FINITE ELEMENT MODELLING OF NON-LINEAR MAGNETIC CIRCUITS USING COSMIC NASTRAN

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ABSTRACT

The general purpose Finite Element program COSMIC NASTRAN currently has the ability to model magnetic circuits with constant permeablilities. An approach has been developed which, through small modifications to the program, allows modelling of non-linear magnetic devices including soft magnetic materials, permanent magnets and coils. Use of the NASTRAN code results in output which can be used for subsequent mechanical analysis using a variation of the same computer model. Test problems have been found to produce theoretically verifiable results.

INTRODUCTION

Several computer programs exist for the modelling of Magnetic Scalar or Vector Potential by the Finite Element Method [1,2,3], although most are not well-suited for applications to magneto-mechanical design. The close analogy between the equations of Steady-State Heat Transfer and Magnetostatics has been noted [4,5] and for the linear (constant permeability) case it has been shown that NASTRAN's Heat Transfer capabilities produce theoretically verifiable solutions to Magnetostatic problems. Several features have already been added to NASTRAN to take advantage of this [6]. The analogy between the equations of Heat Transfer and Magnetostatics are not exact, however, in the non-linear case, and existing Rigid Formats cannot be used. In this paper a method is described wherein, using DMAP ALTER statements and new NASTRAN modules, non-linear Magnetostatic problems are solved iteratively.

THEORY

There are several formulations of Magnetostatic equations. The most appropriate for this analysis is also the most familiar:

$$B = \mu . H \qquad (1)$$

where B is the Magnetic Flux Density, H is the Magnetic Field Strength and μ the permeability. H is the Magnetic Scalar Potential Gradient where V is the Magnetic Potential

$$H = - \ grad(V) \qquad (2)$$

With this formulation the analogy with Static Heat Transfer is

apparent

$$B = -\mu. \text{ grad}(V) \tag{3}$$

$$\dot{q} = -k. \text{ grad}(T) \tag{4}$$

(where $\dot{q}$ is the normalized heat flow, k the Thermal Conductivity and T the Temperature).

By use of the Thermal analogues of the terms in (3) linear Magnetostatic problems can be solved for V, and the derived quantities B and H obtained by differentiation using the NASTRAN DMAP sequence for Static Heat Transfer Analysis. Table (1) shows the analogies and differences between the two cases. In the non-linear case the permeability, μ, is not constant and varies not as a function of potential, but of potential gradient

$$B = = - \mu \text{ (grad}(V)) \text{ .grad}(V) \tag{5}$$

Problems of this type are solved iteratively; initial values are assigned to μ and a solution obtained in V. The derived quantity H is used to assign new permeability values to each element of the model from a reference table of B vs. H and the process is repeated until the desired degree of convergence is obtained. This has been done by a modification to the Static Heat Transfer Analysis DMAP sequence of NASTRAN and use of two new modules. It is noted that the Nonlinear Static Heat Transfer Analysis DMAP is less suitable as the iteration is carried out in the modules rather than the DMAP listing, and the non-linear cases are not analogous since k depends on T rather than grad(T)

$$\dot{q} = - k \text{ (T) . grad}(T) \tag{6}$$

IMPLEMENTATION

The Static Heat Transfer Analysis DMAP sequence [7] can be considered to have three segments: (1) Matrix Formulation , (2) Matrix Solution, (3) Result Interpretation. In order to minimize execution time in an iterative modification of the Rigid Format it is required to repeat as little of segment (1) as possible. The iterative process requires that, as new permeability values are obtained for each element, the Global Stiffness Matrix (HKGG) be updated. HKGG is not ordered by element but is generated from the element-ordered Element Stiffness Matrix (HKELM). HKELM is generated immediately prior to HKGG in the DMAP sequence. The effect of changes in permeability can be applied to HKELM by multipling all element records in HKELM by the ratio of old and new permeabilities, after which HKGG is reformulated by a linear combination of terms from HKELM. The bulk of the Matrix Formulation operation are eliminated. This reduces execution times by approximately 40%. In practice it is convenient to give unit permeability (conductivity) values to all material in the Bulk Data File and this create a reference HKELM with unit properties. This file is used by the dummy module MODA to generate an initial

HKELM using data from an external file. After a solution is obtained the module MODC obtains new permeablilty values and creates a new HKELM. The program then loops to the statement forming the HKELM block. Fig(1) shows the sequence and Fig.(2) is a listing of the required DMAP alter statements.

ITERATION METHOD

Successive iterations are performed with new permeability values obtained from linear interpolation of a table of B vs. H for each material type. After a solution is found and H calculated the corresponding value of B is obtained and μ calculated for the next iteration. To avoid instability a damping coefficient of 0.05 to 0.10 is applied in the case of soft materials and of 0.75 to 0.90 for permanent magnets. The large factor is necessary in permanent magnets as, in certain conditions, μ tends to infinity. This condition is unlikely to be a valid physical solution but the large damping factor is required to prevent the iterative process from overshooting the correct solution and approaching the condition. Fig.(3) shows a generalized Magnetic Hysteresis curve. The broken line is an initial magnetization curve while the solid line is the Hysteresis loop. The permeability anywhere on the line is the value of B/H. In the second and fourth quadrants where B/H < 0 the value is refered to as B/H rather than μ. In a soft material such as iron values of μ are very large and the coercive force Hc as shown in Fig.(3) is very small. In this case a curve such as Fig.(4) adequately models the material. Fig. (5) shows the second quadrant of a permanent magnet hysteresis curve. This is refered to as a "demagnetization curve" as the magnet is being demagnetized by a negative value of H, and B/H is negative. It is possible to operate a permanent magnet in the first quadrant, but for it to fulfill the purpose of a magnet (ie to produce flux) it must operate in the second or fourth quadrant. The second and fourth quadrants are physically indistinguishable, and the algorithms used for soft materials are also usable for the fourth quadrant of the Magnetization curve, so data on permanent magnets are entered as positive H values and negative B values as in Fig.(6). Materials enclosed by coils may be considered to be subject to an additional magnetizing force which shifts the axis of the Magnetization curve in one direction or other as in Fig.(7). In either case the result is that the Magnetization curve looks like that of a permanent magnet, and the coil may be modelled as such.

VERIFICATION

For verification purposes a simple model on a plate of material in air subect to an external field or potential difference was used. More complex models are not verifiable analytically for realistic material properties in the non-linear case. It has already been shown [4,5] that NASTRAN produces verifiable results for more complex geometries in the linear case, and the non-linear solution method is simply an iteration of linear solutions. In each case tested the solution has been checked for agreement with the equations of Magnetostatics. The

first model discussed here consists of two dissimilar soft magnetic materials in air, subject to an externally applied flux level of 1490 Gauss as in Fig. (8). The magnetic properties are listed in table (2). Convergence to the correct values of B in both materials occurs in about ten iterations with a 10 % damping coefficient as shown in Fig(9). In the absence of damping the iterations oscillate about the correct solution. The second model (Fig(10)) is of a permanent magnet in air subject to a fixed potential difference. Table (3) lists the demagnetization curve. In this case convergence occurs in six iterations with 90 % damping as shown in Fig. (11).

CONCLUSIONS

An iterative method has been demonstrated for the application of NASTRAN to non-linear magnetostatic problems. The method is shown to work for simple cases. Refinement is required in the modelling of anisotropic materials, and in the modelling of hysteresis effects by means of restarts with varying loads. The method as developed thus far is comparable with some specialized programs and has the advantage of commonality with the NASTRAN program and the inherent flexibility thereof.

REFERENCES

[1]: J. Simpkin and C. W. Trowbridge, IEE Proc., Vol. 127, 368-374 (1980)
[2]: J. L. Brauer, Research and Development, 71-73 (Dec. 1984)
[3]: M.V.K. Chari, A. Konrad, M.A. Palmo and J. D'Angelo, ,IEEE Trans. Magn., Vol MAG-18, 436-446, (1982)
[4]: F.E. Baker, S.H. Brown, J.L. Brauer and T.R. Gerhardt, Int. J. Num. Meth. Eng., Vol.19, 271-280 (1983)
[5]: T.W. McDaniel, R.B. Fernandez, R.R. Root and R.B. Anderson, Ibid, Vol.19, 725-737 (1983)
[6]: M. Hurwitz, NASTRAN users' manual, 1.15-1 - 1.15-9
[7]: NASTRAN users' manual, 3.17-1 - 3.17-10

TABLE 1:   ANALOGIES AND DIFFERENCES BETWEEN
HEAT TRANSFER AND MAGNETOSTATICS

HEAT TRANSFER QUANTITY		MAGNETOSTATIC QUANTITY	
k	Thermal conductivity	μ	Magnetic Permeability
$k = f(T)$		$\mu = f(H)$	
$\dot{q}$	Heat Flux per unit area	B	Magnetic Flux Density
grad (T)	Temperature Gradient	H	Magnetic Field Strength or Potential Gradient
T	Temperature	V	Magnetic Potential

TABLE (2): MAGNETIC PROPERTIES OF SOFT MATERIALS MODELLED

MATERIAL 1 = AIR : B = H

MATERIAL 2 = SILICON STEEL

H (OERSTEDS)	B (GAUSS)
0.0	0.0
0.1	1750.0
0.2	6600.0
0.3	12000.0
0.4	13000.0
0.5	13700.0
1.0	15400.0
10.0	17750.0
100.0	19250.0
1000.0	19500.0
2000.0	20500.0

MATERIAL 3 = SUPERMENDURE

H(OERSTEDS)	B (GAUSS)
0.00	0.0
0.01	4500.0
0.10	7200.0
0.50	7750.0
1.00	7800.0
10.00	7900.0
100.00	8000.0
200.00	8200.0
2000.00	10000.0

TABLE 3: MAGNETIC PROPERTIES OF PERMANENT MAGNET MODELLED

H (OERSTEDS)	B (GAUSS)
0.0	-800.0
200.0	-600.0
400.0	-300.0
500.0	0.0

FIG (1)

```
          ┌──────────────┐
          │   DMAP  1    │
          │  THRU  27    │
          └──────┬───────┘
                 │
          ┌──────┴───────┐
          │   OUTPUT2    │────▶ HKELM
          │   HKELM      │      REFERENCE
          └──────┬───────┘
                 │
          ┌──────┴───────┐         INITIAL
          │    MODA      │◀──── PERMEABILITIES
          │   HKELM      │         HKELM
          │   INITIAL    │◀──── REFERENCE
          └──────┬───────┘
                 │
          ┌──────┴───────┐
    ┌────▶│  LABEL  L1   │
    │     └──────┬───────┘
    │            │
    │     ┌──────┴───────┐
    │     │   DMAP  28   │
    │     │  THRU  79    │
    │     └──────┬───────┘
    │            │
    │     ┌──────┴───────┐◀──── HKELM  REF.
    │     │    MODC      │
    │     │              │◀──── PERMEABILITY
    │     │              │      TABLES
    │     │              │◀──── DAMPING
    │     └──────┬───────┘      COEFFICIENTS
    │            │
    │     ┌──────┴───────┐
    └─────│ REPT L1,NTIMES│
          └──────┬───────┘
                 │
          ┌──────┴───────┐
          │   DMAP  80   │
          │  THRU  96    │
          └──────────────┘
```
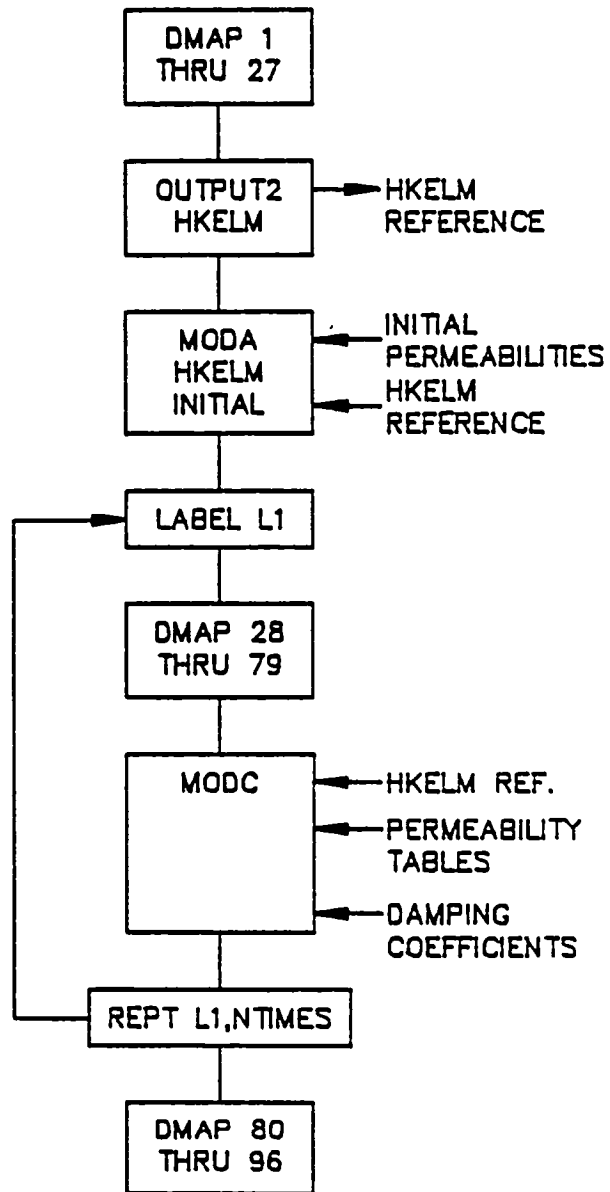
Fig.(2): NASTRAN DMAP ALTERS

```
NASTRAN TITLEOPT=-1
ID MAG1A,NASTRAN
APP HEAT
TIME 10000
SOL,1,1
ALTER 27
OUTPUT2 HEST,;,, // 0 / 18 $
OUTPUT2 HKELM,,,, // 0 / 15 $
MODA // -1 $
LABEL L1 $
INPUTT2 / HKELM,,,, / 0 / 19 $
ALTER 79
OUTPUT2 HOEF1,,,,// 0 / 14 $
MODC // -1 $
PURGE HKGG,GPST/HNOKGG $
EMA HGPECT,HKDICT,HKELM/HKGG,GPST $
REPT L1,2 $
ENDALTER
CEND
```

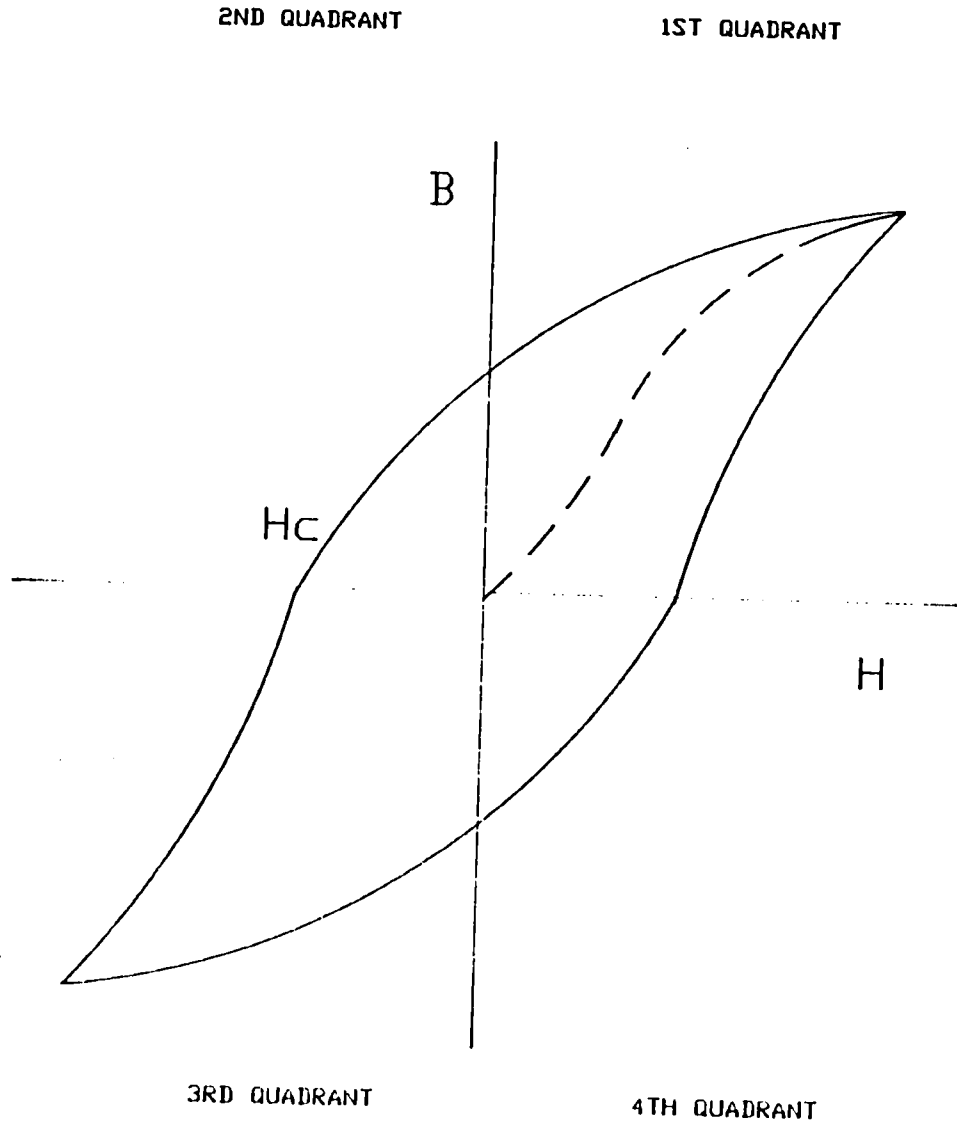# Fig.(3): MAGNETIC HYSTERESIS CURVE

2ND QUADRANT   1ST QUADRANT

B

Hc

H

3RD QUADRANT   4TH QUADRANT

Fig.(4): SOFT MATERIAL MAGNETIZATION CURVE
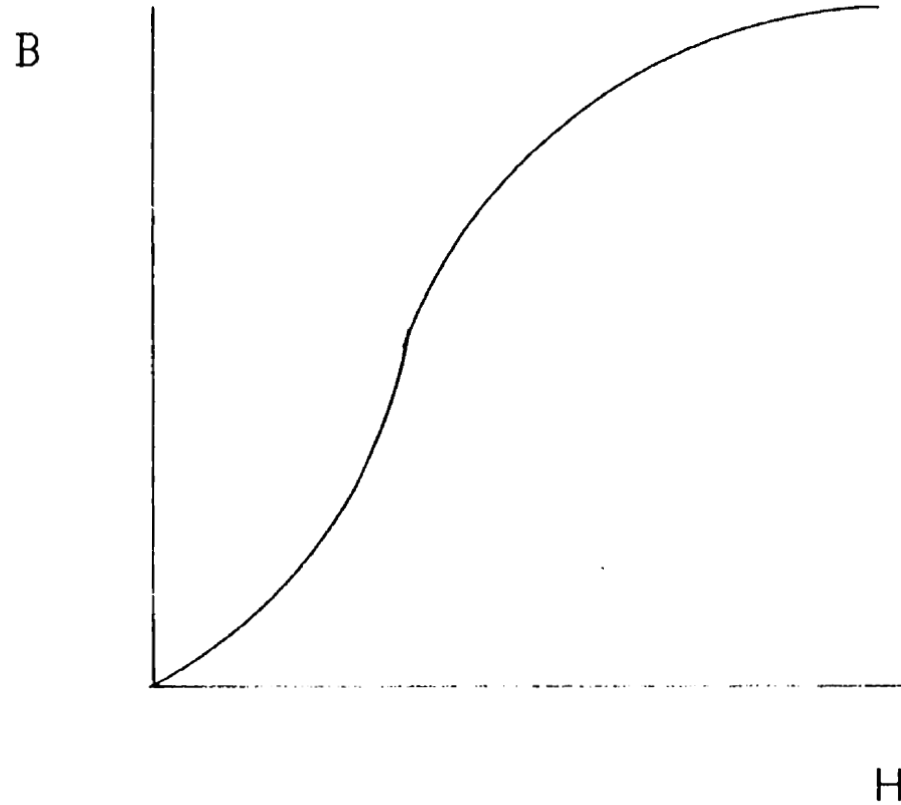
Fig.(5): PERMANENT MAGNET DEMAG. CURVE

MAGNETIC FLUX DENSITY B

DEMAGNETIZATION FIELD H
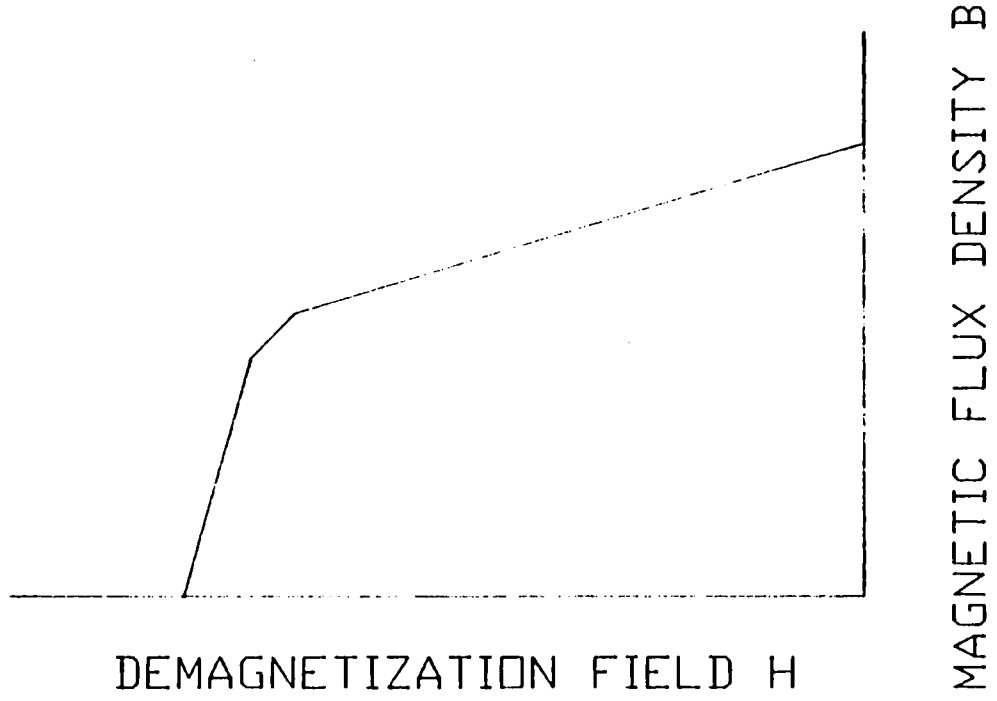
Fig.(6): FOURTH QUADRANT DEMAG. CURVE
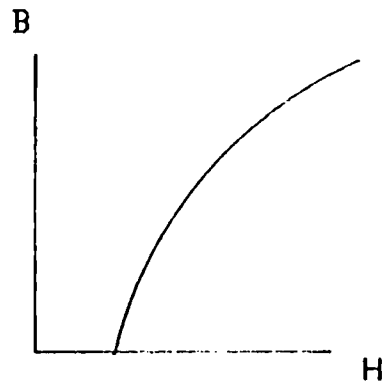
MAGNETIC POTENTIAL GRADIENT H

MAGNETIC FLUX DENSITY B
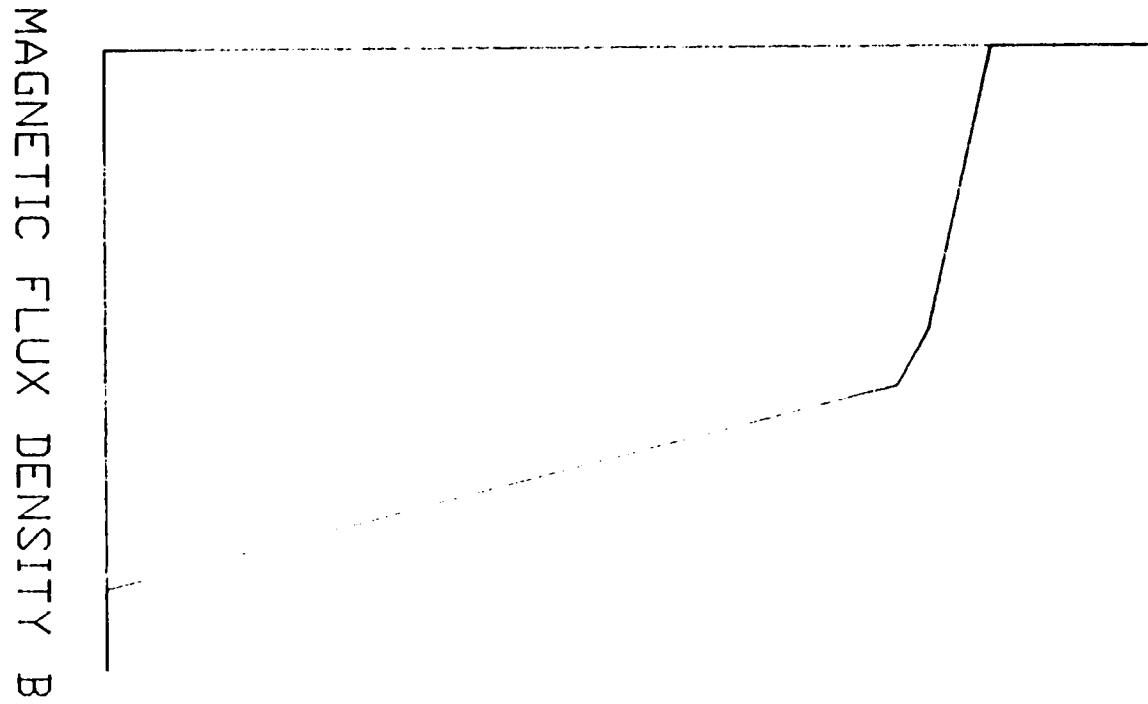
Fig.(7): EFFECTS OF COIL ON MAGNETIZATION
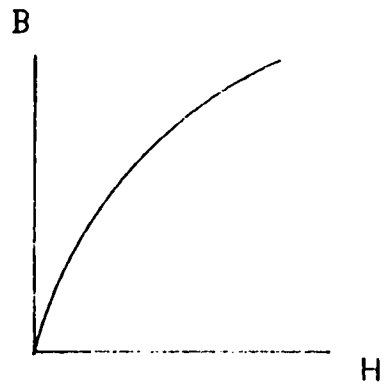
NATIVE CURVE

REVERSE BIAS

H = Hext − Hcoil
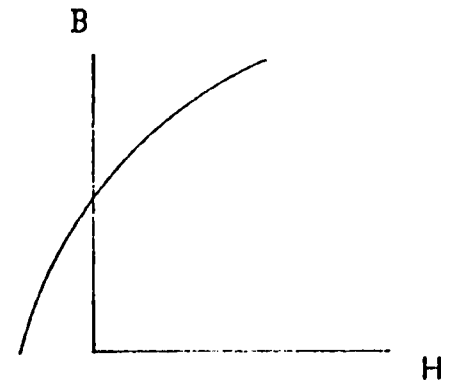
FORWARD BIAS

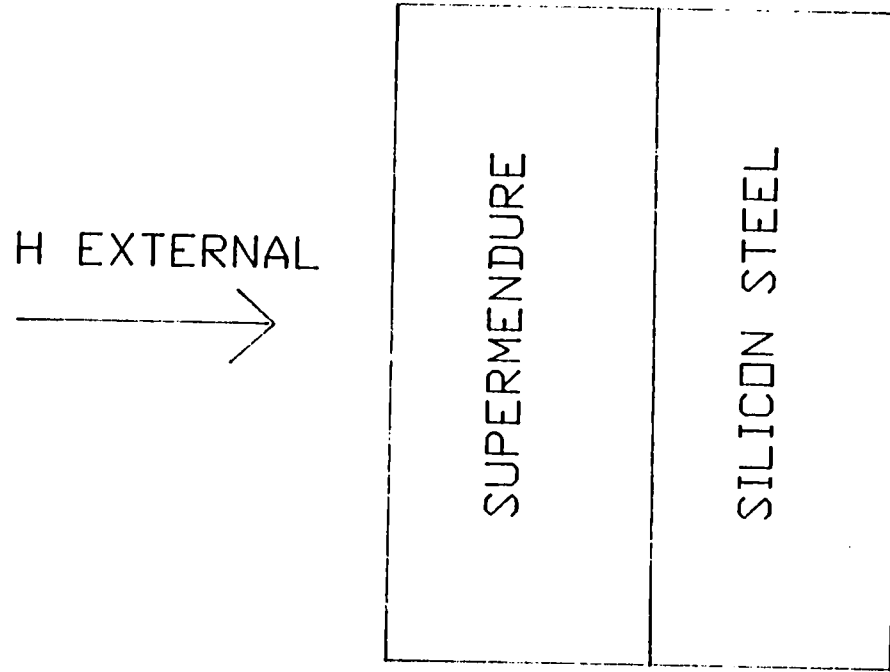H = Hext + Hcoil

Fig.(8): MODEL OF DISSIMILAR STEELS IN AIR

H EXTERNAL

$\longrightarrow$

SUPERMENDURE

SILICON STEEL

Fig.(9): Iteration of Soft Material

B (Gauss) (Thousands)

No. of Iterations

□   SILICON STEEL   +   PERMENDURE

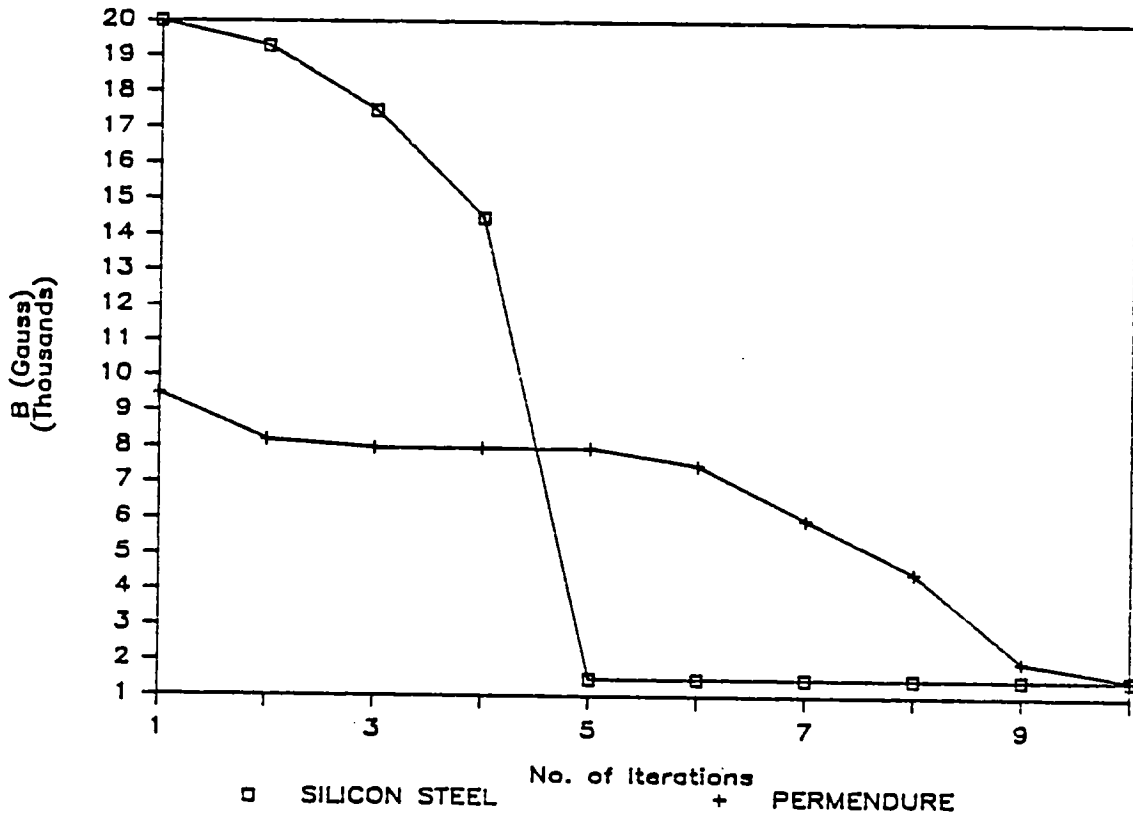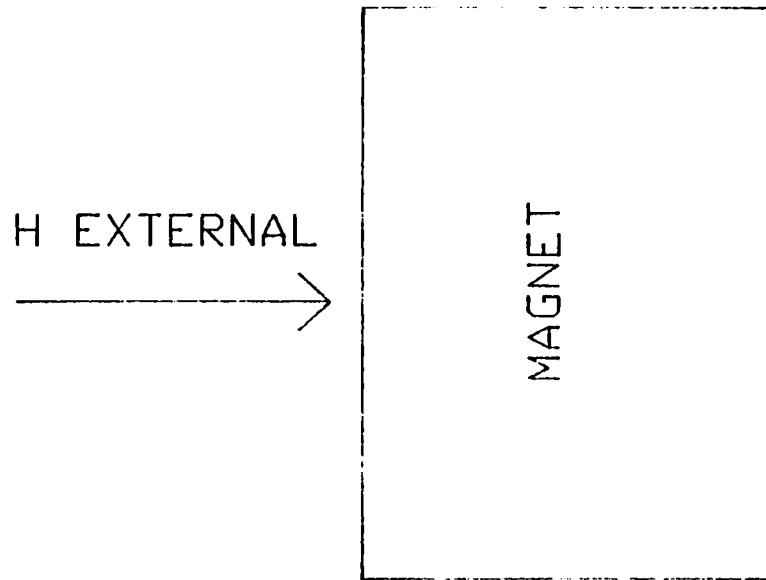Fig.(10): MODEL OF PERMANENT MAGNET IN AIR

H EXTERNAL

MAGNET

Fig.(11): Iteration of Magnet in Air