

## NONLINEAR CONTROL OF AIRCRAFT

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## ABSTRACT

In recent work at the NASA Ames Research Center, transformations of nonlinear systems have been used to design automatic flight controllers for vertical and short take off aircraft. Under the assumption that a nonlinear system can be mapped to a controllable linear system, we motivate by partial differential equations a method to construct approximate transformations in cases where exact ones cannot be found. We also present an application of the design theory to a rotorcraft, the UH-1H helicopter.

## I. INTRODUCTION

There are two related problems of interest in this paper. Assuming that a nonlinear system is transformable to a controllable linear system, we first derive a technique for constructing an approximation of such a mapping. Second, we want to present test results of the application of nonlinear transformations to the design of automatic flight controllers for aircraft. Thus the unifying thread is the theory of transformations of nonlinear systems to linear systems, and we shall provide a brief review of the results in that area.

If a nonlinear system can be transformed to a controllable linear system, then the transformation is presented as a solution to a system of partial differential equations (which can be reduced to ordinary differential equations). However, it is not always possible to solve these equations exactly in order to find the desired mapping. By examining the partial differential equations, we derive an interesting linear system, called the modified tangent model, and indicate how an approximating transformation (about a point in state space) can be found from this model. Results concerning

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this modified tangent model are presented in [3] and [8].

The second author first outlined his nonlinear to linear transformation approach in [12], and it has been applied to several aircraft of increasing complexity. The completely automatic flight control system was first tested on a DHC-6 [19], and the reference trajectory for the flight test exercised a substantial portion of the operational envelope of the aircraft. Next, the technique was applied to the Augmentor Wing Jet STOL Research aircraft with the successful flight tests reported in [13]. Methods for providing pilot inputs and application to an A-7 aircraft for carrier landing and testing in manned simulation are summarized in [15], [16] and [18]. Current work on the UH-1H helicopter and results are contained in this paper.

## II. PRELIMINARIES

Before moving to definitions and basic theory concerning transformations of nonlinear systems to linear systems, we mention related references in the literature and the types of systems under consideration. Brunovsky [2] indicated a method for transforming controllable linear systems to canonical form. The second author together with Cicolani in [12] and [13] considered transformations of block triangular nonlinear systems. Krener [10] and Brockett [1] proved results concerning equivalence of systems under various type of coordinate changes. Closely related to the theory found in [5],[6],[7] and [17] is the work of Jakubczyk and Respondek [9].

We examine nonlinear systems of the type (with controls entering linearly)

$$(1) \quad \dot{x}(t) = f(x(t)) + \sum_{i=1}^m u_i(t) g_i(x(t)),$$

where  $f, g_1, \dots, g_m$  are  $C^\infty$  vector fields on  $\mathbb{R}^n$  (or on an open set of  $\mathbb{R}^n$ ). For our canonic linear system we take

$$(2) \quad \dot{y}(t) = A_0 y(t) + B_0 v$$

in Brunovsky canonical form with Kronecker indices  $\kappa_1 \geq \kappa_2 \geq \dots \geq \kappa_m$  with  $\kappa_1 + \kappa_2 + \dots + \kappa_m = n$ .

The Lie bracket of two smooth vector fields  $f$  and  $g$  is

$$[f, g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g,$$

where  $\frac{\partial g}{\partial x}$  and  $\frac{\partial f}{\partial x}$  are Jacobian matrices. Successive Lie brackets like  $[f, [f, g]] = (\text{ad}^2 f, g)$ ,  $[f, [f, [f, g]]] = (\text{ad}^3 f, g)$ , etc. can be introduced. A set of vector fields is involutive if the Lie bracket of any two of them is a linear combination of the elements in the set.

We are interested in the following sets, and note that the vector fields  $g_1, g_2, \dots, g_m$  may be reordered.

$$(3) \quad C = \{g_1, [f, g_1], \dots, (\text{ad}^{\kappa_1-1} f, g_1), g_2, [f, g_2], \dots, (\text{ad}^{\kappa_2-1} f, g_2), \dots, g_m, [f, g_m], \dots, (\text{ad}^{\kappa_m-1} f, g_m)\}$$

$$(4) \quad C_j = \{g_1, [f, g_1], \dots, (\text{ad}^{\kappa_j-2} f, g_1), g_2, [f, g_2], \dots, (\text{ad}^{\kappa_j-2} f, g_2), \dots, g_m, [f, g_m], \dots, (\text{ad}^{\kappa_j-2} f, g_m)\} \text{ for } j = 1, 2, \dots, m.$$

The following local result is proved in [7].

**Theorem 1.** The system (1) is transformable to the system (2), where the state variables  $x_1, x_2, \dots, x_n$  lie in a sufficiently small open neighborhood  $W$  of the origin in  $\mathbb{R}^n$ , if and only if the following three properties hold on  $W$ ;

- i) the  $n$  vector fields in  $C$  are linearly independent,
- ii) the sets  $C_j$  are involutive for  $j = 1, 2, \dots, m$ , and
- iii) the span of  $C_j$  equals the span of  $C_j \cap C$  for  $j = 1, 2, \dots, m$ .

For simplicity we assume that  $W = \mathbb{R}^n$ , avoiding having to say for  $(x_1, x_2, \dots, x_n) \in W$ . We also define a new set of vector fields by adding  $(\text{ad}^{\kappa_1} f, g_1), (\text{ad}^{\kappa_2} f, g_2), \dots, (\text{ad}^{\kappa_m} f, g_m)$  to  $C$  and reordering to obtain

$$(5) \quad D = \{(\text{ad}^{\kappa_1} f, g_1), (\text{ad}^{\kappa_1-1} f, g_1), \dots, (\text{ad}^{\kappa_2} f, g_1), (\text{ad}^{\kappa_2} f, g_2), (\text{ad}^{\kappa_2-1} f, g_1), (\text{ad}^{\kappa_2-1} f, g_2), \dots, (\text{ad}^{\kappa_3} f, g_1), (\text{ad}^{\kappa_3} f, g_2), (\text{ad}^{\kappa_3} f, g_3), (\text{ad}^{\kappa_3-1} f, g_1), (\text{ad}^{\kappa_3-1} f, g_2), (\text{ad}^{\kappa_3-1} f, g_3), \dots, g_1, g_2, \dots, g_m\}.$$

Before forming this set we should check  $\kappa_1 \geq \kappa_2 \geq \dots \geq \kappa_m$ . If  $\kappa_1 = \kappa_2$ , then  $(\text{ad}^{\kappa_1-1} f, g_1)$  appears after  $(\text{ad}^{\kappa_2} f, g_2) = (\text{ad}^{\kappa_1} f, g_2)$ , etc.

For the remainder of this paper we assume that system (1) satisfies the hypotheses of Theorem 1. Next we consider the problem of constructing transformations.

### III. MODIFIED TANGENT MODEL

To build a transformation taking system (1) to system (2) we must solve the partial differential equations (here  $\sigma_1 = \kappa_1, \sigma_2 = \kappa_1 + \kappa_2, \sigma_3 = \kappa_1 + \kappa_2 + \kappa_3, \dots, \sigma_m = \kappa_1 + \kappa_2 + \dots + \kappa_m = n$ ).

$$\begin{aligned} \langle dT_1, (\text{ad}^j f, g_i) \rangle &= 0, j = 0, 1, \dots, \kappa_1 - 2 \text{ and } i = 1, 2, \dots, m, \\ \langle dT_{\sigma_1+1}, (\text{ad}^j f, g_i) \rangle &= 0, j = 0, 1, \dots, \kappa_2 - 2 \text{ and } i = 1, 2, \dots, m, \\ &\vdots \\ \langle dT_{\sigma_{m-1}+1}, (\text{ad}^j f, g_i) \rangle &= 0, j = 0, 1, \dots, \kappa_m - 2 \text{ and } i = 1, 2, \dots, m, \end{aligned}$$

$$(6) \quad \langle dT_k, f \rangle = T_{k+1}, k = 1, 2, \dots, n; k \neq \sigma_1, \sigma_2, \dots, \sigma_m,$$

$$\langle dT_{\sigma_1}, f \rangle + \sum_{i=1}^m u_i \langle dT_1, (\text{ad}^{\kappa_1-1} f, g_i) \rangle = T_{n+1},$$

$$\begin{aligned} \langle dT_{\sigma_2}, f \rangle + \sum_{i=1}^m u_i \langle dT_{\sigma_1+1}, (\text{ad}^{\kappa_2-1} f, g_i) \rangle &= T_{n+2}, \\ &\vdots \\ \langle dT_n, f \rangle + \sum_{i=1}^m u_i \langle dT_{\sigma_{m-1}+1}, (\text{ad}^{\kappa_m-1} f, g_i) \rangle &= T_{n+m} \end{aligned}$$

with the matrix

$$(7) \quad \begin{bmatrix} \langle dT_1, (\text{ad}^{\kappa_1-1} f, g_1) \rangle \dots \langle dT_1, (\text{ad}^{\kappa_1-1} f, g_m) \rangle \\ \langle dT_{\sigma_1+1}, (\text{ad}^{\kappa_2-1} f, g_1) \rangle \dots \langle dT_{\sigma_1+1}, (\text{ad}^{\kappa_2-1} f, g_m) \rangle \\ \vdots \\ \langle dT_{\sigma_{m-1}+1}, (\text{ad}^{\kappa_m-1} f, g_1) \rangle \dots \langle dT_{\sigma_{m-1}+1}, (\text{ad}^{\kappa_m-1} f, g_m) \rangle \end{bmatrix}$$

being nonsingular. Here + is for  $\kappa_i$  odd and - is for  $\kappa_i$  even. The important equations in (6) to consider are the first  $m$ , the others following by easy Lie differentiation. Of course, it is not always possible to solve the ordinary differential equations in closed form, and that is the reason for the method to be introduced shortly. We want to emphasize a related set of partial differential equations that a transformation  $T$  must solve. These can be deduced from (5) and a Leibniz formula (see [7]).

$$\begin{aligned} \langle dT_1, (\text{ad}^j f, g_i) \rangle &= 0, \quad j = 0, 1, \dots, \kappa_1-2 \text{ and } i = 1, 2, \dots, m, \\ \langle dT_2, (\text{ad}^j f, g_i) \rangle &= 0, \quad j = 0, 1, \dots, \kappa_1-3 \text{ and } i = 1, 2, \dots, m, \\ &\vdots \\ \langle dT_{\sigma_1-1}, g_i \rangle &= 0, \quad i = 1, 2, \dots, m, \\ \langle dT_{\sigma_1+1}, (\text{ad}^j f, g_i) \rangle &= 0, \quad j = 0, 1, \dots, \kappa_2-2 \text{ and } i = 1, 2, \dots, m, \\ (8) \quad \langle dT_{\sigma_1+2}, (\text{ad}^j f, g_i) \rangle &= 0, \quad j = 0, 1, \dots, \kappa_2-3 \text{ and } i = 1, 2, \dots, m, \\ &\vdots \\ \langle dT_{\sigma_2-1}, g_i \rangle &= 0, \quad i = 1, 2, \dots, m \\ &\vdots \\ \langle dT_{\sigma_{m-1}+1}, (\text{ad}^j f, g_i) \rangle &= 0, \quad j = 0, 1, \dots, \kappa_m-2 \text{ and } i = 1, 2, \dots, m \end{aligned}$$

$$\langle dT_{\sigma_{m-1}+2}, (ad^j f, g_i) \rangle = 0, \quad j = 0, 1, \dots, \kappa_m - 3 \text{ and } i = 1, 2, \dots, m$$

$$\vdots$$

$$\langle dT_{\sigma_m - 1}, g_i \rangle = 0, \quad i = 1, 2, \dots, m,$$

and we again want the matrix (7) to be nonsingular.

With these equations in mind we turn to our development of the system called the modified tangent model. A comparison of this model and the tangent model [11] will be made later in this section.

Suppose we take a point  $x_0$  in state space and linearize a transformation  $T$  about  $x_0$  and denote the linear part by  $T^L$ . Then  $T^L$  solves equations (8) where each  $(ad^j f, g_i)$  is replaced by  $(ad^j f, g_i)(x_0)$ . The motivation behind the modified tangent model is to find a linear system

$$(9) \quad \dot{x} = f(x_0) - Ax_0 + Ax + Bu$$

so that the Lie brackets of  $Ax$  and  $B = (b_1, b_2, \dots, b_m)$  agree with the corresponding brackets of  $f$  and  $g_1, g_2, \dots, g_m$  at  $x_0$ .

Definition 2. The system (9) is called the modified tangent model at  $x_0$  for system (1) if  $A$  and  $B$  satisfy (take + for  $k$  even and - for  $k$  odd)

$$(10) \quad \begin{aligned} A^k b_1 &= \pm (ad^k f, g_1)(x_0), \quad k = 0, 1, \dots, \kappa_1 \\ A^k b_2 &= \pm (ad^k f, g_2)(x_0), \quad k = 0, 1, \dots, \kappa_2 \\ &\vdots \\ A^k b_m &= \pm (ad^k f, g_m)(x_0), \quad k = 0, 1, \dots, \kappa_m \end{aligned}$$

A transformation taking  $Ax + Bu$  into Brunovsky canonical form (2) satisfies equations (8) where each  $(ad^j f, g_i)$  is evaluated at  $x_0$ .

There is an interesting geometrical interpretation of the modified tangent model. Letting  $C(x_0)$  and  $C_j(x_0)$  denote the evaluations of the sets in (3) and (4) at  $x_0$ , we find that for every  $j, 1 \leq j \leq m$ ,  $C_j(x_0) \cap C(x_0)$  spans the tangent space at  $x_0$  of the integral manifold of  $C_j \cap C$  guaranteed by the Frobenius Theorem. Since

$$C(x_0) = \{b_1, -Ab_1, \dots, \pm A^{\kappa_1 - 1} b_1, b_2, -Ab_2, \dots, \pm A^{\kappa_2 - 1} b_2, \dots, \\ b_m, -Ab_m, \dots, \pm A^{\kappa_m - 1} b_m\}$$

$$C_j(x_0) = \{b_1, -Ab_1, \dots, \pm A^{\kappa_j - 2} b_1, b_2, -Ab_2, \dots, \pm A^{\kappa_j - 2} b_2, \dots, b_m, -Ab_m, \dots, \pm A^{\kappa_j - 2} b_m\}$$

$j = 1, 2, \dots, m.$

the same is true for the corresponding brackets generated by equations (8).

We remark that it is extremely easy to calculate  $A$  and  $B$  from (10) despite the nonlinearities of the equations involved. We form the  $(n+m)(n+m)$  matrix whose first column is the first vector in (5) plus  $m$  zeros, second column is the second vector in (5) plus  $m$  zeros, etc.

Use orthogonal coordinate changes on  $\mathbb{R}^n$  to take this matrix to "generalized lower Hessenberg form" as in [4]. The linear independence of  $b_m, b_{m-1}, \dots, b_1, Ab_m, Ab_{m-1}, \dots, Ab_1, \dots, A^{k_1-1} b_1$  (i.e. the vectors in  $C(x_0)$ ) allow us to trivially solve in order for  $b_m, b_{m-1}, \dots, b_1$ , the last column of A, the (n-1)th column of A) ..., the first column of A.

The purpose of the theoretical part of this paper is to construct an approximate transformation taking equations (1) to equations (2). This approximate mapping is found by giving appropriate solutions to equations (8), where each Lie bracket is evaluated at  $x_0$ . As remarked earlier, the transformation that moves  $Ax+Bu$  in the modified tangent model to Brunovsky form is designed to solve those equations. A simple algorithm for taking a controllable linear system to canonical form is provided in [4]. It involves taking the modified tangent model to a "block triangular system" (see [4]), and the orthogonal coordinate changes mentioned earlier accomplish this. Next, "Lie differentiation" completes the linear transformation for  $Ax+Bu$ . For the approximate transformation, this Lie differentiation is done with respect to  $f(x_0)-A(x_0)+Ax$ .

The tangent model approach was introduced in [11] and applied on a trajectory autopilot for a helicopter. For the tangent model we write equations in the same form as (9)

$$(11) \dot{x} = f(x_0) - Ax_0 + Ax + Bu,$$

except that A is the Jacobian matrix of  $f(x)$  at  $x_0$ , and  $B=(b_1, b_2, \dots, b_m)$  is an  $n \times m$  matrix with  $g_1(x_0)$  as its first column,  $g_2(x_0)$  as its second column, ...,  $g_m(x_0)$  as its last column. The first step in constructing an approximate transformation by use of (11) is to take  $Ax+Bu$  to linear canonical form. However, it is not in general true that  $-[f, g_1](x_0) = Ab_1, [f, g_2](x_0) = Ab_2$ , etc. (an example is provided in [3]). Hence, a transformation derived in this manner may not have the desirable geometric interpretation possessed by the modified tangent model. However, the tangent model seems to provide an adequate transformation in many cases, as illustrated by its use on the simulator for a tail-sitter aircraft.

#### IV. APPLICATION TO A HELICOPTER

The helicopter will be represented by a rigid body moving in three-dimensional space in response to gravity, aerodynamics, and propulsion. The state,

$$(12) x = (r, v, C, \omega)^T \in X \subset \mathbb{R}^3 \times \mathbb{R}^3 \times SO(3) \times \mathbb{R}^3$$

where  $r$  and  $v$  are the inertial coordinates of body center-of-mass position and velocity, respectively, and  $C$  is the direction cosine matrix of the body-fixed axes relative to the runway-fixed axes (taken to be inertial). The attitude  $C$  moves on  $SO(3)$ . The body coordinates of angular velocity are represented by  $\omega$ .

The controls are

$$(13) \quad u = (u^M, u^P)^T \in U \subset \mathbb{R}^3 \times \mathbb{R}$$

where  $u^M$  is the three-axis moment control, that is, roll cyclic and pitch cyclic, which tilt the main-rotor thrust, and the tail-rotor collective, which controls the yaw moment; and  $u^P$  is the main-rotor collective, which controls the main-rotor thrust.

The effectively 12-dimensional state equation consists of the translational and rotational kinematic and dynamic equations:

$$(14) \quad \left. \begin{aligned} \dot{r} &= v \\ \dot{v} &= f^F(x, u) \\ \dot{C} &= S(\omega)C \\ \dot{\omega} &= f^M(x, u) \end{aligned} \right\}$$

where  $f^F$  and  $f^M$  are the total force and moment generation processes, and  $(x, u)$  are defined by Eqs. (12) and (13).

In general, the moment generation process  $f^M$  is invertible with respect to the pair  $(\dot{\omega}, u^M)$ , and for the restricted class of maneuvers being considered in this experiment (i.e., no  $90^\circ$  rolls),  $f^F$  is invertible with respect to the pair  $(\dot{v}_3, u^P)$ . So the four controls  $u$  may be related to four accelerations  $(\dot{\omega}, \dot{v}_3)$  by a nonsingular transformation, say,

$$(15) \quad u = h^M(x; (\dot{\omega}, \dot{v}_3))$$

If  $(\dot{\omega}, \dot{v}_3)$  are chosen to be the new independent control variables  $(\alpha, a_3)$  to replace the natural controls  $(u^M, u^P)$ , then the state equation (14) becomes the following:

$$(16) \quad \left. \begin{aligned} \dot{r} &= v \\ \begin{pmatrix} \dot{v}_1 \\ \dot{v}_2 \end{pmatrix} &= f^0(r, v, C) + f^1[r, v, C, \omega, (\alpha, a_3)] \\ \dot{v}_3 &= a_3 \\ \dot{C} &= S(\omega)C \\ \dot{\omega} &= \alpha \end{aligned} \right\}$$

The function  $f^1$  in (15) represents (nonlinear) zeros:

$$(17) \quad f^1 = \epsilon_1(r, v, C)\omega + \epsilon_2(r, v, C)\alpha + \epsilon_3(r, v, C)a_3 + f^2$$

where  $f^2$  is second or higher order in  $(\omega, \alpha, a_3)$ . In the case of the actual helicopter,  $f^2$  is very small and completely negligible. Consequently, the first condition for linearizability, which requires that controls  $(\alpha, a_3)$  enter linearly into the state equation is in fact satisfied. With the new controls, the state equation becomes,

$$\begin{aligned}
 \dot{r} &= v \\
 \begin{pmatrix} \dot{v}_1 \\ \dot{v}_2 \end{pmatrix} &= f^0(r, v, C) + \epsilon_1(r, v, C)\omega + \epsilon_2(r, v, C)\alpha + \epsilon_3(r, v, C)a_3 \\
 \dot{v}_3 &= a_3 \\
 \dot{C} &= S(\omega)C \\
 \dot{\omega} &= \alpha
 \end{aligned}
 \tag{18}$$

The primary means for controlling the horizontal motion of the helicopter is through  $f^0$ . The  $\epsilon_i$  teams are parasitic and negligible for regulator bandwidths below 0.5 rad/sec. The dominant team  $f^0$  is invertible with respect to the pair  $((\dot{v}_1, \dot{v}_2, E_3), C)$  in which  $E_3$ , a rotation (angle  $\psi$ ) about the vertical, defines the heading of the helicopter. So, the helicopter attitude  $C$  can be related to horizontal acceleration and heading  $(a_1, a_2, E_3)$  by a nonsingular transformation, say,

$$C = h^F(r, v, (a_1, a_2, E_3)). \tag{19}$$

From (18) with  $\epsilon_i=0$  and (19) it follows that angular velocity

$$\omega = h_\omega^F(r, v, a, E_3)(\dot{a}_1, \dot{a}_2, \dot{\psi})^T \tag{20}$$

where  $h_\omega^F$  is nonsingular and  $a=(a_1, a_2, a_3)^T$ ,  $T$  now denoting transpose.

In the actual implementation of the control scheme, the inverse transformations  $h^M$  and  $h^F$  do not appear explicitly. Instead the Newton-Raphson algorithm is used to compute  $u$  from  $f^M$  and  $C$  from  $f^F$ . The Jacobian matrix  $h_\omega^F$  in (19) is available as a by-product of the algorithm.

The canonic model has the Kronecker index set  $\{4, 4, 2, 2\}$ . The canonic coordinates  $y$  and control  $y^5$  are chosen to be the following

$$\begin{aligned}
 y^1 &= (r_1, r_2)^T \\
 y^2 &= (v_1, v_2)^T \\
 y^3 &= (a_1, a_2, r_3, E_3)^T \\
 y^4 &= (\dot{a}_1, \dot{a}_2, v_3, \dot{\psi})^T \\
 y^5 &= (\ddot{a}_1, \ddot{a}_2, a_3, \dot{\psi})^T
 \end{aligned}
 \tag{21}$$

The map from canonic controls  $y^5$  back to natural controls  $u$  is given in two steps. First, commanded angular acceleration is computed by taking the time derivative of (20) along the model trajectory,  $y(t)$ , and neglecting  $\dot{h}^F$ ,

$$\alpha = h_\omega^F(r, v, a, E_3)(\ddot{a}_1, \ddot{a}_2, \dot{\psi})^T \tag{22}$$

Then the controls  $u$  are computed from (15),

$$u = h^M(x, (\alpha, a_3)) \tag{23}$$

where  $x$  is the natural state defined by (12).



Extensive testing indicates that, at least in the present application of the particular helicopter, the two approximations  $f^1=0$  and  $h_w^F=0$ , which greatly reduce the computing load on the flight computer, cause insignificant errors. The design is robust with respect to modeling errors such as weight and location of the center of mass. However, in high speed flight there is too much sensitivity to errors in wind estimates. The problem is due to the independent control of heading, which, in high speed, should point the helicopter in the direction of the relative air velocity vector. The uncontrolled helicopter has a natural tendency to point into the total wind. This and other benign built-in characteristics are removed by the transformation to Brunovsky form. A control system structure that allows the retention of the benign characteristics of the plant is shown in Figure 1.

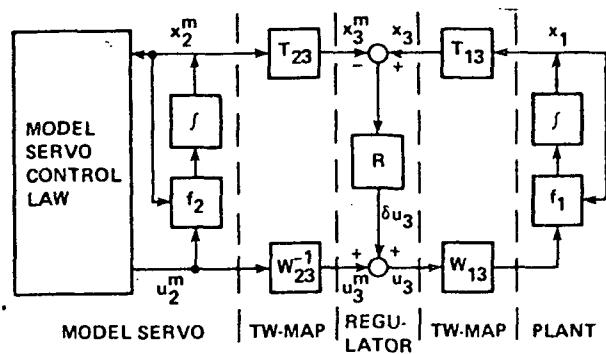


Figure 1. Structure of the Generalized Model Follower.

The transformations  $T_{13}, T_{23}, W_{13}, W_{23}$  are nonsingular. The usual configuration (see [13]) is then a special case in which the model dynamics  $u_2^m \rightarrow x_2^m$  and regulator dynamics  $u_3^m \rightarrow x_3^m$  are both Brunovsky (i.e.  $T_{23}$  and  $W_{23}^{-1}$  are identities). With the additional freedom provided by the new structure, it was possible to retain the benign weather cock stability of the helicopter and thereby reduce the system sensitivity to wind estimator errors to tolerable level.

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