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Leon W. Zelby, John B. Mathes,
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University of Oklahoma
Norman, Oklahoma

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TRANSMISSION LINE DESIGN FOR A POWER DISTRIBUTION SYSTEM AT 20 kHz FOR AIRCRAFT

Leon W. Zelby, John B. Mathes, and John W. Shawver
University of Oklahoma
Norman, Oklahoma 73019

1.0 INTRODUCTION

The purpose of this work is to design a transmission line for a power distribution system in aircraft. The line is to operate at 20 kHz, have very low inductance and, consequently, a low characteristic impedance; it must withstand 440 V at an altitude of 15 km (50 000 ft) and 1 kV at 300 km (about 100 000 ft), and must be capable of carrying 100 A. The connectors and supports must have a minimum creepage path of 0.635 cm (1/4 in.).

Additional considerations of the line design include connectors for parallel loads, number of such connectors, mechanical strength and heat dissipation, dc resistance, and weight.

The fundamental problem in determining the inductance per unit length, and the characteristic impedance of the line is the calculation of inductance or capacitance because $LCv^2 = 1$, with L and C representing the inductance and capacitance per unit length, respectively; and v representing the velocity of wave propagation along the line (in the case of the fundamental mode, the TEM mode, the velocity is governed by the properties of the material between the wires). At a frequency of 20 kHz, the skin depth for copper (resistivity 1.7×10^{-8} Ω-m conductivity = 5.88×10^{7} mho/m) is 0.46 mm $\approx$ 0.5 mm, and the wavelength in free space, is 15.0 km and about 9.0 km in many commercially used insulating materials (relative dielectric constant on the order of 3). Thus, the total length of the line is on the order of 0.02 to 0.033 wavelength which justifies the quasi-static approximations (refs. 1 to 5). The close spacing among the individual conductors strengthen the validity of the relation $LCv_2 = 1$ and of the "quasi-TEM" mode dominance. [Strictly speaking, the TEM mode in this case is not absolutely pure because of two factors: there are two dielectrics between the wires, the insulation and air; and the wires are lossy. As a consequence, there will be an axial component of the electric field, but it is so small that the assumption about TEM is more than satisfactory for practical purposes. Furthermore, with reasonable insulation the shunt conductance (between the wires) will be on order of 10^{-17} mho/m (ref. 6). As a consequence, the term $RG/\omega^2 LC$ will be on the order of $10^{-14} \ll 1$, so that the propagation constant will very nearly be equal to $\omega\sqrt{LC}$, so that the relation $LCv^2 = 1$ will be satisfied in any practical system.]

The SI system of units will be used throughout this report.

2.0 DETERMINATION OF PARAMETERS

2.1 Introduction

In order to design a suitable transmission line, several parameters need to be determined for different configurations. The parameters are L, the inductance per unit length; C, the capacitance per unit length; Z_0, the characteristic impedance; and R, the resistance per unit length. (G, the shunt conductance per unit length is too small to be included.) L and C are related as indicated above through the relation $LCv^2 = 1$; also, for a lossless line, $Z_0 = (L/C)^{1/2}\ \Omega$. At a frequency of 20 kHz, the skin depth in copper is 0.467 mm, with the current distributed in a circular cylindrical conductor as shown in figure 1. The distribution is governed by the relation (ref. 7)

$$\frac{I_z}{I_o} = \frac{\mathrm{Ber}\left(\frac{r\sqrt{2}}{\delta}\right) + j\mathrm{Bei}\left(\frac{r\sqrt{2}}{\delta}\right)}{\mathrm{Ber}\left(\frac{r_o\sqrt{2}}{\delta}\right) + j\mathrm{Bei}\left(\frac{r_o\sqrt{2}}{\delta}\right)} \tag{1}$$

where $\mathrm{Ber}(x) + j\mathrm{Bei}(x) = J_0(j^{-1/2}x)$, with $J_0(x)$ the Bessel function of the first kind and zeroth order, and $j = (-1)^{1/2}$. The other symbols are: r, radial distance; r_0, the radius of the wire; and δ, the skin depth. In view of this distribution, calculations will be concerned mainly with wires whose diameter does not exceed 1 mm (number 18 AWG wire).

The main parameter is the inductance which should be as low as possible in order to minimize the effects of switching. Consequently, emphasis will be placed on the determination of L which will be calculated two different ways: one method will utilize the definition of inductance (flux linkage per unit current); the other method will utilize the calculation of capacitance from which the inductance will be evaluated from the relation $L = (Cv^2)^{-1} = \mu\epsilon/C = Z_0\sqrt{\mu\epsilon}$.

2.2 Inductance Calculations

In the following calculations, the permeability of all materials will be assumed to be that of free space ($\mu_0 = 4\pi\times10^{-7}$ H/m). The inductance per unit length of a bifilar lead of radius r_a (current in each direction is assumed uniformly distributed over the cross section of the wire, see fig. 1) separated from the return lead of radius r_a' by a distance $D_{aa'}$ is (ref. 8)

$$L_{aa'} = \frac{\mu_o}{2\pi}\left[\frac{1}{2} + \ln\left(D^2_{aa'}/r_a r_{a'}\right)\right] = \frac{\mu_o}{4\pi}\left[1 + 4\ \ln\left(D_{aa}/r_a\right)\right]\ \mathrm{H/m} \tag{2}$$

when the return wire radius is the same as in the first wire (the case here under consideration). For an n wire system (n pairs) one can form an n x n matrix

$$\begin{bmatrix} L_{11} & \cdots & L_{1i} & \cdots & L_{1n} \\ \vdots & \ddots & \vdots & & \vdots \\ L_{i1} & \cdots & L_{ii} & \cdots & L_{in} \\ \vdots & & \vdots & \ddots & \vdots \\ L_{n1} & \cdots & L_{in} & \cdots & L_{nn} \end{bmatrix} \tag{3}$$

where $L_{ij} = L_{ji}$, and calculate the equivalent inductance of the system (refs. 9 and 10)

$$L_{eq} = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{1}{L_{ij}} \right) \right)^{-1} \quad \text{H/m} \tag{4}$$

which can be determined to any given degree of accuracy (ref. 10).

The FORTRAN program for this calculation, CRIMPD.F, which is listed in appendix A, uses a series of menus to facilitate the entry of different two-dimensional conductor geometries. The entries are the coordinates of the centers of the conductors and their respective radii, and they may be entered either in rectangular or polar coordinates, (see figs. 2 and 3 for the polar coordinates).

2.3 Check on Calculations

To check the results obtained from calculations performed according to the above discussion, two additional calculations were performed for several geometries. One check used the extension of equation (2) to multiple parallel conductors

$$L_a = \frac{\mu_o}{2\pi} \left(\frac{1}{2} + 2 \ln \frac{D_{aa'} \cdot D_{ab'} \cdot \cdots \cdot D_{an'}}{r_a \cdot D_{ab} \cdot D_{ac} \cdot \cdots D_{an}} \right) \text{H/m} \tag{5}$$

with the parameters defined in figure 4. The equivalent inductance per unit length was calculated using

$$L_{eq} = \left(\sum_{i=1}^{n} \frac{1}{L_i} \right)^{-1} + \left(\sum_{i=1}^{n} \frac{1}{L_i} \right)^{-1} \quad \text{H/m} \tag{6}$$

Calculations for 4-, 8-, and 16-conductor arrangements agreed exactly with the other calculations.

The other check utilized the expression for capacitance between two infinitely long conductors of circular cross section (ref. 11) (radius r, distance between centers D)

$$C = \frac{\pi\epsilon}{\cosh^{-1}\left(\frac{D}{2r}\right)} = \frac{\pi\epsilon}{\ln\left(\frac{D}{2r} + \sqrt{\left(\frac{D}{2r}\right)^2 - 1}\right)} \quad \text{F/m} \tag{7}$$

so that the external self-inductance of one conductor is

$$L = \frac{\mu}{2\pi} \ln\left(\frac{D}{2r} + \sqrt{\left(\frac{D}{2r}\right)^2 + 1}\right) \quad \text{H/m} \tag{8}$$

because $LC = \mu\epsilon$. Calculations were performed using the above argument of the logarithmic function instead of using the D_{ij}'s. The results, for the ratio of the distance between conductor centers to conductor radius equal to four, are about 7 percent lower than the other calculations. This was to be expected because the internal inductance, $\mu/8\pi$, does not appear in equation (8), but its contribution is negligible for the distance-to-radius ratio of four or more, and the 7 percent discrepancy is well within bounds (ref. 10). Note that the dimensional relations of this report meet this criterion: the ratio of the distance between centers to conductor radius never is less than four. Consequently, the error-if any-should not exceed 7 percent, a value usually acceptable in practice.

3.0 CONFIGURATION DATA

3.1 Circular Cylindrical Arrangements

Calculations were performed for wire arrangements of the type shown in figures 2 and 3. For the single "ring" of wires, as in figure 2, it was found that the best arrangements (i.e., the lowest values of inductance and characteristic impedance) were for alternating current directions in adjacent wires. The values of inductance decrease with increasing number of wires. This circular configuration was not suitable, however, because of the large number of wires needed to reduce the characteristic impedance to acceptable values (on the order of an ohm). Furthermore, with an increasing number of wires, the radius of the "ring" R is increasing, and this leads to inefficient use of space.

A better configuration was of the multiring variety. Tables I and II list values of L per unit length, and of Z_0 for different number of wires (N represents the total number of wires in the line, with $N/2$ in the inner and $N/2$ in the outer ring). The angle θ is defined in figure 3 and represents the shift of the first wire in the outer ring relative to the first wire in the inner ring. As in the case of the single ring, the lowest values were obtained for alternating polarities.

One of the problems with the coaxial arrangement is that the inductances of the wires in the inner ring are lower so that more current is concentrated there relative to the outer ring. This can be compensated by reducing the radii of the wires in the inner loop.

For comparison of the different wire radii, calculations for the arrangement shown in figure 5 were made. The values for the five ring arrangement for the same wire radii (Line 1), and for different radia (Line 2), are listed in Table III. These show that in the latter case, the overall inductance is increased which means that an increase in the number of wires would be required. Clearly, this again suggests not only an inefficient use of space, but also higher L and Z_0 values than comparable rectangular arrangement (see items 1 and 2 of Table IV).

3.2 Rectangular Arrangement

To carry 100 A, #4 AWG wire is recommended (ref. 12) whose cross section is 21.15×10^{-6} m^2. Inasmuch as the skin depth at 20 kHz is approximately 0.5 mm, the largest reasonable wire would be #18 AWG (radius, 0.512 mm; cross-sectional area, 0.8231 mm^2). The minimum number of #18 AWG wires would therefore be 26. In subsequent calculations, a larger number of wires will be used. (Note: the 26 wires do not include the return path which calls for additional 26 wires. The minimum for the line would then be 52 wires #18 AWG.)

Figures 6 to 8 show the results of 80-wire 4 by 20 rectangular arrangement which appears to have the best parameters: L = 10.4 nH/m; C = 2.5 nF/m; Z_0 = 2.1 Ω (the computer printed values were, respectively, 10.4196, 2.5264, 2.0611). As the method to compute these values was the "inductance" method, the numbers represent the upper bound. The wires in calculation were #18 AWG, total cross-sectional area in one direction is 32.68 mm^2.

A calculation for 256 wires #24 AWG in an 8 by 32 arrangement, as shown in figure 9, was also made. As expected, the inductance and characteristic impedance values are lower than for the 80-wire configuration: L = 3.2 nH/m, C = 7.9 nF/m, Z_0 = 0.6 Ω (printout values, respectively, 3.2193, 7.9382, 0.6368); cross-sectional area, 25.1 mm^2. Note that the variations of the self-inductance of the individual wires are greater than for the 80-wire arrangement.

Given the same size wires and the same distances among them, inductances of different size bundles are proportional to the inverse ratio of the number of conductors in the bundle. This was used to arrive at the values listed in Table V for the minimum number of wires required to pass 100-A currents. (if the self-inductances of each of the wires were the same, this relation would hold for any number of wires. Inasmuch as this is not the case (see figs. 7 to 9), the number of wires should be greater than about 20 for the approximation to be acceptable.)

Table V lists several parameters for lines composed of a minimum number of conductors that can carry the rated (100-A) current. The line lengths are 150 m each, so that the dc resistances were calculated for 300-m length to take into account the return path. The total number of wires in each line is twice that listed in the table. The corrections for the ac values were taken from standard tables (ref. 13), and those for proximity effects from Smith (ref. 14). The proximity effects account for about additional 40 percent of resistance (see Table I and fig. 5 of ref. 14) for a distance-to-radius ratio of four (two in ref. 14 because the distance between the centers in this reference is "2c").

To adjust the value of inductance per unit length and characteristic impedance of the 52-wire #18 AWG line listed in Table V, it is only necessary to multiply by 80/52 = 1.5, which would make $9.7 < L < 16$ nH/m, and $1.9 < Z_0 < 3.2\ \Omega$. Similar adjustments can be made for the other lines.

3.3 Miscellaneous Considerations

At the rated current, about 6.5 W will be dissipated per meter of line, which would mean an increase (ref. 15) between 0.08 to 0.13 K, depending upon the type of insulation. Such temperature rise is not sufficiency large to cause concern.

The coefficient of expansion for copper is on the order of 10^{-6} m/K from 25 to 1200 K (ref. 16). At 300 K, the expansion is 16.8×10^{-6} m, which would mean a total of 2.5 mm for the entire 150-m line. This, too, is not considered significant.

For the recommended types of aircraft wire (ref. 17), Mil-W-16878, the dielectric strengths substantially exceed the specification of standing off 1 kV, which - for the recommended configuration - would be on the order 1 kV/mm = 1 MV/m = 25 V/mil. The breakdown strength of polyethylene exceeds 20 kV/mm at 20 kHz, as does that of polystyrene, Teflon, and polypropylene (ref. 18).

The conductance per unit length between the individual conductors can be calculated from the relation

$$C/G = \epsilon/\sigma \qquad (9)$$

which, for the materials under consideration, will be on the order of 10^{-15} mho/m (R of the order of 10^{15} Ω/m), using 10^{17} Ω-m for resistivity (ref. 6). Clearly the losses due to transverse currents can be neglected.

3.4 Connectors and Junction Boxes

A conventional, circular, multipin connector is shown in figure 10 as an example. Junction boxes with connecting wires, with conventional connectors of circular or rectangular shape will fulfill the specifications. The junction box shown in figures 11(a) and (b) would be more suitable, however, because the interconnecting plates substituted for wires would reduce the inductance locally. Connectors could be of the conventional, multipin variety, with soldered or brazed connections internally. Alternatively, internally the pins could be connected to blades for contact with the conductors. The specific configuration is not very critical, but it would be recommended that the connecting surfaces be gold-plated. The reason for this is to prevent increases of contact resistances as a result of fretting (ref. 19).

3.5 Miscellaneous

In addition to the configurations described above, parallel-plate arrangements of the type shown in figures 12 and 13 were analyzed. That of figure 12, in spite of the very low characteristic impedance (0.14 Ω), is thought to be less suitable than the configurations recommended in the next section of the report because the thin plates will very likely buckle. Should the plates be made thicker, the line would be too heavy and too unwieldy. Also, connectors with suitable characteristics would be difficult to design.

The program for the determination of line parameters for the configuration shown in figure 13, for different values of θ, thickness, and spacing, is listed in appendix B. The values of inductances per unit length, and characteristic impedances are at least four times larger than those of the configuration shown in figure 6.

4.0 RECOMMENDATIONS

The line recommendations are made on the basis of the assumption that off-the-shelf availability, or ease for assembly-line production, are secondary to the other attributes such as low impedance, reliability, and the like. As a result, the best choices for the line are listed in Table IV. The total number of wires per 150-m line is listed, with one-half representing the return path. The direction of current alternates between neighboring lines. (The difference between Table IV and Table V is that in the latter one-half of the wires was used in a 300-m long loop for the determination of the total resistance, whereas the number of wires in the former represent two parallel sets each 150 m long.) The alternating current directions have at least two advantages: lowering the inductance of the comparable wire arrangement with adjacent currents in the same direction by at least a factor of four, and by reducing the external magnetic fields which reduces electromagnetic interference. Numbers 1 and 6 in Table IV represent the configurations of figures 6 and 9; the remainder represents the minimum number of wires required for 100-A currents.

Multipin connectors of the type generally available from manufacturers such as Litton, Amphenol, or others, would be satisfactory for the junction boxes shown in figure 11, with gold-plated contacts to prevent increases of resistance due to fretting (platinum plating would also be acceptable).

A general recommendation for any type of line used in an aircraft power distribution system is that instead of one line 150 m long there should be several 5-, 10-, or 15-m sections connected in series via junction boxes. The advantage of this would be that in case of failure of a section, it could be easily replaced-even in flight. Also, the several junction boxes at given intervals would facilitate connections to the respective loads.

```
      PROGRAM CRIMPD.F
*
*
      DOUBLEPRECISION A,E,C
      DOUBLEPRECISION S(1:1024),O(1:1024),X(1:1024),Y(1:1024)
      DOUBLEPRECISION L(0:1024),L0(0:1024),L1(0:1024),LN
      DOUBLEPRECISION D(0:1024),T(1:1024),R(1:1024),B(0:1024)
      DOUBLEPRECISION SA,SD,PI,SLOPEANG
      INTEGER M,K,UUU,JE,ALTPOL
      INTEGER U,J,I,N,RP(1:1024)
      CHARACTER Q1,POL(1:1024)*3
      PARAMETER (PI=3.141592654)
*
*
      Q1='SELECTION IS OUT OF BOUNDS. TRY AGAIN...'
 100  CONTINUE
      DO 140 I=1,64
        PRINT*
        S(I)=0.
        O(I)=0.
        X(I)=0.
        Y(I)=0.
        L(I)=0.
        L0(I)=0.
        L1(I)=0.
        D(I)=0.
        T(I)=0.
        R(I)=0.
        B(I)=0.
      POL(I)='N'
 140  CONTINUE
      N=1
 200  PRINT'(////////)'
      PRINT*,'                   Main Menu'
      PRINT*
      PRINT*,'     1. Polar coordinate input'
      PRINT*,'     2. Rectangular coordinate input'
      PRINT*,'     3. Data file input'
      PRINT*,'     4. Run inductance calculation (Skilling method)'
      PRINT*,'     5. Run capacitance calculation (Cheng method)'
      PRINT*,'     6. Check individual conductors'
      PRINT*,'     7. Exit to shell'
      PRINT*,'     8. Enter new configuration (reset index counter)'
      PRINT'(//)'
*
      PRINT*,'Choose from 1 thru 8...'
      READ*,U
      IF(U .EQ. 7)GOTO 7000
      IF(U .EQ. 6)GOTO 6000
      IF(U .EQ. 5)GOTO 250
      IF(U .EQ. 4)GOTO 4000
      IF(U .EQ. 3)GOTO 250
      IF(U .EQ. 2)GOTO 2000
      IF(U .EQ. 1)GOTO 1000
      IF(U .EQ. 8)GOTO 100
      PRINT*,Q1
      GOTO 200
 250  PRINT*,'ROUTINE NOT OPERATIVE,CHOOSE ANOTHER MODE...'
      GOTO 200
*
*
*     Polar coordinate input
 1000 PRINT*,'       Polar coordinate input'
      PRINT*,'       1. Automatic ring program'
      PRINT*,'       2. Individual input'
      PRINT*,'       3. Exit to main menu'
*
      PRINT*,'Choose one...'
      READ*,U
      IF(U .EQ. 3)GOTO 200
      IF(U .EQ. 2)GOTO 1500
      IF(U .EQ. 1)GOTO 1100
      PRINT*,Q1
      GOTO 1000
*
*     Auto polar ring
 1100 PRINT*,'AUTO POLAR RING'
      PRINT*,'Input number of conductors in this ring...'
      READ*,M
      SA=2.0*PI/REAL(M)
      SD=SA*180.0/PI
```

```
      PRINT*,'Input coordinates, radius and polarity of 1st conductor'
      PRINT*,'in this ring (r(mm),theta(degrees),radius(mm),+ or -)'
      READ(5,1110) S(N),T(N),R(N),POL(N)
1110  FORMAT(F8.4,F8.4,F8.4,A1)
      O(N)=T(N)*PI/180.0
      S(N)=S(N)/1000.0
      R(N)=R(N)/1000.0
      X(N)=S(N)*COS(O(N))
      Y(N)=S(N)*SIN(O(N))
      RP(N)=0
1120  PRINT'(///A)','Choose polarity of wires...'
      PRINT*,'     1. Alternating'
      PRINT*,'     2. All the same as 1st'
      PRINT*,'     3. Individually selected'
      PRINT*,'     4. Exit to last menu'
1160  READ*,U
      IF(U .EQ. 1)GOTO 1200
      IF(U .EQ. 2)GOTO 1220
      IF(U .EQ. 3)GOTO 1180
      IF(U .EQ. 4)GOTO 1000
      PRINT*,Q1
      GOTO 1120
*
1180  UUU=1
      GOTO 1260
1200  UU=0
      GOTO 1260
1220  UU=1
1260  DO 1340 I=N+1,N+M-1
        RP(I)=0
        S(I)=S(N)
        O(I)=O(I-1) + SA
        T(I)=T(I-1) + SD
        X(I)=S(I)*COS(O(I))
        Y(I)=S(I)*SIN(O(I))
        R(I)=R(N)
1280    IF(UUU .NE. 1)GOTO 1300
        PRINT'(/,T2,A,I3,A,F8.5,2X,F8.4)','Conductor',I  ,'with coords.',S(I)*1000.0,T(I)
        PRINT*,'Input polarity (+ or -)...'
        READ'(A)',POL(I)
1300    IF(U .EQ. 2)GOTO 1320
        IF(POL(I-1) .EQ. '+')POL(I)='-'
        IF(POL(I-1) .EQ. '-')POL(I)='+'
        GOTO 1340
1320    POL(I)=POL(I-1)
1340  CONTINUE
*
      N=N+M
      GOTO 200
*
*     Individual polar input
1500  PRINT*,'      Individual Polar Inputs'
      PRINT*,'How many conductors in this series?'
      READ*,K
      DO 1560 I=N,N+K-1
        PRINT*,'Input r(mm),theta(degrees),radius of conductor(mm),polarity(+ or -)...'
        READ(5,1110) S(I),T(I),R(I),POL(I)
        RP(I)=0
        R(I)=R(I)/1000.0
        S(I)=S(I)/1000.0
        O(I)=T(I)*PI/180.0
        X(I)=S(I)*COS(O(I))
        Y(I)=S(I)*SIN(O(I))
1560  CONTINUE
      N=N+K
      GOTO 200
*
*
*
*     rectangular coordinate input
2000  CONTINUE
      PRINT*,'RECTANGULAR COORDINATE INPUT'
2100  PRINT*,'Choose one...'
      PRINT*
      PRINT*,'      1. Individual input'
      PRINT*,'      2. Line input (equally spaced wires)'
      PRINT*,'      3. Return to main menu'
      READ*,U
      IF(U .EQ. 1)GOTO 2120
      IF(U .EQ. 2)GOTO 2200
      IF(U .EQ. 3)GOTO 200
```

```
 2120     PRINT*,'Individual Rectangular Inputs'
          PRINT*,'How many conductors in this series...'
          READ*,K
          DO 2150 I=N,N+K-1
            PRINT*
            PRINT*,'Input coordinates,radius & polarity of cond.',I,'.'
            PRINT*,'(X(mm),Y(mm),r(mm),+ or -)'
            READ(5,1110) X(I),Y(I),R(I),POL(I)
              RP(I)=1
              X(I)=X(I)/1000.0
              Y(I)=Y(I)/1000.0
              R(I)=R(I)/1000.0
              S(I)=SQRT(X(I)**2.0+Y(I)**2.0)
              IF(S(I) .EQ. 0.0)GOTO 2140
              T(I)=180.0/PI*ACOS(X(I)/S(I))
              GOTO 2150
 2140         T(I)=0.0
 2150     CONTINUE
          N=N+K
          GOTO 200
*
*AUTO LINE
 2200     CONTINUE
          PRINT*,'AUTO LINE INPUT'
          PRINT*,'Input coordinates, radius & polarity of 1st conductor...'
          PRINT*,'(X(mm),Y(mm),r(mm),+ or -)...'
          READ(5,1110) X(N),Y(N),R(N),POL(N)
            X(N)=X(N)/1000.0
            Y(N)=Y(N)/1000.0
            R(N)=R(N)/1000.0
            S(N)=SQRT(X(N)**2.0 + Y(N)**2.0)
            IF(S(N) .EQ. 0.0)GOTO 2210
            T(N)=180.0/PI*ACOS(X(N)/S(N))
            GOTO 2230
 2210       T(N)=0.0
 2230       CONTINUE
            RP(N)=1
          PRINT*
          PRINT*,'Input number of conductors on this line...'
          READ*,M
          PRINT*,'Input distance between centers of conds on this line (SPACING)...'
          READ*,SPACING
            SPACING=SPACING/1000.0
          PRINT*,'Input slope angle of the line (positive x-axis=0 degrees)'
          READ*,SLOPEANG
            SLOPEANG=SLOPEANG*PI/180.0
          DO 2300 I=N+1,N+M-1
            X(I)=X(I-1) + SPACING*COS(SLOPEANG)
            Y(I)=Y(I-1) + SPACING*SIN(SLOPEANG)
            R(I)=R(I-1)
            S(I)=SQRT(X(I)**2+Y(I)**2)
            T(I)=ACOS(X(I)/S(I))*180.0/PI
            RP(I)=1
 2300     CONTINUE
 2320     CONTINUE
          PRINT'(///A)','Choose polarity of wires...'
          PRINT*,'      1. Alternating'
          PRINT*,'      2. All the same as the 1st'
          PRINT*,'      3. Individually selected'
          PRINT*,'      4. Return to last menu'
          READ*,U
            IF(U .EQ. 1)GOTO 2340
            IF(U .EQ. 2)GOTO 2380
            IF(U .EQ. 3)GOTO 2420
            IF(U .EQ. 4)GOTO 2000
            PRINT*,Q1
            GOTO 2320
 2340     CONTINUE
            IF(POL(N) .EQ. '+')THEN
              ALTPOL=1
            ELSEIF(POL(N) .EQ. '-')THEN
              ALTPOL=-1
            ENDIF
            DO 2360 I=N+1,N+M-1
              IF(ALTPOL .EQ. 1)THEN
                POL(I)='-'
              ELSEIF(ALTPOL .EQ. -1)THEN
                POL(I)='+'
              ENDIF
              ALTPOL=-ALTPOL
```

```
 2360      CONTINUE
           N=N+M
        GOTO 200
*
 2380   CONTINUE
           DO 2400 I=N+1,N+M-1
              POL(I)=POL(N)
 2400      CONTINUE
           N=N+M
        GOTO 200
*
 2420   CONTINUE
           DO 2440 I=N+1,N+M-1
              PRINT'(/,T2,A,I3,A,F8.5,2X,F8.4)','conductor',I,'with coords. x(mm)=',X(I),'y(mm)=',Y(I)
              PRINT*,'Input polarity (+ or -)...'
              READ*,POL(I)
 2440      CONTINUE
           N=N+M
        GOTO 200
*
*
*
*
 3000   continue
*
*
*
*
 4000   D(0)=1.0
        PRINT*,'Input value of dielectric constant...'
        READ*,E
 4050   print*
 4140   DO 4320 I=1,N-1
           IF(POL(I) .EQ. '-')GOTO 4220
           DO 4200 J=1,N-1
              IF(J .EQ. I)GOTO 4160
              D(J)=SQRT((X(J)-X(I))**2+(Y(J)-Y(I))**2)
              IF(POL(J) .EQ. '-')GOTO 4180
              D(J)=1.0/D(J)
              GOTO 4180
 4160         D(J)=1.0
 4180      D(J)=D(J)*D(J-1)
 4200      CONTINUE
           GOTO 4300
*
 4220   D(0)=1.0
        DO 4280 J=1,N-1
              IF(J .EQ. I)GOTO 4240
              D(J)=SQRT((X(I)-X(J))**2+(Y(I)-Y(J))**2)
              IF(POL(J) .EQ. '+')GOTO 4260
              D(J)=1.0/D(J)
              GOTO 4260
 4240         D(J)=1.0
 4260         D(J)=D(J)*D(J-1)
 4280      CONTINUE
 4300      L(I)=(0.5+2.0*LOG(D(N-1)/R(I)))*100.0
 4320   CONTINUE
*
*
        L0(0)=0.0
        L1(0)=0.0
        DO 4440 I=1,N-1
           IF(POL(I) .EQ. '-')GOTO 4400
           L1(I)=1.0/L(I)
           L0(I)=0.0
           GOTO 4420
 4400      L1(I)=0.0
           L0(I)=1.0/L(I)
 4420      L1(I)=L1(I)+L1(I-1)
           L0(I)=L0(I)+L0(I-1)
 4440   CONTINUE
*
        B(0)=0.0
        DO 4460 I=1,N-1
           B(I)=R(I)*1000.0
           A=(B(I))**2+(B(I-1))**2
           B(I)=SQRT(A)
 4460   CONTINUE
        A=A*PI/2.0
        IF(ABS(L0(N-1)) .LE. 0.0000001)GOTO 4480
        LN=1.0/L1(N-1)+1.0/L0(N-1)
```

```
4470    goto 4490
4480    LN=1.0/L1(N-1)
4490    CONTINUE
        C=E*100.0/9.0/LN
        PRINT'(T2,A,F10.4,A)','INDUCTANCE OF CIRCUIT IS L=',LN,'nH/m'
        PRINT'(T2,A,F10.4,A)','CAPACITANCE IS C=',C,'nF/m'
        PRINT'(t2,a,F10.4,a)','CHAR. IMPED. IS Z=',SQRT(LN/C),'ohms'
        PRINT'(T2,A,T51,F10.4,T62,A//)','TOTAL CROSS-SECT. AREA IN ONE DIRECTION IS A=',A,'sq. mm'
*
        PRINT'(//)'
*
        PRINT'(2X,A,T10,A,T20,A,T30,A,T37,A,T50,A,T60,A,T72,A)','cond','x(mm)','y(mm)','r(mm)','theta(deg)','polarity','radius','L'
        DO 4640 I=1,N-1
        IF(RP(I) .EQ. 1)GOTO 4600
          PRINT'(2X,I3,T25,F10.4,T36,F10.4,T53,A,T56,F10.4,T67,F10.4)',I,S(I)*1000.0,T(I),POL(I),R(I)*1000.0,L(I)
        GOTO 4640
*
 4600   continue
          PRINT'(2X,I3,T7,F10.4,T18,F10.4,T53,A,T56,F10.4,T67,F10.4)',I,X(I)*1000.0,Y(I)*1000.0,POL(I),R(I)*1000.0,L(I)
 4640   CONTINUE
 4700   PRINT*
        goto 200
*
*
 5000   continue
*
*
*
*
 6000   PRINT*,'       Check individual conductors'
        PRINT*,'Input number of 1st conductor to be checked...'
        READ*,J
        PRINT*,'Input number of last conductor to be checked...'
        READ*,JE
        PRINT'(2X,A,T9,A,T18,A,T33,A,T43,A,T50,A,T60,A)','cond','r(mm)','theta(deg)','x(mm)','y(mm)','polarity','radius'
        DO 6200 I=J,JE
        PRINT'(2X,I3,T7,F8.4,T21,F9.3,T31,F8.4,T41,F8.4,T54,A,T58,F8.4)',I,S(I)*1000.0,T(I),X(I)*1000.0,Y(I)*1000.0,POL(I),R(I)*1000
 6200   CONTINUE
 6300   PRINT*,'Check another conductor? (1=yes,0=no)...'
        READ*,U
        IF(U .EQ. 1)GOTO 6000
        IF(U .EQ. 0)GOTO 200
        PRINT*,Q1
        GOTO 6300
*
*
 7000   PRINT*,'Are you sure you want to exit program?'
        PRINT*,'       1=YES'
        PRINT*,'       2=NO'
        READ*,U
        IF(U .EQ. 2)GOTO 200
        IF(U .EQ. 1)GOTO 7500
        PRINT*,Q1
        GOTO 7000
 7500   PRINT*
        END
```

```
*=========================================================================
          PROGRAM LAPLAC
*specifications---------------------------------------------------------
          INTEGER NMIN,NMAX,MMIN,MMAX
          INTEGER P(0:64,0:64),L
          INTEGER N,M,N1,N2,N3,N4,M1,M2,NS,MS
          INTEGER LMAX,LS,OUT,IN,ID,JD,LT
          INTEGER I,J,K,KD,KQ,KT,KMAX,JSL,MM2,MM3,M2SL,RPN1,RPM1
          REAL SL,ERLIM,ACAP,DCAP,RPA,RPB,RPC
          REAL ERMAX,LAMBDA
          REAL F(0:64,0:64),C(0:18,1:5),H,RP1,RP2
          CHARACTER PS(0:18)*1
          COMMON F,P,C,L,H,RP1,RP2
          COMMON/RELAX/LAMBDA,ERMAX,I,J
          COMMON/CONTOR/NMIN,NMAX,MMIN,MMAX
*initializations--------------------------------------------------------
          DATA N,M,NMIN,NMAX,MMIN,MMAX/32,64,0,20,0,48/
          DATA ERLIM,KMAX/1E-4,100/
          DATA N1,N2,N3,N4,M1,M2,NS,MS/4,12,16,4,16,32,6,18/
          DATA (PS(I),I=0,18)/
     :    '.','+','*','1','-',' ','.',
     :    'a','b','c','d','e','f','g','h','i','j','k','l'/
          DATA LAMBDA,LMAX,H/1.4,4,0.125/
          DATA RP1,RP2,RPN1,RPM1,KD,KT/1,1,16,32,0,0/
*function statements----------------------------------------------------
          IRND(F1)=NINT(F1*1E3)
*data inputs------------------------------------------------------------
          PRINT'(A)','OUTPUT TYPE (0=ALL, 1=FINAL, 2=DATA FILE)?'
          READ*,OUT
          PRINT'(A)','INPUT TYPE (0=INTERNAL,1=EXTERNAL,2=DATA FILE)'
          READ*,IN
          IF(IN .EQ. 2)THEN
            READ'(3I3,2F6.3)',N,M,L,H,SL
            DO 2 JD=0,M
              DO 1 ID=0,N
                READ'(2I3,E15.7,I3)',I,J,F(ID,JD),P(ID,JD)
 1            CONTINUE
 2          CONTINUE
            CALL CNTOUR
            GOTO 999
          ENDIF
          IF(IN .LT. 1)THEN
            PRINT'(A)','INPUT SLOPE( 0,0.5,1,2,3=NO FLAP ):'
            READ*,SL
          ENDIF
 3        PRINT'(/T2,A,F6.3,A//,10A5,2A8)',
     :    'BOUNDARY DIMENSIONS( 1 =',H,' mm ):',
     :    'N','M','N1','N2','N3','N4','M1','M2','NS','MS','SLOPE','LAMBDA'
          IF(IN .LT. 1)GOTO 4
          READ*,N,M,N1,N2,N3,N4,M1,M2,NS,MS,SL,LAMBDA
 4        PRINT'(10I5,2F8.3//)',N,M,N1,N2,N3,N4,M1,M2,NS,MS,SL,LAMBDA
          PRINT'(T3,A//,4A6)',
     :    'INPUT CONTOUR LIMITS','NMIN','NMAX','MMIN','MMAX'
          IF(IN .LT. 1)GOTO 5
          READ*,NMIN,NMAX,MMIN,MMAX
 5        PRINT'(4I6/)',NMIN,NMAX,MMIN,MMAX
```

```
*boundary conditions-----------------------------------------------------
        PRINT'(A)','HOMOGENEOUS DIELCTRIC CONSTANT(YES=GE(1),NO=LE(-1))'
        READ*,RP1
        IF(RP1 .GT. 0)THEN
          KD=0
          RP2=RP1
          RP1=1.0
         ELSE
          RP2=ABS(RP1)
          RP1=1.0
          KD=0
          PRINT'(A)','DIELECTRIC CONFIGURATION(1=INSIDE,2=OUTSIDE)'
          READ*,KD
            IF(KD .EQ. 1)THEN
              KT=0
              PRINT'(A)','DISTANCE FROM CENTER=?'
              READ*,KT
              RPM1=M1+KT
              RPN1=N3
             ELSE
              PRINT'(A)','OUTSIDE LAYER THICKNESS=?'
              READ*,KT
              RPN1=N3+KT
              RPM1=M2+KT
            ENDIF
        ENDIF
        RPA=2*RP1
        RPB=2*RP2
        RPC=RP1+RP2
*point type array setup-----------------------------------------------
*homogeneous interior point-----------------------
        DO 7 J=1,5
          C(5,J)=RPA
 7      CONTINUE
        DO 8 J=1,5
          C(6,J)=RPB
 8      CONTINUE
*dielctric in two adjacent quadrants -----------
        C(7,1)=RPC
        C(7,2)=RPC
        C(7,3)=RPA
        C(7,4)=RPB
        C(7,5)=RPC*2
        C(8,1)=RPA
        C(8,2)=RPB
        C(8,3)=RPC
        C(8,4)=RPC
        C(8,5)=RPC*2
        C(9,1)=RPC
        C(9,2)=RPC
        C(9,3)=RPB
        C(9,4)=RPA
        C(9,5)=2*RPC
        C(10,1)=RPA
        C(10,2)=RPB
        C(10,3)=RPC
        C(10,4)=RPC
```

```
      C(10,5)=2*RPC
*dielctric in one quadrant-------------------
      C(11,1)=RPA
      C(11,2)=RPC
      C(11,3)=RPA
      C(11,4)=RPC
      C(11,5)=RPC+RPA
      C(12,1)=RPA
      C(12,2)=RPC
      C(12,3)=RPC
      C(12,4)=RPA
      C(12,5)=RPC+RPA
      C(13,1)=RPA
      C(13,2)=RPC
      C(13,3)=RPA
      C(13,4)=RPC
      C(13,5)=RPC+RPA
      C(14,1)=RPC
      C(14,2)=RPA
      C(14,3)=RPA
      C(14,4)=RPC
      C(14,5)=RPC+RPA
*dielectric in three quradrants----------------
      C(15,1)=RPC
      C(15,2)=RPB
      C(15,3)=RPC
      C(15,4)=RPB
      C(15,5)=RPC+RPB
      C(16,1)=RPC
      C(16,2)=RPB
      C(16,3)=RPB
      C(16,4)=RPC
      C(16,5)=RPC+RPB
      C(17,1)=RPC
      C(17,2)=RPB
      C(17,3)=RPC
      C(17,4)=RPB
      C(17,5)=RPC+RPB
      C(18,1)=RPB
      C(18,2)=RPC
      C(18,3)=RPC
      C(18,4)=RPB
      C(18,5)=RPC+RPB
*initial point type--------------------------------
      DO 10 I=0,N
        DO 9 J=0,M
          F(I,J)=0.0
          DC(I,J)=RP1
 9        P(I,J)=5
 10   CONTINUE
*defined boundaries-------------------------
      DO 600 LS=2,0,-1
        L=(2**LS)
        KQ=KMAX/REAL(L)
*upper boundary (x,y=m)*------------
      DO 15 I=0,N,L
        P(I,M)=2
```

```
 15     CONTINUE
*right boundary (x=n,y)*-------------
        DO 20 J=0,M,L
          P(N,J)=2
 20     CONTINUE
*lower dielectric boundary (x,y=0)*--
        DO 30 I=(N1+L),(N2-L),L
          P(I,0)=3
 30     CONTINUE
        DO 35 I=(N3+L),N,L
            P(I,0)=3
 35     CONTINUE
*left dielectric boundary (x=0,y)*---
        DO 40 J=(M1+L),M,L
          P(0,J)=4
 40     CONTINUE
*defined conductors-------------------------------
*center conductor (x,y)*-------------
        DO 55 I=0,N1,L
          DO 50 J=0,M1,L
            F(I,J)=1.0
 50         P(I,J)=1
 55     CONTINUE
*outer conductor (x,y)*--------------
        DO 65 I=N2,N3,L
          DO 60 J=0,M2,L
 60         P(I,J)=2
 65     CONTINUE
*outer conductor edge(x,y)*----------
        IF(SL .GT. 2)GOTO 80
        MM2=M2-LMAX
        MM3=M2+5*LMAX
        DO 75 J=MM2,MM3,L
          DO 70 I=N3,N4,-L
            JSL=NINT(SL*(N2-I)/L)*L
            M2SL=M2+JSL
            IF(((M2SL-(N3-N2+L)) .LT. J .AND. J .LE. M2SL) .OR.
     :      (N2 .LE. I .AND. J .LT. M2SL)) THEN
              F(I,J)=0.0
              P(I,J)=2
            ELSE
              P(I,J)=5
            ENDIF
 70       CONTINUE
 75     CONTINUE
*second dielectric fill-----------------------------------------
 80     IF(KD .GT. 0)THEN
          DO 140 J=1,(RPM1-L)
            DO 120 I=1,(RPN1-L)
              IF(P(I,J) .GT. 4)THEN
                P(I,J)=6
              ENDIF
 120        CONTINUE
            IF(KD .EQ. 2)THEN
              P(RPN1,J)=8
            ENDIF
 140      CONTINUE
```

```
          DO 160 I=1,(RPN1-1)
            IF(P(I,RPM1) .GT. 4)THEN
              P(I,RPM1)=7
            ENDIF
 160      CONTINUE
          IF(KD .EQ. 2)THEN
            P(RPN1,RPM1)=11
          ENDIF
        ENDIF
*air pocket location---------------------------------
        IF(L .GT. 1)GOTO 180
 170    PRINT'(A)','INPUT AIR POCKET LOCATION[(I,J),(-1,X)=NONE]'
        READ*,I,J
        LT=1
        IF(I .GT. 0)THEN
          P(I,J)=5
          P(I-LT,J)=5
          P(I+LT,J)=5
          P(I,J-LT)=5
          P(I,J+LT)=5
          P(I-2*LT,J)=7
          P(I,J-2*LT)=8
          P(I+2*LT,J)=9
          P(I,J+2*LT)=10
          P(I-LT,J-L)=11
          P(I+LT,J-L)=12
          P(I+LT,J+L)=13
          P(I-LT,J+L)=14
          P(I-2*LT,J-L)=15
          P(I-LT,J-2*L)=15
          P(I+LT,J-2*L)=16
          P(I+2*LT,J-L)=16
          P(I+2*LT,J+L)=17
          P(I+LT,J+2*L)=17
          P(I-LT,J+2*L)=18
          P(I-2*LT,J+L)=18
          GOTO 170
        ENDIF
*point type printout----------------------------------------------------
 180    IF(OUT .EQ. 1 .AND. L .GT. 1)GOTO 200
        IF(OUT .GT. 1)GOTO 200
        PRINT'(/T2,A//,T5,A,I2,/)',
     :  'BOUNDARY CONDITIONS','MESH SIZE =',L
        PRINT'(T5,A/,T5,A/)','+:V=1000,   *:V=0','|:dV/dy=0, -:dV/dx=0'
        DO 190 J=M,0,-L
          PRINT'(T6,33A)',(PS(P(I,J)),I=0,N,L)
 190    CONTINUE
        PRINT'(////)'
 200    CONTINUE
*lambda loop------------------------------------------------------------
*iterate solution---------------------------------------------
        DO 230 K=1,KQ
          IF(K .EQ. 1) THEN
            ERMAX=1
           ELSE
            ERMAX=0
          ENDIF
```

```
          DO 220 J=0,M,L
            DO 210 I=0,N,L
              CALL PTCALC
 210        CONTINUE
 220      CONTINUE
          IF(ERMAX .LE. ERLIM)GOTO 240
 230    CONTINUE
        K=0
 240    CONTINUE
 250    IF(L .LE. 1 .OR. OUT .LT. 1)THEN
          CALL CAPCT(NS,MS,ACAP,DCAP)
        ENDIF
*output analysis------------------------------------------------------------
 300    IF(OUT .EQ. 2)GOTO 500
        IF(L .GT. 1 .AND. OUT .GT. 0)GOTO 600
        IF(K .EQ. 0)THEN
          PRINT'(T2,A,I4,A,/)',
     :    'SOLUTION HAS NOT CONVERGED IN ',KQ,' ITERATIONS'
        ELSE
          PRINT'(T2,A,I4,A,/)',
     :    'SOLUTION CONVERGED AFTER ',K,' ITERATIONS'
        ENDIF
 320    PRINT'(T2,A,F9.6,/)',
     :  'WITH A MAXIMUM ABSOLUTE ERROR OF',ERMAX
        PRINT'(T2,A,I3,A,/,T2,A,F6.3,//)',
     :  'MESH SIZE=',L,' * STEP SIZE','LAMBDA   =',LAMBDA
        PRINT'(T2,A,F10.6,A/)','CORNER CAPACITANCE (AIR) =',ACAP,'pF/m'
        PRINT'(//////////////)'
*solution array printout----------------------------------------------------
        IF(LS .LT. 1) THEN
          PRINT'(T2,A/)','SOLUTION AT n*m POINTS, (x=horiz, y=vert):'
          DO 400 J=M,0,-L
            PRINT'(T2,33(I4))',(IRND(F(I,J)),I=0,N,L)
 400      CONTINUE
        PRINT'(///,T2,A,/,T2,A,/)',
     :  'CONSTANT VOLTAGE CONTOUR PLOT','+:V=1000   *:V=0   .:V<10'
          CALL CNTOUR
        ENDIF
        GOTO 600
*data file output-------------------------------------------------------
 500    IF(L .EQ. 1) THEN
          PRINT'(3I3,2F6.3)',N,M,L,H,SL
          DO 520 J=0,M
            DO 510 I=0,N
              PRINT'(2I3,E15.7,I3)',I,J,F(I,J),P(I,J)
 510        CONTINUE
 520      CONTINUE
          PRINT'(I3)',-1
          GOTO 999
        ENDIF
*termination--------------------------------------------------------
 600    CONTINUE
        PRINT'(//A/,A,////)','CHANGE PARAMETERS? (-1 TO END, 1=FLAP SLOPE,',
     :  '2=LAMBDA, 3=BOUNDARY CONDITIONS)'
        READ'(I3)',I
        IF(I .LT. 0)GOTO 999
        GOTO(620,630,640),I
```

```
 620      READ*,SL
          GOTO 3
 630      READ*,LAMBDA
          GOTO 3
 640      IN=1
          GOTO 3
 999      END
*********************************************************************
          SUBROUTINE PTCALC
*specifications--------------------------------------------------
          INTEGER I,J,PK
          INTEGER P(0:64,0:64),L
          REAL F1,F2,F3,F4,F5,F6,F7,F8
          REAL C1,C2,C3,C4,C5
          REAL PAST,TEMP,ER,FINT,FNORM,FDIEL
          REAL ERMAX,LAMBDA
          REAL F(0:64,0:64),C(0:18,1:5),H,RP1,RP2
          COMMON F,P,C,L,H,RP1,RP2
          COMMON/RELAX/LAMBDA,ERMAX,I,J
*function statements---------------------------------------------
          FINT(F1,F2,F3,F4,F5,F6,F7,F8)=(4*(F1+F2+F3+F4)+(F5+F6+F7+F8))*0.05
          FNORM(F1,F2)=(4*F2-F1)/3
          FDIEL(F1,F2,F3,F4,C1,C2,C3,C4,C5)=(C1*F1+C2*F2+C3*F3+C4*F4)/(2*C5)
*relaxation------------------------------------------------------
          PK=P(I,J)
          GOTO (195,195,120,140,180,180,160,160,160,160),PK
 120      TEMP=FNORM(F(I,(2*L)),F(I,L))
          GOTO 190
 140      TEMP=FNORM(F((2*L),J),F(L,J))
          GOTO 190
 160      TEMP=FDIEL(F((I+L),J),F((I-L),J),F(I,(J+L)),F(I,(J-L)),
     :    C(PK,1),C(PK,2),C(PK,3),C(PK,4),C(PK,5))
          GOTO 190
 180      TEMP=FINT(F(I+L,J),F(I-L,J),F(I,J+L),F(I,J-L),
     :    F(I+L,J+L),F(I+L,J-L),F(I-L,J+L),F(I-L,J-L))
 190      PAST=F(I,J)
          F(I,J)=PAST+LAMBDA*(TEMP-PAST)
          ER=ABS(PAST-F(I,J))
          ERMAX=MAX(ERMAX,ER)
 195      RETURN
          END
*********************************************************************
          SUBROUTINE CAPCT(NS,MS,ACAP,DCAP)
*specifications--------------------------------------------------
          INTEGER NS,MS
          INTEGER P(0:64,0:64),L
          REAL ENORM,PERMFS,ACAP,DCAP
          REAL F(0:64,0:64),C(0:18,1:5),H,RP1,RP2
          COMMON F,P,C,L,H,RP1,RP2
          PARAMETER (PERMFS=8.85419)
*surface integration---------------------------------------------
          ENORM=0.0
          ENORM=(F(NS+L,0)-F(N2-L,0)+F(NS+L,MS)-F(NS-L,MS))*0.5
          ENORM=ENORM+(F(0,MS+L)-F(0,MS-L)+F(NS,MS+L)-F(NS,MS-L))*0.5
          DO 10 J=L,(MS-L),L
            ENORM=ENORM+F(NS+L,J)-F(NS-L,J)
 10       CONTINUE
```

```
      DO 20 I=L,(NS-L),L
        ENORM=ENORM+F(I,MS+L)-F(I,MS-L)
 20   CONTINUE
      ACAP=ABS((5E-4*PERMFS*(NS+MS)*ENORM)/H)
      DCAP=RP2*ACAP
      RETURN
      END
************************************************************************
      SUBROUTINE CNTOUR
*specifications-------------------------------------------------------
      INTEGER IR,IC,NCOL,MROW,MARKX,MARKY
      INTEGER I,J,NO,MO,DN,DM,PD
      INTEGER NMIN,NMAX,MMIN,MMAX
      INTEGER P(0:64,0:64),L
      REAL F(0:64,0:64),C(0:18,1:5),H,RP1,RP2
      CHARACTER DARY(0:132,0:260)*1,CS(0:16)*1
      COMMON F,P,C,L,H,RP1,RP2
      COMMON/CONTOR/NMIN,NMAX,MMIN,MMAX
      COMMON/CHRSET/DARY,CS
      COMMON/INTSET/I,J,NO,MO,DN,DM,PD
*initializations------------------------------------------------------
      DATA NCOL,MROW,MARKX,MARKY/128,256,5,4/
      DATA DN,DM/5,4/
      DATA (CS(I),I=0,16)/
     : '0','1','2','3','4','5','6','7','8','9','0',
     : '+','*','|','-',' ','.'/
*character array clear------------------------------------------------
      DO 20 IR=0,MROW
        DO 10 IC=0,NCOL
          DARY(IC,IR)=CS(15)
 10     CONTINUE
 20   CONTINUE
*character array fill-------------------------------------------------
      DO 40 J=MMIN,MMAX
        MO=(J-MMIN)*DM
        DO 30 I=NMIN,NMAX
          NO=(I-NMIN)*DN
          IF(P(I,J) .LT. 3)THEN
            PD=(-P(I,J))
            CALL CHRFIL
          ELSE
           IF(I .GT. NMIN)THEN
            IF(P(I-L,J) .LT. 3)THEN
              PD=1
              CALL CHRFIL
            ENDIF
           ENDIF
           IF(J .GT. MMIN)THEN
            IF(P(I,J-L) .LT. 3)THEN
              PD=2
              CALL CHRFIL
            ENDIF
           ENDIF
            PD=0
            CALL CHRFIL
          ENDIF
 30     CONTINUE
```

```
 40     CONTINUE
*printing character array-------------------------------------------
        DO 50 IR=((MMAX-MMIN)*DM),0,-L
          PRINT'(132A1)',(DARY(IC,IR),IC=0,((NMAX-NMIN)*DN))
 50     CONTINUE
        RETURN
        END
**********************************************************************
        SUBROUTINE CHRFIL
*specifications-----------------------------------------------------
        INTEGER CF,LI,LJ
        INTEGER N,M
        INTEGER I,J,NO,MO,DN,DM,PD
        INTEGER P(0:64,0:64),L
        REAL FS,RRMIN,RMIN,RMAX,NX,MY,MPN
        REAL F(0:64,0:64),C(0:18,1:5),H,RP1,RP2
        CHARACTER DARY(0:132,0:260)*1,CS(0:16)*1
        COMMON F,P,C,L,H,RP1,RP2
        COMMON/CHRSET/DARY,CS
        COMMON/INTSET/I,J,NO,MO,DN,DM,PD
*initializations----------------------------------------------------
        DATA RRMIN,RMIN,RMAX/0.01,0.05,0.95/
*character array DN*DM segment fill---------------------------------
        LI=L
        LJ=L
        IF(PD .LT. 0)THEN
          CF=(10-PD)
          DARY(NO,MO)=CS(CF)
          GOTO 99
        ELSE
          IF(PD .EQ. 1)THEN
            LI=-L
          ELSE
            IF(PD .EQ. 2)THEN
              LJ=-L
            ENDIF
          ENDIF
        ENDIF
        DO 30 M=0,(DM-1)
          DO 20 N=0,(DN-1)
            IF(PD .GE. 0)THEN
              IF((N+M) .LT. 1)THEN
                FS=F(I,J)
              ELSE
                NX=N/REAL(DN)
                MY=M/REAL(DM)
                MPN=NX*MY
                FS=F(I,J)*(1.0+MPN-NX-MY)+F(I+LI,J)*(NX-MPN)+
     :          F(I,J+LJ)*(MY-MPN)+F(I+LI,J+LJ)*MPN
              ENDIF
              IF(FS .LT. RRMIN)THEN
                CF=16
              ELSE
               IF(FS .LT. RMIN .OR. FS .GT. RMAX)THEN
                 CF=10
               ELSE
```

```
                 IF(FS .GE. 0.999)THEN
                   CF=INT(FS*10)
                 ELSE
                   CF=NINT(FS*10)
                 ENDIF
               ENDIF
             ENDIF
           ENDIF
 10        DARY(NO+N*LI,MO+M*LJ)=CS(CF)
 20      CONTINUE
 30    CONTINUE
 99    RETURN
       END
*=========================================================================
```

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TABLE I. - CHARACTERISTIC IMPEDANCE AND INDUCTANCE OF DOUBLE LAYER MULTICONDUCTOR LINES ("INDUCTANCE" METHOD)

Number of wires	R_1, mm	R_2, mm	R_1, mm	R_2, mm	θ, deg	Z, Ω	L, nH/m
8 8	25 25	50 50	0.5 .5	0.5 .5	0 45	78.5 88.7	396.8 448.2
16 16	25 25	50 50	0.5 .5	0.5 .5	0 25.5	36.2 37.4	182.9 189.2
32 64	25 25	50 50	0.5 .5	0.5 .5	0 0	15.2 5.9	76.9 29.7
8 16	10 10	25 25	0.5 .5	0.5 .5	0 0	65.6 28.6	331.9 144.4
16 32	25 25	50 50	0.5 .5	1.0 2.0	0 0	33.1 11.7	167.2 59.3
32 64	25 25	50 50	1.0 .5	2.0 2.0	0 0	10.3 4.1	51.9 20.8
16 32	10 10	25 25	0.5 1.0	4.0 4.0	0 0	17.2 4.3	87.2 21.8

TABLE II. - Z_o AND L OF DOUBLE RING MULTICONDUCTOR LINES ("CAPACITANCE" METHOD)

R_1, mm	R_2, mm	θ, deg	r, mm	Total number of wires in line, N	Zo, Ω	L, nH/m
25 25 25 25	50 ↓ ↓ ↓	0 45 0 22.5	0.5 ↓ ↓ ↓	8 8 16 16	73.5 83.7 33.7 34.9	371.5 423.0 170.0 176.5
25 25 25 25	50 ↓ ↓ ↓	0 ↓ ↓ ↓	0.5 1 2 4	32 32 32 32	13.9 10.4 6.6 0.22	70.5 52.6 33.3 1.09
25 25 10 10	50 50 25 25	0 ↓ ↓ ↓	1 2 1 .5	64 64 16 32	3.3 .17 18.7 9.6	16.8 .88 94.4 48.6
10 10	25 25	0 0	1 .5	32 64	5.4 3.0	27.6 15.1

TABLE III. - 64-WIRE, 5-RING TRANSMISSION LINES

Ring	Number in ring	r, mm	Coordinates (1st) R, mm	Coordinates (1st) r, deg	Polarity (1st)	Ring polarity
Line 1 (all radii the same #18 AWG)						
1	8	0.51	2.6393	0	+	Alt.
2	8	↓	4.6593	↓	−	↓
3	16	↓	6.6793	↓	+	↓
4	16	↓	8.6993	↓	−	↓
5	16	↓	10.7193	↓	+	↓
Line 2 (five different wire sizes)						
1	8	0.3	2.6393	0	+	Alt.
2	8	.51	4.6593	0	−	Alt.
3a	4	.27	6.6793	0	+	Same
3b	8	.45	↓	22.5	−	Same
3c	4	.51	↓	45	+	Same
4a	4	.45	8.6993	0	−	Same
4b	8	.35	↓	22.5	+	Same
4c	4	.3	↓	45	−	Same
5	16	.51	10.7193	0	+	Alt.

DATA OUTPUT		
	Line 1	Line 2
L, nH/m	15.33	18.70
C, nF/m	1.67	1.37
Z, Ω	3.03	3.70
A, mm^2	26.15	18.95

TABLE IV. - RECOMMENDED WIRE ARRANGEMENTS FOR POWER LINE

Number	Wire size (#AWG)	Total number of wires	Ranges of values inductance, nH/m		Characteristic impedance, Ω		Resistance, Ω	Weight of copper, kg
			High	Low	High	Low		
1	18	80	6.3	10.4	1.2	2.1	0.227	87.8
2	18	52	9.7	16.0	1.8	3.2	.349	57.1
3	21	104	4.8	8.0	0.9	1.6	.342	57.0
4	22	132	3.8	6.3	0.7	1.3	.348	56.8
5	24	208	2.4	4.0	0.5	0.8	.341	56.8
6	24	256	2.0	3.2	0.4	0.7	.277	69.9

TABLE V. - DC AND AC RESISTANCE AND WEIGHT FOR MINIMUM NUMBER OF WIRES PER LINE

[Line Length, 150 m (total length for resistance calculation is 300 m), i.e., wires/direction.]

Number AWG	Minimum number of wires/line	R_{dc}, Ω	R_{ac}, Ω	R_p, Ω*	Weight, kg**
18	26	0.242	0.249	0.349	57.1
21	52	.242	.244	.342	57.0
22	65	.244	.245	.343	56.4
33	833	.244	.244	.342	56.4
36	1670	.244	.244	.342	56.4

*R_p includes proximity effects.
**Weight (copper alone).

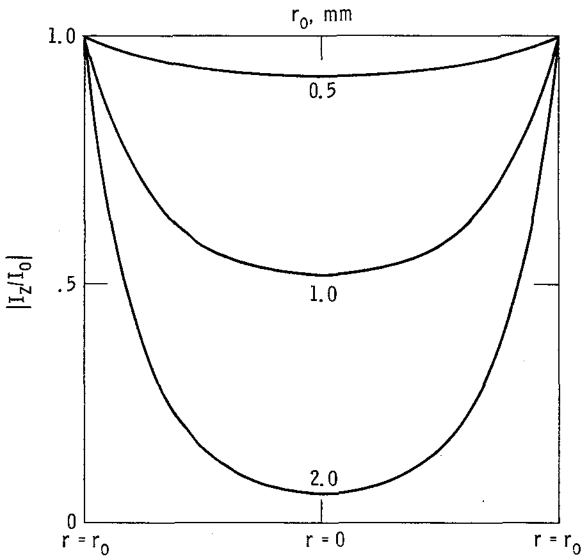


Figure 1. - Current distribution in a circular cylindrical wire of radius r_0 at 20 kHz.

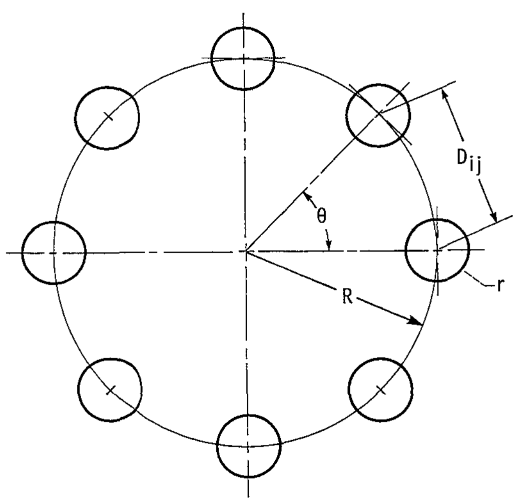


Figure 2. - Polar coordinates for CRIMPD.F, single-ring arrangement.

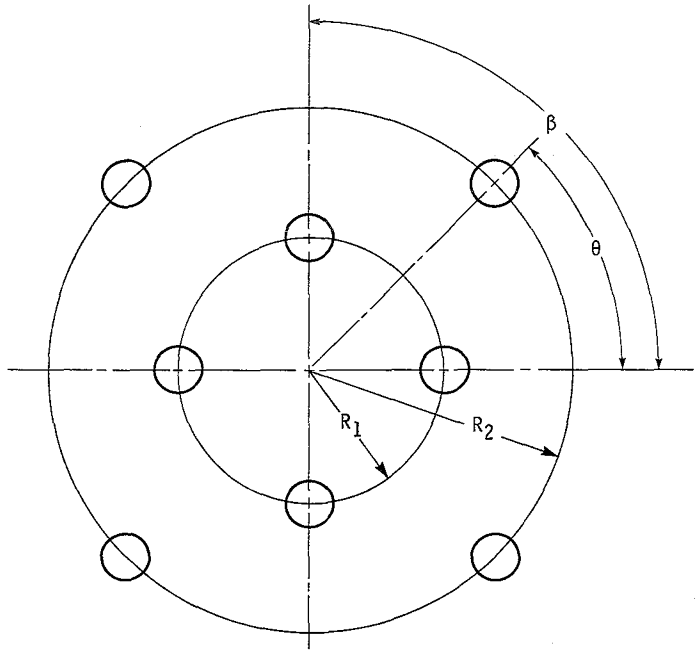


Figure 3. - Polar coordinates for CRIMPD.F, double-ring arrangement.

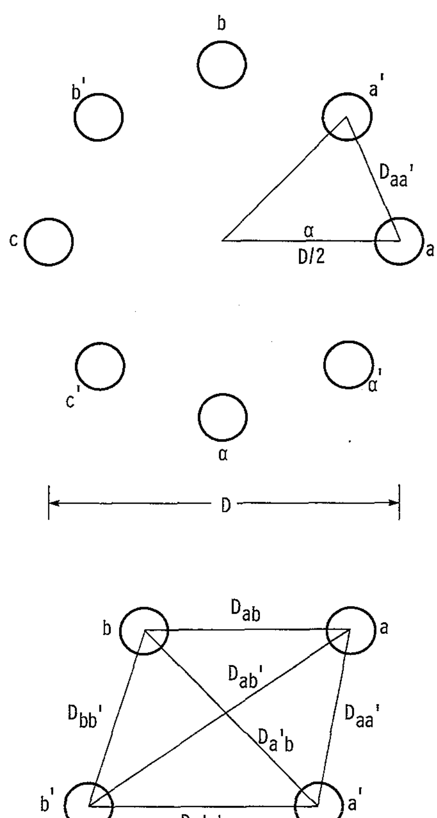


Figure 4. - Parameter definition for equation (5). The direction of currents in the primed conductors is opposite to that in the unprimed.

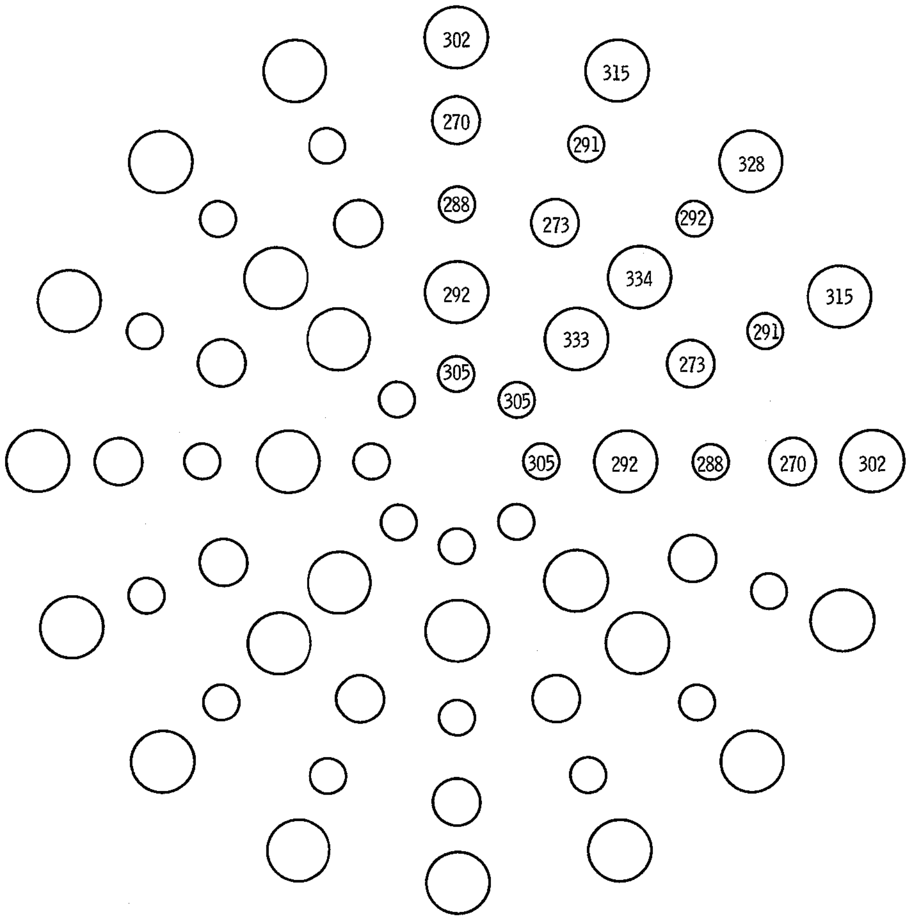


Figure 5. - 64-Wire, 5-ring line 2 (inductance values in nH/m).

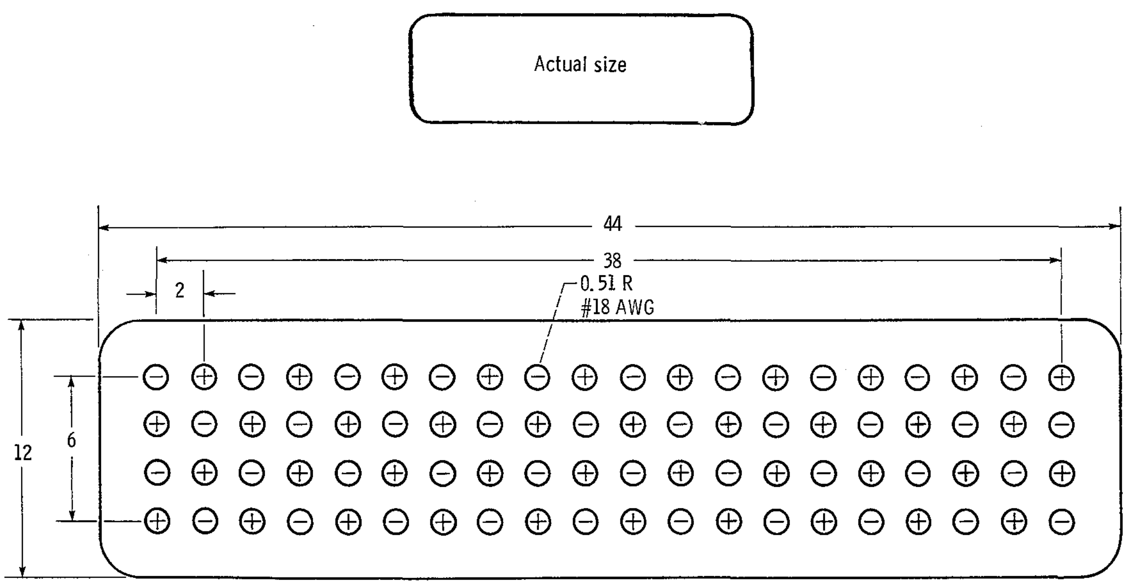


Figure 6. - 80-Wire (4x20) configuration. (All dimensions in inches.) For ε_r = 2.3, L = 10.42 nH/m, C = 2450 pF/m, and Z = 2.06 Ω. Scale: 0.2 inch = 1 mm.

Conditions	Line	x_1, mm	y_1, mm	Polarity	Number of conditions	Spacing, mm	Line polarity
1 to 20	1	0	0	+	20	2	Alt.
21 to 40	2	↓	2	-	↓	↓	↓
41 to 60	3		4	+			
61 to 80	4		6	-			

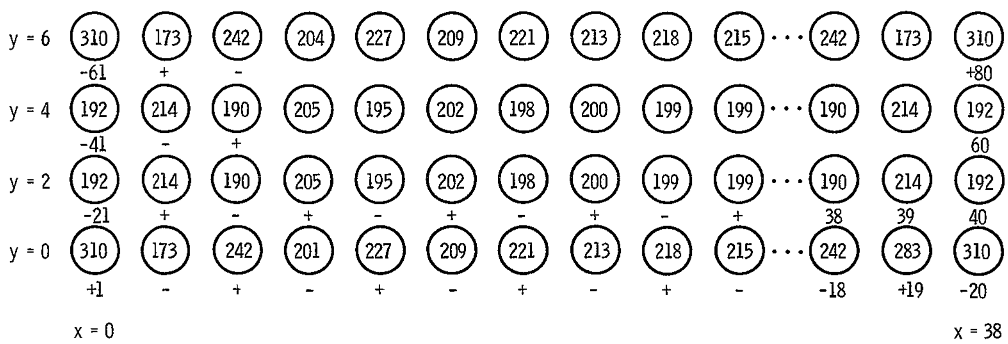


Figure 7. - 80-Wire (4x20) configuration with listing of individual self inductances in nH/m ("Inductance" method). L = 10.4196 nH/m; C = 2.4526 nF/m; Z = 2.0611 Ω; A = 32.6851 mm^2; r = 0.51 mm (#18 AWG).

239 61	93	163	122	148	131	142	135	139	137	. . .	163	93	239 80
113 41	131	107	122	112	118	114	117	116	116	. . .	107	131	113 60
113 21	131	107	122	112	118	114	117	116	116	. . .	107	131	113 40
239 1	93 2	163 3	122 4	148 5	131 6	142 7	135 8	139 9	137 10	. . .	163 18	93 19	239 20

Figure 8. - 80-Wire line ("Capacitance method") individual inductances. L = 6.3115 nH/m; C = 4.0490 nF/m; Z_0 = 1.2485 Ω; A = 32.68 mm^2; r = 0.51 mm (#18 AWG).

Actual size

Condition	x_1, mm	y_1, mm	Spacing, mm	Polarity	Line polarity	Radii, mm
1 to 32	0	0	1.0	+	Alt.	0.25
33 to 64	↓	1.0	↓	-	↓	↓
65 to 96	↓	2.0	↓	+	↓	↓
97 to 128	↓	3.0	↓	-	↓	↓
129 to 160	↓	4.0	↓	+	↓	↓
161 to 192	↓	5.0	↓	-	↓	↓
193 to 224	↓	6.0	↓	+	↓	↓
225 to 256	↓	7.0	↓	-	↓	↓

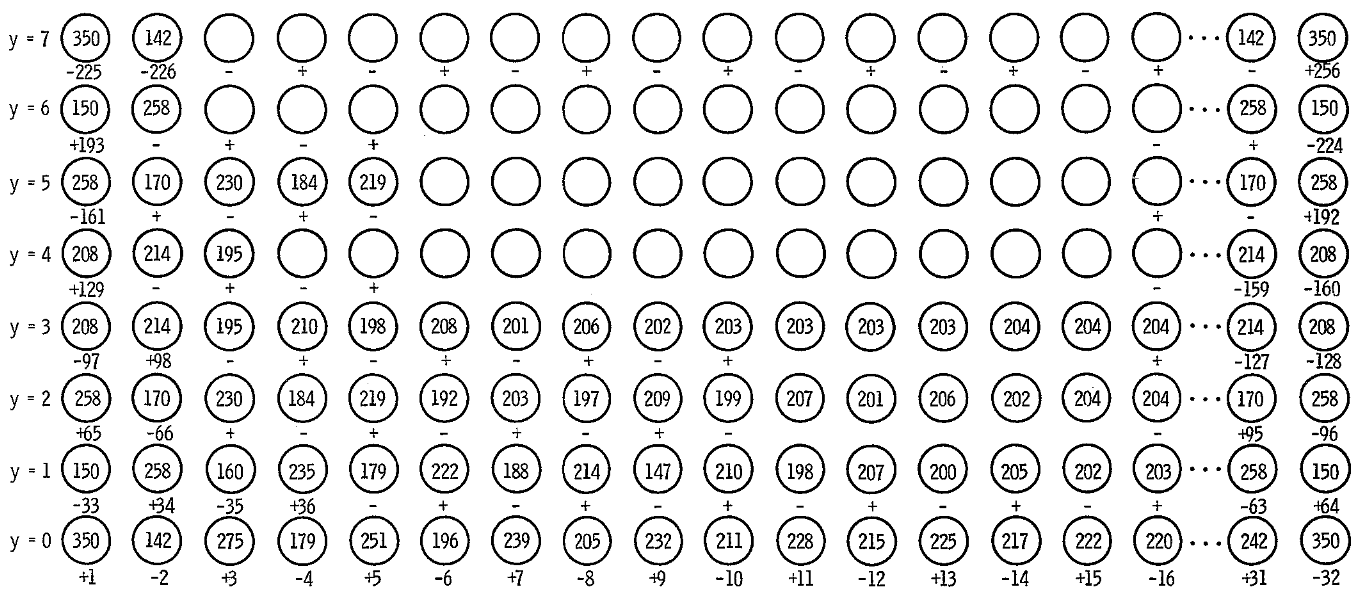


Figure 9. - 256-wire (8x32) arrangement with individual self inductances in nH/m. L = 3.2193 nH/m; C = 7.9382 nF/m; Z = 0.6368 Ω; A = 25.1327 mm^2; r = 0.25 mm; ε_r = 2.3.

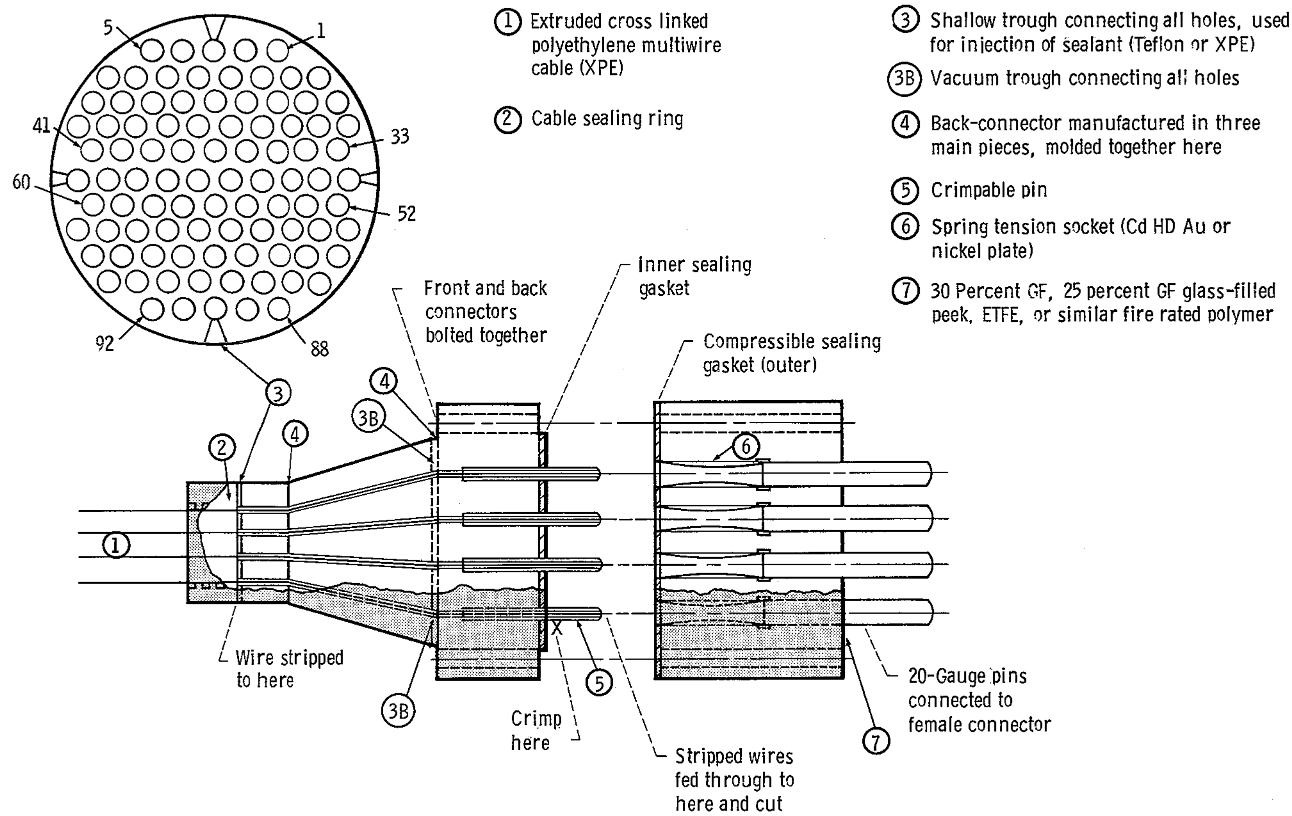
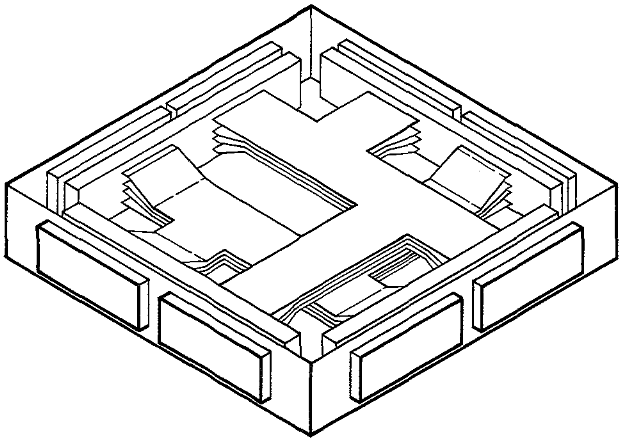
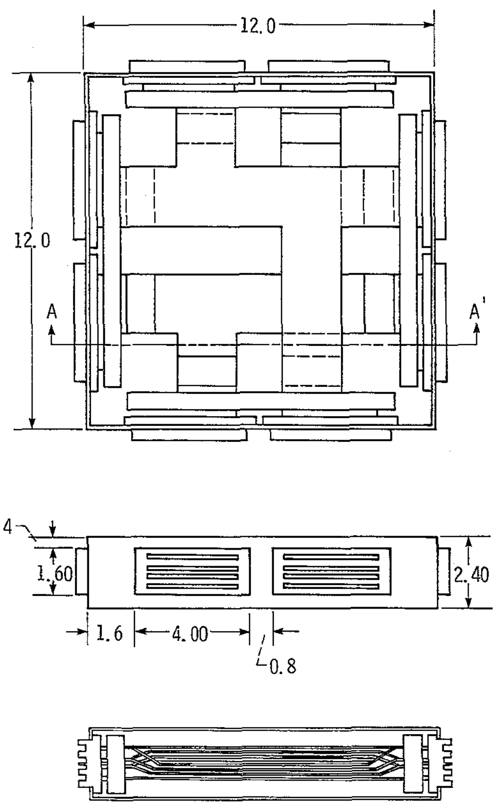


Figure 10. - Male-female junction boxes with conventional outlets.



30° Isometric view

(a) High frequency power distribution buss. Top cover not shown; material: chassis 0.063 Al-Alloy 5053; scale, 1/2.



Section A-A'

(b) Junction-box detail. (All dimensions in inches.)

Figure 11. - Junction box.

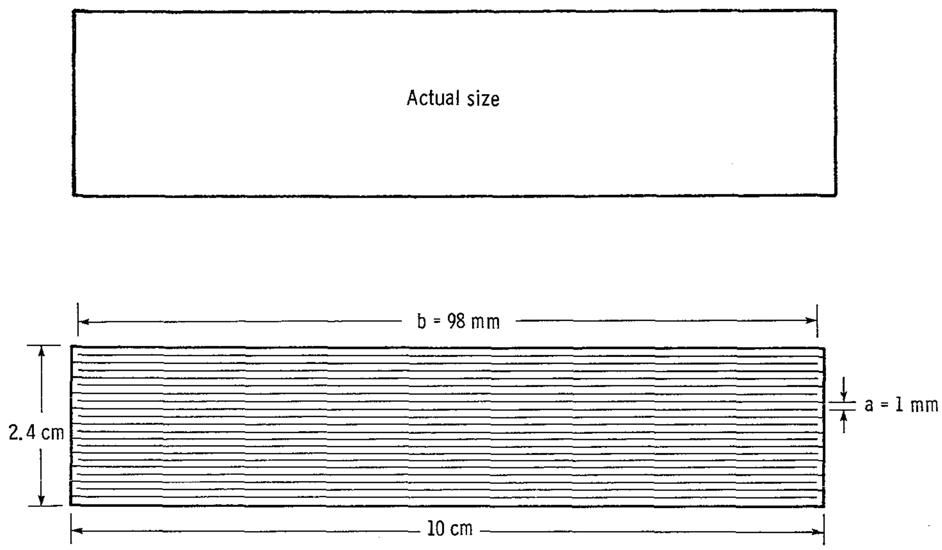


Figure 12. - Cross section of a multiple-plate transmission line. L = 0.66 nH/m; C = 35.3 nF/m; Z_0 = 0.14 Ω; ε_r = 2.1

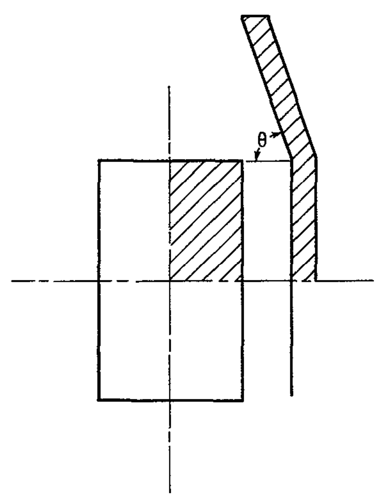


Figure 13. - Stripline configuration.

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15. Supplementary Notes

Final report. Project Manager, Irving Hansen, Power Systems Engineering Division, NASA Lewis Research Center, Cleveland, Ohio 44135.

16. Abstract

A low inductance, low characteristic impedance transmission line was designed for a 20 kHz power distribution system. Several different conductor configurations were considered: strip lines, interdigitated metal ribbons, and standard insulated wires in multiwire configurations (circular and rectangular cylindrical arrangements). The final design was a rectangular arrangement of multiple wires of the same gauge with alternating polarities from wire to wire. This offered the lowest inductance per unit length (on the order of several nanohenries/meter) and the lowest characteristics impedance (on the order of one ohm). Standard multipin connectors with gold-plated elements were recommended with this transmission line; the junction boxes to be internally connected with flat metal ribbons for low inductance; and the line to be constructed in sections of suitable length. Computer programs for the calculation of inductance of multiwire lines and of capacitances of strip lines were developed.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
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