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# MODELING OF ELASTICALLY MOUNTED VERTICAL ROTOR

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The evaluation of the dynamic behavior of a rotating system is possible by means of modal parameters (Eigenvalues and Eigenvectors). A mixed analytical and experimental approach is used to identify the modal parameters of a specially-designed test rig. The modal identification is done both for nonrotating as well as rotating systems. These modal parameters are used to validate a developed Finite Element Model.

## INTRODUCTION

In the design of a rotating turbomachine, there is a great deal of concern about the stability and the level of vibration. Unstable vibration may occur caused by different effects, like internal damping, oil film forces, etc. There is also a big discussion about the number of bearings used to guarantee a safe level of vibration during operation.

There are several difficulties to identify the dynamic characteristic of large turbomachinery. The problem is very complex. Normally there are few points at the rotor where it is possible to excite the system; also the positions of the point to measure the response of the system are not arbitrary. In operating conditions, it is not always easy to identify the system. Turn on and off procedures may be very fast, and in steady state, it is frequently difficult to estimate the disturbing forces.

As an attempt to understand and to analyze specific problems related to vertical shaft Francis Hydrogenerators, a special test rig was designed and constructed, which is described by Eiber (Ref. 1). The test rig is designed in a way that facilitates changing the parameters of the rotor (stiffness, damping, mass distribution). The study of the dynamic behavior of the rig gives important information on how to proceed measuring the real machine.

The stability of a linear rotor-system is given by the real part of the complex eigenvalues which correspond to the damping constants. Natural frequencies are obtained from the imaginary part. Together with the natural modes (Eigenvectors), they allow the evaluation of the dynamic behavior in free as well, as in forced, vibration. Eigenvalues and natural modes are called the "modal parameters" of a system.

The classical "modal analysis," a combined experimental and analytical method which was developed some years ago, is used to identify modal parameters of nonrotating systems. The method makes assumption of symmetric matrices for the analytical model. Although the method is not admissible in the case of rotating systems (nonsymmetrical matrices), it is a very useful tool to identify the modal parameters while the system is on rest.

It is the aim of this paper to identify the modal parameters of the rig in several stages of its mounting on the test stand. In this case, the modal parameters provide us the necessary information to adjust gradually the mathematical model.

It has been shown in Reference 2 that a generalized modal analysis, in connection with an expansion in series of the right eigenvectors and the left eigenvectors, allows the identification of modal parameters of a system. This method will be applied to the rotor test rig. Some experimental data are presented.

## ROTOR TEST RIG

The rotor system is shown schematically in Figure 1. The vertical shaft represents a body with a rigid upper and an elastic lower part. On the rigid section, there is axially clamped the rotor of an asynchronous motor with 4 kw power. The stator itself is hinged with leaf springs on swinging platforms. These leaf springs are adjustable and, therefore, allow us to vary the stiffness in a very wide range. The total suspension of the stator is carried out by eight platforms.

Because of noncontact transducers, an inductive displacement measuring system allows the recording of vibration of the rotating shaft. This system is used in one horizontal plane to measure the state variables of the lower disc.

## MODAL ANALYSIS OF THE UNDAMPED NONROTATING SYSTEM

There are different possibilities in the description of the mentioned rotor with continuous mass and stiffness distribution. The exact formulation is described by partial differential equations together with appropriate boundary conditions, as described by Wauer (Ref. 3).

For practical calculations, however, a Finite Element Model is employed with finite number of coordinates. Basically, the bending vibration of the rotor shaft is the focus of interest. Making use of scalar energy quantities in a variational form, the Hamilton's Principle leads us directly to the equations of motion. The unknown deflection functions are substituted by deflection shapes formed by third-order polynomials with free parameters. In this model, the free parameters are the deflection and the angle at the boundary of the beam elements.

The Finite Element model for the rig is shown in Figure 2. The model has 6 nodes. It consists of 5 beam elements, 4 spring elements, and 6 inertia elements. All the elements which are related to the stator and its spring leafs, and also the upper discs, are condensed at the node number 1. For making the model more flexible for parameter variation, the rotating part of the stator (rotor) and the rotating parts of the intermediate bearing are considered as separate inertia elements. The system is allowed to vibrate in the XY plane and also in the XZ plane. The rotation of the system about X axis, and also its translation in the same direction, is suppressed. As a result, the system has 24 degrees number of freedom. Having the system matrices [M], [K], the solution of the ordinary eigenvalue problem gives the natural frequencies and the corresponding modes.

The modal analysis of the rig is done in three steps. In the first step, the "vertical shaft," a body with rigid upper and an elastic lower part, is considered as a free-free body in space.

In the second step, the rotor is modeled as a free-free body in space. The rotor consists of the vertical shaft, the upper disc, the lower disc, and the intermediate bearing.

Finally, the model considers the rotor mounted on the rigid test stand by leaf springs.

## IDENTIFICATION OF MODAL PARAMETERS OF THE NONROTATING SYSTEM

Usually a combined experimental and analytical method identifying modal parameters of nonrotating elastic systems has been applied in various fields. The aim of the method is to analyze a structure in its elementary modes and to determine its characteristics.

In this case, the first nine measurement points along the X axis of the rotor in the XY plane are chosen. The rotor is excited by applying an impulse on the measurement points, and the response is measured in a specific point. After transformation (FFT) of the input and the output signals into frequency domain, different frequency response functions are determined. Analytical functions are fitted to the measured functions by variation of the modal parameters. The results of this iterative fitting procedure are the modal parameters.

The same as theoretical analysis, identification of modal parameters is done in the three steps as already was explained. The same method was also applied for identifying the modal parameters of the rotor in the XZ plane.

## COMPARISON OF THE RESULTS

The results obtained by the F.E.M. and the identification of modal parameters are presented as corresponds to different stages. Table 1 shows the natural frequencies of the free-free "vertical shaft." The natural modes for the first three frequencies are shown in Figure 3.

Table 2 shows the natural frequencies of the free-free rotor. The corresponding modes are shown in Figure 4.

Figure 5 shows the natural modes of the mounted rotor on XY plane. The corresponding natural frequencies are shown in Table 3. Also the natural frequencies of the mounted rotor on XZ plane are shown in Figure 5. As the natural modes in XZ plane are almost the same as in XY plane, the modes are not presented.

The results obtained for natural frequencies of different stages by Finite Element Model are very close to experimental ones. This gives us a model to be used in predicting the frequencies while the rotor is running.

## MODAL ANALYSIS OF ROTATING SYSTEM

Due to gyroscopic effect and asymmetry in damping and stiffness matrices, the classical method for modal analysis fails to decouple the equations of motion of the rotor while running. The modal parameters are also speed dependent in this case.

However, working with eigenvectors and natural modes of the nonconservative system leads to the desired decoupling as shown by Nordmann in Reference 2.

## MODAL PARAMETERS OF ROTORS

The first step in modal analysis is always the determination of eigenvalues and eigenvectors. These parameters can be calculated by the homogenous equation:

$$[M]\{\ddot{x}\} + [D]\{\dot{x}\} + [K]\{x\} = 0 \tag{1}$$

where [M], [D], [K] are respectively mass, damping, stiffness matrices with order N×N. The solution of equation (1) is of the form:

$$\{x\} = \{\phi\}e^{\lambda t} \tag{2}$$

Substitution yields the quadratic eigenvalue problem

$$(\lambda^2[M] + \lambda[D] + [K])\{\phi\} = 0 \tag{3}$$

with 2N eigenvalues $\lambda j$ and corresponding modes $\{\phi\}j$. The eigenvalue, as well as the eigenvectors, mainly occur in conjugate complex pairs (real eigenvalues and eigenvectors are not considered).

$$\text{Eigenvalues} \quad \lambda j = \alpha j+i\omega j \quad ; \quad \bar{\lambda} j = \alpha j-i\omega j \tag{4}$$

$$\text{Eigenvectors} \quad \phi j = sj+itj \quad ; \quad \bar{\phi} j = sj-itj \tag{5}$$

The part of the solution which belongs to such a conjugate complex pair can be written as:

$$xj(t) = Bje^{\alpha jt}\ \{sj \sin(\omega jt+\partial j)+tj \cos(\omega jt+\partial j)\} \tag{6}$$

where $\omega j$ is the circular frequency and $\alpha j$ the damping constant. The damping constant $\alpha j$ (real part of >j) determines whether the solution xj(t) is stable ($\alpha j<0$) or unstable ($\alpha j>0$).

The constants Bj and the phase angle $\partial j$ depend on the initial conditions. Normally all of the conjugate complex pairs contribute to the solution of the natural vibrations. To explain the natural modes is not so easy as in the case of conservative systems.

The expression in parentheses in Equation (6) can be defined as natural modes representing a time dependent curve in space.

## IDENTIFICATION OF MODAL PARAMETERS OF THE RUNNING ROTOR

The natural frequencies of the rotor are identified by the same method as Reference 2. Some experimental data are shown in Fig. 6. The figure shows the variation of the first three eigenvalues (only the imaginary part), due to rotational speed of the rotor. It is clearly shown that the natural frequencies are speed dependent. It is worth mentioning that the identification process becomes more difficult as the running speed increases. It is necessary to develop special hammers which could excite the rotor at higher speeds.

## REFERENCES

1. Eiber, A. Weber, H.I. "Experiments in Rotordynamic," VII Cobem, December 1983, Uberlandia -- MG, Brasil.

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3. Wauer, J., "Modal Analysis of Non-Conservative Distributed Parameter Rotor Systems. Proceedings of the XIVth Conference on Dynamics of Machines," page 199, September 1983. Prague, Liblice, Czechoslovakia.

The Natural Frequencies of The Free-Free Vertical Shaft

Table 1

| | F1 Hz | F2 Hz | F3 Hz |
|---|---|---|---|
| Finite Element Model | 36.42 | 153.2 | 412.5 |
| Identification Of Modal Parameters | 36.18 | 153.1 | 410.5 |

The Natural Frequencies of The Free-Free Rotor

Table 2

| | F1 Hz | F2 Hz | F3 Hz |
|---|---|---|---|
| Finite Element Model | 14.74 | 51.27 | 92.29 |
| Identification Of Modal Parameters | 14.65 | 49.81 | 97.66 |

The Natural Frequencies of The Mounted Rotor on XY Plane

Table 3

| | F1 Hz | F2 Hz | F3 Hz |
|---|---|---|---|
| Finite Element Model | 8.224 | 14.11 | 27.43 |
| Identification Of Modal Parameters | 8.380 | 13.01 | 22.11 |

The Natural Frequencies of The Mounted Rotor on XZ Plane

Table 4

| | F1 Hz | F2 Hz | F3 Hz |
|---|---|---|---|
| Finite Element Model | 8.199 | 13.34 | 24.71 |
| Identification Of Modal Parameters | 8.508 | 12.27 | 21.87 |

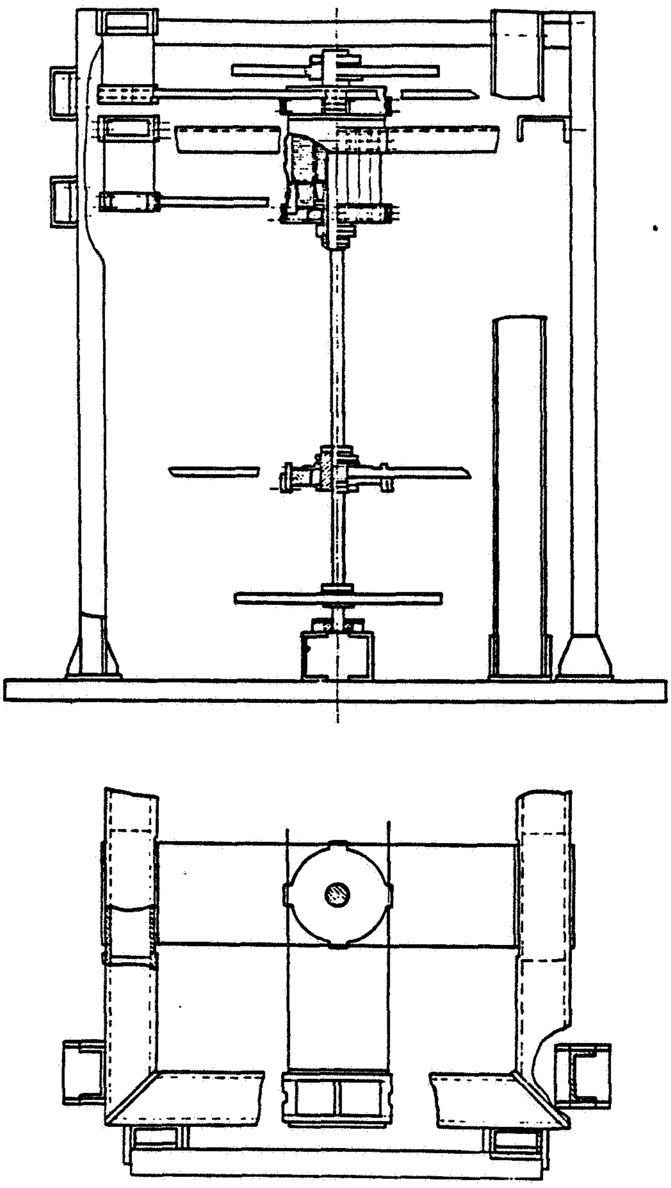

Figure 1. - Test rig (scale 1:20).

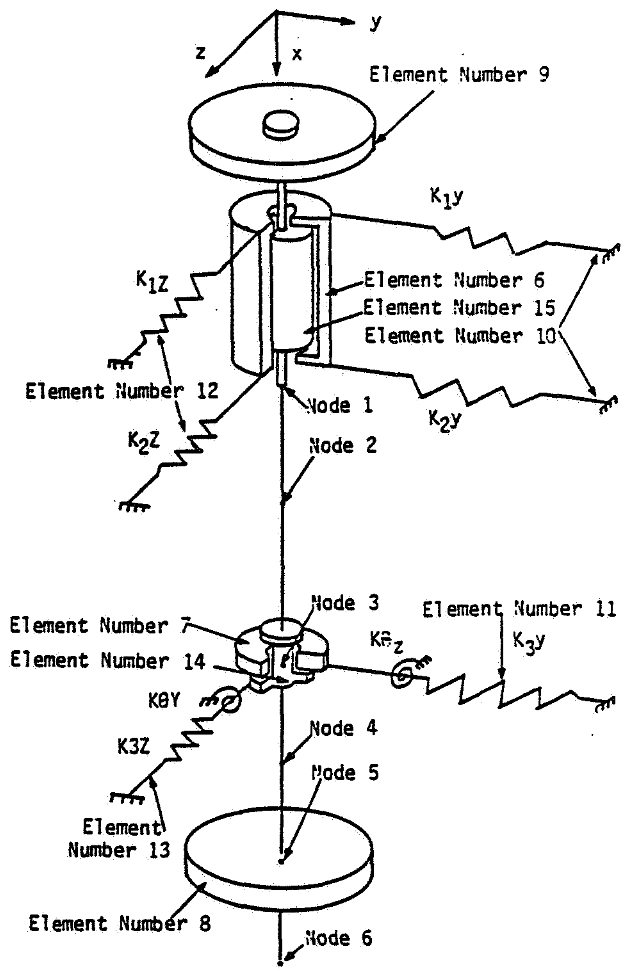


Figure 2. - Finite element model for rig.

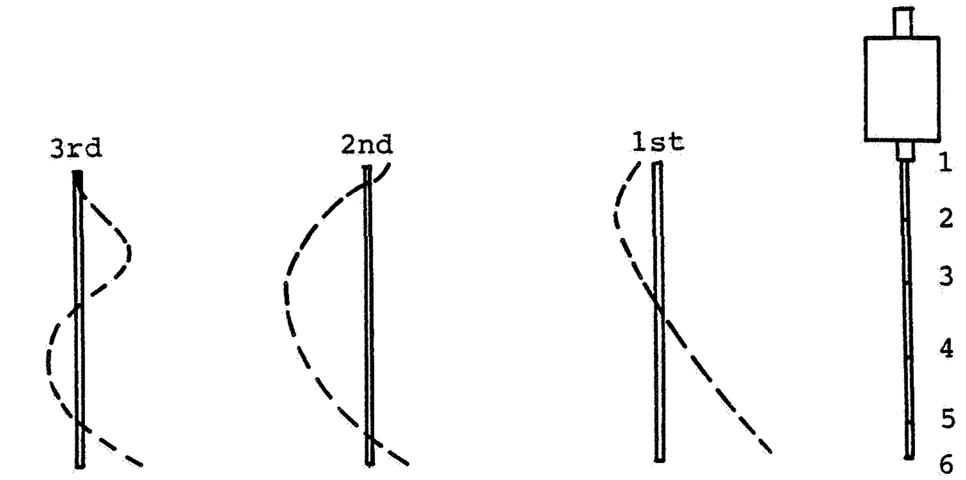


Figure 3. - Natural modes for free-free vertical shaft.

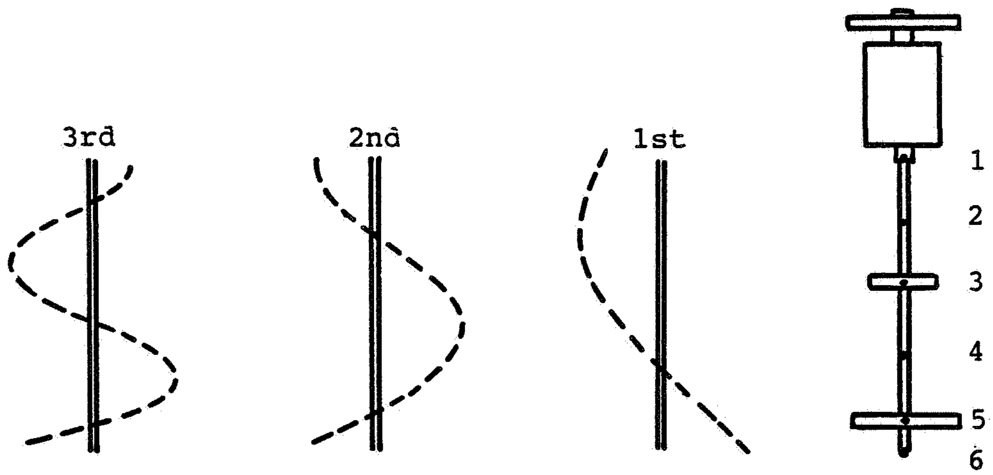


Figure 4. - Natural modes for free-free rotor.

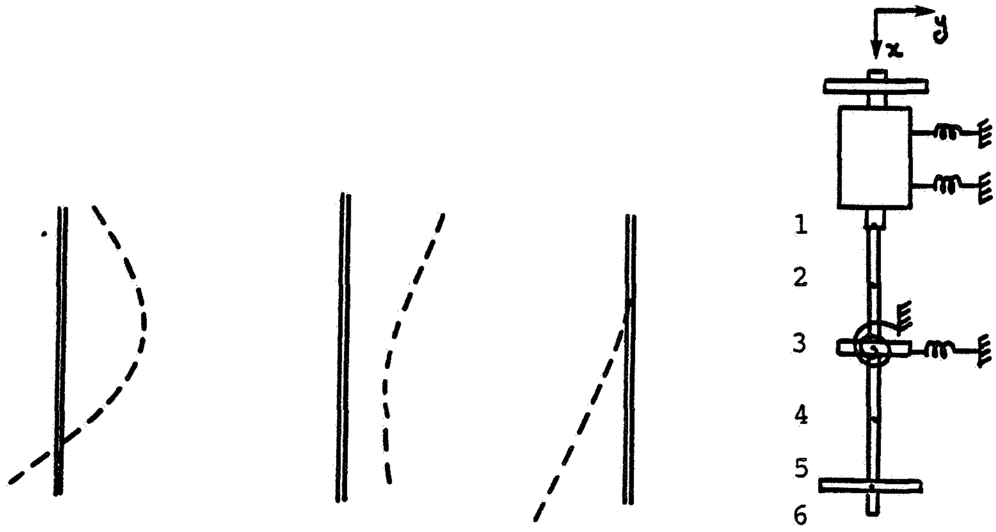


Figure 5. - Natural modes for mounted rotor in XY plane.

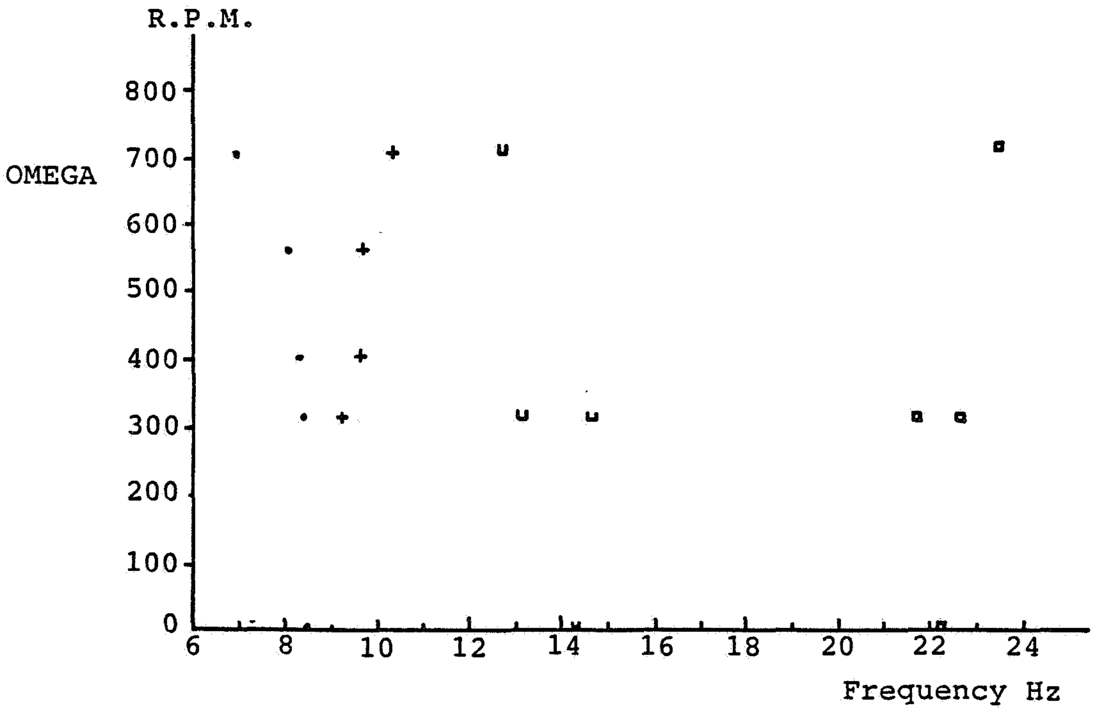


Figure 6. - Variation of eigenvalues of mounted rotor with rotor running speed.