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SUMMARY

The Denavit-Hartenberg parameters define the relative location of joint axis systems in a robot arm. A recent criticism is that one of these parameters approaches infinity as the rotational axes of two successive joints approach a parallel condition. This causes an ill-conditioned transformation matrix and locates a joint axis system far away from the joint itself. This paper introduces a simple modification of these parameters to easily overcome this criticism. This modification (which entails the constraint that a transverse vector between successive joint rotational axes be normal to one of the rotational axes, instead of both) leads to modified Denavit-Hartenberg parameters that more favorably locate successive joint axis systems.

An example is given with respect to the rotational axes of the elbow and shoulder joints in a robot arm. The regular and modified Denavit-Hartenberg parameters that locate the axis system of the elbow joint relative to the axis system of the shoulder joint are extracted in the example by an algebraic method with simulated measurements of three different locations of a point on a robot arm. For small misalignments of the elbow- and shoulder-joint rotational axes from a parallel condition, the regular Denavit-Hartenberg parameters, unlike the modified parameters, were found to be extremely sensitive to measurement accuracy.

INTRODUCTION

The motion of a robot hand is the result of joint movements in the robot arm. To transform operator commands to the robot hand into joint movements and to pass sensor information along the arm, the relative location of successive joint axis systems must be known. By far, the most popular way to describe the relative location and orientation of one joint axis system with respect to another is to use the well-known Denavit-Hartenberg parameters (ref. 1), which define the elements in transformation matrices. However, these parameters have been criticized recently (refs. 2 and 3) as unsuitable in the case where successive rotational axes approach a parallel condition. Specifically, several of the elements in the transformation matrix approach infinity, and a joint axis system is located far away from the joint itself.

The purpose of the present paper is to modify the Denavit-Hartenberg parameters to overcome the aforementioned criticism. The algebraic method in reference 4 is extended to extract the modified parameters.

SYMBOLS

$\vec{A},\vec{B},\vec{C}$	measurement vectors from world coordinate system to a point on robot arm
A_{i-1}^{i}	homogeneous transformation matrix from coordinate system i to $i - 1$
a_i	length of $\vec{a}_i$
$\vec{a}_i$	common normal vector between Z_{i-1} and Z_i; intersects Z_i at O_i

$\vec{C}_i$ vector from world coordinate system to center of circular trajectory of a point on robot arm about line of rotation of joint i

$\vec{D}_i$ constrained transverse vector from Z_{i-1} to Z_i

E elbow of robot arm

$\vec{e}_i$ unit vector normal to both Z_{i-1} and Z_i that, by definition, points along X_i

H hand of robot arm

$\vec{h}_i$ location of O_i from O_{i-1} in coordinate system $i - 1$

i integer

$\vec{\ell}_i$ vector from world coordinate system to a point on line of rotation for joint i

N neck of robot arm

$\vec{N}$ vector normal to circular trajectory plane of point on robot arm

O base of robot arm

O_i, O_w origin of coordinate system i for joint $i + 1$ and of world coordinate system, respectively

$Q, Q_1, Q_2, Q(x,y,z)$ points in three-dimensional space

$\vec{R}_i$ vector from O_w to O_i

R^i_{i-1} rotational transformation matrix from coordinate system i to $i - 1$

r_i coordinate along Z_{i-1} of O_i

$\vec{r}_i$ vector from O_{i-1} to coordinate along Z_{i-1} of O_i

S shoulder of robot arm

$\vec{u}_i$ unit vector along positive rotational axis of joint i

$\vec{v}^*_2$ normal vector between lines of rotation for joints 2 and 3

W wrist of robot arm

X_i axis directed along common normal between Z_{i-1} and Z_i

X_w, Y_w, Z_w world coordinate axes

Y_i axis directed to complete right-hand axis system with X_i and Z_i

Z_i axis of positive rotation of joint $i + 1$

α_i angle between Z_{i-1} and Z_i, measured positive about X_i

β_i constant bias angle, which yields joint angle θ'_i when summed with joint angle θ_i

$\theta_A, \theta_B, \theta_C$ joint angle corresponding to measurement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$, respectively

θ_i joint angle with initial value (0°) corresponding to initial position of robot arm

θ_i' joint angle between X_{i-1} and X_i, measured positively about Z_{i-1}

$\Delta\theta_i$ incremental positive change in θ_i

λ_i real variable in vector line equation for joint i

ρ_i radius of circular trajectory of point on robot arm about line of rotation of joint i

ξ_i, η_i, ζ_i components of $\vec{D}_i$ with respect to coordinate system $i - 1$ when $\theta_i = 0°$

Mathematical notations:

$\cdot$ dot or scalar product

$\times$ cross or vector product

T superscript to indicate transpose

$| \ |$ length of a vector or absolute value

ANALYSIS

The regular Denavit-Hartenberg parameters are first described and then the difficulties encountered with their use in practical robotic applications are discussed. The basic set of parameters is then modified such that the criticism is no longer valid. Next, an algebraic approach, adapted from reference 4, is extended to extract the modified set of parameters. Parameters are then extracted for near-parallel rotational axes in a simulated robot arm.

Denavit-Hartenberg Parameters

In figure 1, a_i (component of $\vec{a}_i$ along X_i), r_i (component of $\vec{r}_i$ along Z_{i-1}), α_i, and θ_i' referred to as the Denavit-Hartenberg parameters, completely characterize the relative location of successive joint axis systems.

Joint axis systems.- Figure 1 illustrates the axis systems associated with joints i and $i + 1$. By convention, joint i is associated with the coordinate system $i - 1$. Hence, joint i corresponds to the axis system with origin at O_{i-1}, whereas joint $i + 1$ corresponds to the adjoining axis system with origin O_i. By definition, the axis of rotation for joint i always lies along the associated Z_{i-1}. The vector $\vec{a}_i$, directed toward Z_i, is the normal vector between Z_{i-1} and Z_i. The intersection point of $\vec{a}_i$ with Z_i locates the origin O_i. The axis X_i originates from O_i in the same direction as $\vec{a}_i$. In the event that Z_{i-1} and Z_i intersect (fig. 1(b)), $\vec{a}_i$ is the zero vector, and X_i is then directed from this intersection in the direction of the cross product obtained by multiplying a unit vector along Z_{i-1} by a unit vector along Z_i. The vector $\vec{r}_i$ is from the origin O_{i-1} to the intersection of $\vec{a}_i$ with Z_{i-1} (fig. 1(a)); for intersecting lines of rotation, $\vec{r}_i$ is a vector along Z_{i-1} from O_{i-1} to O_i (fig. 1(b)). The angle α_i is the angle between a line parallel to Z_{i-1} through the origin O_i and Z_i, being measured positive about X_i (fig. 1). Finally, the joint angle θ'_i is the angle between X_{i-1} and a line parallel to X_i through O_{i-1} and is measured positive about Z_{i-1} (fig. 1). For clarity, the axes Y_i and Y_{i-1}, which simply complete right-hand axis systems, are omitted.

Relationship between joint angles θ_i and θ'_i.- In general, the joint angle θ'_i is not equal to the joint angle θ_i, which is referenced to the initial position of the robot arm (e.g., the position of the robot arm in fig. 2). Corresponding values of θ_i and θ'_i are assumed to be related by the linear equation

$$\theta'_i = \theta_i + \beta_i \tag{1}$$

where β_i is a constant bias, reflecting an initial offset in θ'_i, because θ'_i and θ_i are generally measured from different starting positions. The joint angle θ_i is measurable prior to establishing the joint axis systems, but θ'_i is not yet measurable.

Basic coordinate transformation.- The relative joint geometry dictates the basic transformation equations between adjacent joints. The coordinates of an arbitrary point $Q(x,y,z)$ with respect to the coordinate system i can be transformed to coordinates of Q with respect to the coordinate system $i - 1$ by using the relation

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{i-1} = A^{i}_{i-1} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{i} \tag{2}$$

with

$$A^{i}_{i-1} = \left[\begin{array}{ccc|c} & R^{i}_{i-1} & & \vec{h}_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \tag{3}$$

where

$$R^{i}_{i-1} = \begin{bmatrix} \cos\theta'_i & -\cos\alpha_i \sin\theta'_i & \sin\alpha_i \sin\theta'_i \\ \sin\theta'_i & \cos\alpha_i \cos\theta'_i & -\sin\alpha_i \cos\theta'_i \\ 0 & \sin\alpha_i & \cos\alpha_i \end{bmatrix} \tag{4}$$

is the rotational transformation matrix that relates coordinate system i to $i - 1$ with respect to orientation, and where

$$\vec{h}_i = \begin{pmatrix} a_i \cos \theta_i' \\ a_i \sin \theta_i' \\ r_i \end{pmatrix} \tag{5}$$

is the location of O_i from O_{i-1} in coordinate system $i - 1$. As can be seen, the elements in the basic transformation matrix A_{i-1}^{i} are expressed in terms of the Denavit-Hartenberg parameters.

Criticism of Denavit-Hartenberg Parameters

A criticism of the Denavit-Hartenberg parameters is that $|\vec{r}_i| \to \infty$ as $\alpha_i \to 0°$. (See fig. 1(b).) Indeed, this criticism is justified in that this behavior causes: (1) Sensitivity of the parameter r_i to errors in misalignment from a parallel condition ($\alpha_i = 0°$); (2) Ill-conditioned transformation matrices (r_i terms and matrix products approach infinity as $\alpha_i \to 0°$); and (3) Excessive displacement of the axis system from the robot arm. The intent in this paper is to eliminate these weaknesses by modifying the Denavit-Hartenberg parameters.

Modified Denavit-Hartenberg Parameters

In this paper, the regular Denavit-Hartenberg parameters are modified by only insisting that a transverse vector between successive joint rotational axes be normal to one of the axes instead of to both axes. This simple modification leads to a more favorable location of successive joint axis systems.

As with the regular Denavit-Hartenberg parameters, let Z_i be the axis of rotation for joint $i + 1$, let X_i point in a direction that is normal to both Z_{i-1} and Z_i, and let Y_i complete the right-hand coordinate system. The angle between Z_{i-1} and Z_i is α_i, measured positive for rotation about X_i. The difference between the regular and modified parameters is in how O_i is located from O_{i-1}.

In figure 3, $\vec{u}_i$ and $\vec{u}_{i+1}$ are unit vectors along the lines of rotation for joints i and $i + 1$, respectively. Also in figure 3, $\vec{C}_{i+1}$ (determined later) is a vector from O_w to a point on Z_i; $\vec{R}_{i-1}$ and $\vec{R}_i$ locate O_{i-1} and O_i from O_w, respectively. The vector $\vec{D}_i$ is from a point on Z_{i-1} to a point on Z_i. The origin O_i is removed from O_{i-1} by

$$\vec{h}_i = r_i \vec{u}_i + \vec{D}_i \tag{6}$$

where

$$\vec{D}_i = \vec{R}_i - \vec{R}_{i-1} - r_i\vec{u}_i \tag{7}$$

and where r_i is the coordinate of O_i along Z_{i-1}.

When used in equation (3), $\vec{h}_i$ is expressed in the coordinate system $i - 1$. When $\theta_i = 0°$,

$$\vec{h}_i = \begin{pmatrix} \xi_i \\ \eta_i \\ \zeta_i + r_i \end{pmatrix} \tag{8}$$

where (using $\vec{D}_i$ and $\vec{e}_{i-1}$ for the condition $\theta_i = 0°$)

$$\xi_i = \vec{D}_i \cdot \vec{e}_{i-1} \tag{9}$$

$$\eta_i = \vec{D}_i \cdot \left(\vec{u}_i \times \vec{e}_{i-1}\right) \tag{10}$$

and

$$\zeta_i = \vec{D}_i \cdot \vec{u}_i \tag{11}$$

denote the projections of $\vec{D}_i$ along X_{i-1}, Y_{i-1}, and Z_{i-1}, respectively, and $\vec{e}_{i-1}$ is a unit vector along X_{i-1}. The parameters ξ_i, η_i, ζ_i, and r_i now take the place of the regular Denavit-Hartenberg parameters a_i and r_i. As joint i rotates, the location of O_i from O_{i-1} changes with θ_i as

$$\vec{h}_i = \begin{pmatrix} \xi_i \cos\theta_i - \eta_i \sin\theta_i \\ \xi_i \sin\theta_i + \eta_i \cos\theta_i \\ \zeta_i + r_i \end{pmatrix} \tag{12}$$

which is used in equation (3). Equation (12) reduces to equation (5) if $\xi_i = a_i$, $\eta_i = \zeta_i = 0$, and $\theta_i = \theta_i'$.

Transverse vector normal to both Z_i and Z_{i-1} for regular Denavit-Hartenberg parameters.- When constrained to be normal to both Z_{i-1} and Z_i, $\vec{D}_i$ is located from Q_1 to Q_2 in figure 3. Then, r_i becomes the location of the point Q_1 along Z_{i-1}. The direction of $\vec{D}_i$ is along X_i, and its component is a_i. Since X_i is simply X_{i-1} rotated by θ_i' about Z_{i-1}, $\vec{D}_i$ with components in coordinate system $i - 1$ is

$$\vec{D}_i = \begin{pmatrix} a_i \cos \theta_i' \\ a_i \sin \theta_i' \\ 0 \end{pmatrix} \tag{13}$$

where θ_i' is θ_i measured from an initial zero position where X_{i-1} and X_i are parallel. Then, from equation (6), $\vec{h}_i$ is the same as equation (5) for the regular Denavit-Hartenberg parameters.

The problem with constraining $\vec{D}_i$ to be normal to both Z_{i-1} and Z_i is that r_i can shift O_i far away from its associated joint on the robot arm. Consequently, two alternate constraints are imposed on $\vec{D}_i$ in this paper. They are (1) Only constraining $\vec{D}_i$ to be normal to Z_{i-1} and (2) Only constraining $\vec{D}_i$ to be normal to Z_i.

Transverse vector normal to Z_{i-1}.- The transverse vector $\vec{D}_i$ is given by equation (7), with

$$\vec{R}_i = \vec{C}_{i+1} + \lambda_{i+1} \vec{u}_{i+1} \tag{14}$$

where $\vec{C}_{i+1}$ (discussed subsequently) is a point on Z_i. The value of λ_{i+1}, which simply moves the terminal point of $\vec{R}_i$ along Z_i, is chosen to satisfy the constraint

$$\vec{D}_i \cdot \vec{u}_i = 0 \tag{15}$$

That is, $\vec{D}_i$ is normal to Z_{i-1}. Equations (7), (14), and (15) show that

$$\lambda_{i+1} = \frac{\left(\vec{R}_{i-1} + r_i \vec{u}_i - \vec{C}_{i+1}\right) \cdot \vec{u}_i}{\vec{u}_{i+1} \cdot \vec{u}_i} \tag{16}$$

Equations (9) to (12) then yield $\vec{h}_i$. Since $\vec{D}_i$ is normal to Z_{i-1}, $\zeta_i = 0$. The parameter r_i is now arbitrary and is chosen to shift O_i to a favorable location.

Transverse vector normal to Z_i.- The transverse vector $\vec{D}_i$ is given by equation (7), with $\vec{R}_i$ given by equation (14), where λ_{i+1} is chosen to satisfy the constraint

$$\vec{D}_i \cdot \vec{u}_{i+1} = 0 \tag{17}$$

That is, $\vec{D}_i$ is normal to Z_i. Equations (7), (14), and (17) yield

$$\lambda_{i+1} = \left(\vec{R}_{i-1} + r_i\vec{u}_i - \vec{C}_{i+1}\right) \cdot \vec{u}_{i+1} \tag{18}$$

Equations (9) to (12) then yield $\vec{h}_i$, and r_i is a free parameter that is chosen to shift O_i to a favorable location.

Algebraic Method To Extract Modified Denavit-Hartenberg Parameters

Measurement data needed to extract the relative joint parameters is generated by individually varying the joint angles in the robot arm and measuring the location of a point on the robot hand or other extension. (See ref. 4.) Each joint angle is measured relative to a selected zero position; for example, see the position of the robot arm in figure 2.

Any point on the robot arm (except a point on the line of rotation) generates a circular trajectory as the joint angle θ_i is varied, and the location of a point on the robot arm is measured relative to a world reference axis system. As subsequently shown, three locations of the point on the circular trajectory, along with the corresponding values of θ_i, are enough to determine the unit vector $\vec{u}_i$ along the line of rotation for joint i and the vector $\vec{C}_i$ from the world axis system to the center of the circular trajectory of the point. With $\vec{u}_i$ and $\vec{C}_i$ for successive joints, the relative joint parameters are computed. First, joint i is rotated to obtain $\vec{u}_i$ and $\vec{C}_i$. Then, with $\theta_i = 0°$, joint $i + 1$ is rotated to obtain $\vec{u}_{i+1}$ and $\vec{C}_{i+1}$. The process is initiated by assuming a fictitious joint with a rotational axis along Z_w.

Circular trajectory center.- Let $\vec{A}$, $\vec{B}$, and $\vec{C}$ denote three positions of a point on a robot arm as the point moves in a circular trajectory about the line of rotation for joint i (fig. 4), and let θ_A, θ_B, and θ_C be the corresponding values of θ_i. The objective is to compute the vector $\vec{C}_i$ from the world axis system to the center of the circular trajectory. Let the radius of the circle be ρ_i, then

$$\left(\vec{A} - \vec{C}_i\right) \cdot \left(\vec{A} - \vec{C}_i\right) = \rho_i^2 \tag{19}$$

$$\left(\vec{B} - \vec{C}_i\right) \cdot \left(\vec{B} - \vec{C}_i\right) = \rho_i^2 \tag{20}$$

$$\left(\vec{C} - \vec{C}_i\right) \cdot \left(\vec{C} - \vec{C}_i\right) = \rho_i^2 \tag{21}$$

That is, all points are located at a distance ρ_i from the center of the circle. Equations (19) to (21) constitute three equations in four unknowns (ρ_i and the three components of $\vec{C}_i$). A fourth equation (mistakenly neglected in ref. 4) results from the constraint that the tip of $\vec{C}_i$ lies in the plane of the data points; that is, one of the equations

$$\left(\vec{C}_i - \vec{A}\right) \cdot \vec{N} = 0 \tag{22}$$

$$\left(\vec{C}_i - \vec{B}\right) \cdot \vec{N} = 0 \tag{23}$$

$$\left(\vec{C}_i - \vec{C}\right) \cdot \vec{N} = 0 \tag{24}$$

where

$$\vec{N} = (\vec{C} - \vec{A}) \times (\vec{B} - \vec{A}) \tag{25}$$

is a normal vector to the circular trajectory plane.

Eliminating ρ_i from equations (19) and (20) by using equation (21) results in the following equations:

$$\left(\vec{A} - \vec{C}_i\right) \cdot \left(\vec{A} - \vec{C}_i\right) = \left(\vec{C} - \vec{C}_i\right) \cdot \left(\vec{C} - \vec{C}_i\right) \tag{26}$$

$$\left(\vec{B} - \vec{C}_i\right) \cdot \left(\vec{B} - \vec{C}_i\right) = \left(\vec{C} - \vec{C}_i\right) \cdot \left(\vec{C} - \vec{C}_i\right) \tag{27}$$

Equations (22), (26), and (27) are three linear equations in three unknowns (the components of $\vec{C}_i$). An equivalent matrix equation for computing the components of $\vec{C}_i$ is

$$\begin{bmatrix} N(1) & N(2) & N(3) \\ C(1) - A(1) & C(2) - A(2) & C(3) - A(3) \\ C(1) - B(1) & C(2) - B(2) & C(3) - B(3) \end{bmatrix} \begin{pmatrix} C_i(1) \\ C_i(2) \\ C_i(3) \end{pmatrix} = \begin{pmatrix} \vec{N} \cdot \vec{A} \\ \frac{1}{2}(\vec{C} \cdot \vec{C} - \vec{A} \cdot \vec{A}) \\ \frac{1}{2}(\vec{C} \cdot \vec{C} - \vec{B} \cdot \vec{B}) \end{pmatrix} \tag{28}$$

Once $\vec{C}_i$ is known, any one of equations (19) to (21) yields ρ_i^2.

Unit vector $\vec{u}_i$.- Figure 5 shows the circular trajectory of a point on the robot arm and two position vectors $\vec{A}$ and $\vec{B}$, along with the incremental joint angle

$$\Delta\theta_i = \theta_B - \theta_A \tag{29}$$

between these position vectors. A unit vector normal to the plane of the circular trajectory and passing through point C_i (whose coordinates are the components of the vector $\vec{C}_i$) is

$$\vec{u}_i = \frac{\left(\vec{A} - \vec{C}_i\right) \times \left(\vec{B} - \vec{C}_i\right)}{\rho_i^2 \sin \Delta\theta_i} \tag{30}$$

With $0° < \Delta\theta_i < \pi$, $\vec{u}_i$ in equation (30) is in the same direction as the rotational vector of joint i. An average $\vec{u}_i$ should be used to reduce the effects of measurement errors.

Axis X_i.- The direction of X_i is defined by a unit vector $\vec{e}_i$ (fig. 3) that is normal to both the line of rotation for joint i and the line of rotation for joint i + 1. Such a unit vector is given by

$$\vec{e}_i = \left(\vec{u}_i \times \vec{u}_{i+1}\right)/\left|\vec{u}_i \times \vec{u}_{i+1}\right| \tag{31}$$

if $\vec{u}_i \times \vec{u}_{i+1} \neq 0$, or by

$$\vec{e}_i = \vec{D}_i/\left|\vec{D}_i\right| \tag{32}$$

if $\vec{u}_{i+1} \times \vec{u}_i = 0$. The vector cross product in equation (31) gives a vector which is normal to both $\vec{u}_i$ and $\vec{u}_{i+1}$ if the lines of rotation for joints i and i + 1 are not parallel. If the rotational lines are parallel, equation (32) is used.

Alternatively, $\vec{e}_i$ is any nonzero solution to the simultaneous linear homogeneous equations

$$\vec{u}_i \cdot \vec{e}_i = 0 \tag{33}$$

and

$$\vec{u}_{i+1} \cdot \vec{e}_i = 0 \tag{34}$$

whether or not $\vec{u}_i$ and $\vec{u}_{i+1}$ are parallel.

Axes Z_i and Y_i.- By convention, Z_i points in the direction of positive rotation for joint $i + 1$, that is, in the direction of $\vec{u}_{i+1}$. Y_i completes a right-hand axis system with X_i and Z_i.

Equation for $\tan \theta_i'$.- The equation for $\tan \theta_i'$ is

$$\tan \theta_i' = \frac{(\vec{e}_{i-1} \times \vec{e}_i) \cdot \vec{u}_i}{\vec{e}_{i-1} \cdot \vec{e}_i} \tag{35}$$

The numerator in equation (35) shows the cross product of a vector $\vec{e}_{i-1}$ along X_{i-1} and a vector $\vec{e}_i$ along X_i. Forming the dot product of this result and a unit vector $\vec{u}_i$ along Z_{i-1} produces $\sin \theta_i'$ with the correct sign for a positive rotation about Z_{i-1} (or equivalently $\vec{u}_i$). The denominator is equivalent to $\cos \theta_i'$. Hence, the fraction represents $\tan \theta_i'$, where $0° < \theta_i' < 2\pi$. The joint angle θ_i' in equation (35) corresponds to the fixed position of joint i after $\vec{u}_i$ has been determined and when joint $i + 1$ is being varied to obtain $\vec{u}_{i+1}$. The bias β_i in equation (1) is just the calculated value of θ_i' when $\theta_i = 0°$.

Tan α_i with X_i in direction of $\vec{e}_i$.- The appropriate equation is

$$\tan \alpha_i = [(\vec{u}_i \times \vec{u}_{i+1}) \cdot \vec{e}_i]/\vec{u}_i \cdot \vec{u}_{i+1} \tag{36}$$

The right-hand side of this equation shows the cross product of a vector along Z_{i-1} (or $\vec{u}_i$) and a vector along Z_i (or $\vec{u}_{i+1}$), and then the dot product of this result and a unit vector along X_i (or $\vec{e}_i$) gives $\sin \alpha_i$. The dot product in the denominator yields $\cos \alpha_i$.

Parameters ξ_i, η_i, ζ_i, and r_i.- At $\theta_i = 0°$, the components of $\vec{D}_i$ in coordinate system $i - 1$ are ξ_i, η_i, and ζ_i in equations (9) to (11), respectively, and r_i is a free parameter chosen for favorable location of O_i. The transverse vector $\vec{D}_i$ is computed in equation (7) with $\vec{R}_i$ from equation (14), where λ_{i+1} depends on how $\vec{D}_i$ is constrained. If $\vec{D}_i$ is constrained normal to Z_{i-1}, equation (16) is used for λ_{i+1}, but if $\vec{D}_i$ is constrained normal to Z_{i-1}, equation (18) is used. In use, the constant parameters ξ_i, η_i, ζ_i, and r_i appear in equation (12) for $\vec{h}_i$, which only varies with the joint angle θ_i (or $\theta_i' - \beta_i$, eq. (1)). The vector $\vec{h}_i$ is then used in equation (3).

EXAMPLE

For the robot arm depicted in figure 2, this example focuses on determining the relative location of the elbow-joint axis system with respect to the shoulder-joint axis system. A transverse vector between the rotational axis Z_2 of the elbow joint and the rotational axis Z_1 of the shoulder joint is used in locating the elbow-joint axis system.

Transverse-Vector Constraints

Three ways to locate the axis system result from three separate constraints on the transverse vector, which is constrained to be either (1) Normal to both rotational axes Z_1 and Z_2, (2) Normal to the elbow-joint rotational axis Z_2, or (3) Normal to the shoulder-joint rotational axis Z_1. The first constraint leads to the well-known Denavit-Hartenberg parameters.

Transverse vector normal to both Z_1 and Z_2.- The rotational axes of the shoulder and elbow joints in figure 2 are parallel. The Denavit-Hartenberg parameters are $\alpha_2 = 0°$, $r_2 = SN$ along Z_1, $a_2 = ES$ in the X_2 direction, which is normal to both Z_1 and Z_2, and $\theta_2' = \theta_2 + \beta_2$, where $\beta_2 = 90°$ is the angle between X_1 and X_2. These parameters locate the elbow-joint axis system as shown at the point E. Such is not the case, however, when the axes are not parallel. For example, suppose Z_2 is rotated counterclockwise by 0.1° about Y_2. The Denavit-Hartenberg parameters then become $\alpha_2 = 0.1°$ (angle between Z_1 and Z_2), $r_2 = -9734$ in. (intersection of Z_2 with Z_1 along Z_1), and $a_2 = 0$ (since Z_2 intersects Z_1). Thus, the elbow-joint axis system is moved away (by r_2) from the robot arm by about 811 ft. This does not happen with the modified parameters introduced in this paper. Indeed, the elbow-joint axis system remains located at the desired point E.

Transverse vector normal to Z_1.- When the transverse vector is normal to Z_1 (fig. 6(a)), the point E is located by distances ξ_2 along X_1, η_2 along Y_1, and $r_2 + \zeta_2$ along Z_1, where r_2 is now the desired location of the elbow-joint axis system. For this example, in which $\alpha_2 = 0.1°$, these parameters are as follows: $\xi_2 = 0$, $\eta_2 = ES = 17$ in., $\zeta_2 = 0$ (by the constraint on the transverse vector, ξ_2 is always zero and need not be determined), and $r_2 = SN = 6$ in. (by choice).

Transverse vector normal to Z_2.- When the transverse vector is normal to Z_2 (fig. 6(b)), the parameters that locate the elbow-joint axis system are as follows: $\xi_2 = 0$, $\eta_2 = ES = 17$ in., $\zeta_2 = -ES/\tan \alpha_2 = -0.030$, and $r_2 = SN - \zeta_2 = 6.060$ in. Hence, $r_2 + \zeta_2 = SN = 6$ in.

Simulated Measurements

In figure 7, assume that the waist- and shoulder-joint axis systems have been determined. Now the problem is to determine the elbow-joint axis system by using simulated locations of a point on the robot arm for three distinct elbow-joint angles. Specifically, locations of the point W are assumed to be measured with respect to the waist-joint axis system (which, in this example, coincides with the world reference axis system) for three distinct values of θ_3. The simulated measurements are based on the following dimensions (in inches): NO = 26, SN = 6, and ES = WE = 17. Examples of simulated measurements are as follows:

$$\theta_A = 0, \qquad \vec{A} = \begin{pmatrix} 0.000 \\ 6.000 \\ 60.000 \end{pmatrix}$$

$$\theta_B = 45°, \qquad \vec{B} = \begin{pmatrix} 12.021 \\ 6.009 \\ 55.021 \end{pmatrix}$$

$$\theta_C = 90°, \qquad \vec{C} = \begin{pmatrix} 17.000 \\ 6.030 \\ 43.000 \end{pmatrix}$$

These measurements give the location of point W to the nearest one-thousandth of an inch as θ_3 takes on the values of θ_A, θ_B, and θ_C. In computing the simulated vector positions $\vec{A}$, $\vec{B}$, and $\vec{C}$ (see appendix), the fact that $\alpha_2 = 0.1°$ is assumed to be unknown. The following vectors are known:

$$\vec{R}_1 = \begin{pmatrix} 0 \\ 0 \\ NO \end{pmatrix}$$

$$\vec{C}_2 = \vec{R}_1$$

$$\vec{e}_1 = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{u}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

These vectors are expressed in the waist (or world) axis system. The vector from 0 to N is $\vec{R}_1$; $\vec{C}_2$ is the vector from the center of the circular trajectory of point W as the shoulder joint is rotated; $\vec{e}_1$ is a unit vector normal to Z_0 and Z_1 that defines X_1; and $\vec{u}_2$ is a unit vector in the direction of rotation of joint 2 (shoulder) in base coordinates.

Also, based on the simulated measurement data,

$$\vec{C}_3 = \begin{pmatrix} 0.000 \\ 6.030 \\ 43.000 \end{pmatrix}$$

and

$$\vec{u}_3 = \begin{pmatrix} 0.000 \\ 1.000 \\ .002 \end{pmatrix}$$

where $\vec{C}_2$ is a vector to the center of the circular trajectory from the waist coordinate system and $\vec{u}_3$ is a unit vector along Z_2 (the line of rotation of the elbow joint) in the waist coordinate system.

Location of Origin of Elbow-Joint Axis System by Denavit-Hartenberg Parameters

For the regular Denavit-Hartenberg parameters, the transverse vector, which is $\vec{D}_2$ in the notation of the text, is normal to both of the lines of rotation of the elbow and shoulder joints. Specifically (from ref. 4, pages 9 to 11, with $i = 2$ and $\vec{D}_2 = \vec{v}_2^*$),

$$\vec{D}_2 = \vec{\ell}_3 - \vec{\ell}_2 \tag{37}$$

where

$$\vec{\ell}_2 = \vec{C}_3 + \lambda_3\vec{u}_3 \tag{38}$$

and

$$\vec{\ell}_1 = \vec{C}_2 + \lambda_2\vec{u}_2 \tag{39}$$

with

$$\lambda_1 = \frac{(\vec{C}_3 - \vec{C}_2) \cdot [\vec{u}_2 - (\vec{u}_3 \cdot \vec{u}_2)\vec{u}_3]}{1 - (\vec{u}_3 \cdot \vec{u}_2)^2} \tag{40}$$

$$\lambda_2 = \frac{-(\vec{C}_3 - \vec{C}_2) \cdot (\vec{u}_3 - (\vec{u}_3 \cdot \vec{u}_2)\vec{u}_2}{1 - (\vec{u}_3 \cdot \vec{u}_2)^2} \tag{41}$$

That is, λ_1 and λ_2 are such that $\vec{D}_2 \cdot \vec{u}_2 = 0$ and $\vec{D}_2 \cdot \vec{u}_3 = 0$.

The regular Denavit-Hartenberg parameter r_2 is the component of

$$\vec{r}_2 = \vec{\ell}_2 - \vec{R}_1 \tag{42}$$

along Z_1; that is,

$$r_2 = \vec{r}_2 \cdot \vec{u}_2 \tag{43}$$

The regular Denavit-Hartenberg parameter a_2 is the component of

$$\vec{a}_2 = \vec{\ell}_3 - \vec{\ell}_2 \tag{44}$$

along X_2, that is,

$$a_2 = \vec{a}_2 \cdot \vec{e}_1 \tag{45}$$

Based on the simulated measurement data, $r_2 = -9789.719$ in. and $a_2 = 0.298$ in. The true values are $r = -9734.273$ in. and $a_2 = 0$ in. Hence, the calculated values of r_2 and a_2 are in absolute error by 55.446 in. and 0.298 in., respectively.

Different values of α_2 are shown in the first column of table I, and the Denavit-Hartenberg parameters r_2 and a_2 that locate the origin of the elbow-joint axis system are shown in the second and third columns, respectively. Note that $a_2 = 0$ because the lines of rotation of the elbow and shoulder joints intersect. For measurements (components of $\vec{A}$, $\vec{B}$, and $\vec{C}$) rounded off to the nearest ten-thousandth, one-thousandth, and one-hundredth of an inch, table I shows the absolute errors in the computed values of the parameters r_2 and a_2. As the elbow and shoulder joints approach a parallel condition ($\alpha_2 \to 0°$), two important observations from table I are as follows: (1) The elbow-joint axis system is located far off the robot arm (large value of r_2), and (2) The error in the computed value of r_2 is excessively large, even for very accurate measurements.

Location of Origin of Elbow-Joint Axis System by Modified Parameters

In this example for the modified parameters, the transverse vector from the elbow-joint axis of rotation to the shoulder-joint axis of rotation is normal to the elbow-joint axis of rotation. By choice, r_2 = SN = 6 in. in this example. The true modified parameters are shown in table II for different values of α_2 (angle between rotational axes of the elbow and shoulder joints) to locate the elbow-joint axis system at point E in figure 7. Errors in the calculated parameters using measurement data rounded off to the nearest ten-thousandth, one-thousandth, and one-hundredth of an inch are also shown. In addition to locating the elbow-joint axis system at the desired point E in figure 7, the modified parameters can be accurately calculated even as the lines of rotation of the elbow and shoulder joints approach a parallel condition ($\alpha_2 \rightarrow 0°$).

CONCLUDING REMARKS

At present, the most popular way to describe the relative location of successive joint axis systems in a robot arm is to use the Denavit-Hartenberg parameters. However, a recent justifiable criticism is that one of these parameters approaches infinity when two successive joints have nearly parallel rotational axes. Geometrically, this parameter removes the joint axis an excessive distance from the robot arm; computationally, this large parameter leads to an ill-conditioned transformation matrix. In this paper, a simple modification in the location of this axis system easily overcomes this criticism. This modification results by insisting that a transverse vector between successive joint rotational axes be normal to one of the rotational axes instead of to both axes. This simple modification leads to modified Denavit-Hartenberg parameters that favorably locate successive joint axis systems.

An example is given with respect to the rotational axes of the elbow and shoulder joints in a robot arm. The regular and modified Denavit-Hartenberg parameters (that locate the elbow-joint axis system relative to the shoulder-joint axis system) are extracted by an algebraic method via simulated measurement data.

A point near the robot hand (off the line of rotation) generates a circular trajectory as the elbow joint is rotated. Three position vectors to this point and the corresponding three elbow joint angles (referenced to an initial position) are simulated for the extraction process.

For small misalignments of the shoulder and elbow joints away from a parallel condition (i.e., parallel rotational axes), the Denavit-Hartenberg parameters locate the elbow-joint axis system far away from the robot arm; the modified parameters locate the axis system at the desired place on the robot arm. In addition, for a given accuracy of the measurements used in the parameter-extraction process, the extracted values for the Denavit-Hartenberg parameters yielded considerably larger errors than did the extracted values for the modified parameters. It appears that the modified parameters provide a more natural location of successive joint axis systems and are useful in the industrial calibration of robot arms.

NASA Langley Research Center
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March 13, 1986

APPENDIX

EQUATIONS FOR SIMULATED MEASUREMENTS

By Euler's theorem (ref. 5), a vector $\vec{r}$ rotated by an angle θ about a line (or unit vector $\vec{\omega}$) becomes

$$\vec{r}\,' = \cos\theta\,\vec{r} + \vec{r}\cdot\vec{\omega}(1 - \cos\theta)\vec{\omega} - (\vec{r}\times\vec{\omega})\sin\theta \tag{A1}$$

This equation is applied to obtain an expression for the location of point W on the robot arm in figure A1 with respect to the waist axis system as the elbow joint θ_3 is rotated for a specified value of α_2.

For the position of the robot arm in figure A1, and with respect to the X, Y, and Z axes, let $\vec{r} = (0, WE, 0)^T$ be a vector from E to W, and let $\vec{\omega} = \vec{u}_3 = (\sin\alpha_2, 0, \cos\alpha_2)^T$ be a vector along the rotational axis of the elbow joint. Moreover, let $\theta = \theta_3$ be the elbow-joint angle. Then $\vec{r}\,'$ gives the new location of W as a function of θ_3 and α_2. The location of W with respect to the waist axis system is then simply

$$\vec{W} = \begin{pmatrix} 0 \\ SN \\ NO + ES \end{pmatrix} + \begin{pmatrix} r'_Y \\ r'_Z \\ r'_X \end{pmatrix} \tag{A2}$$

where r'_X, r'_Y, and r'_Z are the components of $\vec{r}\,'$ (or coordinates of W) along the X, Y, and Z axes.

Simulated measurement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are the same as $\vec{W}$, except θ_3 is replaced by θ_A, θ_B, and θ_C, respectively. For the example in the text of this report, $\theta_A = 0°$, $\theta_B = 45°$, and $\theta_C = 90°$.

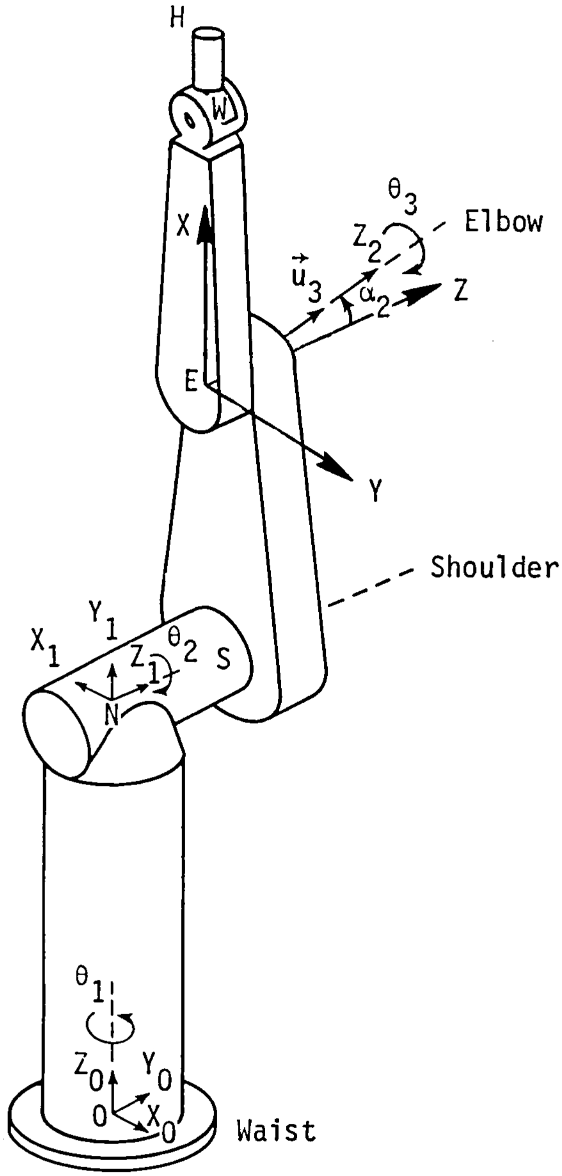

Figure A1.- Elbow-joint rotational axis, with respect to an auxiliary axis system, for simulating measurement data.

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TABLE I.- ERRORS IN CALCULATED DENAVIT-HARTENBERG PARAMETERS THAT LOCATE ORIGIN OF ELBOW-JOINT AXIS SYSTEM FOR ROBOT ARM

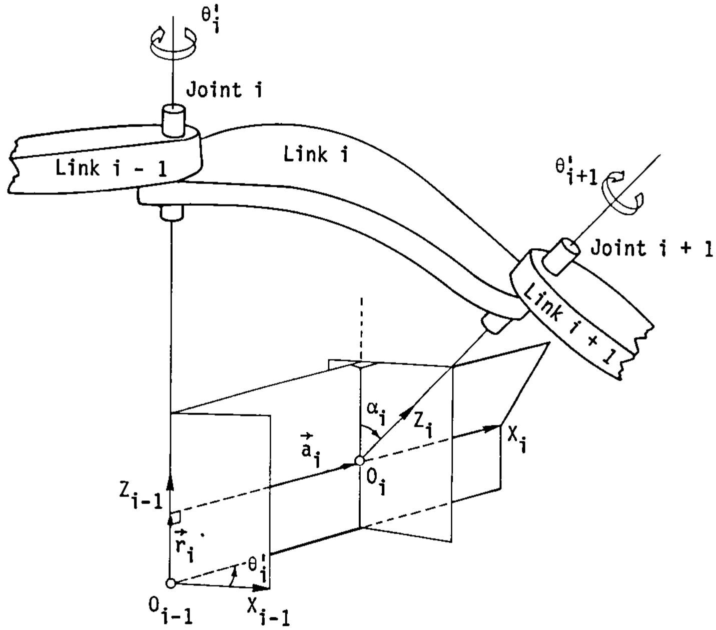
[Simulated measurements to point on robot arm correspond to elbow-joint angles of 0°, 45°, and 90°]

True simulated value of -			Simulated point measurements rounded off to nearest -	Absolute error in computed value of -	
α_2, deg	r_2, in.	a_2, in.		r_2, in.	a_2, in.
0.01	-97396.82	0.00	10^{-4}	11.22	0.30
.01	-97396.82	.00	10^{-3}	6410.25	1.83
.01	-97396.82	.00	10^{-2}	97406.82	6.51
0.10	-9734.27	0.00	10^{-4}	9.58	0.00
.10	-9734.27	.00	10^{-3}	55.45	.30
.10	-9734.27	.00	10^{-2}	808.07	1.82
1.00	-967.93	0.00	10^{-4}	0.37	0.00
1.00	-967.93	.00	10^{-3}	1.10	.00
1.00	-967.93	.00	10^{-2}	4.46	.28
10.00	-90.41	0.00	10^{-4}	0.00	0.00
10.00	-90.41	.00	10^{-3}	.04	.01
10.00	-90.41	.00	10^{-2}	.25	.03

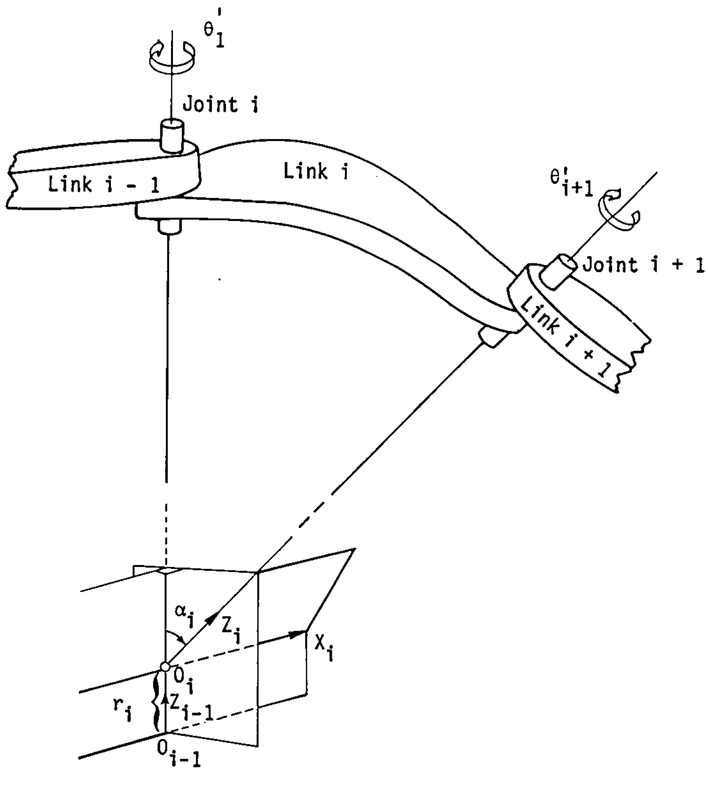
TABLE II.- ERRORS IN CALCULATED MODIFIED PARAMETERS THAT LOCATE ORIGIN OF ELBOW-JOINT AXIS SYSTEM FOR ROBOT ARM WHEN TRANSVERSE VECTOR IS CONSTRAINED NORMAL TO ELBOW-JOINT ROTATIONAL AXIS

[Simulated measurements to point on robot arm correspond to elbow-joint angles of 0°, 45°, and 90°]

True simulated value of -				Simulated point measurements rounded off to nearest -	Absolute error in computed value of -	
α_2, deg	ξ_2, in.	η_2, in.	r_2, in.		ξ_2, in.	η_2, in.
0.01	0.00	17.00	6.00	10^{-4}	0.0005	0.0005
.01	.00	17.00	6.00	10^{-3}	.0006	.0006
.01	.00	17.00	6.00	10^{-2}	.0028	.0028
0.10	0.00	17.00	6.00	10^{-4}	0.0001	0.0001
.10	.00	17.00	6.00	10^{-3}	.0006	.0006
.10	.00	17.00	6.00	10^{-2}	.0028	.0028
1.00	0.00	17.00	6.00	10^{-4}	0.0002	0.0002
1.00	.00	17.00	6.00	10^{-3}	.0003	.0003
1.00	.00	17.00	6.00	10^{-2}	.0041	.0067
10.00	0.00	17.00	6.00	10^{-4}	0.0001	0.0001
10.00	.00	17.00	6.00	10^{-3}	.0004	.0006
10.00	.00	17.00	6.00	10^{-2}	.0022	.0035



(a) Nonintersecting lines of rotation.



(b) Intersecting lines of rotation; $a_i = 0$.

Figure 1.- Denavit-Hartenberg parameters.

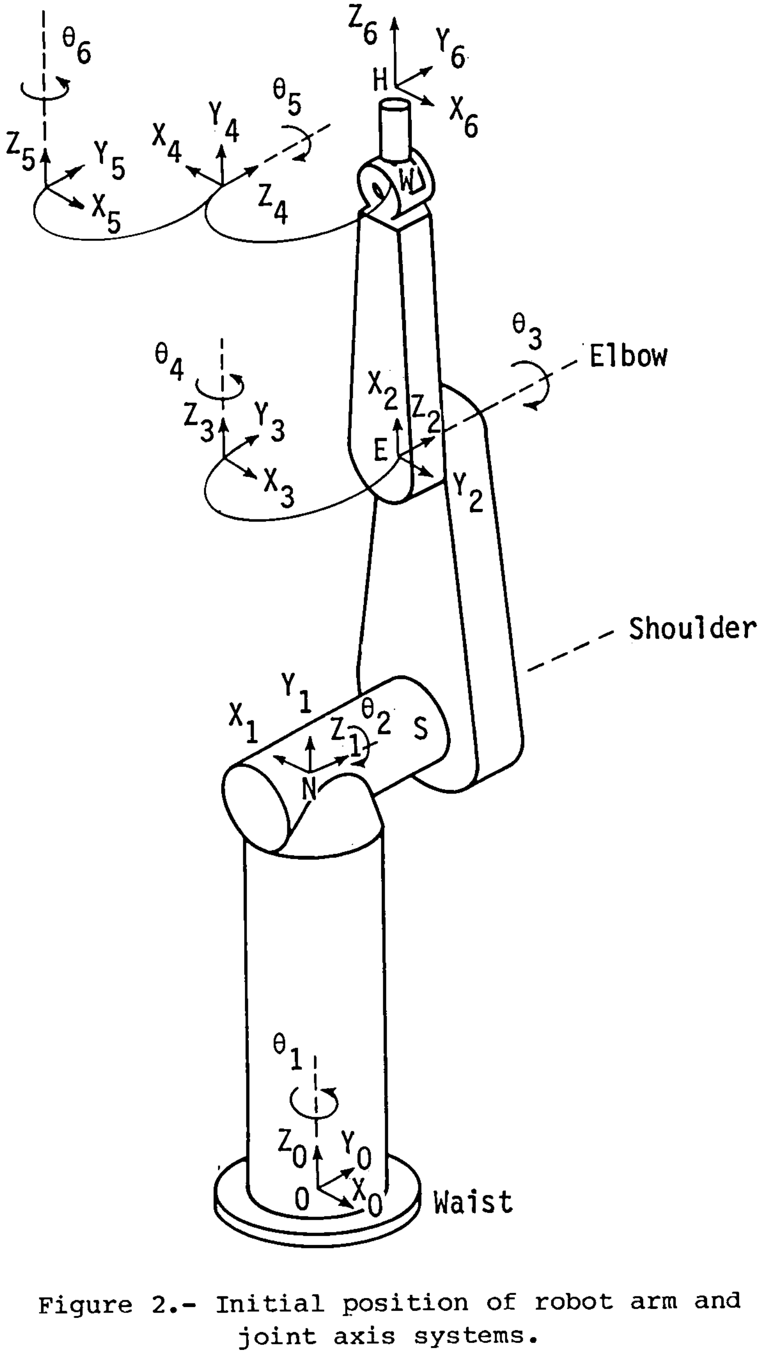


Figure 2.- Initial position of robot arm and joint axis systems.

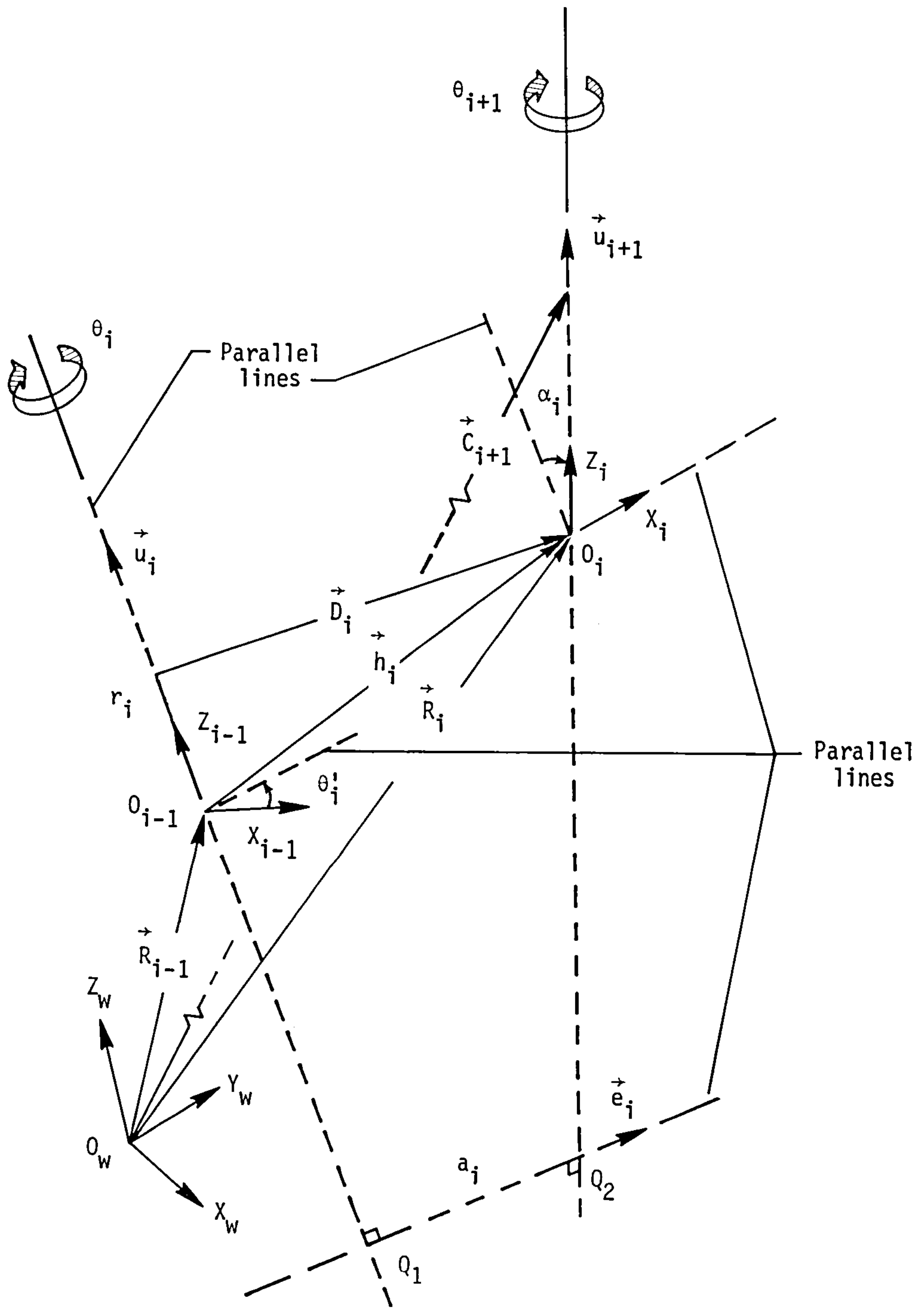


Figure 3.- Lines of rotation for successive joints.

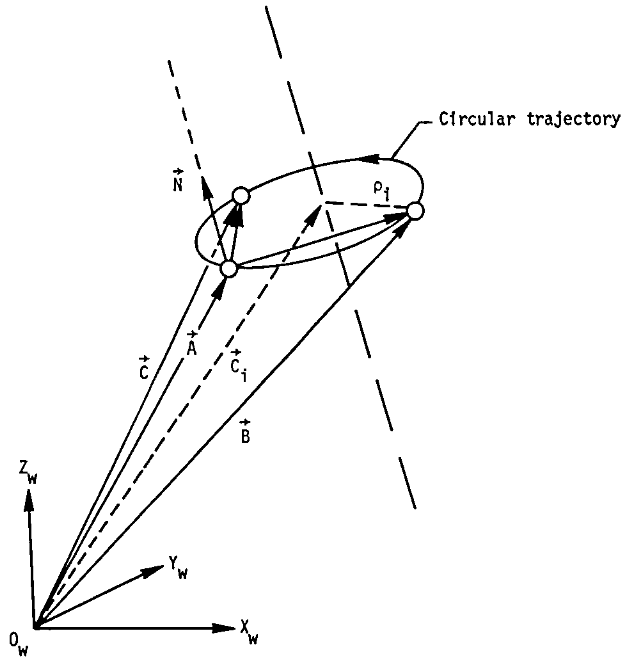


Figure 4.- Three locations on a circular trajectory of a point on robot arm about line of rotation for joint i.

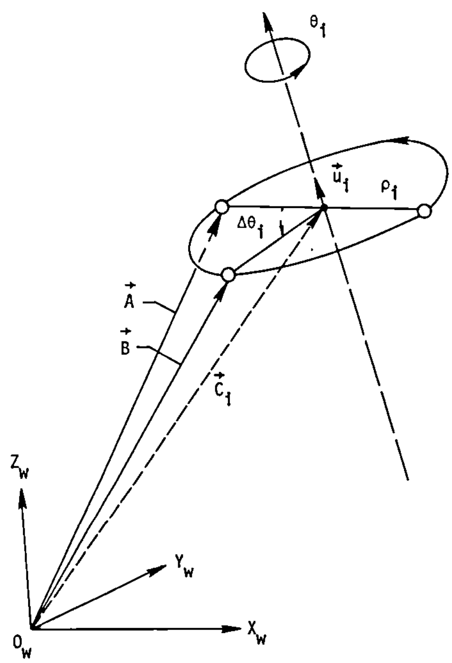
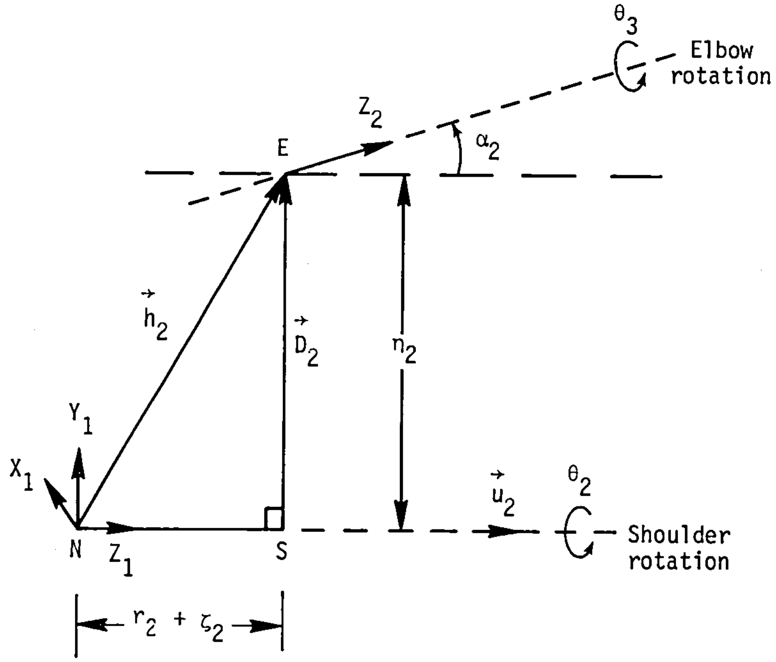
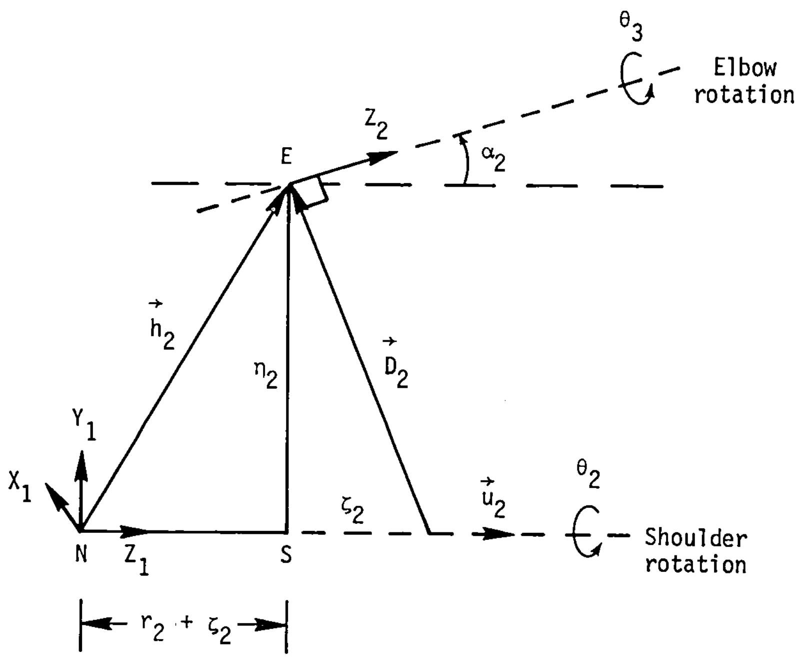


Figure 5.- Unit vector in same direction as joint rotational vector.



(a) $\vec{D}_2$ normal to shoulder rotational axis Z_1.



(b) $\vec{D}_2$ normal to elbow rotational axis Z_2.

Figure 6.- Location of point E from N by $\vec{h}_2$.

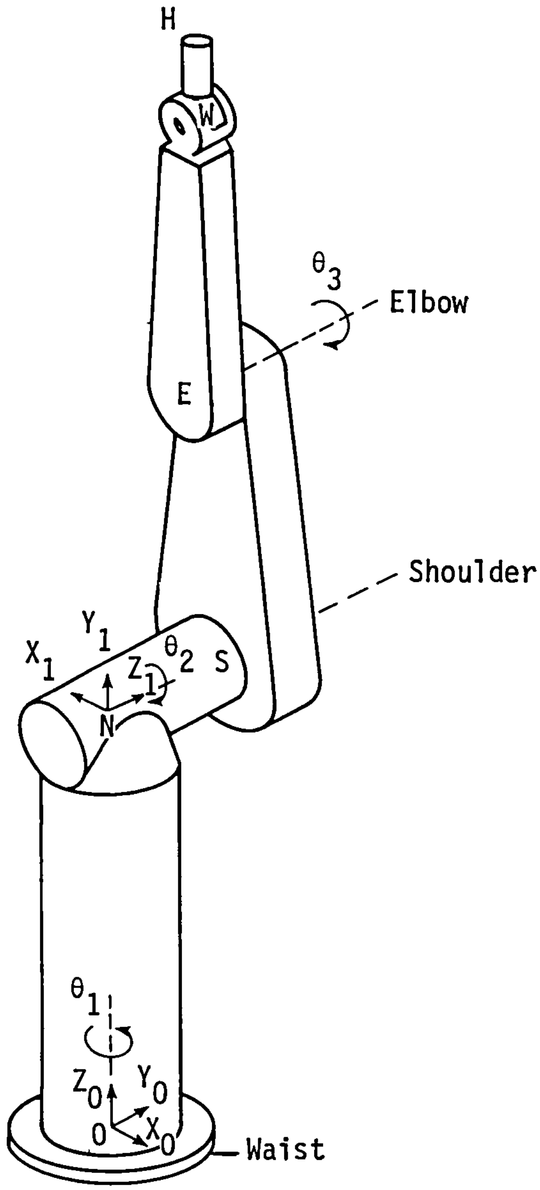


Figure 7.- Geometry used in locating elbow-joint axis system.

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16. Abstract

The Denavit-Hartenberg parameters define the relative location of successive joint axis systems in a robot arm. A recent justifiable criticism is that one of these parameters becomes extremely large when two successive joints have near-parallel rotational axes. Geometrically, this parameter then locates a joint axis system at an excessive distance from the robot arm and, computationally, leads to an ill-conditioned transformation matrix. In this paper, a simple modification (which results from constraining a transverse vector between successive joint rotational axes to be normal to one of the rotational axes, instead of both) overcomes this criticism and favorably locates the joint axis system.

An example is given for near-parallel rotational axes of the elbow and shoulder joints in a robot arm. The regular and modified parameters are extracted by an algebraic method with simulated measurement data. Unlike the modified parameters, extracted values of the regular parameters are very sensitive to measurement accuracy.

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