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RELIABILITY MODELS APPLICABLE TO
SPACE TELESCOPE
SOLAR ARRAY ASSEMBLY SYSTEM

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ABSTRACT

A complex system may consist of a number of sub-
systems with several components in series, parallel, or
combination of both series and parallel. In order to
predict how well the system will perform, it is necessary
to know the reliabilities of the subsystems and the
reliability of the whole system. The objective of the
present study is to develop mathematical models of the
reliability which are applicable to complex systems.
The models are determined by assuming k failures out of
n components in a subsystem. By taking k = 1 and k = n,
these models reduce to parallel and series models;
hence, the models can be specialized to parallel,
series combination systems. The models are developed
by assuming the failure rates of the components as
functions of time and as such, can be applied to processes
with or without aging affects. The reliability models
are further specialized to Space Telescope Solar Array
(STSA) System. The STSA consists of 20 identical solar
panel assemblies (SPA's). The reliabilities of the SPA's
are determined by the reliabilities of solar cell
strings, interconnects, and diodes. The estimates of
the reliability of the system for one to five years
are calculated by using the reliability estimates of
solar cells and interconnects given in ESA documents.
Aging effects in relation to breaks in interconnects
are discussed.

1. Introduction

If events occur in time and the outcomes are countable numbers, the outcomes of the process can be described by a continuous time discrete stochastic process $N(t)$. The failures of various components in a complex system are countable events occurring in time, and hence, form a continuous time discrete stochastic process. Let $N_1(t)$, $N_2(t)$ ..., $N_n(t)$ be the outcomes of various failure event processes of various components in a system, and T_i be the random variable denoting the failure time of the ith component, then the reliability of this component is denoted by $R_i(t)$ and is defined as the probability that the component has not failed during time t. The reliability of the overall system $R(t)$ is the probability that the system has not failed during time t. This reliability $R(t)$ can be determined from the various mathematical models for the reliabilities of the components $R_i(t)$ and possible assembly of the components in parallel and series combination.

In this report, we shall develop mathematical models for systems which form continuous time discrete stochastic processes. We shall obtain the reliability models for the components of subsystems in which the failures occur randomly. Using these models, the reliability model for the system is obtained. The models are obtained for the failure rate $\lambda(t)$, which is a function of time t, and therefore, can be applied to a process in which failure rate is constant as well as time dependent failure rate which takes into account any aging effect. The models are formulated for series and parallel combination of components for which the reliabilities may be independent or nested.

The mathematical models developed will be applied in determining the reliability of the Space Telescope Solar Array (STSA) System. STSA System consists of two wings with two blankets in each wing. Each blanket is made up of five Solar Panel Assemblies (SPA's). The SPA consists of several strings. Each string consists of an array of solar cells in series and parallel, which are connected by interconnects and diodes. Assuming a reliability model for the solar cells and diodes, a reliability model will be built up for the whole STSA System. The reliability estimates for the STSA will be calculated for some models using the failure rates of solar cells and diodes given in European Space Agency (ESA)

documents. The estimates for the reliability of
STSA System are obtained under different power losses
in the STSA System. Also, the effects of breaks in
solar cell interconnects will be discussed.

2. Stochastic Process, Failure Time and Reliability

A stochastic process is defined in Karlin and Taylor
(1975) as a family of random variables determined by a
process. A realization of a stochastic process is
denoted by $N(t)$, $t \in (0, \infty)$. If t denotes the time and
$N(t)$ corresponds to some outcome of the process, then
$N(t)$ is called the time dependent stochastic process.
If the random variable takes on countable values, 0, 1, 2,
3, ... the $N(t)$ process is called a discrete stochastic
process. Many physical processes are time dependent
discrete stochastic processes. If a complex system is
working in time, then the failures of the components of the
system occur randomly and the number of failures form a
time dependent stochastic process $N(t)$. If $N(t_1)$,
$N(t_2)$, ... denote the outcomes of a time dependent
stochastic process $N(t)$, corresponding to a complex
system at times t_1, t_2, ... t_n... then $T = t_n - t_{n-1}$,
is a random variable and T is called the interevent or
interarrival time of the process. If the outcomes of
the stochastic process $N(t)$ corresponds to failure
events of the process then T is called the failure time
of the process and is a random measure of the time
between two successive failures. Since T is random, it
has a distribution function denoted by $F(t)$ and is
defined by

$$F(t) = Pr \{ T \leq t \} . \tag{1}$$

Here pr { } is abbreviated for probability. The relia-
bility of the system is the probability that the system
will not fail at least until time t, and is denoted by
$R(t)$ and can be expressed by

$$R(t) = Pr \{ T > t \} = 1 - F(t) . \tag{2}$$

Since time T is a continuous random variable, its distri-
bution function $F(t)$ is continuous and also, differenti-
able. The derivative of $F(t)$ is called the probability
density function (p.d.f.) of T and is denoted by $f(t)$.
The p.d.f. $f(t)$ can be written as

$$f(t) = \frac{d}{dt} F(t) \tag{3}$$

For a given distribution of a process $N(t)$, there is an unique p.d.f. $f(t)$, of T. Hence, the process $N(t)$ can be characterized by the p.d.f. of T.

The reliability $R(t)$ can be expressed in terms of $f(t)$ as

$$R(t) = \int_t^\infty f(t)\, dt \quad . \tag{4}$$

and conversely,

$$f(t) = -\frac{dR(t)}{dt} \quad . \tag{5}$$

From (4) the reliability can be obtained from the p.d.f of T. The theory of reliability and historical prospective is discussed by Barlow (1984), also in the paper, a large list of references on the topic is given.

3. Failure Rates

The failure rate $\lambda(t)$ is the rate of failure at which the components fail and the failure rate can be defined by

$$\lambda(t) = \lim_{dt \to o} Pr\,(t < T < t + \Delta t \mid T > t) \quad .$$

$$= \lim_{dt \to o} P\{(t < T < t + \Delta t, T > t)\}/P(T > t)$$

$$= f(t)/(1 - F(t)) = -\frac{d\,R(t)/\,d(t)}{R(t)} \tag{6}$$

From the relation (6), the failure rate can also be expressed in terms of $R(t)$ as

$$\int_0^t \lambda(u)\, du = -\ln R(t) \tag{7}$$

and
$$R(t) = e^{-\int_0^t \lambda(u)\,du} \tag{8}$$

The failure rate is also called the hazard function. The hazard function may be increasing function, constant function or decreasing function of t.

3.1 Constant Failure Rate

If the events occur in time randomly and in a short interval of time, at most one event can occur and the events in nonoverlapping intervals occur independently, then it can be shown that the resulting process is a poisson process and the number of events occurring in the process in given time, t can be shown to be

$$\Pr\ \{\ N(t)\ \leq n\} = \sum_{j=0}^{n} e^{-\lambda t}\ \frac{(\lambda t)^j}{j!},\ n=0,1,2\ldots \tag{9}$$

In this case, it can be shown that the waiting time for the process has a gamma distribution and the distribution of interevent time, T has the exponential distribution with p.d.f. f(t) given by

$$f(t) = \lambda e^{-\lambda t} \qquad t > o,\ \lambda > o\ . \tag{10}$$

The corresponding reliability function R(t) is given by

$$R(t) = e^{-\lambda t}\ . \tag{11}$$

and the failure rate function $\lambda(t)$ reduces to

$$\lambda(t) = \frac{d}{dt}\ \ln\ \{R(t)\} = \lambda e^{-\lambda t}/e^{-\lambda t} = \lambda.$$

In this case, the failure rate $\lambda(t)$ is a constant function.

3.2 Variable Failure Rate

The rate of failures in a small interval may not be constant, and the failure rate may be either increasing or decreasing. In the case of system components, the

components may have an aging effect and the hazard may
increase with time, in this case $\lambda(t)$ is an increasing
function of time and failure may occur more often such
a system is called by Asher (1983) as a "sad" system.
On the other hand, the process may be a learning process,
in this case the hazard rate $\lambda(t)$ is a decreasing function
of t, and the system in this case is called a "happy"
system. There can be many different hazard functions
associated with different phenomena. The failure rate
which varies directly with a power of t can be defined by

$$\lambda(t) = \lambda \beta t^{\beta -1} \quad , t > o. \tag{12}$$

For $\beta=1$ the hazard rate is constant, for $\beta<1$ the hazard
rate is decreasing and for $\beta>1$ the hazard rate is increasing.
The model with the failure rate given in (12) is called
Weibull model. Using equation (8) the reliability can
be written as

$$R(t) = e^{-\lambda t^{\beta}} . \tag{13}$$

and the p.d.f. of T is given by

$$f(t) = \lambda \beta \, t^{\beta-1} e^{-\lambda t^{\beta}} \quad , t>o, \lambda>o. \tag{14}$$

It should be noted that the failure rate contains two
parameters λ, β. Often $\beta=2$ is used, in this case failure
rate is linear function of t.

Other failure rate models could be used in different
situations. If the failure rate decreases sharply with
time, then

$$\lambda(t) = e^{-\lambda t} . \tag{15}$$

is suitable.

If the failure rate decreases initially, then
increases, the failure rate in such situation is dis-
cussed in Shooman (1968) is given by

$$\lambda(t) = \begin{cases} K_0 - K_1 t, & 0 < t \le \dfrac{K_0}{K_1} \\ 0 & \dfrac{K_0}{K_1} < t \le t_0 \\ K(t-t_0), & t_0 < t < \infty \end{cases} \qquad (16)$$

4. Reliability Models

Suppose a system S consists of number of subsystems and each subsystem consists of number of components. The components may work in series, parallel or combination, also components may work independently or dependently. Different models would be used for different situations. Two types of model are discussed below.

4.1 Independent Components

Suppose a subsystem has m components and the components work independently and the system works satisfactory if at least k of the m components work. If all the components are different then from Basu and Mawaziny (1978) if $R_{\alpha i}(t)$ is the reliability of αith component then reliability of the subsystem $R_{si}(t)$ can be written as

$$R_{si}(t) = \sum_{j=k}^{m} \sum_{\alpha i} \prod_{i=1}^{j} (R_{\alpha i}(t)) \prod_{i=j+1}^{m} (1 - R_{\alpha i}(t)) . \qquad (17)$$

Where $\sum_{\alpha i}$ is taken over $\binom{m}{j}$ distinct values of αi.

If all the components are identical then R(t) reduces to

$$R_{si}(t) = \sum_{j=k}^{m} \binom{m}{j} (R(t))^{j} (1 - R(t))^{m-j} . \qquad (18)$$

The system could be called k redundant system, for k=1 the system is called completely redundant or parallel system, for k=n the system is completely nonredundant and system works if each component works and the system reduces to a series system.

If there are s subsystems in a total system and
the $R_i(t)$ is the reliability of components in ith
subsystem and the subsystems are independent then the
reliability of the system using (18) can be obtained as

$$R_S = \prod_{i=1}^{s} R_{si} = \prod_{i=1}^{s} \left\{ \sum_{j=k}^{m} \binom{m}{j} (R_i(t))^j (1-R_i(t))^{m-j} \right\}. \quad (19)$$

The subsystems in a system may not be completely indepen-
dent and the system might work with one or more subsystems
working, in this case, the nested model may be useful.

4.2 Nested Model

In the nested models we consider the subsystems
which work independently and the system may work with one
or more subsystems, further, the probability of success
of a subsystem may dependent on sub-subsystems and so on.
Suppose there are y stages of a subsystem connected
in strings, the system could be represented with a tree
diagram. As an example, we consider a system with 3
stages with 2 components in 1st string, 4 components
in 2nd string and 3 components in 3rd string. The
system can be represented by the tree diagram as follows:
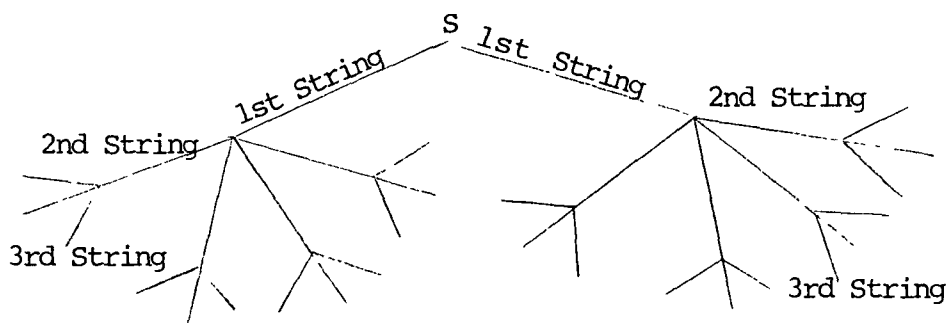
Suppose the components in each string are identical and
the reliabilities of the upper string depend on the
reliabilities of the lower branches. Suppose the system
works if at least 2 components work in 3rd string, at
least 2 in the 2nd string, and at least 1 in 1st string.

Then the reliability of the system is determined as

$$R_s = \sum_{j=1}^{2} \binom{2}{j} R_1^j (1-R_1)^{2-j} , \qquad . \qquad (20)$$

$$R_1 = \sum_{j=2}^{4} \binom{4}{j} R_{12}^j (1-R_{12})^{4-j} . \qquad (21)$$

$$R_{12} = \sum_{j=2}^{3} \binom{3}{j} R_{123}^j (1-R_{123})^{3-j} . \qquad (22)$$

The reliability R_{123} is either estimated or given then R_s can be obtained successive using (22), (21), and (20).

More generally, if the system consists of y stages and there are n_i identical components in ith stage and ith stage works if at least k_i components work and the system is completely nested then

$$R_s = \sum_{j=k_1}^{n_1} \binom{n_1}{j} R_1^j (1-R_1)^{n_1-j} . \qquad (23)$$

$$R_1 = \sum_{j=k_2}^{n_2} \binom{n_2}{j} (R_{12})^j (1-R_{12})^{n_2-j} . \qquad (24)$$

$$R_{12...y-1} = \sum_{j=k_y}^{n_y} \binom{n_y}{j} (R_{12...y})^j (1-R_{12...y})^{n-j} . \qquad (25)$$

The reliability of the system can be completely determined knowing the reliability of $R_{12...y}$ and using equations (25), (24), and (23). Such models are used often in complex systems. The Space Telescope Solar Array could be based on such a model. Also, it is noted that if $k_i = n_i$, i=1, 2, y then each subsystem becomes series model and the whole system reduces model with no loss allowance.

5. Estimation of Parameters

In this section we discuss estimation of parameters which appear in the reliability expressions. We shall restrict to exponential and Weibull models as these are the models which are used often and we have given expressions for the reliabilities involving these models. We consider the maximum likelihood estimators (m.l.e), these are most widely used and have good statistical properties like asymptotic normality and consistency.

5.1 The Exponential Model

If t_1 t_2, ...t_n is a sample from the exponential distribution with p.d.f. in (10). The m.l.e. for λ is denoted by $\hat{\lambda}_t$ and is given by

$$\hat{\lambda}_T = n/\sum_{i=1}^{n} t_i \quad . \tag{26}$$

The estimator is biased but is consistent.

However, if the observations are made on the counting process, if $n_1...n_k$ are failures in k independent trials in given time t from the homogenous poisson distribution, then the maximum likelihood estimator for λ can be written as

$$\hat{\lambda}_N = \sum_{i=1}^{k} n_i/kt \quad . \tag{27}$$

This estimator is the minimum variance unbiased estimator for λ and its variance can be found. Both estimators can be used to find interval estimators for λ.

5.2 Weibull Model

If $t_1, t_2...t_n$ is a sample from the distribution with p.d.f given in (14), if β is known, then taking the likelihood of the sample the m.l.e of λ can be found as

$$\hat{\lambda}_T = n/\sum_{i=1}^{n} t_i^{\beta} \quad . \tag{28}$$

If the observations are made on the number of failures, for given time $t, n_1, n_2 \ldots n_k$ are failures in k independent trials from the nonhomogenous poisson process with intensity function $\lambda(t) = \lambda\beta t^{\beta-1}$ then the m.l.e of λ can be obtained as

$$\hat{\lambda}_N = \sum_{i=1}^{k} n_i \big/ (kt^{\beta}) . \qquad (29)$$

If the shape parameter β is not known, then the maximum likelihood estimators of λ and β can not be obtained explicitly, however, the estimating equations for λ and β can be obtained. If $t_1, t_2 \ldots t_n$ are n independent failure time observations from the Weibull p.d.f in (14) then the m.l.e for $\hat{\beta}$ is obtained by solving

$$1/\hat{\beta} - (\sum_{i=1}^{n} t_i^{\hat{\beta}} \ln t_i) / (\sum_{i=1}^{n} t_i^{\hat{\beta}}) + (\sum_{i=1}^{n} \ln t_i)/n = 0. \qquad (30)$$

and the m.l.e of λ is obtained as

$$\hat{\lambda} = n / (\sum_{i=1}^{n} t_i^{\hat{\beta}}) . \qquad (31)$$

The m.l.e's of $\hat{\lambda}$, $\hat{\beta}$ and their properties are discussed by Cohen (1965).

The confidence interval estimators of λ can be obtained by using distributions of $\hat{\lambda}$. Large sample confidence intervals are discussed by Abernethy et al. (1983).

Crow (1974) has obtained explicit expressions for m.l.e's of β and λ based ordered observations for a repairable system.

6.   Application to Space Telescope Solar Array System

The methodology developed in section 2-5 is now applied to Space Telescope Solar Array (STSA) System. First, we describe the STSA System.

6.1 Description of STSA System

Space Telescope is made up two identical wings. Each wing consists of two identical blankets. Each blanket is made up of five identical Solar Panel Assembly (SPA). There are 20 identical SPA's in STSA System. These SPA's are connected by 40 connecting diodes with 2 on each SPA. Each SPA is built up of three strings of solar cells. Each string has a length of 106 solar cells. Two end strings have eight cells in parallel and the middle string has seven cells in parallel. Strings 1 and 3 are built by 848 cells each whereas the middle string is made up of 742 cells. Each string is further broken into 7 substrings of 14, 15, or 16 cells so the substring consists of either 14, 15, and 17 long and 8 or 7 cells wide. There are seven shunting diodes. Each SPA has 21 shunting diodes and 2,438 solar cells. The cells are connected by Cell Interconnects (CIC's). There are the same number of CIC's as the cells. The STSA System consists of 48,760 cells, same number of CIC's, 420 shunting diodes, 40 connecting diodes and same number of solders. The description of STSA System is given in a number of ESA documents. Two of these are GL-SA-B002 and AN-1367-108.

Each individual cell is made up of silicon and 20 mms. wide, 40 mms. long, and generates .349 volts and carries current .300 amp. at 55°. However, total voltage at SA/SSM interface is approximately 34 volts and working current is .27 amps. Approximate power without any loss of cells diodes and CIC's is 4.18 watts per string. Since 460 parallel strings in STSA System, STSA produces approximately 4,222.8 watts. The losses due to solder and other connectors in current and voltages are not clear. The estimates mentioned are taken from GL-SA-B002.

6.2 The Reliability of STSA System

The reliabilities of photovoltaic devices and system are discussed in a special issue of IEEE Transaction (1982) Vol. R-31. The papers of some interest on the topic are, "Photovoltaic Module Reliability Improvement Through Application Testing and Failure Analysis" by Dumas and Shumk (1982). "Reliability Terminology and Formulae for Photovoltaic Power System" by Lauftenburger and Anderson (1982), and "A Methodology for Photovoltaic System Reliability and Economic Analysis" by Stember Huss and Bridgman (1982).

The reliability of STSA System depends on the reliabilities of the various components in the system, the design of the components, the definition of the failure,

and the mathematical models used in determining the
reliabilities of the components. Most of the estimates for
failure rate are based on the constant failure rate,
which arises out of the exponential models as these models
have been used by ESA. However, estimates based on Weibull
model could be obtained if some estimates or knowledge
of shape parameter is assumed.

First, we discuss the reliabilities of blocking
and shunted diodes. For connecting diodes, since there are
40 blocking diodes, if all of them have to work, then
we find

$$R_0 = (R_{BD})^{40} ,$$ (32)

R_1 is the reliability of individual connecting diode.

If the success is defined with k (k is an integer
close to 40) diodes working the R_1 reduces to

$$R_1 = \sum_{j=k}^{40} \binom{40}{k} R_{BD}^j (1-R_{BD})^{40-j} .$$ (33)

For shunting diodes, the reliability, when each
diode is working, is

$$R_0 = (R_{SD})^{420} .$$ (34)

For k diodes working, the reliability reduces to

$$R_1 = \sum_{j=k}^{420} \binom{420}{j} (R_{SD})^j (1-R_{SD})^{420-j} .$$ (35)

The reliabilities R_{SD} for the exponential for given
time t can be evaluated by

$$R_{SD} = e^{-\lambda_1 t} ,$$

Where λ_1 is the failure rate of the diodes to be evaluated
from the data.

Next, we consider the reliabilities associated with
CIC. Since there are 20 SPA's and each SPA has 3 strings
with width 8 and 1 with width 7. Here, for calculating

reliability, we assume width 8. Each string has 106
CIC's in series. The reliabilities of CIC's by nested
design can be written as

$$R_{CIC} = \sum_{j=k_1}^{60} \binom{60}{j} R_1^j (1-R_1)^{620-j} , \qquad (36)$$

$$R_1 = \sum_{j=k_2}^{8} \binom{8}{j} R_{12}^j (1-R_{12})^{8-j} , \qquad (37)$$

and

$$R_{12} = R_{123}^{106} . \qquad (38)$$

Here k_1, k_2 are integers close to upper limits and R_{123}
is the reliability of interconnect can be obtained from

$$R_{123} = e^{-\lambda_c t} ,$$

λc being failure rate of an interconnect.

We now discuss the reliability models associated
with the solar cell arrays. In the first model, we assume
that a substring consists of an average of 15 cells in
series and each string has 7 substrings and there are 8
parallel strings associated with each string. Since
there are 3 strings per SPA, out of 21 subgroups any
number of subgroups might fail, a nested model could
be used. Let $R_c = R_{1234}$ be the reliability of a cell, then

$$R_{123} = R_{1234}^{15} \text{ is the reliability of a substring, and}$$

$$R_{12} = \sum_{j=k_3}^{21} \binom{21}{j} (R_{123})^j (1-R_{123})^{21-j} . \qquad (39)$$

R_{12} gives the reliability with k_3 substrings working.
Since there are 8 strings in parallel, if some of these
fail, then the power of the system would be affected.
Hence, using k_2 parallel strings working the reliability
of SPA is determined by

$$R_1 = R_{SPA} = \sum_{j=k_2}^{8} \binom{8}{j} (R_{12})^j (1-R_{12})^{8-j} . \qquad (40)$$

Since there are 20 SPA's, the reliability of STSA System from cell failures can be reduced to

$$R_S = \sum_{j=k_1}^{20} \binom{20}{j} R_1^j (1-R_1)^{20-j} \quad . \tag{41}$$

If there is no loss of power due to any cells in the entire system, then the reliability of the system reduces to

$$R_S = (R_c)^{48760} . \tag{42}$$

There are other models considered for the reliability of solar cell system. In AN-1367-108, it is suggested that each string should be considered as one unit. Since there are 460 strings of 106 cells long in the STSA System, the reliability R_S can be written as

$$R_S = \sum_{j=k}^{460} \binom{460}{j} R_1^j (1-R_1)^{460-j} \quad , \tag{43}$$

where R_1 is the reliability of the string and is given by

$$R_1 = R_c^{106} \quad . \tag{44}$$

R_c is the reliability of a single cell and is determined from

$$R_c = e^{-\lambda_c t}$$

in the exponential case and for Weibull case R_c is given by

$$R_c = e^{-\lambda_c t^\beta} \quad ,$$

$\lambda_c t^\beta$ could be determined from a set of data.

7. Estimates of Reliabilities

In this section, we calculate the reliabilities associated with diodes, CIC's and solar panels. These reliabilities are then used to calculate the reliabilities of STSA System. In calculating the reliabilities, we are using various models proposed in section 6. The estimates are based on failure rate estimates given in ESA documents.

Since these estimates are based on the exponential models, we have to resort to these models. The reliabilities are calculated from one to five years and presented in a tabular form as well as in graphical form. First, we calculate the reliabilities of the diodes.

7.1 Reliability Estimates of Diodes

For blocking diodes, the failure rate given in TN-SA-B147 is $\lambda = 1.2 \times 10^{-9}$/hr., hence, the reliabilities are calculated from $R_{BD}(t) = \bar{e}^{\lambda t}$. Since there are 40 diodes, if all of them have to work reliabilities are calculated from

$$R_o(t) = (R_{BD})^{40} . \tag{45}$$

If we allow one of them to fail then

$$R_1(t) = (R_{BD})^{40} + 40 \ (R_{BD})^{39} \ (1-R_{BD}) \tag{46}$$

Using these formulae, we find the reliabilities assuming one year is 8,760 hours. The results are presented in table I.

Table I.   Reliabilities of Blocking Diodes

Time t (in years)	$R_{BD}(t)$	$R_o(t)$	$R_1(t)$
1	.99998948	.99957961	.99999991
2	.99997897	.99915939	.99999965
3	.99996846	.99873935	.99999922
4	.99995795	.99831949	.95999862
5	.99994744	.99789981	.99999785

Next, we find the reliabilities for the shunting diodes. Since there are 420 shunting diodes and failure rate of each diode is $\lambda = 1 \times 10^{-9}$, the reliabilities for all diodes working are obtained from

$$R_D(t) = \bar{e}^{\lambda t}$$

$$R_o(t) = (R_{SD})^{420} . \tag{47}$$

If we allow in our success criteria one diode to fail, then the reliabilities are calculated from

$$R_1(t) = (R_{SD})^{420} + 420 (R_{SD})^{419}(1-R_{SD}). \tag{48}$$

The reliabilities for different years are given in table II.

Table II. Reliabilities of Shunting Diode

Time t (years)	$R_{SD}(t)$	$R_o(t)$	$R_1(t)$
1	.99999124	.99632760	.99999328
2	.99998248	.99266860	.99997314
3	.99997372	.98902309	.99993969
4	.99996496	.98539096	.99989302
5	.99995620	.98177217	.99983327

7.2 Reliability Estimates of Solar Cells

We now discuss the reliabilities of solar cells. First, we use the nested model discussed in section 6. For the series model

$$R_o(t) = R_S = (R_C)^{48760}. \tag{49}$$

Also, taking $K_3 = 21$, $K_2 = 8$ and $K_1 = 20$ in equations for solar cells in section 6, we find

$$R_{123} = R_C^{15}. \tag{50}$$

$$R_{12} = R_{123}^{21} + (21) (R_{123})^{20} (1-R_{123}). \tag{51}$$

$$R_{SPA} = R_{12}^8 + 8 R_{12}^7 (1-R_{12}). \tag{52}$$

$$R_1(t) = R_S = R_1^{20}. \tag{53}$$

This model allows a loss of up to 1 substring and up to 1 parallel string to fail in each of the 20 SPA's. This amounts to 4.9% power loss in STSA System. The loss figure is taken from TN-SA-B151.

Alternative model with .22% of power loss is considered. This is based on 460 total strings (counting parallel strings) in the STSA System. The loss corresponds to a loss of one string of 106 cells in series. The reliabilities are obtained by taking K = 459 in equation (43). The reliability of STSA System is calculated from

$$R_1 = R_C^{106}. \tag{54}$$

$$R_1'(t) = R_1^{460} + 460 \, R_1^{459} \, (1-R_1) \tag{55}$$

For the cell, the failure rate from TR-STSA-42 is 1×10^{-9}/hr. Using this estimate and the above equations for various t's, the reliabilities $R_0(t)$, $R_1(t)$, and $R_1'(t)$ are calculated and presented in table III.

Table III. Reliabilities of Solar Cells in STSA System

Time t	$R_C(t)$	$R_0(t)$ (Zero Loss)	$R_1(t)$ (1 String Loss)	$R_1'(t)$ (.22% Loss)
1	.99999124	.65237696	= .99999999	.93115794
2	.99998248	.42559154	= .99999989	.78950169
3	.99997372	.27764476	= .99999941	.63391835
4	.99996496	.18112816	= .95999815	.49117021
5	.99995620	.11816326	= .99999550	.37110988

The reliabilities of the CIC's could be calculated from the formulae for CIC's. However, the reliabilities for a CIC as functions of time are not available. Since there are the same number of CIC's as the cells and their configuration is similar to those of the cells, it is possible to combine the reliabilities of CIC's and cell and find the reliabilities of CIC's with cells in STSA System. The estimate of reliability for a CIC from AN-1367-108 is R= .9999976714. The reliability of CIC and cell is given by

$$R_C'(t) = (.9999976714) \, e^{-\lambda t} . \tag{56}$$

We use this formula to calculate R_C and the formulae $R_0(t)$, $R_1(t)$, and $R_1'(t)$ given in equation (49) to equation (55). The reliabilities for different t are given in table IV.

Table IV. Reliabilities of Solar Cells With CIC in STSA

Time t (in yrs.)	$R_C'(t)$	$R_0(t)$ (Zero Loss)	$R_1(t)$ (1 String Loss)	$R_1'(t)$ (.22% Loss)
1 yr.	.99998891	.58234352	.99999998	.89739874
2 yrs.	.99998015	.37990379	.99999982	.74797591
3 yrs.	.99997139	.24784048	.99999917	.59409527
4 yrs.	.99996263	.16168463	.99999760	.45687567
5 yrs.	.99995387	.10547829	.99999445	.34330636

7.3 The Reliabilities of STSA System

The STSA System consists of diodes, CIC's and solar cells. The reliability of STSA System R_S is calculated from

$$R_S(t) = R_D^{(t)} R_{Cell}^{(t)} \qquad\qquad (57)$$

For zero loss $R_{S_O}(t)$ is calculated by multiplying $R_O(t)$ from tables I, II, and IV. For .22% power loss $R_{S_1}(t)$ is tabulated from $R_1(t)$ in tables I and II and R_1 in table IV, and these are tabulated in table V. Also, at the beginning there is no loss, hence, at time 0, the reliability is 1.

Table V. STSA System Reliabilities

Time t	$R_{S_O}(t)$ (No Power Loss)	$R_{S_1}(t)$ (.22% Power Loss)
0	1	1
1	.57996100	.89739874
2	.37680155	.74795555
3	.24481094	.59405897
4	.15905482	.45682516
5	.10333816	.34324838

The graphs of the reliabilities of STSA System are presented for zero power loss and .22% power loss.

8. Discussion and Conclusion

After a search of literature on the failure rate estimates and reliabilities of solar cells, most of the identifiable sources on the subject were found in the ESA documents. However, none of the documents contained any original data. The failure rate of the solar cells was determined from the failure rate of the solar cell diodes. The failure rate of solar cells is taken as the same as that of the diode (STSA B151) because both have the same base material, doping, contact system, cover slides, and adhesive. The data on failure rate of diodes is also not available. It appears that the failure rate of diode is determined from the estimates of the components in the diode. The failure rate on CIC's is estimated from a confidence procedure on the binomial distribution, which is independent of time.
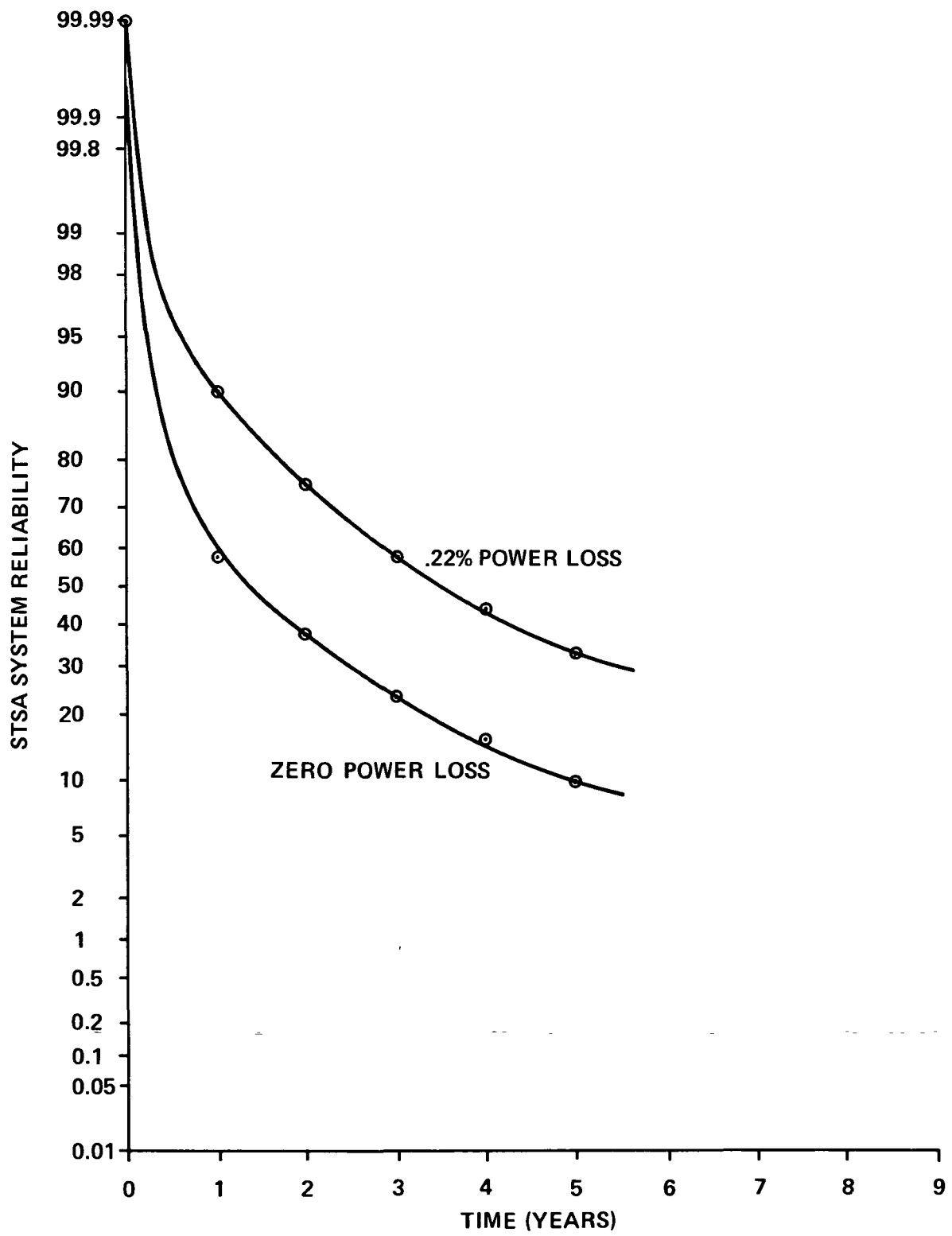
There are no explicit results available on the effect on the failure rate due to the temperature change of the solar cells, although the cells go through a change of temperature from -80° to 180° centigrade. The effect of temperature on the cell voltage is discussed by Rajeswaren et al. (1982). Also, Anderson and Kim (1978) give the relation between open voltage and temperature. There may not be an immediate impact on the performance of the solar cell, but long term effect is not known. In the (1978) paper Anderson and Kim state that there was a degradation after the crack appeared in the cell. This suggests that the constant failure rate models may not be suitable for systems working for a long time.

Also recently, Alexander (1985) has run some tests on the solar cells. The tests were run on only 12 cells. In the study, he found some breaks in the strands on interconnects. The total number of unbroken strands on the interconnects is not clear. Also, there were not any complete breaks in interconnects. It is difficult to determine failure rate on the interconnects.

Although general models applicable to STSA System are developed, the models are not tested for any other model except the constant failure rate model. There is a need to test aging models. From the available failure rate estimates, the reliability estimate of STSA System is obtained based on the maximum likelihood estimators of λ. The maximum likelihood estimators of the reliability are biased, hence, the biases and variances need to be investigated. And the confidence intervals on the reliabilities need to be developed to gain further information on the reliability of STSA System.

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