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NASA Technical Memorandum 87724 NASA-TM-87724 19860019249

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July 1986

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NF01615

TRANSPORT MODEL OF NUCLEON-NUCLEUS REACTION

Abstract

A simplified model of nucleon-nucleus reaction is developed and some of its properties are examined. Comparisons with proton production measured for targets of $^{27}A\ell$, ^{58}Ni, ^{90}Zr, and ^{209}Bi show some hope for developing an accurate model for these complex reactions. It is suggested that binding effects are the next step required for further development.

N86-28721#

Introduction

The understanding of the interaction of energetic charged particles with matter is of importance to radiation protection in space as well as in astrophysics and radiotherapy (ref. 1). Fundamental to proton transport is the understanding of the nuclear reactions involved. Considered herein is a simplified model of the nuclear induced reactions with the intent of developing a simple formalism which is accurate enough for transport calculations.

Theory

Many years ago, Serber proposed that high-energy nuclear reactions consist of two distinct phases when viewed on a time scale (ref. 2) First, the passage of the initiating event is on the order of the transit time across the nuclear volume requiring only

$$t_1 \approx 5 \text{ fm/c} = 1.7 \times 10^{-23} \text{ sec}$$

and is short compared to the internal motion within the nuclear interior (ref. 2). The second phase of the reaction, which involves the residual excitation of the target nucleus after the passage of the projectile, is characteristic of the internal nuclear structure and occurs on a much longer time scale ($\sim 10^{-15}$ sec).

To treat the first phase, an approximate Boltzmann equation will be applied to nuclear matter (refs. 3 and 4). Particle spectra at the end of the first step will be evaluated. These may be used in a compound nucleus model to further define the Serber second phase.

The transport equation in nuclear matter is

$$[\Omega \cdot \nabla + \rho\sigma]\psi(x,\Omega,E) = \int \rho\sigma f(E,\Omega,E',\Omega')\psi(x,\Omega',E')d\Omega' dE' \quad (1)$$

where σ is the two-body cross section for interaction of the field ψ with individual constituents of the nuclear matter, ρ is the constituent density, $f(E,\Omega,E',\Omega')$ is the secondary production spectra in the interaction events, and $\psi(x,\Omega,E)$ is the field function of the transported particles.

Past studies have shown that the straight ahead approximation is fully justified in space radiation shielding applications (ref. 1). This encourages us to ignore the angular dependence in the nuclear problem as it is ignored in the transport problem. All particles are then assumed to be produced in the forward direction as would be expected at high energies due to relativistic effects as well as the dynamical nature of the two-body force. This causes simplifications in equation (1) as

$$\left[\frac{\partial}{\partial Z} + \rho\sigma\right]\psi(\vec{b},Z,E) = \int_E^\infty \rho f(E,E')\psi(\vec{b},Z,E')dE' \tag{2}$$

where the direction of motion is along $+\hat{Z}$, and $\vec{b}$ is the impact parameter vector. The integral form of equation (2) may be written as

$$\psi(\vec{b},Z,E) = e^{-\int_{-\infty}^{Z} \rho(\vec{b},y)\sigma\, dy}\, \psi_i(\vec{b},E)$$

$$+ \int_{-\infty}^{Z} e^{\int_{-\infty}^{Z'} \rho(\vec{b},y)dy}\, \rho(\vec{b},Z')dZ' \int_E^\infty f(E,E')\psi(\vec{b},Z,E')dE' \tag{3}$$

where $\psi_i(\vec{b},E)$ is the incident particle fluence at impact parameter $\vec{b}$, and its coefficient in equation (3) is the appropriate attenuation factor. In what follows, the incident fluence is taken as

$$\psi_i(\vec{b},E) = \delta(E-E_0) \tag{4}$$

corresponding to a unit fluence with energy E_0.

Attenuation of the Primary Beam

The attenuation of the primary beam is given by the inhomogenous term in equation (3) as

$$\psi_0(\vec{b},Z,E) = e^{-\int_{-\infty}^{Z} \sigma\rho(\vec{b},y)dy} \psi_i(\vec{b},E) \tag{5}$$

which are those particles passing through to a position Z without collision. The number which suffer collision before reaching Z are

$$- \quad -\int^{Z} \sigma\rho(\vec{b},y)dy \quad) \; \psi_i(\vec{b},E) \tag{6}$$

from which the absorption cross section is obtained by integrating the absorption factor over all impact parameters as

$$\sigma_{abs} = 2\pi \int_0^\infty (1 - e^{-\int_{-\infty}^{\infty} \sigma\rho(\vec{b},y)dy})bdb \tag{7}$$

in the limit as Z becomes large. Evaluating for a uniform nuclear density yields

$$\sigma_{abs} = 2\pi \int_0^\infty (1 - e^{-2\rho_0\sigma\sqrt{R^2 - b^2}})bdb$$

$$= \pi R^2 - \frac{2\pi}{(2\rho_0\sigma)^2} \hat{\alpha}_1(2\rho_0\sigma R) \tag{8}$$

where ρ_0 is the nucleon density, R is the nuclear radius, and $\hat{\alpha}_n(Z)$ is related to the exponential integrals. In the limit as $\rho_0\sigma \to \infty$ yields

$$\sigma_{abs} \simeq \pi R^2 \tag{9}$$

as represented by an absorbing disk, and in the limit as $\rho_0\sigma \to 0$ yields

$$\sigma_{abs} \approx A_t\sigma(1 - \frac{3}{4}\rho_0\sigma R) \tag{10}$$

where the first term results from scattering from each individual constituent and the second term is the first shadow correction.

Two-body Scattering Cross Sections

The nucleon-nucleon differential cross section is represented as

$$\frac{d\sigma}{dt} = Ae^{+\beta t} \tag{11}$$

where β is the usual slope parameter and t is the square of the four-momentum transfer. A is related to the total cross section and to α, the ratio of the real-to-imaginary part of the scattering amplitude (ref. 5) as

$$A = 2\pi\left(\frac{\sigma_{tot}}{4\pi\hbar c}\right)^2 (1 + \alpha^2) \tag{12}$$

The factor of 2 in equation (12) approximately accounts for the backward peak in the cross section. The energy transfer cross section is then

$$\frac{d\sigma}{dE} = 2mc^2Ae^{-2mc^2\beta(E_o - E)} \tag{13}$$

since

$$t = -2mc^2(E_o - E) \tag{14}$$

where m is the rest mass of a nucleon and $(E_o - E)$ the energy transfer. Considering the target nucleon at rest yields the struck nucleon recoil spectrum as

$$\frac{d\sigma_T}{dE} = 2mc^2A\ e^{-2mc^2\beta E} \tag{15}$$

Thus the spectrum of scattered particles is given by

$$f(E,E') = 2mc^2A[e^{-2mc^2\beta(E' - E)} + e^{-2mc^2\beta E}] \tag{16}$$

for use in equation (3). Since the struck nucleon is immersed in a "sea" of nucleons, not all energy transfers are allowed. Only those for which the struck particle is lifted above the fermi surface may scatter. The appropriate cross section is then

$$\sigma(E_o) = 2mc^2A \int_{E_F}^{E_o} e^{-2mc^2\beta(E_o - E)} dE$$

$$= \frac{A}{\beta}\,\hat{\alpha}_o(2mc^2\beta(E_o - E_F)) \qquad (17)$$

where the contribution from $E_F = o$ is the Pauli blocking factor.

Multiple Scattering Series

Now consider the transport equation neglecting the recoiling target particles

$$\left[\frac{\partial}{\partial Z} + \rho(\vec{b},Z)\sigma\right]\psi(\vec{b},Z,E) = \rho(\vec{b},Z)\int_{E}^{\infty} 2mc^2A\, e^{-2mc^2\beta(E' - E)}\psi(\vec{b},Z,E')dE' \qquad (18)$$

This is solved by treating the left side of equation (18) as a perturbation with the first term

$$\psi_o(\vec{b},Z,E) = e^{-\int_{-\infty}^{Z}\sigma\rho(\vec{b},y)dy}\,\psi_i(\vec{b},E) \qquad (19)$$

and assuming boundary condition in equation (4). The first perturbation term is

$$\left[\frac{\partial}{\partial Z} + \rho(\vec{b},Z)\sigma\right]\psi_1(\vec{b},Z,E) = \rho(\vec{b},Z)\, e^{-\int_{-\infty}^{Z}\sigma\rho(\vec{b},y)dy}\, 2mc^2Ae^{-2mc^2\beta(E_o - E)} \qquad (20)$$

for which

$$\psi_1(\vec{b},Z,E) = \int_{-\infty}^{Z} e^{-\int_{-\infty}^{Z} \sigma\rho(b,y)dy} \rho(\vec{b},Z')2mc^2A\, e^{-2mc^2\beta(E_o - E)} dz' \tag{21}$$

The second perturbation is then

$$\psi_2(\vec{b},Z,E) = \int_{-\infty}^{Z} e^{-\int_{-\infty}^{Z} \sigma\rho(\vec{b},y)dy} \rho(\vec{b},Z')$$

$$\times \int_{-\infty}^{Z'} \rho(\vec{b},Z')dZ''dZ'(2mc^2A)^2(E_o - E)\, e^{-2mc^2\beta(E_o - E)} \tag{22}$$

and the third iteration as

$$\psi_3(\vec{b},Z,E) = \int_{-\infty}^{Z} e^{-\int_{-\infty}^{Z} \sigma\rho(\vec{b},y)dy} \rho(\vec{b},Z')\int_{-\infty}^{Z} \rho(\vec{b},Z'')\int_{-\infty}^{Z} \rho(\vec{b},Z''')dZ'dZ''dZ'''$$

$$\times \frac{1}{2}\,(2mc^2A)^3\,(E_o - E)^2\, e^{-2mc^2\beta(E_o - E)} \tag{23}$$

The general term may be written as

$$\psi_n(\vec{b},Z,E) = \phi_n(\vec{b},Z)\, F_n(E) \tag{24}$$

where

$$\phi_n(\vec{b},Z) = \frac{\rho_o{}^n(Z + \sqrt{R^2 - b^2})^m}{\Gamma(n + 1)}\, e^{-2\rho_o(Z + \sqrt{R^2 + b^2})} \tag{25}$$

The $F_n(E)$ is

$$F_n(E) = (2mc^2A)^n \frac{(E_o - E)^{n-1}}{\Gamma(n)} e^{-2mc^2\beta(E_o - E)} \qquad (26)$$

The even smaller perturbations from the target constituent recoil spectrum will now be treated.

Multiple Production Series

The target recoil constituents are of inferior energy compared to the incident beam and are assumed to be reabsorbed by the surrounding nuclear material if they further interact after initial production. For each of the above terms there is a production term, the first of which is given by

$$\left[\frac{\partial}{\partial Z} + \rho(\vec{b},Z)\sigma\right] \psi_1'(\vec{b},Z,E) = \rho(\vec{b},Z), \; e^{-\int_{-\infty}^{Z} \sigma\rho(\vec{b},y)dy} \; 2mc^2Ae^{-2mc^2\beta E} \qquad (27)$$

The solution is

$$\psi_1'(\vec{b},Z,E) = \phi_1(\vec{b},Z) \; 2mc^2A \; e^{-2mc^2\beta E} \qquad (28)$$

$$= \phi_1(\vec{b},Z) \; F_1'(E)$$

Similarly

$$\psi_n'(\vec{b},Z,E) = \phi_n(\vec{b},Z) \; F_n'(E) \qquad (29)$$

where

$$F_n'(E) = 2mc^2A \; e^{-2mc^2\beta E}\left(\frac{A}{\beta}\right)^n \frac{\hat{\alpha}_{n-2}(2mc^2\beta(E_o - E))}{\Gamma(n-1)} \qquad (30)$$

Multiparticle Production

All particles escaping the nuclear reaction may now be calculated by summing terms and allowing Z to approach infinity as

$$\psi(\vec{b},E) = \psi_1(\vec{b},E) + \psi_2(\vec{b},E) + \psi_3(\vec{b},E) + \dots$$

$$+ \psi_1'(\vec{b},E) + \psi_2'(\vec{b},E) + \psi_3'(\vec{b},E) + \dots \qquad (31)$$

The particle production cross section is then given by

$$\frac{d\sigma_{prod}}{dE} = 2\pi \int_0^{\infty} \psi(\vec{b},E) b \, db$$

$$= \sum_i \hat{\sigma}_i [F_i(E) + F_i'(E)] \qquad (32)$$

where the $\hat{\sigma}_i$ for a uniform nuclear model is

$$\hat{\sigma}_i = 2\pi \int_0^R \phi_i(\vec{b}) b \, db$$

$$= \frac{2\pi \rho_0^{\,i} \, 2^i}{\Gamma(i + 1)(2\rho_0\sigma)^{2 + i}} \quad \hat{\alpha}_{i+1}(2\rho_0\sigma R) \qquad (33)$$

The total production cross section is found by integrating (32) over energy to obtain

$$\sigma_{prod} = \sum_i \hat{\sigma}_i [\hat{F}_i + \hat{F}_i'] \qquad (34)$$

where

$$\hat{F}_i = \int_{E_F}^{E_o} F_i(E)dE$$

$$= \left(\frac{A}{\beta}\right)^i \frac{\hat{\alpha}_{i-1}(2mc^2\beta(E_o-E_F))}{\Gamma(i)} \quad (35)$$

and

$$\hat{F}_i' = \int_{E_F}^{E_o} F_i'(E)dE$$

$$= \hat{F}_i \; e^{-2mc^2\beta E_F} \quad (36)$$

It follows from conservation of particles that

$$\sum \hat{\sigma}_i \hat{F}_i = \sigma_{abs}$$

The multiplicity of the transport event as herein approximated is

$$m = \sigma_{prod}/\sigma_{abs}$$

$$= 2 + (1 - e^{-2mc^2\beta E_F}) \quad (37)$$

which is an indication that the recoil particles need to be treated in more detail since intranuclear cascade codes indicate $2 \leq m \lesssim 2.5$.

Proton Production in Nuclear Reaction

It may be easily shown that the proton production spectrum is approximated by

$$\frac{d\sigma_p}{dE} = \frac{Z\sigma_p}{A\sigma} \sum_i \hat{\sigma}_i [F_i(E) + \hat{F}_i(E)] \tag{38}$$

where Z is the atomic number and σ_p the appropriate proton cross section for the projectile. Results of equation (38) are compared with the experiments of W_u et al. in figure 1. Quite good agreement is obtained at the highest energies (>40 MeV). The low energy portion of the spectrum is adversely affected by high order multiple scattering terms adding to the production of protons. These terms would be limited in their contributions if the bulk binding potential are properly treated. A more complete theory should improve the agreement.

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An Appendix of Useful Functions

Several functions that are closely related to exponential integrals and gamma functions are herein defined.

$$\hat{\alpha}_n(Z) = \int_0^Z t^n \, e^{-t} dt \tag{A1}$$

with special value

$$\hat{\alpha}_o(Z) = 1 - e^{-Z} \tag{A2}$$

and recurrence relation

$$\hat{\alpha}_n(Z) = n \, \hat{\alpha}_{n-1}(Z) - Z^n e^{-Z} \tag{A3}$$

A useful integral is

$$\hat{A}_n(Z) = \int_0^Z \hat{\alpha}_n(t) dt \tag{A4}$$

with special value

$$\hat{A}_o(Z) = Z \, \hat{\alpha}_o(Z) \tag{A5}$$

and recurrence relation

$$\hat{A}_n(Z) = n\hat{A}_{n-1}(Z) - \hat{\alpha}_n(Z) \tag{A6}$$

Useful functions related to $\hat{\alpha}_n(Z)$ are

$$\hat{B}_n(Z) = \int_0^Z e^t \hat{\alpha}_n(t)dt \tag{A7}$$

$$\hat{C}_n(Z) = \int_0^Z te^t \hat{\alpha}_n(t)dt \tag{A8}$$

with special values

$$\hat{B}_o(Z) = e^Z - (1+Z) \tag{A9}$$

$$\hat{C}_o(Z) = (Z-1)\, e^Z + 1 - Z^2/2 \tag{A10}$$

and recurrence relations

$$\hat{B}_n(Z) = n\hat{B}_{n-1}(Z) - Z^{n+1}/(n+1) \tag{A11}$$

$$\hat{C}_n(Z) = n\hat{C}_{n-1}(Z) - Z^{n+2}/(n+2) \tag{A12}$$

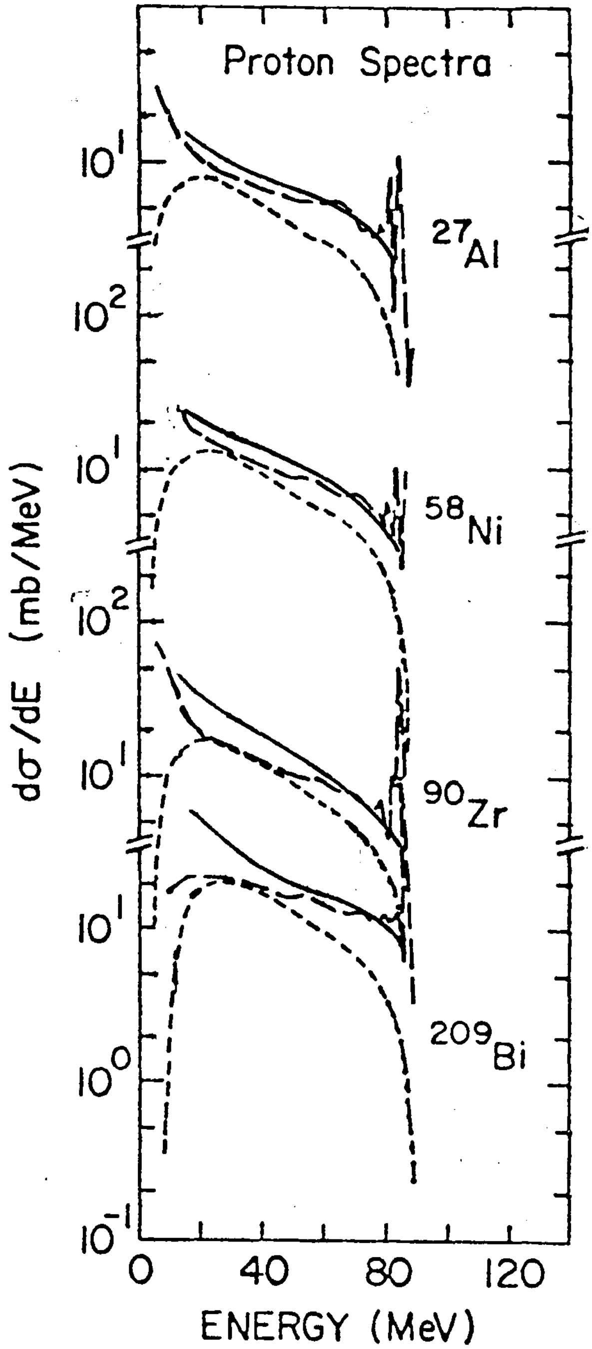

Figure 1-Proton spectra from proton impact on nuclei at 90 MeV. Short dash is the calculations of Wu et al., long dash is the experiments of Wu et al. and the full line is the transport theory.

Standard Bibliographic Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-87724		

4. Title and Subtitle
TRANSPORT MODEL OF NUCLEON-NUCLEUS REACTION

5. Report Date
July 1986

6. Performing Organization Code
506-41-41-01

7. Author(s)
John W. Wilson, Lawrence W. Townsend, and Francis A. Cucinotta

8. Performing Organization Report No.

9. Performing Organization Name and Address
Langley Research Center
Hampton, Virginia 23665-5225

10. Work Unit No.

11. Contract or Grant No.

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Washington, DC 20546

13. Type of Report and Period Covered
Technical Memorandum

14. Sponsoring Agency Code

15. Supplementary Notes

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17. Key Words (Suggested by Authors(s))
Proton; Nuclear Interaction; Cross Sections

18. Distribution Statement
UNCLASSIFIED - Unlimited

Subject Category - 73

19. Security Classif.(of this report)	20. Security Classif.(of this page)	21. No. of Pages	22. Price
UNCLASSIFIED	UNCLASSIFIED	16	A02

For sale by the National Technical Information Service, Springfield, Virginia 22161

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