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MAGNETIC BEARINGS FOR VIBRATION CONTROL

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The paper presents a survey on the research of the Institute of Mechanics of the ETH in the field of vibration control with magnetic bearings. It shows a method for modelling an elastic rotor so that it can be represented by a low order model amenable to control techniques, it deals with the control law and spill-over effects, and it discusses experimental results for an active resonance damper.

INTRODUCTION

Magnetic bearings can exert forces on a body without any physical contact. This makes it a very useful device to influence the position and the motion especially of a spinning rotor. Magnetic bearings are quite capable of supporting even a heavy rotor. Its application to turbomachinery, machine tools, and in the vacuum techniques is described for example by Habermann (ref. 1). Such a bearing, or more precisely such a bearing system, can be designed in a systematic way (ref. 2). By suitably designing the control loop the magnetic actuator can be adjusted to a variety of applications and dynamic requirements. Primary design goals have been to support a rigid rotor, because the rigidity of the rotor facilitates the control design essentially. The magnetic forces can be made to be a function of the rotor motion in such a way that the actuator usually has spring and damper characteristics which suitably depend on the excitation frequency.

The freedom in assigning dynamic characteristics to the actuator can be used not only for supporting a rigid rotor but for vibration control as well. Some efforts have already been undertaken to control various kinds of vibrations. Pietruszka and Wagner (ref. 3) show how an unbalanced rigid rotor can be made to rotate about its principal axis of inertia, thus avoiding vibrational unbalance forces on the magnetic bearings. In reference 4 magnetic bearings are used to shield a measuring platform from residual vibrations of its base. Gondhalekar et al. (ref. 5,6) investigate vibration control problems of an elastic rotor. Even the active damping of self-excited vibrations by internal friction and of parametric vibrations due to rotor asymmetry have been looked into. It shows that pragmatic solutions for individual problems can be found, general methods and the answers to some basic questions, however, are not yet available.

This paper will give a survey on our research in the field of vibration control. At first it presents the modelling of an elastic rotor so that the resulting low order model is amenable to control techniques. Another aspect concerns the control law and the effects of spill-over, and finally design requirements and experimental results for an active resonance damper are discussed. The survey is based on the work of the research group of the Institute of Mechanics at the ETH, especially on the contributions of Bucher, Salm and Traxler.

FUNCTIONAL PRINCIPLE

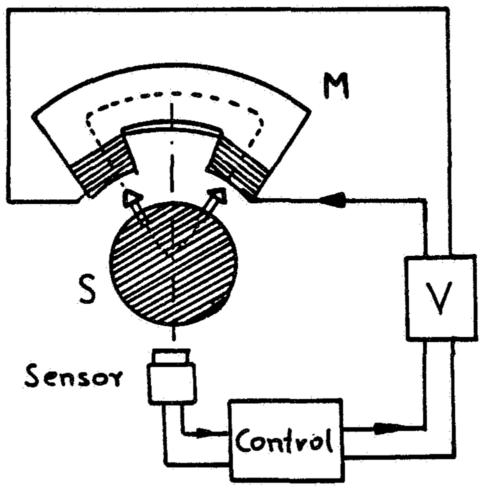


Fig. 1: Magnetic Suspension

At first the functional principle of the magnetic suspension will be explained shortly and demonstrated by examples.

Figure 1 shows a hovering shaft S. Its position is measured by a sensor, the measured signal is processed in a controller, the controller output controls via an amplifier the current through the coils of the magnet M in such a way that the magnetic force stabilizes and firmly keeps the shaft in its hovering state. This simple example shows that obviously the size of the magnetic force depends on the size of the air gap, and that the dynamics of the suspension, its stiffness and damping, to a large extent is determined by the design of the controller.

For suspending a full rotor the simple loop will not be sufficient. Figure 2 shows the radial suspension of a rigid rotor. For each degree of freedom a magnetic actuator has to be controlled independently. In general each bearing force will depend on each sensor signal, leading to a typical multivariable control.

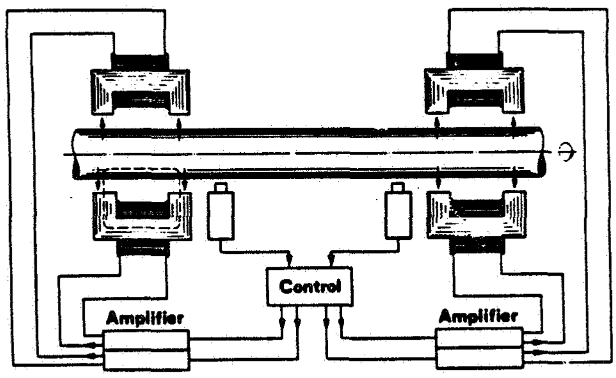


Fig. 2: Block-diagram for the radial suspension of a rotor in one plane

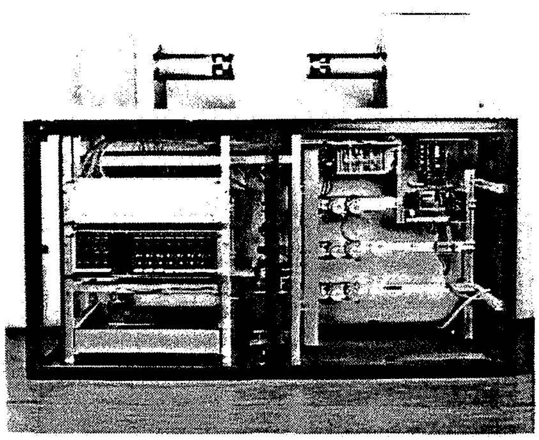


Fig. 3: Magnetic bearing system, front view with control unit, power supply and drive

Figure 3 shows the hardware setup for such a bearing system. The rotor has a length of about 1 m, a diameter of 12 cm and a mass of 12 kg. The air gap of 1 cm is extremely large for technical purposes. The device was used for an exposition. The signals of the optical CCD-sensors are processed by a microprocessor and fed to switched power amplifiers.

MODELLING OF AN ELASTIC ROTOR

For most of the current applications the rotor is considered to be rigid. The derivation of its equations of motion under the action of magnetic control forces shouldn't cause any difficulties. For an elastic structure the modelling procedure is somewhat more complex, as will be outlined in the following.

The model of the elastic rotor has to be such that it can be incorporated into the model of the closed control loop, and it has to be of low order so that the complexity of designing the control law will be reduced and a simulation of the closed loop becomes reasonable.

For the derivation of such a model, following Bucher (ref. 7), let us start with a finite element model of the elastic rotor

(1) $$\underline{A}_2\, \underline{\ddot{q}} + \underline{A}_1\, \underline{\dot{q}} + \underline{A}_o\, \underline{q} = \underline{p}$$

where $\underline{q}$ is a n x 1 vector of generalized displacements, $\underline{p}$ is the vector of generalized forces, and the $\underline{A}_i$ are structural coefficient matrices, characterizing inertia and elasticity as well as gyroscopic and nonconservative properties. Equation 1 may be transformed into the complex frequency domain and expressed by

(2) $$\left[s^2\, \underline{A}_2 + s\, \underline{A}_1 + \underline{A}_o\right] \underline{Q}(s) = \underline{P}(s)$$

The matrix in brackets is termed dynamical stiffness matrix and its inverse is the dynamical flexibility matrix

(3) $$\underline{H}(s) = \left[s^2\underline{A}_2 + s\underline{A}_1 + \underline{A}_o\right]^{-1}$$

The elements of $\underline{H}(s)$, the dynamical flexibility transfer functions, are well-known in measurement and modal analysis techniques. A general way of reducing the large set of finite element equations is to truncate the modal representation of equation 1. In order to find that representation we have to solve the eigenvalue problem of equation 1. For the sake of simplicity - all the subsequent derivations can be extended to the general system (equ. 1) as well (ref. 7) - let us assume a simple elastic structure

(4) $$\underline{M}_o\ddot{\underline{q}} + \underline{K}_o\underline{q} = \underline{p}$$

From the solution of the eigenvalue problem we obtain a set of eigenvalues $\omega_1,\ldots\omega_n$ and the corresponding real normalized eigenvectors $\underline{u}_k$ with the modal matrix $\underline{U} = (\underline{u}_1,\ldots,\underline{u}_k,\ldots,\underline{u}_n)$.

Then the modal expansion of $\underline{q}$

(5) $$\underline{q} = \sum_k \underline{u}_k z_k = \underline{U}\,\underline{z}\ , \qquad k = 1,\ldots,n$$

leads to the modal representation of equation 4

(6) $$\underline{M}\,\ddot{\underline{z}} + \underline{K}\,\underline{z} = \underline{U}^T\underline{p}$$

with $\underline{M} = \underline{U}^T\,\underline{M}_o\,\underline{U} = \underline{\operatorname{diag}}(m_k)$, $\underline{K} = \underline{U}^T\,\underline{K}_o\,\underline{U} = \underline{\operatorname{diag}}(k_k)$, $\omega_k^2 = k_k/m_k$

This modal representation is a time domain representation of the partial fraction expansion of the corresponding flexibility matrix $\underline{H}(s)$. Translating equations 5 and 6 into the s-domain, yields

(7) $$\underline{H}(s) = \underline{U}(\underline{M}\,s^2 + \underline{K})^{-1}\,\underline{U}^T = \sum_{k=1}^{n} \frac{\underline{u}_k\,\underline{u}_k^T}{m_k(s^2+\omega_k^2)}$$

For the design of the controller, a model of the flexible structure, that relates the displacements and velocities at the location of the sensors to the excitation forces of the actuators, is required. By analyzing the closed loop system with respect to the actuator and sensor coupling points, it can be checked whether the design objectives have been reached, i.e., whether the system is stable and shows up with good damping performance. Also, test measurements for the closed loop system are easily carried out by exciting the structure with the available actuators. Hence, for both control design and closed loop analysis, we may confine the selection of coordinates to the coupling points of the actuators and sensors with the structure. A modal representation of the dynamical flexibility matrix of the elastic structure, with respect to the cited coordinates, provides the desired description, as will be shown subsequently.

The n_A actuator forces form the vector $\underline{w}$, the n_D measured displacements are given by $\underline{y}_D$, and the n_V measured velocities by $\underline{y}_V$. These vectors are related by appropriate incidence matrices to the nodal forces and displacements by

(8) $$\underline{w} = \underline{T}_A^T \underline{p} \qquad \underline{y}_D = \underline{T}_D \underline{q} \qquad \underline{y}_v = \underline{T}_v \dot{\underline{q}} \qquad \underline{y}^T = (\underline{y}_D^T, \underline{y}_v^T)$$

The elements of the incidence matrices are zero or one, when discrete actuators and sensors are located at nodal points. We assume that we can truncate the number of modes retaining only m essential modes, so that instead of equation 5 we now have

(9) $$\underline{q} \approx \sum_i \underline{u}_i z_i = \underline{U}_m \underline{z}_m , \quad i = 1,\ldots,m , \quad m < n$$

with the truncated n x m modal matrix $\underline{U}_m$ and the truncated modal vector $\underline{z}_m$. This leads to the truncated set of modal equations

(10) $$\underline{M}_m \ddot{\underline{z}}_m + \underline{K}_m \underline{z}_m = \underline{U}_m^T \underline{p}$$

With the subset of retained coordinates (equ. 8) we have in the s-domain a transfer matrix representation

(11) $$\underline{Y}(s) = \underline{H}_G \underline{W}(s) \qquad \underline{H}_G = \begin{bmatrix} \underline{T}_D \underline{U}_m (s^2 \underline{M}_m + \underline{K}_m)^{-1} \underline{U}_m^T \underline{T}_A \\ s \underline{T}_v \underline{U}_m (s^2 \underline{M}_m + \underline{K}_m)^{-1} \underline{U}_m^T \underline{T}_A \end{bmatrix}$$

Here the transfer matrix $\underline{H}_G$ is a generalized dynamical flexibility matrix, since it relates a force excitation (actuator forces) to nodal displacements and velocities (sensor signals).

By introducing the state vector

$$\underline{x}^T = (\underline{z}_m^T , \dot{\underline{z}}_m^T)$$

an equivalent state space representation is obtained

(12) $$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{w}$$
$$\underline{y} = \underline{C}\,\underline{x}$$

$$\underline{A} = \begin{bmatrix} \underline{0} & \underline{E} \\ -\underline{M}_m^{-1} \underline{K}_m & \underline{0} \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} 0 \\ \underline{U}_m^T \underline{T}_A \end{bmatrix} \qquad \underline{C} = \begin{bmatrix} \underline{T}_D \underline{U}_m & 0 \\ 0 & \underline{T}_v \underline{U}_m \end{bmatrix}$$

All of the representations (equ. 10,11,12) will be suitable for the control design. The method can be extended to include other structural elements as well, i.e. foundation dynamics, by applying the building block approach. Problems left over and to be discussed in the next section are that of truncation and the control design.

CONTROL OF AN ELASTIC ROTOR

When the control design is based on a reduced-order model of the rotor (equ. 10) the question arises whether the real rotor will indeed be controlled correctly by such a simplified controller. Actually there may be detrimental effects, and we can explain and classify them in the following way (ref. 6).

Let us partition the high-dimensional modal coordinate vector $\underline{z}$ into two parts. The important part with the low dimension n_m will be the one to be controlled, the other residual part of dimension n_r is the one to be neglected. Then equation 6 and the measurement equations 8 can be arranged in the following form

$$\text{(13)} \quad \begin{bmatrix} \underline{M}_m & \underline{0} \\ \underline{0} & \underline{M}_r \end{bmatrix} \begin{bmatrix} \ddot{\underline{z}}_m \\ \ddot{\underline{z}}_r \end{bmatrix} + \begin{bmatrix} \underline{K}_m & \underline{0} \\ \underline{0} & \underline{K}_r \end{bmatrix} \begin{bmatrix} \underline{z}_m \\ \underline{z}_r \end{bmatrix} = \begin{bmatrix} \underline{U}_m^T \\ \underline{U}_r^T \end{bmatrix} \underline{p}$$

$$\underline{y}_D = \underline{T}_D \underline{U}_m \underline{z}_m + \underline{T}_D \underline{U}_r \underline{z}_r , \qquad \underline{y}_v = \underline{T}_v \underline{U}_m \dot{\underline{z}}_m + \underline{T}_v \underline{U}_r \dot{\underline{z}}_r$$

For design purposes it will be assumed that the controller is based on the reduced-order model only, being a subset of equations 13

$$\text{(14)} \quad \underline{M}_m \ddot{\underline{z}}_m + \underline{K}_m \underline{z}_m = \underline{U}_m^T \underline{p} = \underline{U}_m^T \underline{T}_A \underline{w}$$

$$\underline{y}_{Dm} = \underline{T}_D \underline{U}_m \underline{z}_m , \qquad \underline{y}_{vm} = \underline{T}_v \underline{U}_m \dot{\underline{z}}_m$$

The relation between the control vector $\underline{w}$ and the measurement follows from the chosen control law, which preferably will be a linear one and can be determined in the usual ways. Such a control design will result in the desired and "good" control for the reduced-order model (equ.14), but when it is applied to the real full-order-system (equ. 13) the system qualities can alter essentially (fig. 4). In reality the measurements $\underline{y}$ do not only consist of the modelled part $\underline{z}_m$ but they also depend on the residual vector $\underline{z}_r$, causing the socalled "observation spillover".

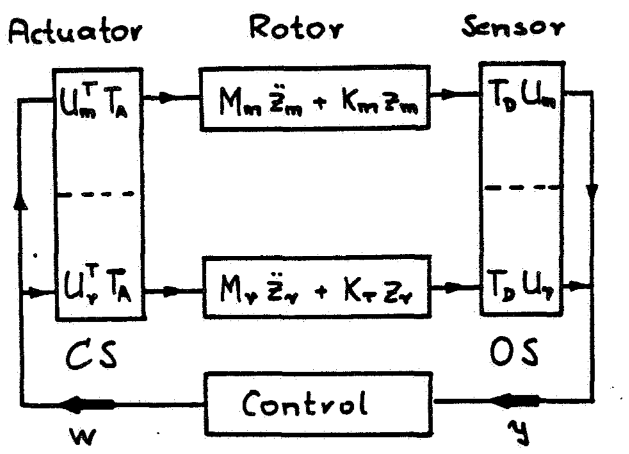


Fig. 4: Reduced order control applied to the real full-order system, demonstrating the observation spillover (OS) and the control spillover (CS)

Furthermore the control vector $\underline{w}$ does not only act on the modelled part but on the real full-order system itself which obviously contains the unmodelled part as well. This influence is called "control spillover". These spillover terms can change and deteriorate the behaviour of the real system and even destabilize it.

The objective of the control design now is to derive the control on the basis of the reduced-order system without exact knowledge about the residual, unmodelled part, and to produce a design that provides suspension and damping for the mechanical system and that is easy to implement as well.

A direct output feedback is known to have these properties and to show robustness qualities with respect to parameter errors and truncation effects (ref. 8). Then the control forces depend linearly on the displacement and velocity measurements

(15) $$\underline{w} = - \underline{G}_D \underline{y}_D - \underline{G}_v \underline{y}_v$$

with the yet unknown gain matrices $\underline{G}_i$. Introducing this control law and the measurement equations 8 into the full-order system equations 6 leads to the full-order closed-loop system (FOCL)

(16) $$\underline{M}\,\ddot{\underline{z}} + \underline{U}^T \underline{T}_A \underline{G}_v \underline{T}_v \underline{U}\,\dot{\underline{z}} + (\underline{K} + \underline{U}^T \underline{T}_A \underline{G}_D \underline{T}_D \underline{U})\underline{z} = \underline{0}$$

In order to avoid the spillover effects the solutions of equation 16 should be stable. Then the FOCL is stable despite the fact that its gain matrices will be based on the reduced-order model. The stability conditions, derived from definiteness properties of the coefficient matrices (ref. 6), are certainly fullfilled,

- if the sensors and actuators are arranged in collocated pairs, i.e. if $\underline{T}_A^T = \underline{T}_D = \underline{T}_v$. And when the actuators have to support the rotor additionally to controlling its vibrations the number of reasonably placed sensors and actuators has to be at least equal to the number of not stable rigid body modes.
- if the gain matrices $\underline{G}_i$ are symmetric and positive definite.

The procedure how to construct such gain matrices will be explained shortly. The reduced-order model, including the rigid-body modes, and all other vibration modes that should be controlled, is given by equation 14. Introducing the control equation

$$\underline{w} = -\underline{G}_D \underline{y}_{Dm} - \underline{G}_v \underline{y}_{vm}$$

into equation 14 leads to the closed-loop equation of the reduced-order model

(17) $$\underline{M}_m \ddot{\underline{z}} + \underline{D}_m \dot{\underline{z}} + (\underline{K}_m + \underline{S}_m)\underline{z} = 0$$

(18) $$\underline{D}_m = \underline{U}_m^T \underline{T}_A \underline{G}_v \underline{T}_v \underline{U}_m , \qquad \underline{S}_m = \underline{U}_m^T \underline{T}_A \underline{G}_D \underline{T}_D \underline{U}_m$$

The terms $\underline{D}_m$, $\underline{S}_m$ are the additional modal damping and stiffness introduced by the control feedback. The required symmetry and definiteness properties of the yet unknown gains $\underline{G}_i$ will be guaranteed by a design approach of Salm (ref. 6). First we assign desired additional modal damping and stiffness values and thereby choose the diagonal matrices $\underline{D}_m$ and $\underline{S}_m$, and then we solve equations 18 for the unknown gains. However, as most often the number of n_A, n_D of actuators and sensors will be smaller than the number m of modelled modes, it is not possible to solve equation 18 uniquely. An approximation for the gains $\underline{G}_i$, in the sense of minimized quadratic differences using Pseudo-Inverses, is feasible and leads to the following design rules:

The reduced-order control of large, and even infinite dimensional vibration systems will result in a robust and certainly stable FOCL, if

- the vibration system is not unstable (rigid body modes are allowed),
- the reduced-order model contains at least all rigid-body modes,
- the sensors and actuators are collocated and their number is at least equal to the number of rigid-body modes.

This control approach includes the technically most interesting case where a real flexible rotor is suspended by actuators which at the same time have to control its elastic vibrations. An example and experimental results will be shown in the next section.

EXAMPLES AND EXPERIMENTAL RESULTS

Active suspension and vibration control of a flexible beam (ref. 6). The transverse vibrations of a flexible beam, supported and controlled by actuators on both ends, are described by the modal equation 6. The mode shapes (equ. 5) used in this case are the free-free modes, the rigid body modes are included in the modal transformation matrix $\underline{U}$.

As a design goal the two actuators will have to control the two rigid-body modes, and additionally two elastic modes should be strongly damped, too. Therefore the reduced-order model is chosen to consist of the two rigid-body modes and the first two elastic modes. Displacement and velocity signals are coming from sensors at the actuator locations. The gain matrices $\underline{G}_i$ for the direct-output control are derived from equation 18, where in our case the assigned modal stiffness and damping, that we want to obtain, is characterized by

$$\underline{S}_m = \underline{\text{Diag}}\ \omega_{mi}^2 \qquad \underline{D}_m = \underline{\text{Diag}}\ (2D\ \omega_{mi})$$

The stiffness of the suspension was assigned by prescribing ω_1, ω_2. The damping value D was varied between 0 and 1. Several general and anticipated results could be corroborated numerically:

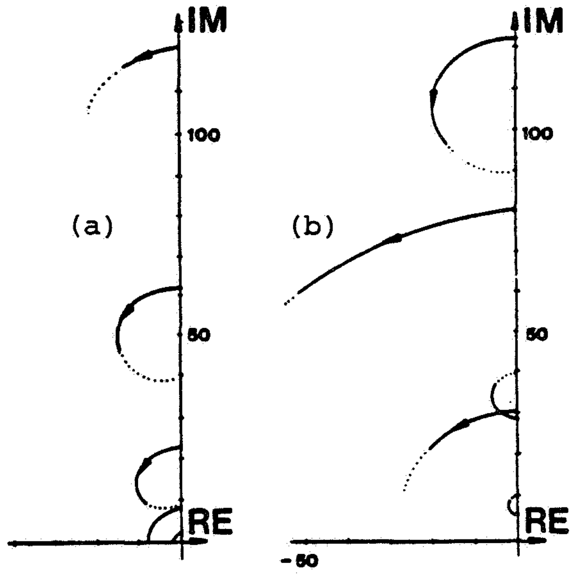


Fig. 5: Typical eigenvalue-curves of the controlled full-order system with increasing damping values D assigned to the modes

- For "low" bearing stiffness (Fig. 5a) the eigenvalues of the controlled system have a typical behaviour when we increase the assigned damping value D. For very high damping the modal frequencies of a fixed-fixed beam are approximated.
- For a "stiff" bearing design (Fig. 5b), where we want the bearing stiffness to be near the first elastic frequency of the beam, the behaviour is similar, but some eigenvalues now go upwards to higher eigenfrequencies of the fixed-fixed beam. The

poor controllability of these modes comes from the fact that high control forces tend to "clamp" the beam, and to shift modal nodes into the actuator location itself.

- Limiting values for the attainable stiffness are given by the lowest frequencies of an equivalent beam with fixed ends.
- The control design always produces a stable system. It provides stability and damping for the modelled and the unmodelled modes.

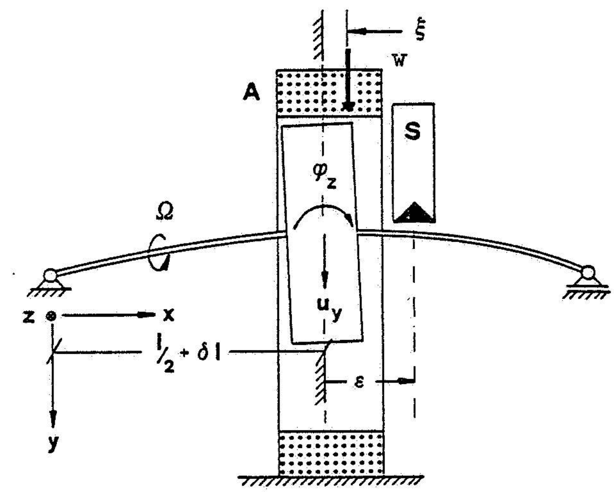


Fig. 6: Rotor model, demonstrating the parameter deviations ε,ξ and δl from a reference position

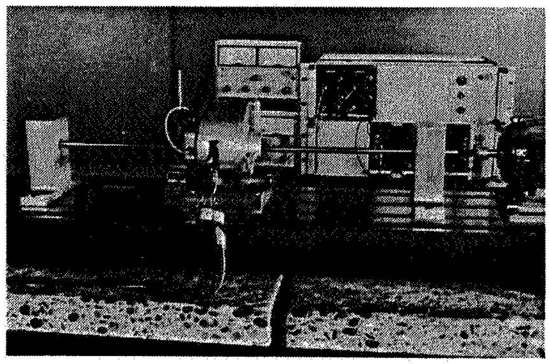


Fig. 7: Experimental setup

Vibration damping and spill-over effects.
A simple rotor model (Fig. 6) demonstrates the active damping of resonance vibrations (ref. 6). The experimental setup (fig. 7) consists of a conventionally supported, flexible rotor shaft with a disc in the "middle" of the shaft. The "exact" model for the rotor of figure 6 considers the imperfection parameters ε,ξ,δl. Now the control design is based on a reduced-order model, taking only the displacements u_y, u_z into account. The motion of the "exact" rotor, however, will be affected by the coupling between translation and inclination of the disc. It is the imperfection parameter ε that causes observation spillover, and ξ provides control spillover. The eigenvalue shifts caused by these spillover effects are demonstrated in figure 8. The unmodelled modes can even become unstable. When actuator and sensor are collocated (ε = ξ) then the "exact" model is stable even with the "simple" control. The calculations have been confirmed by experiments. Near the stability boundary however the above modelling was not sufficient: the small imperfections due to the phase/frequency response of the sensor had to be taken into account (ref. 7).

When the elastic rotor and the control are modelled according to the above rules, the active damper can reach good performance. Measurement results (fig. 9) demonstrate the considerable reduction of the resonance amplitudes for the experimental setup.

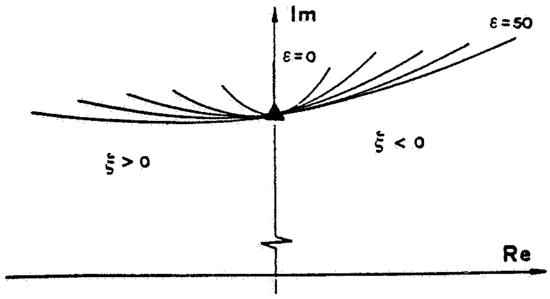
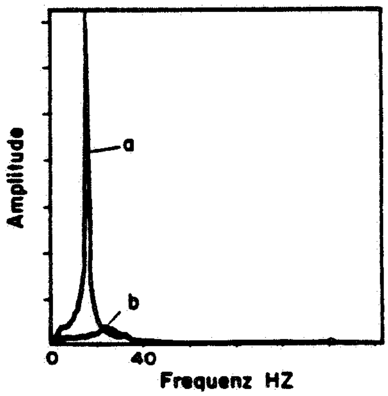


Fig. 8: Eigenvalue shift of the unmodelled modes depending on the parameters ε and ξ. Without control the eigenvalue is at ▲.



CONCLUSION

In order to control vibrations in flexible mechanical systems and to solve special suspension tasks it can be advantageous to use magnetic actuators. For the design of the multivariable control loop a suitable model of the mechanical system is derived. Starting from a FE-model of high order the reduced order model finally only retains the actuator and sensor coordinates and the modes considered to be important. In the control design the spillover aspects are discussed. A method for determining a control based on the reduced-order model and robust with respect to the number of unmodelled modes is demonstrated. Examples show the active suspension of a flexible beam, spillover effects, and measured resonance curves for an actively damped rotor running through its critical speed.

Fig. 9: Measured amplitude/frequency response, (a) uncontrolled, (b) controlled

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