# GEOSTROPHIC ADJUSTMENT IN A SHALLOW-WATER NUMERICAL MODEL AS IT RELATES TO THERMOSPHERIC DYNAMICS 

M. F. Larsen ${ }^{1}$ and I. S. Mikkelsen ${ }^{2}$<br>${ }^{1}$ Dept. of Physics and Astronomy Clemson University<br>Clemson, SC 29631<br>${ }^{2}$ Geophysical Division<br>Meteorological Institute<br>Copenhagen, Denmark

The theory of geostrophic adjustment and its application to the dynamics of the high-latitude thermosphere have been discussed by ourselves (Larsen and Mikkelsen, JGR, 1983; Mikkelsen and Larsen, JGR, 1983) and Walterscheid and Boucher (JAS, 1984) in previous papers based on a linearized treatment of the fluid dynamical equations. However, a linearized treatment is only valid for small Rossby numbers given by $\mathrm{Ro}=\mathrm{V} / \mathrm{fL}$, where V is the wind speed, f is the local value of the Coriolis parameter, and L is a characteristic horizontal scale for the flow. For typical values in the auroral zone, the approximation is not reasonable for wind speeds greater than 25 $\mathrm{m} / \mathrm{s}$ or so. We have developed a shallow-water (one layer) model that includes the spherical geometry and full nonlinear dynamics in the momentum equations in order to isolate the effects of the nonlinearities on the adjustment process. A belt of accelerated winds between $60^{\circ}$ and $70^{\circ}$ latitude was used as the initial condition. We found that the adjustment process proceeds as expected from the linear formulation, but that an asymmetry between the response for an eastward and westward flow results from the nonlinear curvature (centrifugal) terms. In general, the amplitude of an eastward flowing wind will be less after adjustment than a westward wind. For instance, if the initial wind velocity is $300 \mathrm{~m} / \mathrm{s}$, the linearized theory predicts a final wind speed of $240 \mathrm{~m} / \mathrm{s}$, regardless of the flow direction. However, the nonlinear curvature terms modify the response and produce a final wind speed of only $200 \mathrm{~m} / \mathrm{s}$ for an initial eastward wind and a final wind speed of almost $300 \mathrm{~m} / \mathrm{s}$ for an initial westward flow direction. Also, less gravity wave energy is produced by the adjustment of the westward flow than by the adjustment of the eastward flow. The implications are that the response of the thermosphere should be significantly different on the dawn and dusk sides of the auroral oval. Larger flow velocities would be expected on the dusk side since the plasma will accelerate the flow in a westward direction in that sector.

## REFERENCES

Larsen, M. F., and I. S. Mikkelsen, The dynamic response of the high latitude thermosphere and geostrophic adjustment, J. Geophys. Res., 88, 3158-3168, 1983.
Mihaljan, J. M., The exact solution of the Rossby adjustment problem, Tellus, 15, 150-154, 1963.
Mikkelsen, I. S., and M. F. Larsen, An analytic solution for the response of the neutral atmosphere to the high-latitude convection pattern, J. Geophys. Res., 88, 8073-8080, 1983.
Walterscheid, R. L., and D. S. Boucher, Jr., A simple model of the transient response of the thermosphere to impulsive forcing, J. Atmos. Sci., 41, 1062-1072, 1984.


Figure 1. The zonal wind (left), meridional wind (middle), and height of the fluid surface (right) at 1 hr intervals. The initial condition at 0 hr is a $1 \mathrm{~m} / \mathrm{s}$ eastward wind in the latitude band between $60^{\circ}$ to $70^{\circ}$. The height of the fluid is directly related to the pressure. All subsequent figures will be presented in the same format.


Figure 2. Analytic solution for the adjusted flow based on a linearized set of equations in a Cartesian coordinate system. The initial wind was $100 \mathrm{~m} / \mathrm{s}$, and the final wind is $80 \mathrm{~m} / \mathrm{s}$. The initial and final flow velocity will scale linearly together. Thus, an initial wind of $1 \mathrm{~m} / \mathrm{s}$ will produce an adjusted wind of $0.8 \mathrm{~m} / \mathrm{s}$. This figure can be compared directly with the solutions shown in Figure 1.


Figure 3. An enlarged graph showing the adjusted state after 10 hr . The initial wind is eastward at $1 \mathrm{~m} / \mathrm{s}$ between $60^{\circ}$ and $70^{\circ} \mathrm{N}$. The final zonal velocity agrees closely with the zonal velocity given by the linearized analytic solution. The meridional wind is negligible. The height of the free surface, which is directly related to the pressure, shows an asymmetry across the channel. The spherical geometry causes a focusing at the pole. As a result, the heights are higher on the southern side of the belt and lower on the northern side.


Figure 4. Adjusted state in the shallow-water model after 10 hr for an initial eastward wind of 100 $\mathrm{m} / \mathrm{s}$. The final zonal wind is approximately $75 \mathrm{~m} / \mathrm{s}$ which is less than the final value of $80 \mathrm{~m} / \mathrm{s}$ based on the solution of the linearized set of equations.


Figure 5. The adjusted state for an initial $300 \mathrm{~m} / \mathrm{s}$ eastward wind after 8 hr . The final zonal wind is close to $210 \mathrm{~m} / \mathrm{s}$ which is nearly $30 \mathrm{~m} / \mathrm{s}$ less than the predicted value based on a linear analysis. The oscillations which are evident in all three fields are due to gravity waves generated by the adjustment process. The waves propagate north and are reflected by the boundary condition at the pole. Therefore, the $300 \mathrm{~m} / \mathrm{s}$ case is near the limit of validity of the numerical solution.


Figure 6. The adjusted state for an initial westward wind of $100 \mathrm{~m} / \mathrm{s}$ is shown. The amplitude of the final zonal wind is approximately $90 \mathrm{~m} / \mathrm{s}$, or $10 \mathrm{~m} / \mathrm{s}$ greater than expected based on a linear treatment.


Figure 7. Here the adjusted state for an initial westward wind of $300 \mathrm{~m} / \mathrm{s}$ is shown. The final state also has a zonal wind of $300 \mathrm{~m} / \mathrm{s}$. The pressure gradient is very small compared to the cases shown in the previous figures, and there is almost no gravity wave energy generated, as can be seen in the smoothness and low values of the meridional wind. Thus, a significant asymmetry between the dusk and dawn sides of the auroral oval would be expected. We expect that on the dusk side, where the winds produced by the Lorentz acceleration are westward, less gravity wave energy will be produced by the adjustment process, and the pressure gradients will be smaller.

## INITIAL WIND

## LINEAR

## NONLINEAR

EASTWARD

## $100 \mathrm{~m} / \mathrm{s}$ <br> $200 \mathrm{~m} / \mathrm{s}$ <br> $300 \mathrm{~m} / \mathrm{s}$

WESTWARD
$-100 \mathrm{~m} / \mathrm{s}$
$-200 \mathrm{~m} / \mathrm{s}$
$-300 \mathrm{~m} / \mathrm{s}$
$80 \mathrm{~m} / \mathrm{s}$
$160 \mathrm{~m} / \mathrm{s}$
$240 \mathrm{~m} / \mathrm{s}$ $200 \mathrm{~m} / \mathrm{s}$

Figure 8. Table comparing the initial wind speed, the wind speed after adjustment in a linearized formulation, and the wind speed after adjustment when the nonlinear terms in the momentum equation are included.

