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A THEORY FOR LATERAL WING-TIP BLOWING

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ABSTRACT

The concept of lateral blowing consists in utilizing thin jets of air, which are ejected in the spanwise direction from slots at the tips of straight and swept wings, or along the leading edges of delta wings, to generate aerodynamic forces without the assistance of deflecting solid surfaces. For weak intensities of blowing the so-generated forces could be used for roll and lateral control of aircraft.

In this work a theory for this concept as applied to straight wings is presented, revealing the analytical relationship between blowing and aerodynamic forces. The approach is based on perturbing the span of an elliptically loaded wing. Scaling laws involving blowing intensity, aspect ratio, and angle of attack are derived and compared with experiments. It is concluded that this concept has potential as a novel roll and lateral control device.

NOMENCLATURE

A	aspect ratio, as a variable
A_0	aspect ratio, reference value
a	proportionality factor in change of circulation for symmetrical blowing
a'	proportionality factor in change of local lift coefficient for one-sided blowing
b	wing span, as a variable
b_0	wing span, reference value
c	wing chord
C_L	lift coefficient
C_L'	lift slope
C_L^0	unblown local lift coefficient
C_L^1	local lift coefficient for one-sided blowing
C_L^*	blown local lift coefficient for half-span models
$C_{L_0}^0$	maximum value of C_L^0
$C_{L_0}^1$	maximum value of C_L^1
C_l	rolling moment coefficient
C_l^*	measure of rolling moment from half-span models
C_p	pressure coefficient
C_{p0}	maximum value of C_p
C_μ	jet momentum coefficient
$\tilde{C}_\mu$	non-dimensional jet momentum coefficient
$\hat{C}_\mu$	rescaled jet momentum coefficient
$f(A)$	universal function defining lift slope
$F(A)$	universal function in lift increment

$G(A)$	universal function in rolling moment
$H(A)$	universal function relating half-span model results
k	experimentally determined constant
p_1	pressure on upper side of jet
p_2	pressure on lower side of jet
q_∞	free stream dynamic pressure
R	jet local radius of curvature
U_∞	free stream velocity
v_j	jet velocity
x, y, z	coordinate axis
z	as a function, jet displacement
z_{max}	maximum jet displacement
α	angle of attack
δ_j	jet thickness
ΔC_L	increment of lift coefficient
$\Delta C_{L_{twist}}$	change in lift coefficient due to jet twist
Δb	absolute wing span change
η	variable of integration
ϵ	relative wing span change
γ	local circulation
ρ_j	jet fluid density
ζ	substitution variable
θ	local jet angle with respect to the y axis
θ_0	jet ejection angle

CONTENTS

1. INTRODUCTION

When jets in the form of thin sheets are ejected from the tip of a straight or swept wing, as shown in Fig. 1, it is observed that the lift produced by the wing increases. This fact suggest the possibility of utilizing this concept as a means of generating additional lift in a manner similar to flaps or ailerons, but with some important differences, the most significant being that no deflecting solid surfaces would be involved and the loads imposed on the wing would be distributed differently. Also, the way the additional aerodynamic load responds to the intensity of blowing may constitute an advantage.

Ayers and Wilde[1] reported measurements on a wing of aspect ratio 1.39 and 50° sweep, showing significant gains in lift with lateral blowing, as well a beneficial effects of blowing on stall. Carafoli[2] conducted experiments with a straight wing of aspect ratio 2, and formulated a theoretical approach. His theory was based on an extension of Prandtl's lifting line theory and represented the experimental trends reasonably well, although it failed to establish scaling laws or analytical relationships between blowing intensity and aerodynamic forces. Later, Carafoli and Camaracescu[3] conducted experiments on small aspect ratio wings, observing the fact that lift augmentation due to lateral blowing is enhanced for smaller aspect ratios. Further experimental work on lateral blowing has been reported by White[4], who noticed some beneficial effects in drag under certain conditions. Briggs and Schwind[5] considered the lateral blowing concept as a lift augmentation device for STOL aircraft. Their experiments suggest that a net gain in STOL capabilities would be possible. Hickey[6] tested swept wings of aspect ratios 1.9 and 2.5 and observed that the rate of gain of additional lift was larger for weaker blowing. Wu et al[7,8] have looked at the concept of tip blowing where several discrete thin jets are ejected from wing tips, and inferred similarities with the winglet concept. Tavella et al[10] conducted experiments on a rectangular wing of aspect ratio 3.1, where weak tip blowing as a means of generating roll control forces was investigated.

2. THE PHYSICAL PROBLEM

The effect of lateral blowing on a straight wing can be visualized by thinking of the lateral jet as a fluid extension of the wing itself. Although the way in which this fluid extension affects the aerodynamics of the wing is one of great complexity, it can be characterized by the following facts:

a) The lateral jet tends to drive the tip vortices outwards. This effect taken in isolation would cause the wing to react as if it had increased its span.

b) The fluid extension of the wing is subject to a pressure difference between its lower and upper surfaces which varies in the chordwise direction. This causes a positive twist of the fluid extension, to which the flow about the wing reacts by increasing the loading close to the tip. The tip jet eventually rolls up and merges with the shed vortices.

c) Viscous effects in the fluid extension of the wing will affect the pressure distribution on the wing surface.

d) Entrainment into the tip jet has an effect - plausibly a favorable one - on boundary layer development in the vicinity of the wing tip, thereby contributing to stall delay.

The analysis will be restricted to weak blowing, in which case the fluid extension of the wing is a small fraction of the wing span. Rather than an exact solution, scaling laws will be searched for. In this context the following assumptions will be made:

1. The most important effect of weak lateral blowing is to effectively enlarge the span of the wing. In other words, the only effect to be considered is the removal of the tip vortices to a position farther out in the spanwise direction.

2. The change of span process is dominated by inviscid forces. This approach neglects the viscous effects due to entrainment into the jet. Its validity will be indicated by the type of agreement with experimental results.

3. MATHEMATICAL MODEL

3.1 Span Perturbation Concept

The aerodynamic effect of the change in span due to blowing will be computed by perturbing the span of the wing by an amount dependent on the intensity of the tip jet and on the angle of attack. It will be assumed that the distribution of load on the wing is elliptical, and that the tip slot is aligned with the zero-lift direction of the wing. The perturbed wing is constructed by adding to the unblown wing a small segment of length Δb with chord roughly of the same size as the chord in the neighbourhood of the tip of the unblown wing, as shown in Fig. 2. This implies that the analysis will apply to a jet exiting from a slot of length not too different from the mean wing chord. The lift produced by the perturbed wing is given by

$$C_L = (C'_{L_{C_\mu=0}} + \Delta C'_L)\alpha, \tag{1}$$

where $\Delta C'_L$ is due to a change in the span alone. To compute $\Delta C'_L$ let's express the slift slope as

$$C'_L = 2\pi f(A). \tag{2}$$

The increment of lift slope must be referred to the original span of the unperturbed wing. Defining the relative span change as

$$\epsilon = \frac{\Delta b}{b_0} \qquad (3)$$

we have

$$\Delta C_L' = 2\pi\{f[A_0(1+\epsilon)](1+\epsilon) - f(A_0)\}. \qquad (4)$$

Expanding in series we get

$$\frac{\Delta C_L'}{C_L'} = \left[1 + A_0 \frac{\dot{f}(A_0)}{f(A_0)}\right]\epsilon. \qquad (5)$$

This expression also includes the lift acting on the fluid perturbation of the wing span. However, it can be shown that, as $\epsilon \to 0$, the lift increment obtained by utilizing $\Delta C_L'$ from Eq. (5) also becomes the load acting on the solid part of the wing. To prove this, we compare two uncambered wings of the same chord, with self-similar loads, set at the same angle of attack, with semi-spans b_0 and $b_0 + \Delta b$ each, and with local circulations $\gamma(\eta)$ and $(1+a\epsilon)\gamma(\eta)$ respectively, as shown in Fig. 2. The lift increment represented by the shadowed region, when referred to the original wing, is

$$\Delta C_L \propto \int_{-1}^{1} \left\{(1+a\epsilon)\gamma\left[\frac{y}{b_0(1+\epsilon)}\right] - \gamma\left(\frac{y}{b_0}\right)\right\}\frac{dy}{b_0}. \qquad (6)$$

Expanding $\gamma\left[\frac{y}{b_0(1+\epsilon)}\right]$

$$\Delta C_L \propto -\epsilon \int_{-1}^{1} \dot{\gamma}\left(\frac{y}{b_0}\right) \frac{y}{b_0} \frac{dy}{b_0} + a\epsilon \int_{-1}^{1} \gamma\left(\frac{y}{b_0}\right) \frac{dy}{b_0} + O(\epsilon^2). \tag{7}$$

Integrating the first term in Eq. (7) by parts we find

$$\frac{\Delta C_L}{C_L} = (1+a)\epsilon + O(\epsilon^2). \tag{8}$$

Identifying now a with $A_0 \frac{\dot{f}(A_0)}{f(A_0)}$, we see that to first order in ϵ the lift increment given by Eq. (8) agrees with Eq. (5). Then, under the assumption of self-similar loading the lift supported by the fluid extension of the wing is of order ϵ^2.

Eq. (5) also assumes that any induced lift produced by the contortion or twist of the fluid extension of the wing is less important than the lift induced by the effective change in span due to that fluid extension. A plausibility argument as to why this is expected to be the case can be given along the following lines:

As illustrated in Fig. 1, as the tip jet leaves the wing it curls up more intensively near the leading edge than near the trailing edge. This difference in the curling of the jet sheet constitutes the twist of the fluid extension. Its effect can be visualized in Fig. 3. Introducing the assumption that the lift induced by the twisted tip is of the same order as the lift change that would occur if the angle of attack of the wing with untwisted tip were to be increased by a fraction of the order of

$$\Delta\alpha \propto \frac{\Delta b}{b_0}\frac{z_{max}}{c}, \tag{9}$$

it follows that

$$\Delta C_{L_{twist}} \sim \epsilon \frac{z_{max}}{c}. \tag{10}$$

For sufficiently weak blowing, z_{max} must be proportional to Δb , since there are no other relevant length scales in the vicinity of the tip. Hence,

$$\Delta C_{L_{twist}} \sim \epsilon^2 A_0 \tag{11}$$

Finally, since ΔC_L is proportional to ϵ ,

$$\lim_{\epsilon \to 0} \frac{\Delta C_{L_{twist}}}{\Delta C_L} = 0 \tag{12}$$

3.2 Scaling Laws

To compute the relative change in span ϵ consider the lateral jet to consist of an infinitely thin momentum sheet subject to a pressure difference between its two surfaces as shown in Fig. 4. Under the effect of this pressure difference the sheet changes its inclination with respect to the plane of the wing as the jet extends outwards. Assuming now that this idealized jet represents a typical strip in the spanwise direction, the position where the orientation of the jet with respect to the plane of the wing reaches $\frac{\pi}{2}$ will be taken to be proportional to ϵ. The analytical dependence of ϵ on the jet and wing parameters can then be inferred from the position where the solution to an equation for the lateral jet slope becomes singular.

The balance of pressure and centrifugal forces in a thin inviscid jet sheet is expressed by:

$$\frac{1}{R} = \frac{C_p}{C_\mu}, \tag{13}$$

where R is the jet local radius of curvature. Defining a pressure coefficient

$$C_p = \frac{p_2 - p_1}{q_\infty} \tag{14}$$

where p_1 , p_2 are representative values of the pressure in the spanwise direction. The jet momentum coefficient has dimension of length and is given by

$$C_\mu = \frac{\rho_j v_j^2 \delta_j}{q_\infty}. \tag{15}$$

In differential equation form:

$$\frac{d^2 z}{dy^2} = \left[1 + \left(\frac{dz}{dy}\right)^2\right]^{\frac{3}{2}} \frac{C_p}{C_\mu} \tag{16}$$

This equation can be solved by introducing the transformation

$$\sinh \varsigma = \frac{dz}{dy} \tag{17}$$

With this substitution Eq. (16) becomes

$$\frac{d\varsigma}{dy} = \cosh^2 \varsigma \frac{C_p}{C_\mu} \tag{18}$$

This can now be integrated to give

$$\frac{\dot{z}}{\sqrt{1+\dot{z}^2}} = \int_{b_0}^{y} \frac{C_p(\eta)}{C_\mu} d\eta + \frac{\dot{z}}{\sqrt{1+\dot{z}^2}}\Bigg]_{y=b_0} . \tag{19}$$

Introducing the definition

$$\frac{dz}{dy} = \tan\theta, \tag{20}$$

Eq. (19) can be rewritten

$$\sin\theta = \int_{b_0}^{y} \frac{C_p(\eta)}{C_\mu} d\eta + \sin\theta_0 \tag{21}$$

The position where the solution becomes singular provides an equation for ϵ. This is achieved by setting $\theta = \frac{\pi}{2}$.

$$\int_1^{1+\epsilon} c_p(\mu) dy = \frac{C_\mu}{b_0}(1 - \sin\theta_0) \tag{22}$$

In this expression θ_0 represents the angle that the jet forms with the direction of the span at exit.

To solve this equation we assume an elliptical distribution of pressure on the wing of the form

$$\frac{C_p^2}{C_{p_o}^2} + \frac{y^2}{b_o^2(1+\varepsilon)^2} = 1 \tag{23}$$

which in the neighbourhood of the tip simplifies to:

$$C_p = \sqrt{2}C_{p_o}\sqrt{1+\varepsilon-\frac{y}{b_o}} \quad + O(\varepsilon^2) \tag{24}$$

Substituting this expression in Eq. (22) and performing the integral we get

$$\epsilon = \left(\frac{3}{2\sqrt{2}}\right)^{\frac{2}{3}} \left[\frac{C_\mu(1-\sin\theta_0}{C_{p_0}b_0}\right]^{\frac{2}{3}}, \tag{25}$$

Let's consider first the case when the jet leaves tangentially to the direction of the wing span, in such a case $\theta_0 = 0$, and the lift increment is given by

$$\Delta C_L = 2\pi k\left[f(A_0) + A_0\dot{f}(A_0)\right]\left(\frac{C_\mu}{C_{p_0}b_0}\right)^{\frac{2}{3}}\alpha, \tag{26}$$

with k a constant of order 1.

At this point it appears advantageous to define a non-dimensional form of the jet momentum coefficient such that it becomes a property of the wing tip and the jet intensity alone.

Defining a non-dimensional jet momentum coefficient as

$$\tilde{C}_\mu = \frac{C_\mu}{c} \tag{27}$$

Eq. (26) can be rewritten

$$\triangle C_L = 2\pi k[f(A_0) + A_0\dot{f}(A_0)]\left(\frac{2\tilde{C}_\mu}{C_{p_o}A_0}\right)^{\frac{2}{3}}\alpha, \tag{28}$$

Making use of the proportionality

$$C_{p_0} \propto C_L'\alpha \tag{29}$$

Eq. (26) can finally be written as

$$\triangle C_L = kF(A_0)\tilde{C}_\mu^{\frac{2}{3}}\alpha^{\frac{1}{3}}, \tag{30}$$

where k has been redefined as a constant to be determined from experiments and $F(A)$ is a universal function of aspect ratio defined by

$$F(A) = 2\pi^{\frac{1}{3}}\left\{\left[\frac{f(A)}{A^2}\right]^{\frac{1}{3}} + \dot{f}(A)\left[\frac{A}{f^2(A)}\right]^{\frac{1}{3}}\right\}. \tag{31}$$

This expression reveals that the scalings for angle of attack and blowing intensity are given through simple power laws. On the other hand, the dependance on the aspect ratio enters in a complicated fashion through $F(A_0)$, which can only be known approximately for an arbitrary aspect ratio. However, for very small and very large aspect ratios, simplifications are possible that allow us to find algebraic scaling laws for such cases as well. For the case of infinitely small aspect ratio, the following expression holds

$$f(A) = \frac{A}{4}. \tag{32}$$

Substituting in Eq. (31)

$$\lim_{A_0 \to 0} \Delta C_L = (16\pi)^{\frac{1}{3}} k \tilde{C}_\mu^{\frac{2}{3}} \left(\frac{\alpha}{A_0}\right)^{\frac{1}{3}}. \tag{33}$$

For the case of very large aspect ratio, the asymptotic expansion of the lift slope for elliptical loading leads to

$$f(A) = 1 - \frac{2}{A}. \tag{34}$$

Substituting in Eq. (31),

$$\lim_{A_0 \to \infty} \Delta C_L = 2\pi^{\frac{1}{3}} k \left(\frac{\tilde{C}_\mu}{A_0}\right)^{\frac{2}{3}} \alpha^{\frac{1}{3}} \tag{35}$$

To find an expression for the function $F(A)$ for an arbitrary aspect ratio, an equation for the lift slope uniformly valid for any aspect ratio is required. Such an expression has been calculated by Germain[9]

$$f(A) = \left[1 + \frac{2}{A} + \frac{16}{(\pi A)^2} \log(1 + \pi e^{-\frac{9}{8}} A)\right]^{-1} \tag{36}$$

A plot of $F(A)$ is shown in Fig. 5.

3.3 Alternative Definition of Angle of Attack

In the derivations above it was assumed that the angle of attack was measured with respect to the direction of zero lift of the airfoil section. If a different definition were to be used, as might be convenient in the case of a cambered wing, a slight reworking of the equations would recast them in a more usable form. In this case a finite lift coefficient C_{L_0} must be added to the right hand side of Eq. (29), and Eq. (28) becomes

$$\Delta C_L = 2\pi k \left[f(A_0) + A_0 \dot{f}(A_0)\right] \left[\frac{2\bar{C}_\mu}{(C_L'\alpha + C_{L_0})A_0}\right]^{\frac{2}{3}} \alpha, \qquad (37)$$

which can also be written

$$\Delta C_L = kF(A_o)\hat{C}_\mu^{\frac{2}{3}}\alpha^{\frac{1}{3}}. \qquad (38)$$

Where $\hat{C}_\mu$ is a corrected jet momentum coefficient given by

$$\hat{C}_\mu = \frac{\bar{C}_\mu}{1 + \frac{C_{L_0}}{2\pi f(A_0)\alpha}} \qquad (39)$$

Hence, with a proper rescaling of the jet momentum intensity, it is possible to use the same formulation as before for an arbitrary definition of the angle of attack.

3.4 Blowing at Small Ejection Angle

In this case, besides the lift increment due to the span perturbation effect, there is a lift increase due to the vertical component of the momentum ejected into the free stream. The latter increment is

$$\Delta C_{L_m} = -2\frac{\rho_j\ v_j^2\ \delta_j c}{qS}\sin\theta_0. \qquad (40)$$

Written in terms of $\tilde{C}_\mu$,

$$\Delta C_{L_m} = -2\frac{\tilde{C}_\mu}{A_0}\sin\theta_0. \qquad (41)$$

The resultant lift increment becomes

$$\Delta C_L = kF(A_0)\left[\tilde{C}_\mu(1-\sin\theta_0)\right]^{\frac{2}{3}}\alpha^{\frac{1}{3}} - 2\frac{\tilde{C}_\mu}{A_0}\sin\theta_0. \qquad (42)$$

This formula reveals a double dependance on the ejection angle: In addition to the contribution of the vertical component of momentum, the jet behaves as if it were stronger. This is due to the fact that it takes a greater distance for the outer pressure field to curl up the jet when it is ejected at a small negative angle to the span direction.

3.5 Rolling Moment

Lateral blowing will produce rolling moments if there is a difference in the intensity of blowing between the two tips of the wing. Here we will concentrate on the analysis of the case when blowing occurs from one of the tips only.

The rolling moment coefficient is defined as

$$C_l = \frac{\text{rolling moment}}{2q_\infty S b_0}.$$

Calling C_L^0 and C_L^1 the lift coefficients per unit of span before and after blowing respectively, the rolling moment coefficient can be expressed

$$C_l = \frac{1}{4}\int_{-1}^{1}(C_L^1 - C_L^0)\eta dy. \tag{43}$$

The resultant lift distribution can be described in terms of two displaced ellipses. The rolling moment is caused by the difference between these two distributions, which are expressed in the following manner:

$$C_L^0 = C_{L_0}^0\sqrt{1-\eta^2} \tag{44}$$

$$C_L^1 = C_{L_0}^1\sqrt{1-\frac{(\eta-\epsilon/2)^2}{(1+\epsilon/2)^2}}. \tag{45}$$

The center values of the local lift for the two cases are related to each other;

$$C_L^1 = C_{L_0}^0(1+a'\epsilon), \tag{46}$$

where a' depends on A and α , and reflects the effect of span perturbation on the center value of the wing loading. The rolling moment can now be written

$$C_l = \frac{C^0_{L_0}}{4}[1 + a'\epsilon] \int_{-1}^{1} \sqrt{1 - \frac{(\eta - \epsilon/2)^2}{(1 + \epsilon/2)^2}}\, \eta d\eta. \tag{47}$$

Performing the integral in Eq. (47) and taking the limit for small ϵ , we find

$$C_l = \frac{C^0_{L_0}}{4}(1 + a'\epsilon)\left[\frac{\pi}{4}\epsilon + O(\epsilon^{\frac{3}{2}})\right], \tag{48}$$

which indicates that the effect of the lateral displacement of the two elliptical loadings is of lower order than the increase in magnitude due to blowing. Hence, ignoring terms of higher order in ϵ , and making use of the proportionality

$$C^0_{L_0} \propto C'_L \alpha \tag{49}$$

we get

$$C_l = k\frac{\pi^2}{8} f(A_0)\alpha\epsilon. \tag{50}$$

From Eq. 25 we can rewrite ϵ in the following way

$$\epsilon = \left(\frac{3}{\pi 2\sqrt{2}}\right)^{\frac{2}{3}} \left[\frac{\tilde{C}_\mu}{f(A_0)A_0\alpha}\right]^{\frac{2}{3}}, \tag{51}$$

substituting in Eq. (50) we get

$$C_l = kG(A_0)\tilde{C}_\mu^{\frac{2}{3}}\alpha^{\frac{1}{3}}, \tag{52}$$

where $G(A_0)$ is a universal function of aspect ratio given by

$$G(A) = \frac{(3\pi^2)^{\frac{2}{3}}}{16}\left[\frac{f(A)}{A^2}\right]^{\frac{1}{3}}. \tag{53}$$

A plot of $G(A)$ based on Germain's expression for $f(A)$ is shown in Fig. 5.

Eq. (52) indicates that the rolling moment due to one-sided blowing follows the same scaling in blowing intensity and angle of attack as the lift increment. For the dependance on aspect ratio in limiting cases we have

$$\lim_{A_0 \to 0} C_l = k\frac{1}{16}\left(\frac{3}{2}\pi^2\right)^{\frac{2}{3}}\tilde{C}_\mu^{\frac{2}{3}}\left(\frac{\alpha}{A_0}\right)^{\frac{1}{3}} \tag{54}$$

and

$$\lim_{A_0 \to \infty} C_l = k\frac{(3\pi^2)^{\frac{2}{3}}}{16}\left(\frac{\tilde{C}_\mu}{A}\right)^{\frac{2}{3}}\alpha^{\frac{1}{3}}. \tag{55}$$

Again, in these limits the scaling laws for aspect ratio are the same as those for the lift increment.

3.6 Measurement of Rolling Moment with Half-Span Model

Since lateral blowing affects the lift distribution over the entire length of the wing, special considerations are needed to interpret the measurements from tests with half-span models in regard to rolling moment. Half-span models simulate simultaneous blowing from both tips, while the phenomenon of interest concerns blowing from one of the tips only. To see to what extent the relevant rolling moment can be inferred from half-span model tests, denote the lift produced by a half-span model by C_L^*, and consider the quantity

$$C_l^* = \frac{1}{4}\int_0^1 \left[C_L^*(\eta) - C_L^0(\eta)\right] \eta d\eta, \tag{56}$$

where $C_L^*(\eta)$ is given by

$$C_L^*(\eta) = C_{L_0}^0(1+a\epsilon)\sqrt{1-\frac{\eta^2}{(1+\epsilon)^2}} \tag{57}$$

Here a is the function of aspect ratio discussed on page 7. Substituting $C_L^*(\eta)$ and $C_L^0(\eta)$ in Eq. (56)

$$C_l^* = \frac{C_{L_0}^0}{4}\left[(1+a\epsilon)(1+\epsilon)^2\int_0^{\frac{1}{1+\epsilon}} \sqrt{1-\eta^2}\,\eta dy - \int_0^1 \sqrt{1-\eta^2}\,\eta dy\right]. \tag{58}$$

Performing these integrals we find

$$C_l^* = \frac{C_{L_0}^0}{4}(2+a)\frac{\epsilon}{3}. \tag{59}$$

We see that in this expression a appears multiplied by ϵ, which implies that in Eq. (56) the vertical stretching of the lift distribution has an effect of the same order of magnitude as the horizontal stretching. It was shown before that the first effect is of higher order than the second on the rolling moment. Hence, if Eq. (56) is to be interpreted as an expression for the rolling moment in the half-span case, a correction factor must be included. Substituting for the value of a in Eq. (51), we find that the rolling moment is related to C_l^* ,

$$C_l^* = H(A_0)C_l \tag{60}$$

where $H(A)$ is a function of the aspect ratio given by

$$H(A) = \frac{4}{3\pi}\left[2 + A\,\frac{\dot{f}(A)}{f(A)}\right] \tag{61}$$

For the case of very small aspect ratio, Eq. (60) takes on the limiting form

$$\lim_{A_0 \to 0} C_l^* = \frac{4}{\pi}C_l \sim 1.27C_l \tag{62}$$

which indicates that in this case the quantity C_l^* overestimates the rolling moment by about 27%.

For the case of very large aspect ratio, Eq. (60) has the limiting form

$$\lim_{A_0 \to \infty} C_l^* = \frac{8}{3\pi}C_l \sim .85C_l \tag{63}$$

suggesting that in this case C_l^* underestimates the rolling moment by about 15%. Fig. 5 shows a plot of the function $H(A)$. We observe that for an aspect ratio of about 5.5 , C_l^* is expected to be a roughly equal to the rolling moment.

These arguments would have to be validated through experiments involving both full-span and half-span models.

4. COMPARISON WITH EXPERIMENTS

As experimental sources the measurements reported by Carafoli[2,3], Hickey[6], and tests conducted at Stanford University[10] were used.

Fig. 6 shows a logarithmic plot of the gain in lift coefficient vs jet intensity. If the analytical values of lift increment given by Eq. (30) were plotted in this manner they would produce a family of straight lines with slope 2/3 with respect to the horizontal axis. We see that the different sources reveal this trend rather well.

Fig. 7 shows a similar plot for the relative lift increment where the independent variable is the angle of attack. Since, according to Eq. (30) the lift increment depends on the 1/3 power of the angle of attack, assuming that the unblown lift is linear in α , the relative lift depends on the -2/3 power of the angle of attack. We see that this trend is followed quite closely.

Fig. 8 depicts the dependence of lift increment on aspect ratio. Eqs. (31) and (32) imply that the lift increment, when plotted logarithmically, should become asymptotic to straight lines with slopes -1/3 and -2/3 for infinitely small and infinitely large aspect ratio respectively. Both asymptotes are indicated with dashed lines on Fig. 8. The agreement with experiments is in this case rather weak, although the Stanford measurements seem to confirm the trend for small aspect ratios. No experiments are available for large aspect ratios.

In Fig. 9 the lift increment, jet intensity and angle of attack are grouped such that they become proportional to the universal function $F(A)$. The constant k depends only on the particulars of the tip, such as ratio of slot length to chord, tip shape, and slot location. Hence experiments conducted with different tips are expected to produce results within bands at some distance from each other. This is observed for the two experimental sources shown in the plot. The collapse of the data seems to deteriorate for smaller aspect ratios. This fact could be explained by observing that the scalings given by Eq. (30) are valid for very weak blowing and small angle of attack. The blowing is considered weak if the distance that the jet penetrates into the free stream is a small fraction of the span of the wing. For constant blowing intensity and angle of attack, deviations from the very weak blowing condition are more prominent for wings of small aspect ratio. The values of k shown in Fig. 9 are rough approximations from the group of points that showed the best collapse, and should be sufficient for first estimates.

5. CONCLUSIONS AND RECOMMENDATIONS

A theory for the problem of lateral blowing from the tips of straight wings has been developed. Based on the assumption that the relevant phenomenon is primarily inviscid, the theory succeeded at providing scaling laws relating the different parameters of the jet and the wing. Experimental results reported in the literature as well as testing at Stanford University appear to confirm the theoretically derived scaling laws. It should be noted that the experimental work available concerned itself with slots symmetrically located at the tip and the efflux direction coincided with the direction of the span. The theory presented here suggests that other efflux angles may have an effect on lift and rolling moments. Further experimental work should take place to assess this aspect.

It is also possible that displacing the slot on a plane parallel to the plane of the wing may lead to a non-symmetrical effect of turbulent entrainment, causing viscous effects to play a significant role. Upcoming experiments should attempt to clarify this question.

An important theoretical conclusion to be validated is the interpretation of C_l^* as a measure of the rolling moment. This would involve full-span measurements, in a way that the universal function $H(A)$ may be checked.

Finally, although the derivations presented in this report dealt with straight wings, the same methodology could be applied to swept wings; including subsonic and transonic flow regimes, provided that the change in effective span can be related analytically to load changes in the vicinity of the tip.

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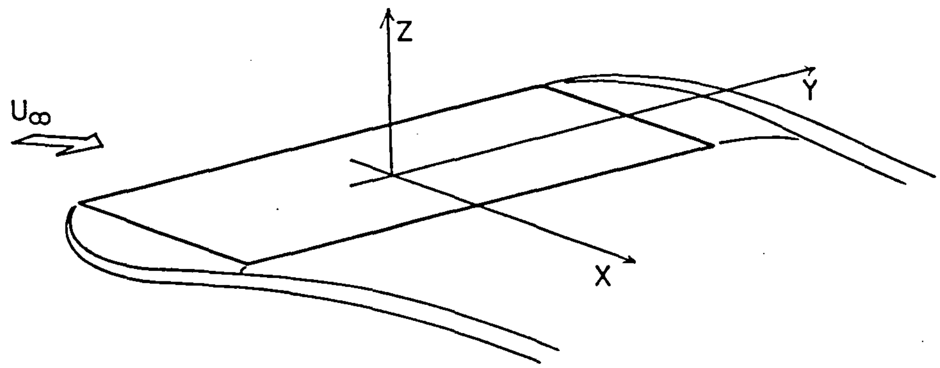

Fig. 1 Rectangular wing with lateral blowing.

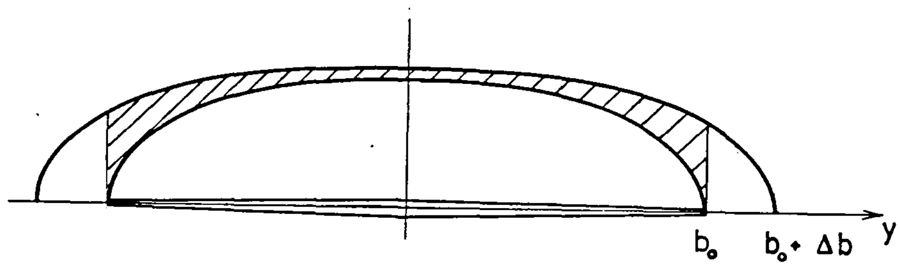

Fig. 2 The span perturbation concept: the shadowed region indicates lift gain.

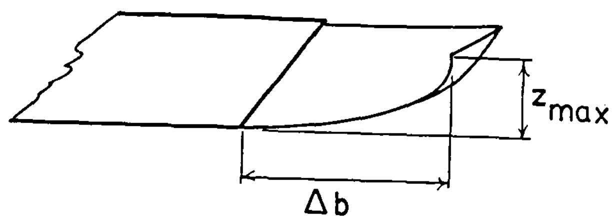

Fig. 3 Definition of effective span increase.

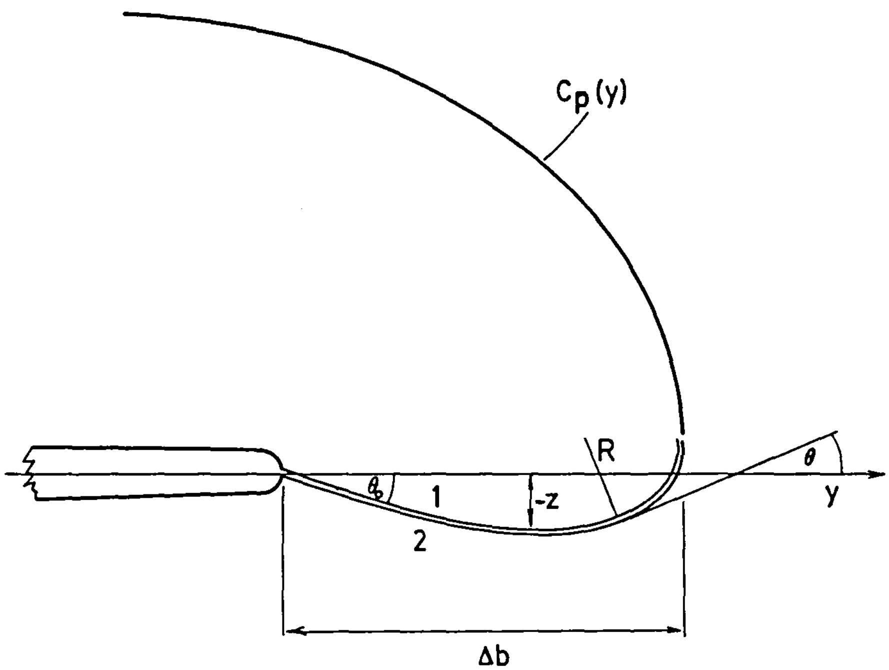

Fig. 4 Tip jet parameters.

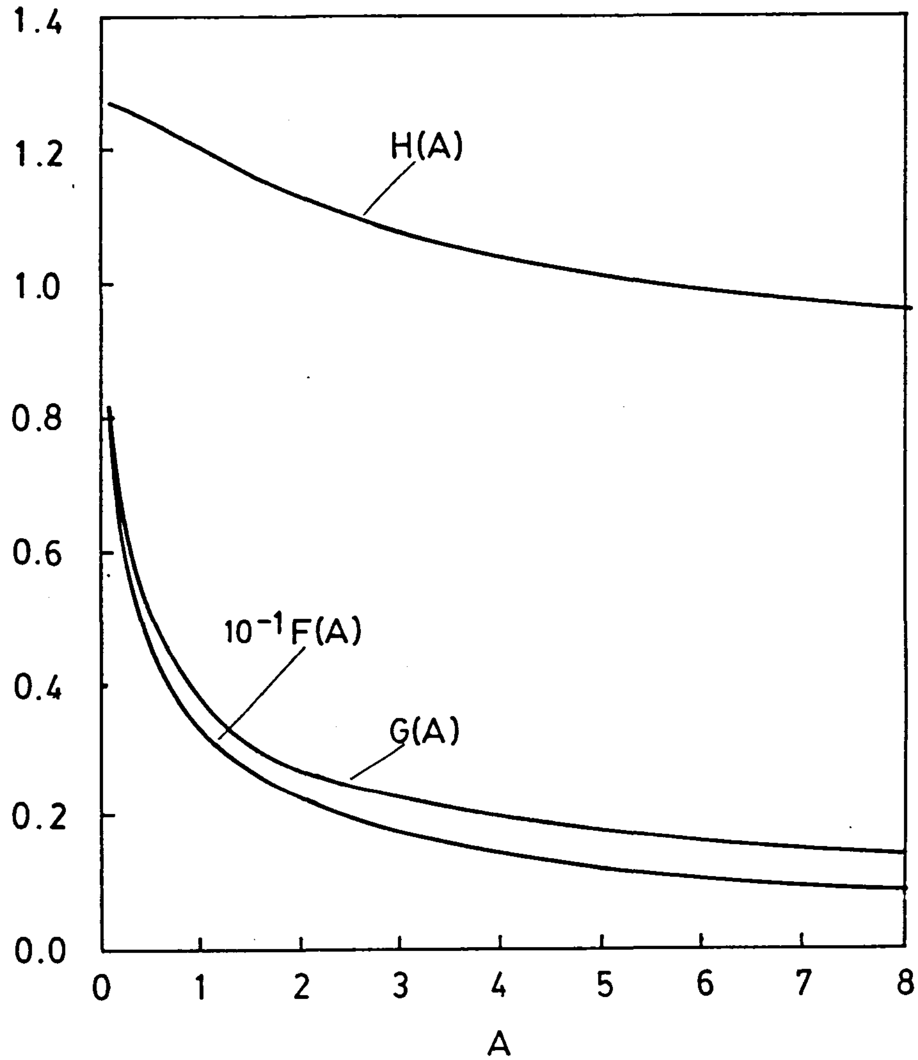

Fig. 5 Universal functions.

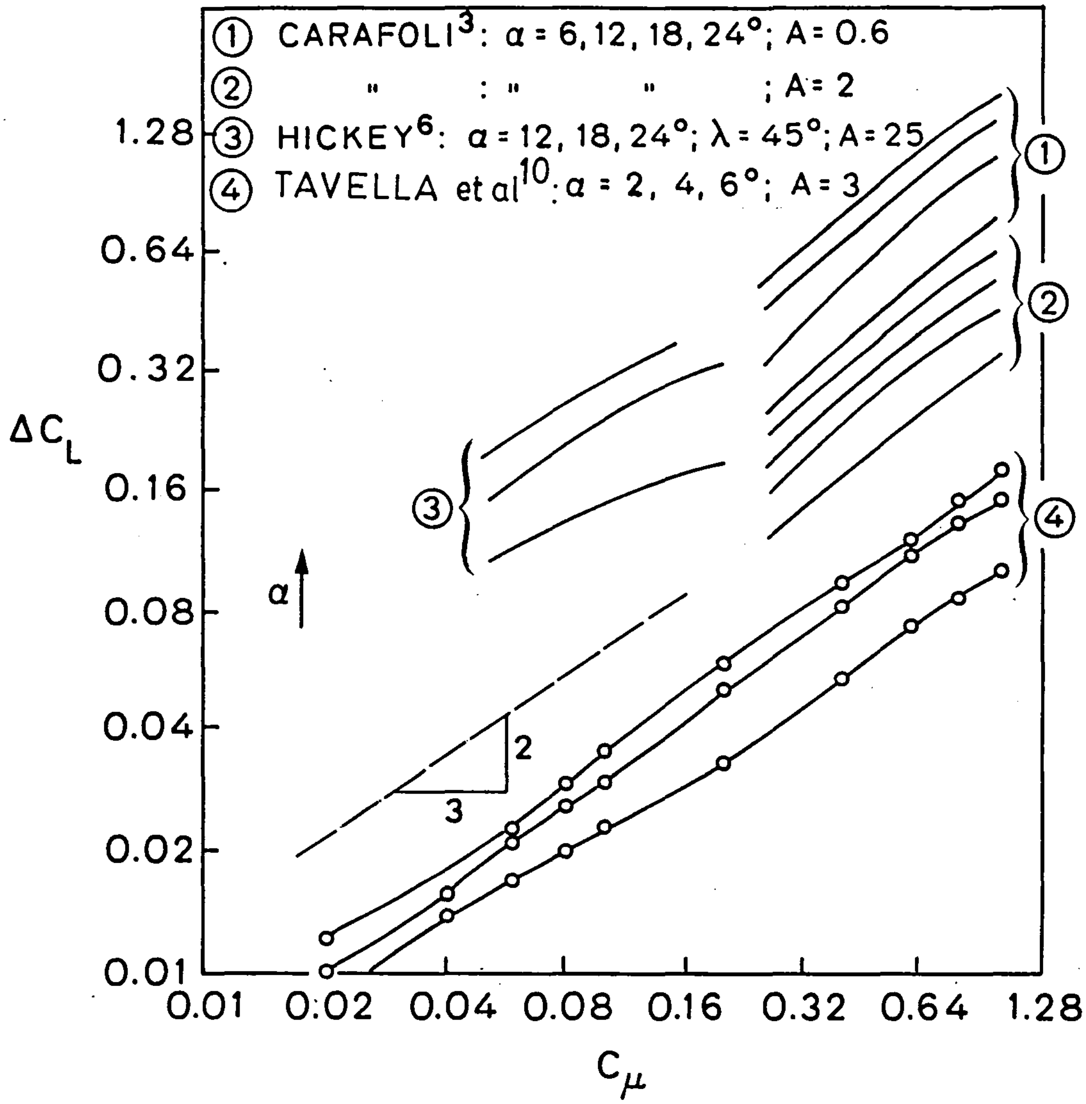

Fig. 6 Logarithmic plot of lift increment vs jet intensity

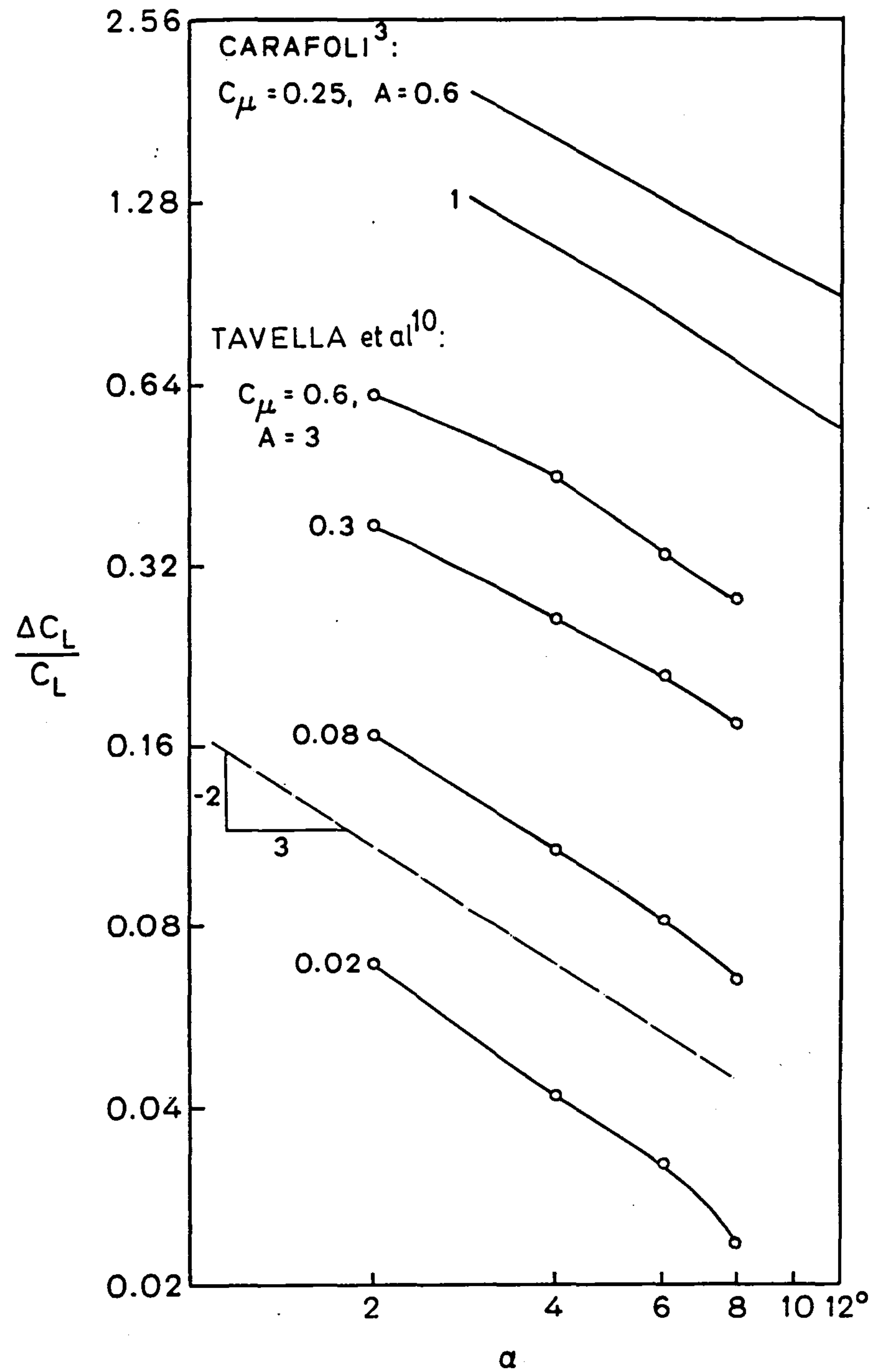

Fig. 7 Logarithmic plot of relative lift gain vs angle of attack.

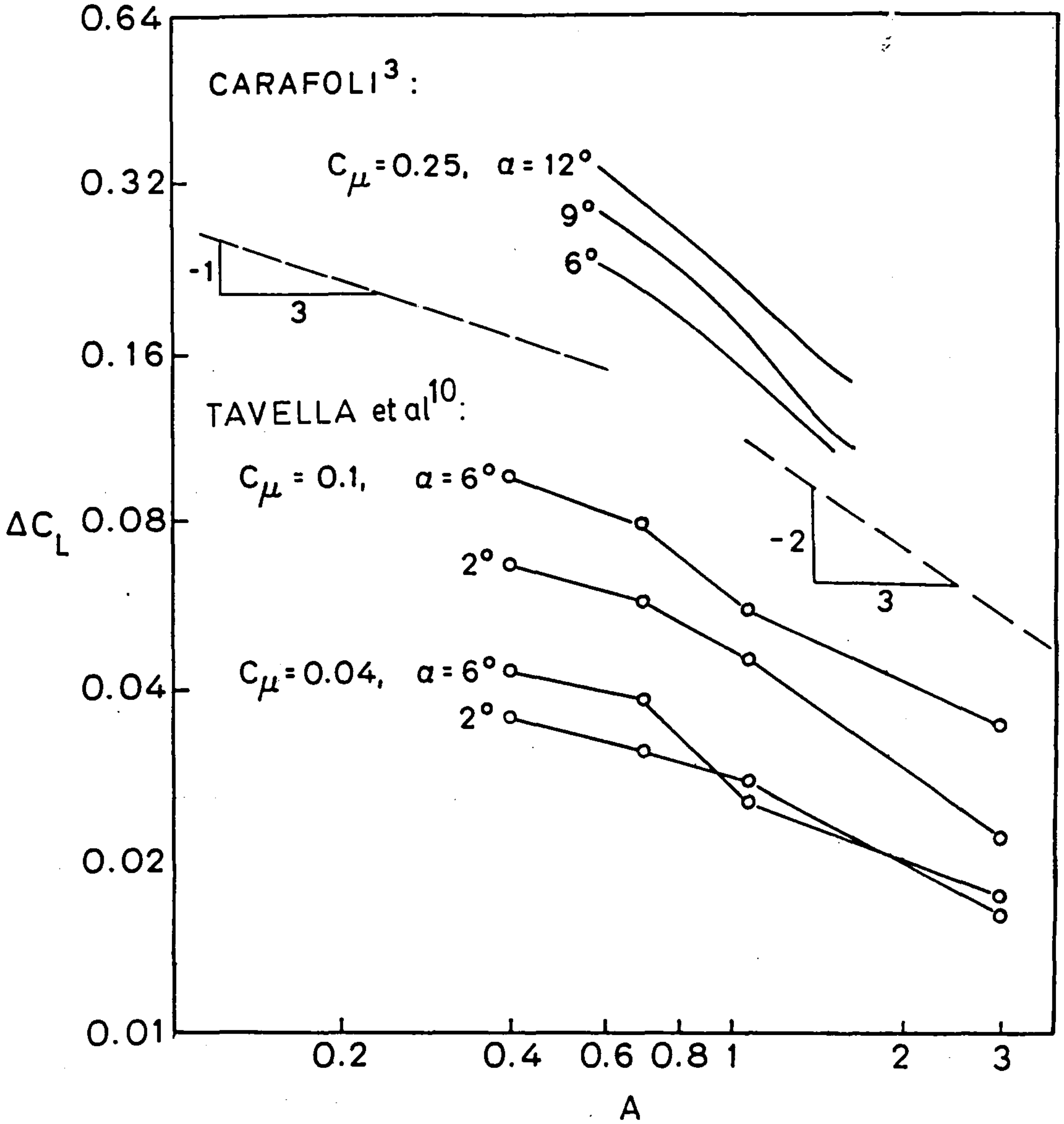

Fig. 8 Logarithmic plot of lift gain vs aspect ratio

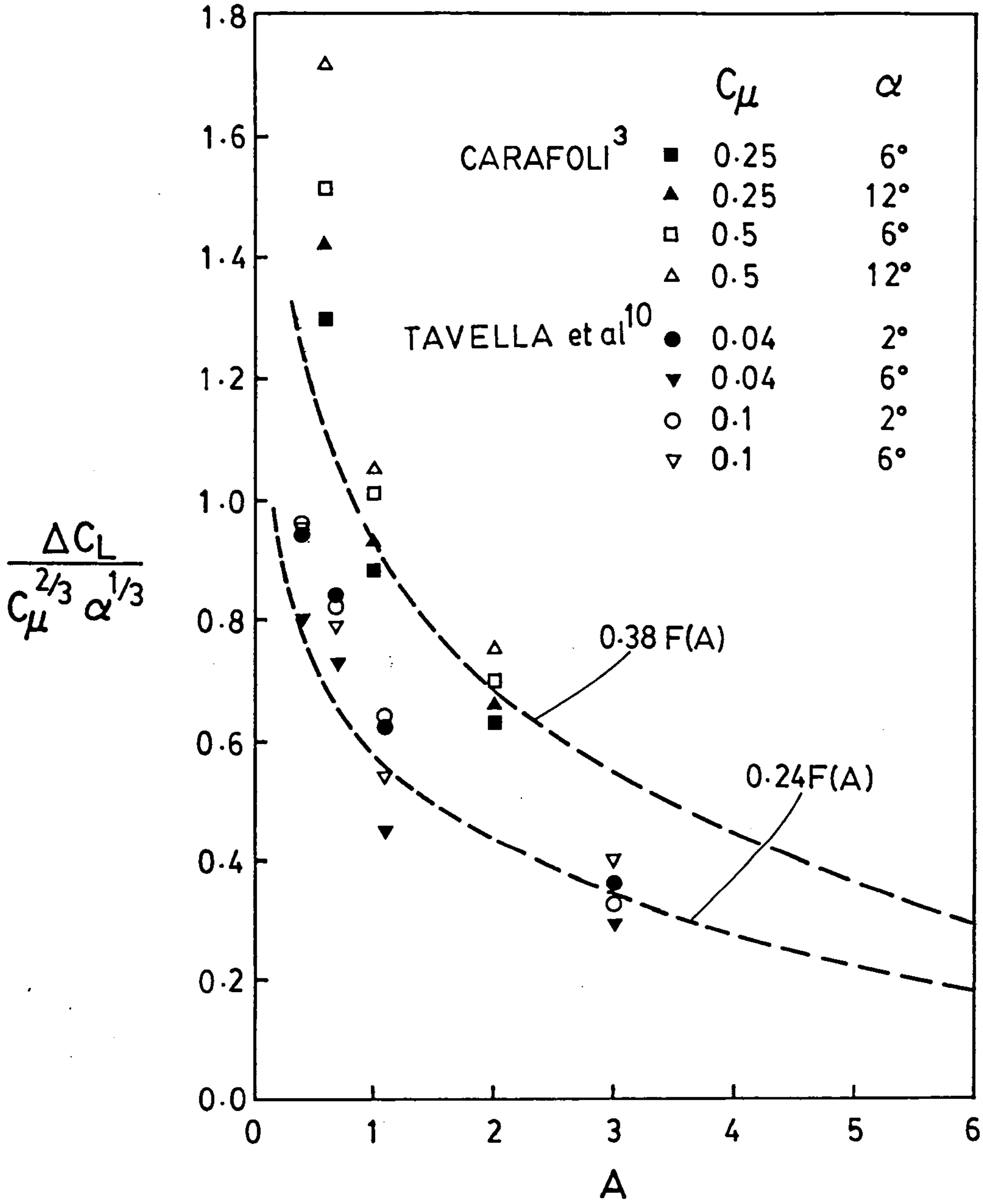

Fig. 9 Collapse of data for rectangular wings.

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