 | 0.009 | 165.79265 | 62.655** |
| 001100 | 149947079 | -98354 | 149.806078 | 171.617 | 0.009 | 16559603 | 62.654 |
| 001200 | 149918497 | -98.344 | 149774174 | 171.625 | 0010 | 165.38231 | 62652 |
| 0.01300 | 149.870565 | -98.334 | 149.723173 | 171.634 | 0010 | 16515844 | 62.650 |
| 0.01400 | 149.803247 | -98321 | 149653156 | 171.646 | 0010 | 16492001 | 62.646 |
| 001500 | 149.716541 | -98.310 | 149564074 | 171.655 | 0010 | 16466658 | 62640 |
| 0.01600 | 149610556 | -98300 | 149456049 | 171.665 | 0010 | 164.39746 | 62634 |
| 0.01700 | 149.485386 | -98.288 | 149.329130 | 171.675 | 0011 | 16411704 | 62627 |
| 0.01800 | 149.340899 | -98.276 | 149183241 | 171.685 | 0011 | 163.81953 | 62.618 |
| 0.01900 | 149177284 | -98.265 | 149.018515 | 171.695 | 0.011 | 163.50879 | 62.609 |
| 0.02000 | 148.994564 | -98253 | 148.835079 | 171.706 | 0011 | 16317743 | 62598 |

*Plane $\phi=180^{\circ}$ (direction of $00114^{\circ}$ beam offset, see Fig 4).
**Beam position for peak gan.

Table 2. Otfset of right circularly polarized beam from antenna boresight*

| Theta | E Theta |  | E Phi |  | Axial <br> Ratio | Ellipse Tilt Angle | RCP Gain, dB |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Volts | Phase | Volts | Phase |  |  |  |
| 0.00000 | 148.897789 | 171534 | 148981325 | 81549 | 0.005 | 166.91653 | 62600 |
| 0.00100 | 149070080 | 171480 | 149.155140 | 81495 | 0.005 | 16819427 | 62610 |
| 000200 | 149223063 | 171427 | 149309898 | 81440 | 0005 | 16951804 | 62.619 |
| 0.00300 | 149356670 | 171375 | 149445599 | 81386 | 0.005 | 17087561 | 62.627 |
| 0.00400 | 149.470888 | 171323 | 149562214 | 81332 | 0006 | 172.24525 | 62633 |
| 000500 | 149565687 | 171271 | 149659662 | 81279 | 0006 | 17360888 | 62639 |
| 0.00600 | 149641045 | 171219 | 149737972 | 81.225 | 0006 | 17495147 | 62643 |
| 0.00700 | 149696924 | 171.167 | 149.797089 | 81.172 | 0.006 | 176.25542 | 62.647 |
| 0.00800 | 149733271 | 171116 | 149.836979 | 81.120 | 0006 | 17750557 | 62.649 |
| 000900 ** | 149750128 | 171065 | 149857647 | 81067 | 0006 | 178.69234 | 62650 ** |
| $001000^{* *}$ | 149747488 | 171015 | 149859093 | 81015 | 0.006 | 179.81087 | 62.650** |
| 0.01100 | 149725349 | 170964 | 149841316 | 80.963 | 0.007 | 085489 | 62.649 |
| 001200 | 149.683687 | 170914 | 149.804310 | 80911 | 0.007 | 182267 | 62.646 |
| 0.01300 | 149622532 | 170864 | 149748096 | 80860 | 0.007 | 271287 | 62643 |
| 0.01400 | 149541880 | 170817 | 149672674 | 80811 | 0.008 | 352796 | 62639 |
| 0.01500 | 149441759 | 170768 | 149578032 | 80760 | 0008 | 4.26989 | 62633 |
| 0.01600 | 149322292 | 170719 | 149.464272 | 80709 | 0008 | 494688 | 62626 |
| 001700 | 149.183455 | 170671 | 149.331474 | 80660 | 0009 | 555535 | 62618 |
| 0.01800 | 149.025164 | 170.623 | 149.179470 | 80.610 | 0009 | 610518 | 62.609 |
| 001900 | 148847649 | 170574 | 149008514 | 80.560 | 0010 | 659810 | 62599 |
| 002000 | 148650909 | 170526 | 148818573 | 80.510 | 0.010 | 704187 | 62.588 |

*Plane $\phi=270^{\circ}$ (direction of $00086^{\circ}$ beam offset, see Fig. 4).
**Beam position for peak gain.


Fig. 1. The 64-m antenna four-reflector antenna system using S/X-band reflex dichroic feed


Fig. 2. Detail of reflex dichroic optics


Fig. 3. Block diagram of physical optics analysis approach


Fig. 4. Dual-frequency dichroic feed RF beam positions

## Appendix A <br> Computer Program List

| Program Name | Usage |
| :--- | :--- |
| HYBRIDHORN | Computes corrugated horn pattern and spherical wave coefficients <br> for vertical $(y)$ polarization. |
| YTXPOL/SWCOE | Obtains horn pattern spherical wave coefficient set for horizontal <br> $(x)$ polarization. Input is spherical wave coefficient set for $y$ <br> polarization. |
| YTORCP/SWCOE | Obtain horn pattern spherical wave coefficient set for RCP. Input <br> is spherical coefficient wave set for $y$ polarization. |
| FSCATT | Ludwig (Ref. 4) asymmetric reflector physical optıcs analysis <br> program. Computes far-field diffraction patterns of ellipsoid, <br> dichroic plate, and hyperboloid. |
| AZ-EXPAND-LG | Programs for computing spherical wave coefficient set of given <br> (near- or far-field) pattern. Performed four times for ellipsoid, |
| SPW-ITER | dichroic, and hyperboloid. |
| CYM-SCAT | Cramer symmetric reflector physical optics analyss program, <br> used in computing far-field pattern of 64-m parabolic main <br> reflector. |
| PHASE-CENT | Determines near-field phase center for given source (near- or far- <br> field), pattern, and distance. |
| FGRID | Produces FSCATT integration grid for ellipsoid. |
| FSURF | Produces FSCATT surface definition for ellipsoid. |
| FGRIDPLT | Provides FSCATT integration grid for dichroic plate. |
| FSVRFPLT | Produces FSCATT surface definition for dichroic plate. |

