A COMPARISION OF NASTRAN (COSMIC) AND EXPERIMENTAL RESULTS FOR THE VIBRATION OF THICK OPEN CYLINDRICAL CANTILEVERED SHELLS*

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SUMMARY

The natural frequencies and associated mode shapes for three thick open cantilevered cylindrical shells were determined both numerically and experimentally. The shells ranged in size from moderately to very thick with length to thickness ratios of 16, 8 and 5.6, the independent dimension being the shell thickness. The shell geometry is characterized by a circumferential angle of 142 degrees and a ratio of length to inner radii arc length near 1.0. The finite element analysis was performed using NASTRAN's (COSMIC) triangular plate bending element CTRIA2, which includes membrane effects. The experimental results were obtained through holographic interferometry which enables one to determine the resonant frequencies as well as mode shapes from photographs of time-averaged holograms.

In all, comparison between experimental and computational results were obtained for a total of 22 cases. In more than 77 percent of the cases the agreement was within 5 percent and for 45 percent of the cases within 2 percent. The largest percent error in frequency occured for all three shells in the first flexural mode, with 8.0, 18.8 and 20.1 percent errors with increasing thickness. There was also a 12.7 percent error in the second flexural mode for the thickest blade. In other cases, the differences between the computed and experimental results did not appear to be a result of changes in shell thickness. A contributing factor to the large error in the flexural modes is the difficulty in providing a true clamped end condition as the shell gets thicker, resulting in lower experimentally determined frequencies.

*Support for this work was provided by the Aero-Propulsion Laboratory, AFWAL/PO, Wright-Patterson Air Force Base.
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INTRODUCTION

The free vibration frequencies and mode shapes were determined both numerically and experimentally for three thick open cylindrical cantilevered shells. The numerical results were computed using NASTRAN's (COSMIC) triangular plate bending element CTRIA2 which includes membrane effects. The experimental results were found using holographic interferometry. The shell dimensions are characterized by length to thickness ratios of 16, 8 and 5.6, with the independent dimensions being the shell thickness.

A review of the literature reveals no results for shells with the dimensional characteristics, ie. short, stubby, thick and nonshallow, of the ones considered in this study. However, the vibration characteristics of cantilevered shallow cylindrical shell segments has been addressed. Walker [1] developed a doubly curved right helicoidal shell finite element which he applied to several thin shallow shells. Gill and Ucmaklioglu [2] applied a three-dimensional isoparametric element to a curved fan blade. Their results were compared with experimental results as well as other finite element solutions. Leissa, Lee and Wang [3] used shallow shell theory and the Ritz Method to solve a range of cantilevered shell problems for a range of aspect, shallowness and thickness ratios. In a recent paper, Lee, Leissa and Wang [4] applied the procedure developed in [3] to the problem of cylindrical shell segments with chordwise taper.

A similar type of investigation was reported in a series of papers by MacBain, Kielb and Leissa [5,6]. Their study considered the vibration of twisted cantilever plates with rectangular planform. The characteristic dimensions for two of their cases are similar to those presented here. The theoretical results included 15 finite element, 2 shell theory and 2 beam theory solutions as well as 3-D elasticity solutions for 2 of the cases. The finite element results included solutions using the NASTRAN (COSMIC) CTRIA2 triangular plate element and the NASTRAN (MSC) CQUAD4 quadrilateral shell element.

EXPERIMENTAL INVESTIGATION

The three cylindrical shells chosen for this study were machined from 50.80 mm diameter steel rods of length 61.91 mm. The shells are only 25.40 mm in length, leaving the remaining portion of the rod to slide into the steel block as shown in Figure 1. The shells, labelled 1, 2 and 3, have the same inner radius of 11.43 mm and an arc length of 142 degrees with thicknesses of 1.59 mm, 3.18 mm and 4.57 mm, respectively. The holes shown on the base of the shell were drilled around the circumference of the cylindrical base at 90 degree intervals. Their purpose being to seat setscrews used in mounting and to provide a consistent way of orienting the shells.

The excitation of the shells was accomplished with a piezoelectric shaker mounted to a steel block of dimensions 76.20 mm x 76.20 mm x 101.60 mm as shown in Figure 1. The block was secured to an optical table by four bolts, one at each corner of the block. A 50.8 mm diameter hole was bored through the top of the block to a sufficient depth to accomodate the base of each shell. Setscrews were then used to secure the shell in place. The total mass driven by the shaker was 232, 109 and 72 times the mass of the shells.

Real time holographic interferometry was used to find the resonant frequencies and mode shapes of the shells. Figure 2 shows the optical setup chosen for the investigation. A description of the procedure can be found in [7].

EXPERIMENTAL RESULTS

The search for the resonant mode shapes was accomplished with an accelerometer attached to the shells. This was done to insure that the observed response was at the first fundamental mode of the excitation, rather than at a harmonic component. Because the accelerometer introduces an added mass (0.35 grams), once the resonant frequencies were identified the accelerometer was removed and the shells retested. As expected, these final frequencies were slightly higher. The accelerometer caused no discernable changes in the mode shapes themselves.

Experimental results for the three shells without an accelerometer are presented in Figures 3-5 in the form of photographs of the time averaged holograms of the concave side of the shells. Resonant modes in the frequency range of 0 to 100 KHZ were sought, but were not found above 45 KHZ. The contour lines on the holographic images represent out-of-plane displacements. Which of the white areas are nodal lines or zones of zero displacement can be deduced from the fact that the lower edge is fixed. The magnitude of the displacement increases as fringes are crossed, moving away from a nodal line. The time averaged hologram does not distinguish between positive and negative displacements, although the relative direction can be determined by using the nodal lines to identify lines of zero displacement and noting that the displacement must have continuous derivatives across those lines.

Upon viewing the photographs of the shells, it is apparent that the far left interior portion of each shell is not visible. This is due to the curvature of the shells and the fact that it was not possible to position the object beam so that this area could be illuminated. The photographs also indicate that the mode shapes are biased toward one side. Since the shells are symmetric about a vertical plane perpendicular to the plane of the paper, the modes should be totally symmetric or anti-symmetric. The reason that the modes are biased is most likely a slight asymmetry in the loading which can be attributed to imperfect clamping and/or misalignment of the shaker.

The first ten mode shapes for Shell 1, the thinnest of the shells, are presented in Figure 3. The frequency for each mode is also given in the figure. Among the ten modes are the first and second torsional modes (1T,2T) and the first flexural mode (1F). Modes at 10,973 HZ and 24,835 HZ represent chordwise bending. The mode at 22,770 HZ appears to be related to the first of these in that it has two veritcal nodal lines with the addition of a second horizontal nodal line. Modes at 24,835 HZ and 39,627 HZ are similarly related. The mode at 23,685 HZ represents an in-plane shear mode. The mode at 36,060 HZ defines a diaphragm type displacement and the mode at 44,300 HZ consists of many localized displacements along the free edges of the shell.

The six modes found for Shell 2 are presented in Figure 4. They include modes 1T, 1F, 2T, the two chordwise bending modes and an in-plane shear mode at 22,965 HZ. The meeting of the fringe pattern for the 1T mode at the line of symmetry means that the mode is not one of pure torsion. Some other component of displacement is also present. A rigid body rotation, caused by a slight looseness in clamping, is suggested. The fact that many less fringes were obtained for each mode of Shell 2 than for Shell 1 or Shell 3 suggests higher energy dissipation, as might be caused by slipping at a less than perfectly clamped end. The 1F mode might be expected to be especially vulnerable to a less than ideal end condition.

Figure 5 shows the six mode shapes found for Shell 3, the thickest of the shells. The observed modes include 1T, 2T, 1F, 2F, a symmetric chordwise bending mode, and an in-plane shear mode at 24,232 HZ. For this blade the lowest mode is the first flexural mode, whereas for the other two shells the fundamental mode was the first torsional. This is believed to be a consequence of the inability to create a completely clamped end. In this case the end may be well clamped, but clamped to a deforming object. This is the only blade for which the second flexural mode was seen.

NUMERICAL INVESTIGATION

A finite element analysis of the cylndrical shells was accomplished using NASTRAN's CTRIA2 element, which is a three node triangular plate bending element that includes membrane effects. The element mesh, Figure 6, used to model Shell 1 consists of 24 and 21 nodes along the circumference and height, respectively. The mesh was refined for Shells 2 and 3 by increasing the number of nodes along the circumference to 26 and 27, respectively. All the eigenmodes should be symmetric or anti-symmetric since the shells are symmetric. However, because of the inherent asymmetry of the triangular elements, the mesh used in the analysis introduced a slight bias in the deformation pattern.

The mode shapes are presented in the form of contour plots of the out-of-plane displacement for comparison with the holographic results. Figures 7-9 show the mode shapes for the three shells. The bottom edges in the figures are the clamped ends.

A comparison of the computed and observed frequencies is given in Tables 1,2 and 3. The modes are listed sequentially according to the numerical results, with the experimental values placed by the corresponding mode shape. Note that all of the modes observed in the experiment were duplicated numerically. Also, the greatest difference in frequency occurs for the 1F modes. This is attributed to imperfect clamping of the shells in the experiments, giving rise to lower frequencies. This is most noticable for Shells 2 and 3, where the differences between computed and observed frequencies are around 20 percent. In general the agreement was very good. For Shells 1 and 2 the errors in the frequency for modes other than the first are all under 5 percent.

CONCLUDING REMARKS

The experimental procedure consisted of a test fixture which provided (approximately) a clamped end condition. The shells were excited by a small shaker exciting them in a direction perpendicular to the planform. The resonant frequencies and mode shapes were then determined using holographic interferometry.

The shells were modelled using NASTRAN's triangular plate bending element (CTRIA2) which includes membrane effects. The resulting mode shapes agree well with the mode shapes obtained by holographic means. The computed and measured frequencies were in very good aggreement, except for the flexural modes. This reflects the difficulty with experimentally providing a perfectly clamped boundary condition.

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Table 1. Comparison of NASTRAN and Experimental Frequencies For Shell 1.

MODE NUMBER	FREQUENCY (HZ)		PERCENT ERROR
	NASTRAN	EXPERIMENT	
1	5,265	5,088	3.5
2	8,609	7,972	8.0
3	11,428	10,973	4.2
4	20,959	20,662	1.4
5	23,017	22,770	1.1
6	23,244	23,685	-1.9
7	25,257	24,835	1.7
8	37,827	36,060	4.9
9	39,892	39,627	0.7
10	45,693	44,300	3.1
11	47,060	------	---
12	48,420	------	---
13	50,922	------	---
14	56,797	------	---

Table 2. Comparison of NASTRAN and Experimental Frequencies For Shell 2.

MODE NUMBER	FREQUENCY (HZ)		PERCENT ERROR
	NASTRAN	EXPERIMENT	
1	7,799	7,948	-1.9
2	10,262	8,635	18.8
3	17,633	18,135	-2.8
4	23,644	22,965	3.0
5	31,466	30,414	3.5
6	35,497	------	---
7	41,331	41,227	0.3
8	45,020	------	---
9	50,950	------	---
10	56,636	------	---
11	62,520	------	---
12	72,007	------	---
13	73,613	------	---
14	77,978	------	---

Table 3. Comparison of NASTRAN and Experimental Frequencies For Shell 3.

MODE NUMBER	FREQUENCY (HZ)		PERCENT ERROR
	NASTRAN	EXPERIMENT	
1	9,963	9,909	0.5
2	11,207	9,330	20.1
3	22,363	22,694	-1.5
4	24,255	24,232	0.1
5	39,365	36,418	8.1
6	42,488	37,690	12.7
7	50,993	------	---
8	51,163	------	---
9	52,850	------	---
10	56,610	------	---
11	75,290	------	---

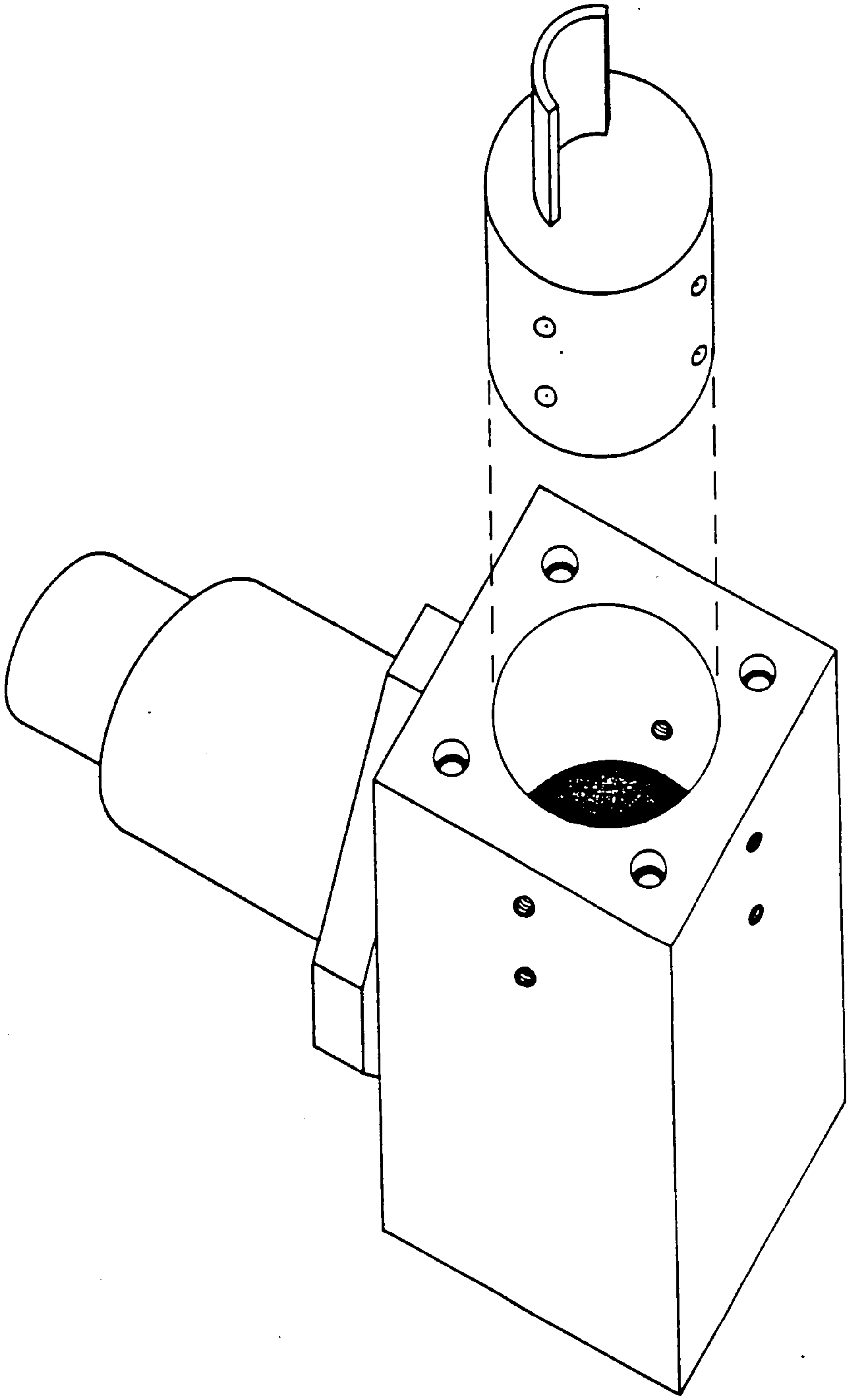


Figure 1. Vibration Test Fixture and Shell Assembly

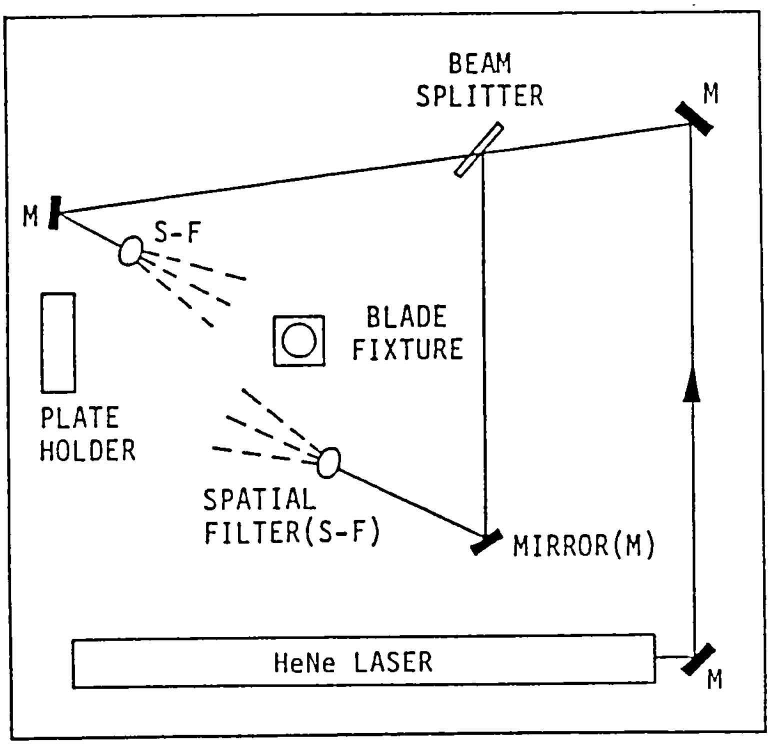


Figure 2. Optical Table Set-Up For Holographic Vibration Tests

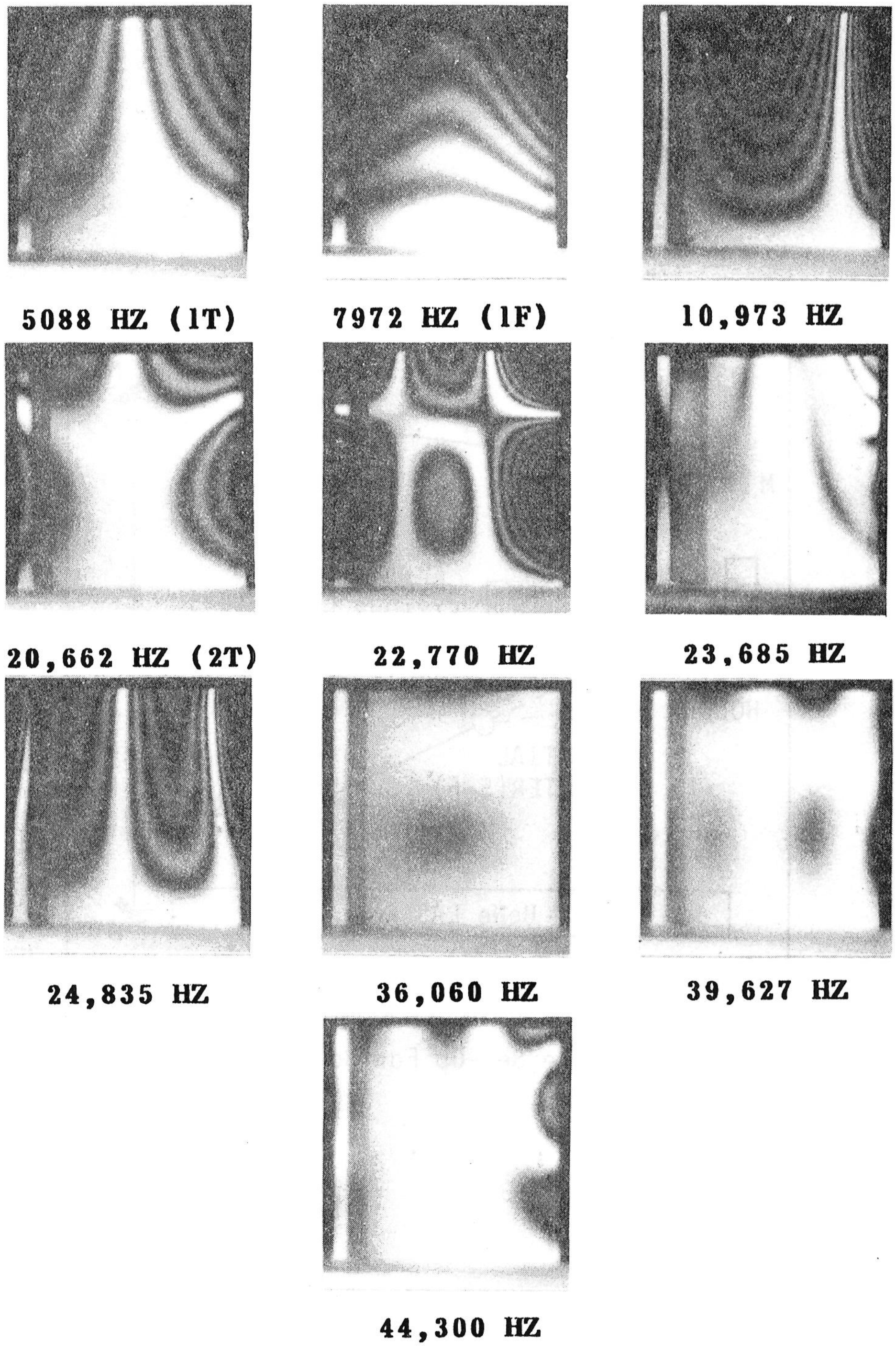


Figure 3. Holographic Vibration Test Results For Shell 1

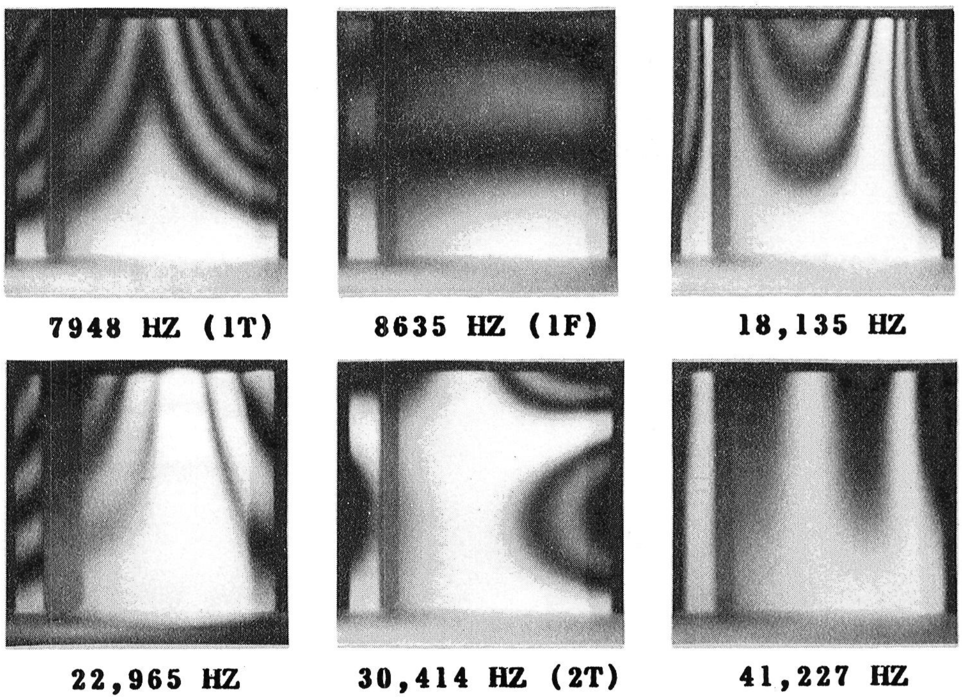


Figure 4. Holographic Vibration Test Results For Shell 2

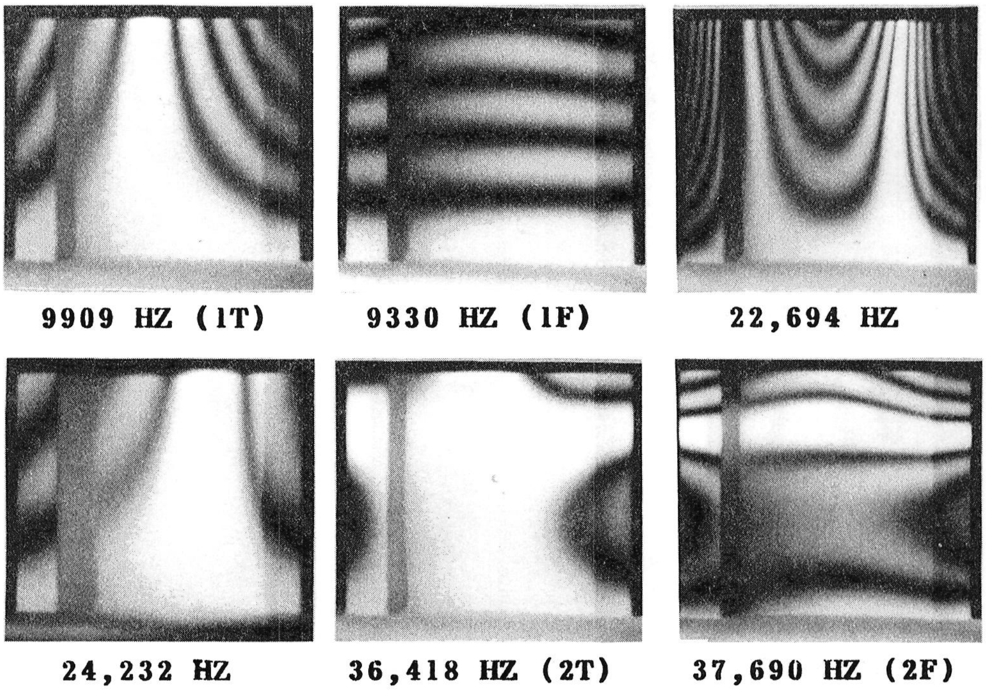
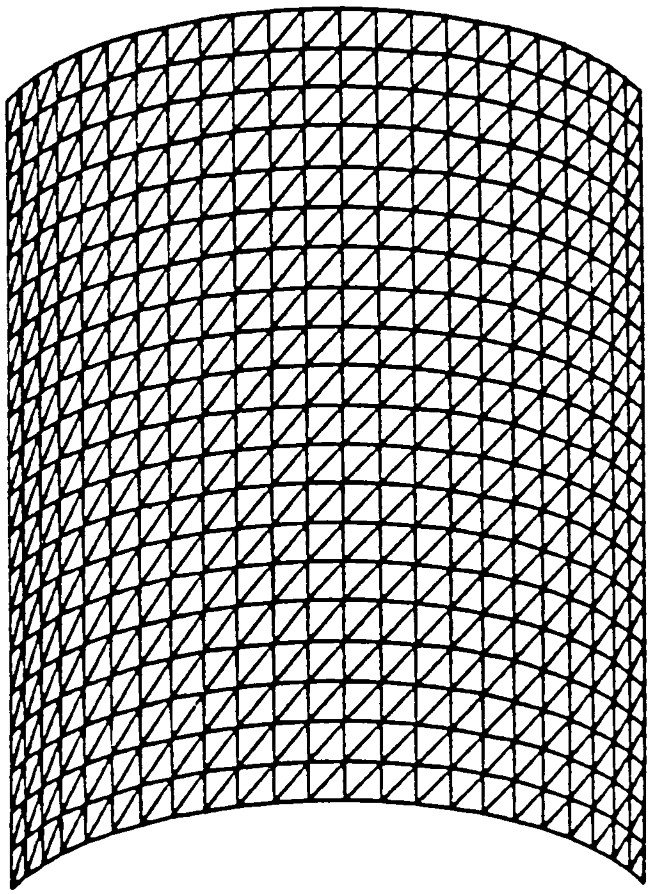


Figure 5. Holographic Vibration Test Results For Shell 3



MASS DENSITY: 7.83×10^3 KG/M^2 (7.32×10^{-4} (LB-SEC^2)/IN^4)
POISSON'S RATIO: 0.3
YOUNG'S MODULUS: 2.07×10^{11} N/M^2 (3.0×10^7 LB/IN^2)

Figure 6. Finite Element Mesh For Shell 1

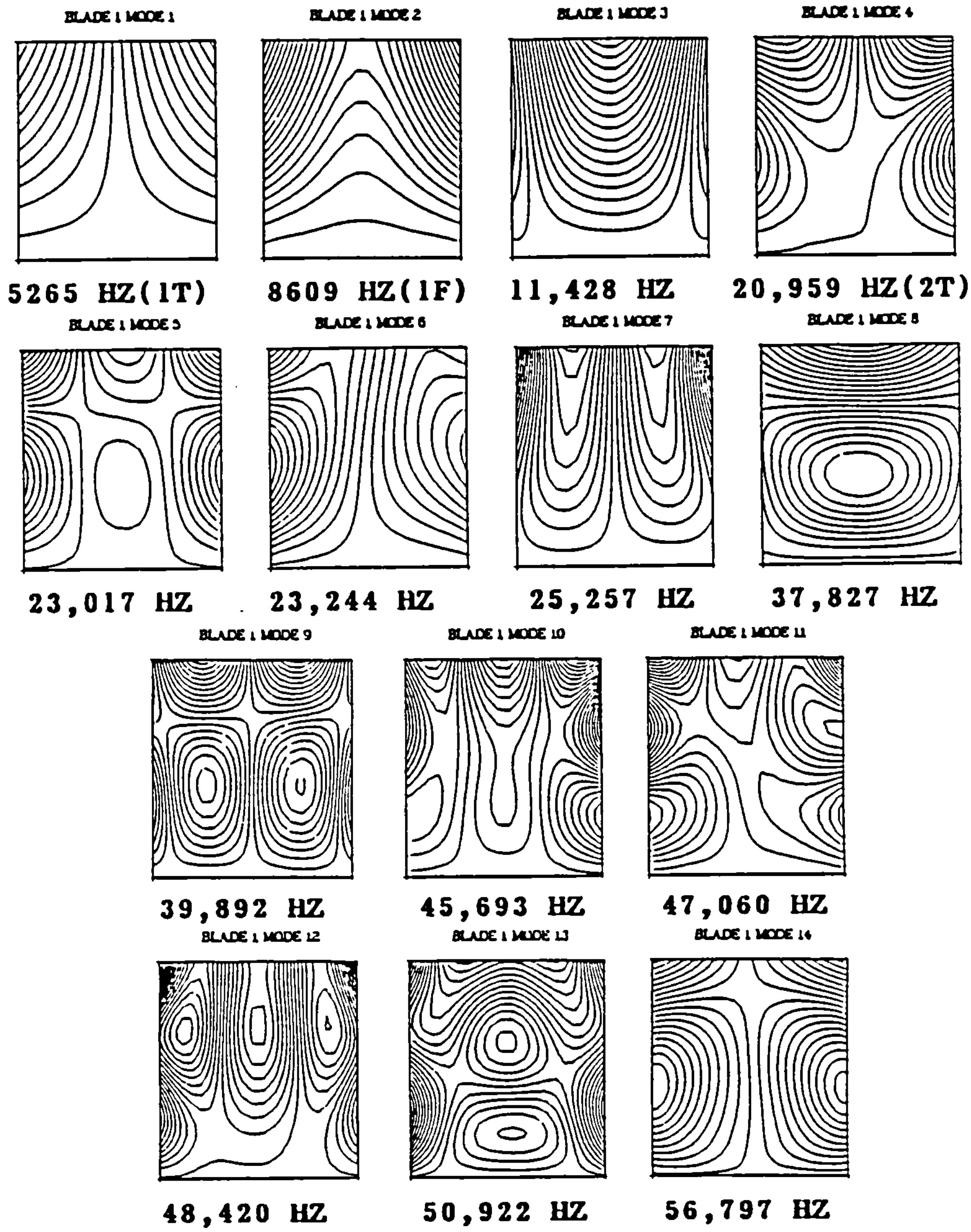


Figure 7. Resonant Mode Shapes From NASTRAN Solution For Shell 1 in the Form of Contour Plots of the Out-of-Plane Displacements

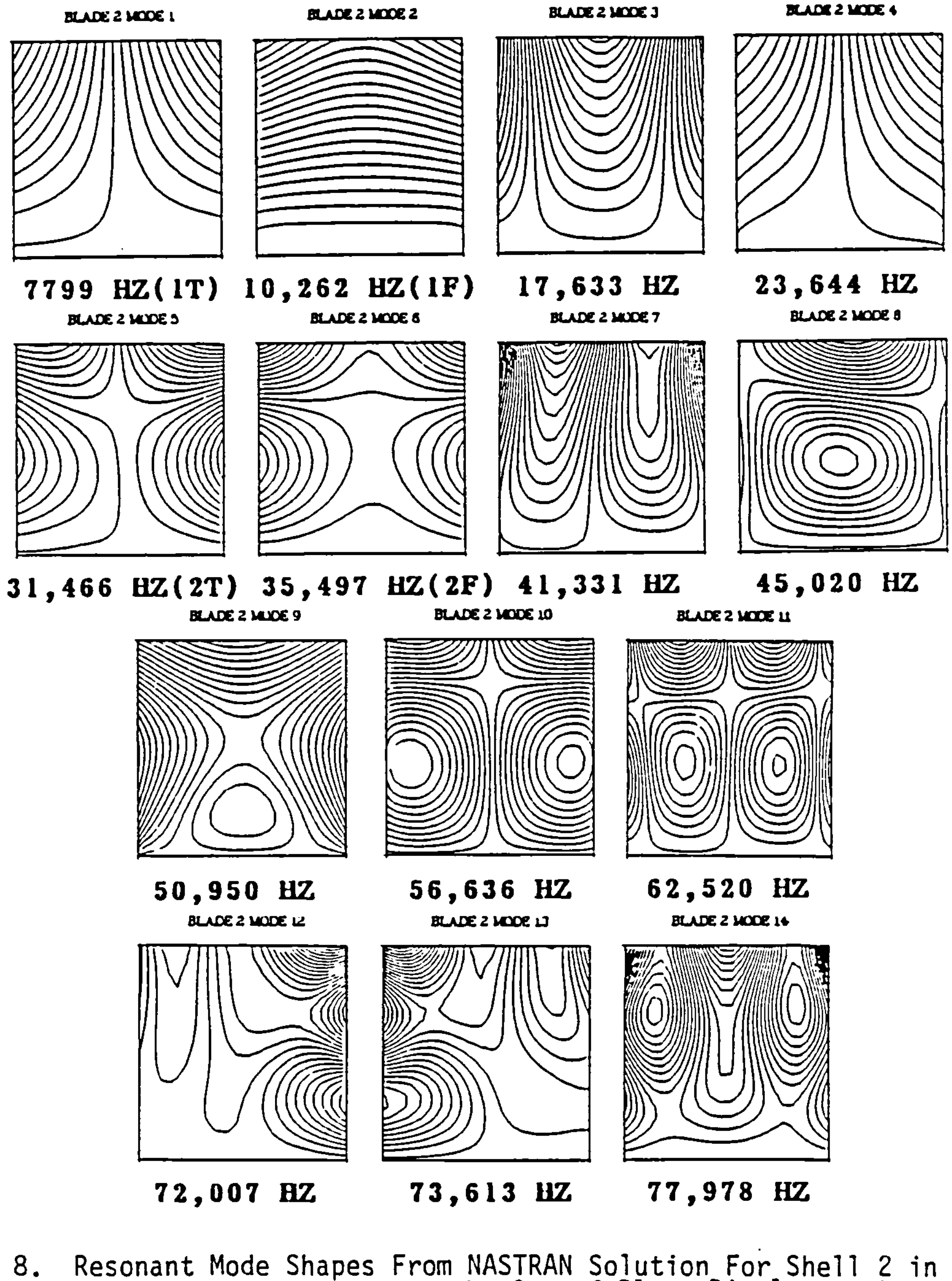


Figure 8. Resonant Mode Shapes From NASTRAN Solution For Shell 2 in the Form of Contour Plots of the Out-of-Plane Displacements

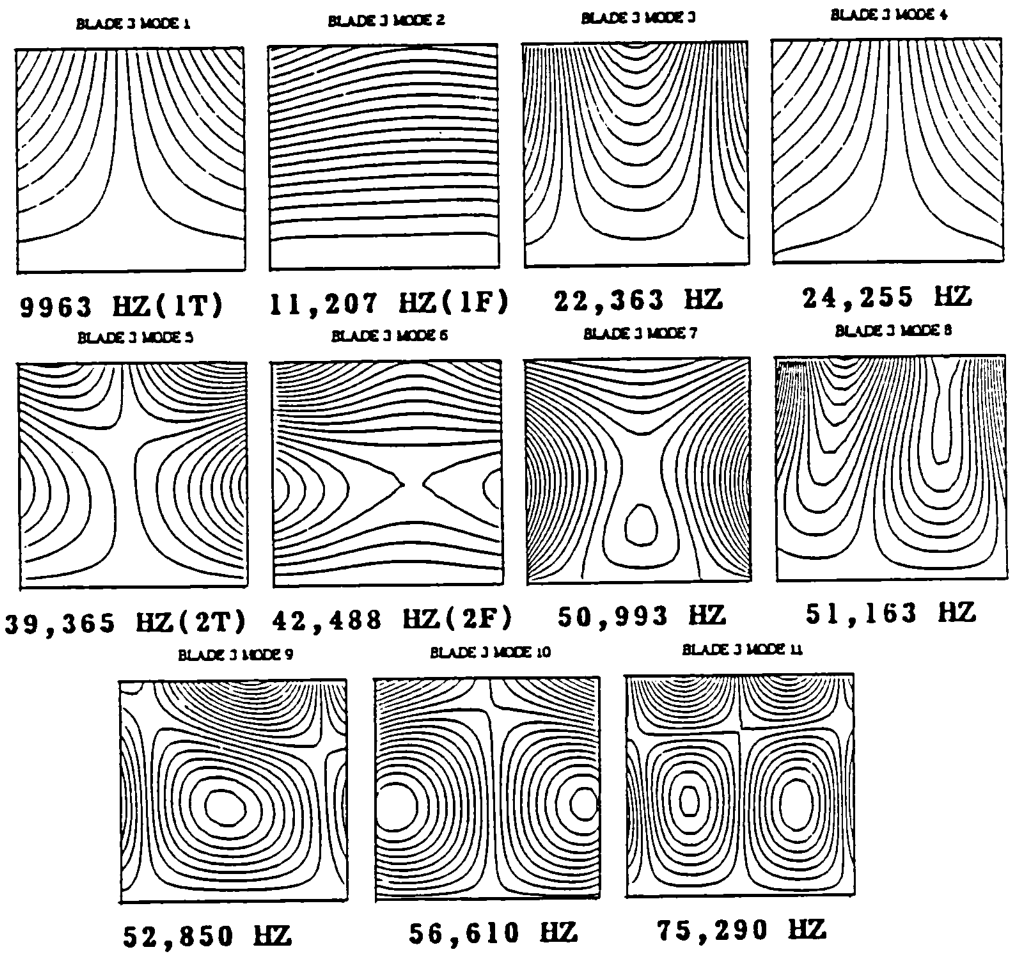


Figure 9. Resonant Mode Shapes From NASTRAN Solution For Shell 3 in the Form of Contour Plots of the Out-of-Plane Displacements