TEST VS ANALYSIS
A DISCUSSION OF METHODS

by

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BUTLER ANALYSES

INTRODUCTION

I write not as a sage with answers but as a confessor with questions. Exposure to this arena has left me with the impression that much needs to be learned about using existing methods, and we need to rely heavily on experience. Some techniques for comparing structural vibration data, determined from test and analysis are discussed. Orthogonality is a general category of one group, correlation is a second, synthesis is a third, and matrix improvement is a fourth. Advantages and short-comings of the methods are explored with suggestions as to how they can complement one another.

OBJECTIVE

The purpose for comparing vibration data from test and analysis for a given structure is to find out whether each is representing the dynamic properties of the structure in the same way. Specifically; whether
   mode shapes are alike;
    the frequencies of the modes are alike;
    modes appear in the same frequency sequence;
and if they are not alike, how to judge which to believe.

PROCEDURE

The first task is to find out which modes from test correspond to ones from analysis. This is no trivial task over a spectral range for complex structures having hundreds or thousands of degrees of freedom. It is tempting to fall into the trap of declaring that two modes correspond when their frequencies are near to one another. It is however, absolutely necessary to determine correspondence based upon their mode shapes, first, and then see how close they are in frequency. The mere fact that their frequencies are not expected to be alike testifies to the notion that there must be variations between two companion mode shapes. The first problem then is learning to recognize likeness. Taking a simple open note of a violin string will illustrate how two eigenvectors may look different but represent the same mode.

$$\frac{h_1}{h_2} = \frac{a_1}{a_2}$$
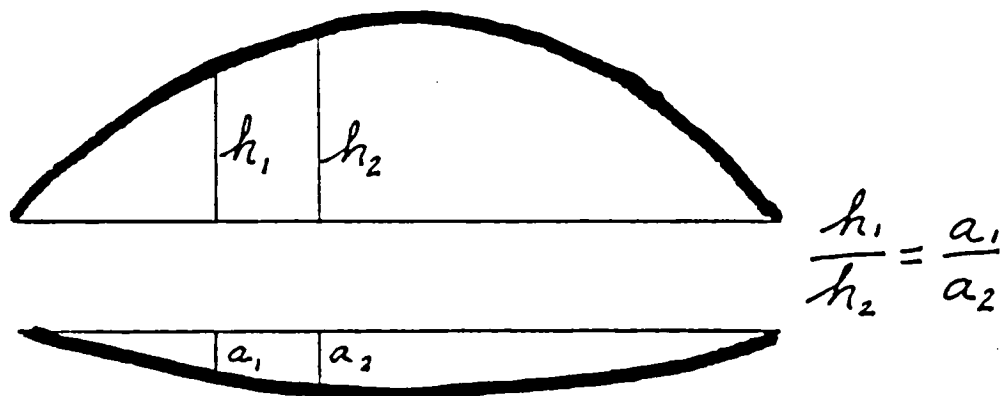
The amplitudes and the phase relationships at the instant of
measurement are different, but they do represent the same mode; they
will both have the same pitch (i.e. their frequencies are the same)
but the top vector will sound louder than the bottom one.
Extrapolating from this simple mode it is evident how necessary it is
to agree on a set of rules as to how to compare modes.

This can be approached mathematically. The eigenvalue problem
has one more unknown than equations, so an additional equation has to
be supplied. A popular approach is to provide a scaling--that is
arbitrarily declaring the magnitude of one displacement in the vector
and then all other displacements in that vector will be scaled to this
arbitrary value. This happens also in test, because one is free to
select how much forcing to apply when exciting a mode. The problem in
comparing results is to put the two sets of arbitrary amplitudes on a
comparable footing. In analysis one usually sets one part to unity.
This is called normalizing. One approach is to canvas a vector for
its largest value and find the ratio of its trial value to 1.0 then
scaling all other terms in the vector by the same ratio. Another
approach is to isolate a reference point then set the trial value of
its modal displacement there to unity followed by a like scaling of
the rest. One that dynamicists often use is to scale a mode's
generalized mass to unity; i.e. if the matrix product for the ith mode
is

$$[\phi_{si}]^T [M_{ss}] [\phi_{sj}] = k,$$

then scaling the ith vector by $\sqrt{k}$ will give the value 1.0 to this
product. The net result is that every eigenvector has its individual
scaling factor regardless of the method of normalizing. For comparing
test with analysis pose the question, "Does the method of
normalization have to be the same for both?" It will for some types
of comparisons and others will have a built-in arbitrator so it might
not.

Only rarely in a complex structure will a test mode match an
analytical mode in every detail. There is a need to arbitrate as to
when any two are comparable. One way, certainly, is to look at their
plots and make a judgement as to whether they are similar enough.
This doesn't quantify anything. Other ways are to compute certain
properties and set ranges for such computed values as to their
comparability. The next section will be devoted to various
computations. The treatment will be organized according to first a
discussion of the methods of making computations, then setting up a
tabulation of (a) the operations that are entailed, (b) the utility of
the computation for helping the analyst to make a judgement, and (c)
the resources involved in the computation.

METHODS

Orthogonality Test-

Modes from test are multiplied into mass from analysis in the
formula for generalized mass:

$$[\phi_e]^T [M_a] [\phi_e] = [G].$$

If the test vectors are normalized to the analytical mass then acceptability can be readily determined by comparing [G] with unity, [I]. It is usual that certain thresholds are assigned for acceptable departures from unity. This technique has been implemented by the author for NASTRAN and is described in reference (1). Raw test data is read from magnetic tape into a processor program called TAP2DMI to convert it into DMI bulk data format. The rest of the computation is done internally in NASTRAN by means of a DMAP ALTER packet. It normalizes the test vector to the analytical mass. Two different quantities are computed. The first is the matrix [G] shown above. The other will be discussed in the succeeding paragraph. The ALTER delivers [G] in standard MATPRN format. The diagonal of [G] will be unity because mass orthogonality forced it to be so, therefore the residue of off-diagonal terms constitutes the test. Ideally non-diagonal terms would be null. When q_{ij} ($i \neq j$) are > a threshold, the test mode is declared to be mis-matched with the analytical model. It does not declare whether test or analysis is at fault, it just declares a mis-match. The value of the threshold is arbitrary. When a threshold is exceeded one needs to consult other data such as plots or correlation data to assess differences.

Cross-Orthogonality-

A product is formed from analytical mass, the matrix of mass normalized test vectors and, the matrix of mass normalized analytical modes.

$$[\phi_e]^T [M_a][\phi_a] = [H].$$

This is implemented in NASTRAN in the same DMAP ALTER packet of reference (1) that was mentioned above in the discussion of the orthogonality test. After the recovery of eigenvectors, $[\phi_a]$, the product of the first two matrices is multiplied into the analytical vectors to obtain [H]. Ideally [H] would be unity. Two criteria are used for acceptability; (a) diagonal terms h_{ii} should lie within a band of unity, i.e. $1-v < h_{ii} < 1+v$, and (b) off-diagonal terms should be less than a threshold c; i.e. $h_{ij} < c$ ($i \neq j$). Failing either of these tests, classifies the test data as mis-matched with respect to analysis data. Once again plots and correlation are helpful in visualizing these differences.

Critique of Orthogonality and Cross-Orthogonality Tests-

Analysis can be condensed to test degrees of freedom in order to produce a mass matrix for normalization that is commensurable with the test vectors. Condensation to only instrumented points could be contrary to good dynamics practice, because points are chosen for measurement in test primarily on the basis of accessibility or on the expectation of being near antinodes, while the needs of analysis are to condense to significant mass locations to preserve kinetic energy. In using NASTRAN without a DMAP ALTER there is no alternative but to select the A-set based on instrumented locations only. If, however, a rational dynamic approach is taken to condensation which includes all instrumented points as a subset, then it would be possible to obtain reliable eigenvectors for the structure based on a generous

number of degrees of freedom. I came upon this idea only while writing this paper, so the idea is only sketched out and has not been checked. The scheme is this. Subsequent to the eigenvalue analysis the eigenvectors could be partitoned down from a reliably implemented eigenvalue analysis to the instrumented set as opposed to a condensation down to the instrumented set. In addition a second Guyan reduction from A-size to I-size (instrumented set size) could be performed using DMAP for partitioning the A-sized stiffness and mass into e-set (for elimination) and i-set (for instrumented) then calculating the $[K_{ee}]$ decomposition in preparation for determining the $[G_e]$ matrix from $[K_{ee}][G_e] = -[K_{ei}]$. Then the second Guyan reduction could be performed from the equation

$$[M_{ii}] = [\bar{M}_{ii}] + [G_e]^T[M_{ei}] + [M_{ei}]^T[G_e] + [G_e]^T[M_{ee}][G_e].$$

The partitioned PHII would need renormalizing with the $[M_{ii}]$ matrix. There is still some question as to how violent an effect this second Guyan reduction would have on the mass matrix; therefore it would be prudent to do an additional orthogonality check on just the analytical I-sized set. If this is acceptable, the I-sized mass matrix is ready to be used to normalize the test vectors and proceed with the orthogonality test. If the I-sized analytical set does not pass the orthogonality check, the I-sized mass matrix condensation should be modified until it does pass the analytical check before applying it to the test vectors. If no satisfactory condensation is achieved, then there should be a renegotiation of the test plan to include instumentation at some necessary mass locations to achieve compatibility between test and analysis.

Test data is not compromised (assuming modes are properly excited) by a relocation of instrumentation unless pick-ups are located too close to node lines. If the test structure is well instrumented and well excited and well mounted, the modal data represents the true vibration properties of the test article. (Aside--this does not imply that the test article necessarily represents the structure as designed.) Normalization of test data with a normalizing factor originating from analysis does not in any way prejudice the test data because each factor is distinct and arbitrary, regardless of origin, so the modal properties are preserved.

The two orthogonality tests diagnose all modes at once with a net result regarding the modes as a whole without any details within the modes. It provides no insight as to which source to suspect if there is a mismatch.

Correlation-

Since in a correlation computation, mode shapes are compared over their entire region with products, point by point - between the two sources, then averaged; a detailed examination is obtained and characterized by a single number. Correlation coefficients computing to 1.0 are exact. Comparable modes can be identified by the high value of their coefficients, and their frequencies can subsequently

be compared. Only shape data are considered, so no mass or
stiffness data get involved explicitly. Theory behind the computation
of correlation and the strategy for the algorithms was developed by
personnel at Goddard Space Flight Center and was published in
reference (2). The implementation of this technique was done at
Goddard. Documentation of its application to a structure is explained
in reference (3). The definition of the correlation coefficient copied
from reference (3) is

$$r_{ab} = \frac{s_{ab}}{s_a \cdot s_b} \qquad \text{where,}$$

$$s_{ab} = (1/n)\sum_{i=1}^{n}(a_i - \bar{a})(b_i - \bar{b})$$

is the covariance between mode a and mode b having n degrees of
freedom to define the mode shapes, and s_a or s_b are standard deviations
which can be obtained by taking the square root of the variance, where
the variance is

$$s_x^2 = (1/n)\sum_{i=1}^{n}(x_i - \bar{x})^2 \qquad \text{and} \qquad \bar{x} = (1/n)\sum_{i=1}^{n}x_i$$

is the mean value of a mode.

Differences-

     As an auxilliary to correlation to find out where and by how much
two sources differ, all points can be scouted in pairs by two methods
as defined on pages 2-2 and 2-3 of reference (3) and repeated here.

     (a) Relative Difference of the ith dof

$$rd_i = \frac{a_i}{s_a} - \frac{b_i}{s_b} \quad , \qquad \text{or}$$

     (b) Scaled Difference of the ith dof

$$D_i = (Ca_i - b_i)/S \quad , \qquad \text{where}$$

$$C = \frac{\sum_{i=1}^{n}a_i b_i}{\sum_{i=1}^{n}a_i^2} \quad , \qquad \text{and}$$

$$s^2 = (1/n)(\sum_{i=1}^{n}b_i^2 - c^2\sum_{i=1}^{n}a_i^2)$$

Critique of Correlation-

Correlation is done as a short-running post processor outside of NASTRAN, but depends on a DMAP ALTER from NASTRAN before it can execute. Full analytical fidelity of modes is preserved by first computing detailed modes before partitioning to instrumented points. Many more analytical modes than test modes may be involved if desired. No scaling of modes is required ahead of computing correlation because the formulas contain self-scaling by their own standard deviations.

The correlation coefficient can give evidence as to which modes are distinct and which have multiple similarities. Point by point comparisons are made. Data is sorted by user prescribed thresholds so the pertinent information is at hand without clutter. Localized evidence of differences allows the analyst to examine a point with respect to G-sized modes to see what structural factors could contribute to local disparities.

Correlation involves only displacement information and does not involve stiffness or mass, but since localized information is given over many anlytical modes, inferences can be drawn from such data as to the type of involvement.

The scaling coefficient C brings unscaled sources within the same average amplitude. The standard error scaling results in magnification of terms and allows separation of coefficients as they near unity.

Synthesis-

Since a large number of analytical modes are usually available and since they constitute an orthogonal set, they can logically be used as a basis for synthesizing test data in analytical terms thus avoiding the difficulties involved in scaling. This method was published by a team from Rockwell International in reference (4).

Expand the observed displacement y_i^k in mode k at instrumented location i in n analytical modes φ_{ij} that have been determined from a large set of points, but have been partitioned down to just the instrumented points. Y_i^k is the approximated expansion.

$$Y_i^k = \sum_{j=1}^{n} \varphi_{ij}\, b_j \qquad \text{where } b_j\text{'s are unknown amplifiers.}$$

Sum the residuals over the m instrumented points i in the kth mode as

$$R^k = \sum_{i=1}^{m} (y_i^k - Y_i^k).$$

Rectify the residuals to develop an expression for the solution of b_j's.

$$RR^k = \sum_{i=1}^{m} (y_i^k - Y_i^k)^2.$$

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Provide for the use of a weighting function such as: just the diagonals of the mass matrix, and substitute in the rectified residuals with the Y_i^k 's expanded.

$$RR^k(M) = \sum_{i-1}^{m} M_i (y_i^k - \varphi_{i1} b_1^k - \varphi_{i2} b_2^k - \ldots\ldots - \varphi_{in} b_n^k)^2.$$

Find the extremals of $RR^k(M)$ with respect to one b_j at a time.

$$\frac{1}{2} \frac{\partial RR^k(M)}{\partial b_j} = \sum_{i=1}^{m} M_i (-y_i^k \varphi_{ij} + \varphi_{ij} \varphi_{i1} b_1^k + \varphi_{ij} \varphi_{i2} b_2^k + \ldots\ldots + \varphi_{ij}^2 b_j^k +$$

$$\ldots\ldots + \varphi_{ij} \varphi_{in} b_n^k) = 0,$$

which when taken for all n b's compresses in matrix notation to

$$[\varphi_{ij}]^T [M_i] [\varphi_{ij}] \{b_j^k\} = [\varphi_{ij}]^T [M_i] \{y_i^k\}.$$

Now $\{b_j^k\}$ can be solved for, because all else is known.

1. The b_j^k tell us how much of each of the n modes are going into simulating the kth test mode. Substitue b's into the original expansion to obtain the intended approximation Y_i's for every instrumented point of the kth test mode. Then construct R^k and $RR^k(M)$.

2. A simple mass weighted correlation coefficient differs from the Goddard one.

$$R_{k\ell} = \frac{\sum_{j=1}^{m} M_j (\varphi_{kj} - \bar{\varphi}_k)(\varphi_{\ell j} - \bar{\varphi}_\ell)}{\{[\sum_{j=1}^{m} M_j (\varphi_{k\ell} - \bar{\varphi}_k)^2][\sum_{j=1}^{m} M_j (\varphi_{\ell j} - \bar{\varphi}_\ell)^2]\}^{1/2}}$$

where the φ's can be either analytical or test modes.

3. Multiple correlation coefficient in the kth mode of approximated vector to test vector. This has no corrolary with the Goddard approach.

$$R_k^2 = \sum_{j=1}^{m} M_j (Y_j^k - \bar{y}^k)^2 / \sum_{j=1}^{m} M_j (y_j^k - \bar{y}^k)^2$$

4. Compute the standard error S^k and use it to scale modal amplitudes b.

$$S^k = \left\{ \sum_{j=1}^{m} M_j (Y_j^k - y_j^k)^2 \right\}^{1/2}$$

$$T^k = b_j^k / S^k.$$

5. Compute spread of amplitudes over sampling points for an analytical mode W_j and the similar spread for the test mode W_j'. Determine their relative influence

$$X(I) = \frac{b_j' \, W_i'}{W_j} \; .$$

6. Orthogonality with synthesized modes. First construct a matrix of all k of the synthesized modes.

$$[Y_{ik}] = [\varphi_{ij}][b_{jk}].$$

$$[G] = [Y_{ik}]^T [M_{ii}][Y_{ik}] = [b_{jk}]^T [\varphi_{ij}]^T [M_{ii}][\varphi_{ij}][b_{jk}]$$

but since the analytical modes were normalized to mass this test reduces to

$$[G] = [b_{jk}]^T [I][b_{jk}].$$

This synthesized [G] can be compared to [I] as to how well analysis compares with test both in diagonal and off-diagonal terms.

Critique of Synthesis-

This is by far the most complete and most versatile of available codes for making comparisons. It would be worth the investment of purchasing the DUMMOD from Cosmic and spending time to sysgen it into one's NASTRAN executable. It operates entirely within a NASTRAN execution. Two kinds of local behavior are reported -- how much an analytical mode is participating in a test mode, and how much an approximation misses its test counterpart. Local behavior is further focused by the T value and the relative influence X(I). The simple correlation is quite similar to the Goddard one except its scaling, but it has the added advantage of diagnosing analysis alone. The multiple correlation is unique in that it gives well magnified measures for one mode at a time. Its generalized mass is more versatile than the usual orthogonality or cross-orthogonality tests and it is more efficient.

Matrix Improvement-

The premise in this technique is that test data has been certified, but analysis doesn't match. Analytical stiffness and mass matrices Kcomputed and Mcomputed are are assumed to be not too divergent but do need improvement. A method of applying incremental values to the analytical matrices was developed at Kaman Aircraft according to reference (5).

Given: $\bar{\Phi}_T$ & Ω^2_T from test data and K_c & M_c from analysis, where c > T dof's.

Objective: Apply corrections to K_c & M_c to arrive at synthesized K_S & M_S.

Develop mass corrections first under these constraints:

$$[\bar{\Phi}_S]^T[M_S][\bar{\Phi}_S] = [I] \qquad \text{and} \qquad [K_S][\bar{\Phi}_S] = [M_S][\bar{\Phi}_S][\Omega^2_T]$$

Step 1. Expand $[\bar{\Phi}_T]$ to c-size by setting up the eigenvalue equation in c-size for just one frequency at a time and partition it between test size and oversize = complement of c with respect to t.

$$\left[\begin{bmatrix} K_{tt} & K_{to} \\ \hline K_{ot} & K_{oo} \end{bmatrix} - \omega_i^2 \begin{bmatrix} M_{tt} & M_{to} \\ \hline M_{ot} & M_{oo} \end{bmatrix} \right] \begin{Bmatrix} \varphi_{Ti} \\ \varphi_{Oi} \end{Bmatrix} = 0$$

Solve for the remnants of $\{\bar{\Phi}_S\}$ mode by mode from

$$\{\varphi_{Oi}\} = -[K_{oo} - \omega_i^2 M_{oo}]^{-1} [K_{to}^T - \omega_i^2 M_{to}^T]\{\varphi_{Ti}\}$$

Step 2. Find correction to $[M_c]$ by minimizing differences of $[M_c - M_S]$ while enforcing orthogonality. The resulting expresssion is based on approximating the correction for diagonal terms.

$$\epsilon = ||[M_c]^{-\frac{1}{2}}[M_S - M_c][M_c]^{-\frac{1}{2}}||.$$

Set up equations in La Grangian multipliers

$$\psi = \epsilon + \sum_{i=1}^{m} \sum_{j=1}^{m} \lambda_{ij} [\bar{\Phi}_S^T M_S \bar{\Phi}_S - I]_{ij}$$

Differentiate with respect to elements of unknowns $[M_S]$ and set to zero then solve for values of λ_{ij} which minimize ϵ. The resulting equation is

$$[M_S] = [M_c] + [M_c][\bar{\Phi}_S][m_c]^{-1} [I - m_c][m_c]^{-1} [\bar{\Phi}_S M_c] ,$$

where $[m_c] = [\bar{\Phi}_S]^T[M_c][\bar{\Phi}_S]$.

Step 3. Find correction to $[K_c]$ by applying the constraint equations to develop $[K_S]$.

$$[K_S][\bar{\Phi}_S] - [M_S][\bar{\Phi}_S][\Omega^2_T] = 0$$

$$[\bar{\Phi}_S]^T[K_S][\bar{\Phi}_S] - [\Omega^2_T] = 0$$

$$[K_S] - [K_S]^T = 0.$$

The resulting quantity to be minimized is

$$\epsilon = \| [M_s]^{-\frac{1}{2}} [K_s - K_c][M_s]^{-\frac{1}{2}} \|$$

Set up three sets of La Grangian multipliers for each of the three constraints. The result is

$$\psi = \epsilon + \sum_{i=1}^{n}\sum_{j=1}^{m} \Lambda_{K_{ij}} [K_s \Phi_s - M_s \Phi_s \Omega_T^2]_{ij}$$

$$+ \sum_{i=1}^{m}\sum_{j=1}^{m} \Lambda_{o_{ij}} [\Phi_s]^T [K_s][\Phi_s] - \Omega_T^2] + \sum_{i=1}^{n}\sum_{j=1}^{n} \Lambda_{s_{ij}} [K_s - K_s^T]_{ij}$$

Differentiating and setting the result to zero produces

$$[K_s] = [K_c] + [\Delta + \Delta^T] \quad \text{where}$$

$$[\Delta] = 1/2 [M_s \Phi_s [\Phi_s^T K_c \Phi_s + \Omega_T^2] \Phi_s^T M_s - K_c \Phi_s \Phi_s^T M_s].$$

The resulting synthesized $[M_s]$ and $[K_s]$ satisfy all constraints and the increments in change can be tabulated element by element with respect to the original computed $[M_c]$ and $[K_c]$.

Critique of Matrix Improvement-

If the only object were to provide a systems analyst with a matrix that could act for a given component structure for the dynamic behavior of a total complex, this method would have good applications. Many times the need is for more than providing a surrogate, but to provide corrections to an existing model such that the resulting improved model will properly predict stresses and internal load paths and deformation behavior in the data recovery process after the results of the systems response is obtained. The interpretation of the incremental changes to the physical model is sometimes impossible, so that in spite of having an improvement it will not serve as a physical guide to model correction. With my limited experience in this area the one suggestion that I might make is to impose a further constraint on the admissable terms for applying the corrections. Null terms of $[K_c]$ and $[M_c]$ should be forced to remain null. I found extensive coupling in the $[K_s]$ and $[M_s]$ terms that defied physical justification. I also feel that this technique is workable during the very early stages of comparative analysis. For instance, it might be applied to the mass only and be tested for physical meaning so that possibly by increments it would act as a guide.

The following is a tabulation of the items described above giving a precis of the operation involved, its utility and its demand for resources.

OPERATION	UTILITY	RESOURCES
Orthogonality	Gives net modal check on test modes or analysis mass by severity in a single simple test.	Preprocess test vectors, into DMI format, then calculation is by DMAP ALTER. In public domain.
Cross-Orthogonality	Gives net modal checks on both test & analysis in a single simple test.	Continues with DMAP ALTER for 2 more steps. In public domain.
Cross-Correlation	Gives quantitative measures of net correspondence between test & analytical modes.	DMAP ALTER followed by a post processor program. Short running. In public domain.
Differences	Gives measures of isolated differences between test and analysis modes. Relates directly to actual positions in a model. Scaled differences give greater spread of results near unity.	An option of cross-correlation program. Short running.
Synthesis. Modal Amplifiers.	Gives measure of how much an analytical mode can behave like a test mode.	DUMMOD available from Cosmic. High memory requirements.
Synthesis. Residuals.	Can give individual also cumulative differences in shapes; test vs. analysis.	An option of synthesis.
Synthesis. Rectified Mass weighted residual	Gives magnified differences Weighting can help distinguish importances.	An option of synthesis. Part of another calculation so is neglible computer time.
Synthesis. Simple Correlation.	Single number to measure one mode with another. Helps evaluate analysis vs analysis; test vs analysis; and test vs test. Helps check self consistency of analytical model.	Correlates without averaging. DUMMOD must be sysgened into NASTRAN executable. The code is Cosmic catalog number (TU 1/80)
Synthesis. Multiple Correlation.	Single number measure one mode at a time. Gives greater spread near 1.0.	Relates to average test value of mode. Distinct differencing operations.

Synthesis. T value.	Gives greater focus of analytical similarity to test at individual points.	Scaling is fast after calculation of standard error function.
Synthesis. Relative Difference X(I)	Refined localized variation	It must canvas spreads overall instrumented points and all modes.
Synthesis. Generalized Mass	Gives equivalent of orthogonality test and cross-orthogonality test in a single matrix.	Quite direct and efficient.
Matrix Improvement. Dilated Test Vector.	Can be used for comparison with analytical mode to check on assumption of whether small changes can correct computed matrices.	Expensive Decomposition. Proprietary. Available only as service.
Matrix Improvement. Mass increments.	If Eqns 5, 6 & 7 were constrained to maintain null values, the lesser coupling might be easier to relate to model. Could be used in early liaison with test.	Simple multiplication.
Matrix Improvement. Stiffness increments.	Would be useful if a way were found to process new increments through data recovery modules so as to give direct connection to individual model elements.	Simple multiplication.

APPLICATION

These are the tools. Plots, Orthogonality, Cross-Orthogonality, Cross- Correlation, Differences, Relative Differences, Scaled Differences, Synthesized Modal Amplifiers, Residuals, Rectified Residuals, Simple Correlation, Multiple Correlation, T value, Relative Difference X(I), Generalized Mass, Improved Vector Dilation, Improved Mass, Improved Stiffness. How and what should be applied when?

The situation is usually this. Analysis has gone on for quite a while and a test plan has been drawn up during design development. So, frequencies, shapes, and plots of analytical eigenvectors are at hand. Test has been set up and liaison has established the set of corresponding instrument locations. The situation with respect to Orthogonality is this. Generally the analytical model has been condensed down to a logical A-set and not to the instrumented set. As soon as test liaison is established, the A-set should be modified to

include the complete set of instrumented freedoms as a sub-set. When
this is done DMAP ALTER's should be considered for applying a second
Guyan reduction for condensing the MAA matrix to MII size.

It makes good sense then to apply correlation as a first step
after test results start to come in. DMAP ALTER packet, as defined in
either TM 86081 or TM 86044, can conveniently be included in NASTRAN
runs to create a TESET vector and to have modes partitioned to PHITE
freedoms. This permits the identification of which analytical modes
correspond to which test modes. It is a shock when correlation
results are viewed for the first time. One has a stereotyped notion
that there will be a few values in the .90 to .98 range and a cluster
of values in the 0.0 to .08 range. The predominance of values in the
.4 to .8 range takes one aback. The first impulse is to condemn
correlation as being useless. It takes a fair amount of study to
begin to realize the implications that are revealed by this plethora
of data. Nothing is clear cut. Develop judgement as to relative
magnitudes and remedies needed to home in on the anomolous parts of
the model. If one analytical mode correlation coefficient $\approx$1.0 and
others are high, this can imply the one near unity is a match and the
other modes with large coefficients (say > 0.4) have defects and
should be flagged for location as to where model should be investigated.
I have yet to talk to any structures man who considers himself to be
an expert in assessing correlation results. One needs to develop
experience by making interpretations; taking actions based on the
initial interpretations; then revise the original interpretation by
reacting to results produced by actions. I have never used synthesis,
but I would expect that multiple correlation will help to isolate some
effects. Test and analysis people should look over the correlation
results together to see what is revealed. For instance, look for
frequency disparities in the modes and check secondary correlation
results for finding anomolous local involvements that might be
corrected. Each discipline can then ask its own set of questions,
such as

Analysis   Test

Are any moments of inertia wrong?   Is the structure being excited in
Do any joints need to be remodeled?   a poor place?
Is there a wrong exponent in a   Are instruments reading in the
modulus of elasticity?   right amplitude range?
Do any BAR elements have misplaced   Is the structure being supported
offsets?   improperly?
Are any of the modes spurious due   Do pick-ups need to be remounted?
to inadequate constraints?   Are any modes not being excited?

After test and analysis have applied remedies based on the first
correlation results, another correlation check ought to be made based
on analytical and test reruns. When all the obvious adjustments have
been made after reruns, the orthogonality ALTER packet should be

included in a succeeding run in conjuction with a correlation process. If one uses orthogonality alone, information is too condensed to home in on discrepencies. With correlation, specific locations can be obtained for applying remedies. The two approaches can be included in a single run to take advantage of simultaneous data. Note should be made, immediately, as to whether a difference in correlation resulted from condensing the analytical model to the instumented points. If there is a great difference then no particular meaning can be gleaned from the orthogonality results. If the shift in correlation is acceptable, the orthogonality and cross-orthogonality results will show which modes are within threshold specification, and how far other modes are out of specification.

SUMMARY

Good tools for comparing vibration data from test and analysis are available in the public domain. The Goddard package is easy to get and quick to run. The Rockwell package is the best. It takes planning to get it operational. The Kaman service can be used as a guide or a position of last resort. In all cases, it takes much practice to use these tools well.

REFERENCES

(1)   Internal OAO Report No. 4-85

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