# SPACE INFRARED TELESCOPE POINTING CONTROL SYSTEM 

Automated Star Pattern Recognition<br>(NASA-CR-176911) SPACE INFREFED TELESCOPE N86-30608<br>POINTING CONTKOL SYSTEM. ADTONAIED STAR<br>PATTERN EECOGNITICN Final Report :Stanford<br><br>submitted to<br>NASA AMES RESEARCH CENTER Moffett Field, Calif. 94035<br>by<br>J. David Powell, Principle Investigator Roel W.H. Van Bezooijen, Research Assistant

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# SPACE INFRARED TELESCOPE POINTING SYSTEM: AUTOMATED STAR PATTERN RECOGNITION 

## 1. INTRODUCTION

The Space Infrared Telescope Facility (SIRTF), depicted in Fig. 1, will be a free flying spacecraft carrying a 1 meter class cryogenically cooled infrared telescope nearly three orders of magnitude more sensitive than the current generation of infrared telescopes. This improvement in performance is primarily due to state-of-the-art instrumentation at the focal plane, a very precise pointing and control system, and the effectiveness of the cryogenic cooling system which keeps the entire telescope at a temperature below $10^{\circ} \mathrm{K}$. SIRTF needs to be pointed with an accuracy of 1 arcsec, while the image at the focal plane is to remain stable to 0.1 arcsec.

The primary attitude data will be provided by a star tracker (STR), integrated into the detector bay of the telescope. It will have a 15 arcmin diameter field of view (FOV), an accuracy of .1 arcsec, and will be capable of processing up to 20 stars simultaneously. The small FOV will require the tracker to be sufficiently sensitive so that stars with a visual magnitude $m_{v}$ up to 14.5 can be used. It will be possible to select guide and tracking stars from a catalog that is being generated for use with the Space Telescope which will contain stars up to $m_{0}=15$ and is expected to be 90 to $95 \%$ complete at that magnitude.


FIGURE 1 SPACE INFRARED TELESCOPE FACILITY (SIRTF)

The basic attitude information that is required for pointing control about all three axes can be obtained by tracking two or more stars inside the star tracker FOV. These stars need to have a large enough mutual angular separation to allow computation of the angle about the boresight with sufficient precision. The attitude accuracy can be enhanced by using additional stars. Simultaneous tracking of more than one star has become possible with the use of Charge Coupled Device (CCD) or Charge Injection Device (CID) type detectors (Salomon and others, 1981; Kollodge and Sand, 1983).

For space astronomy missions where continuous ground contact is not available, it is necessary to automate the process by which the tracking stars are acquired by the star tracker. Even if continuous ground contact would allow the acquisition to be performed manually by using a computer generated image of a large enough area of sky around the target and the image of the part of that area that is observed by the star tracker, automation would still be preferred. This is because an automated scheme can perform the recognition extremely reliably and virtually instantaneously, thereby maximizing the time available for scientific data taking. It is questionable whether a human operator could even perform star recognition at all in a situation where the attitude uncertainty is large compared to the size of the FOV of the star tracker.

Star identification can be performed on the basis of just one star, or a group of stars. 'The planetary spacecraft used Canopus as a single guide star that, by virtue of its brightness, was unique within the region of sky searched. The

Netherlands Astronomical Satellite (ANS) (Christis, 1974) is an example of recognition based upon the use of star pairs, while star triplets are planned to be used for the Galileo Mission (Wong and Lai, 1981). Singley (1984) describes the use of fuzzy clustering algorithms to identify large clusters of stars.

The problem of automated star pattern recognition is rather similar to that of recognizing two dimensional industrial parts, where known local features of the parts, such as holes and corners are used for the identification process (Bolles and Cain, 1982). The location of these features relative to each other is fixed for each of the parts as is true for the stars in a given group. The signature of each feature, such as the diameter of a hole, is analogous to the magnitude of the stars. Partial obscuration of a part due to another part is analogous to the unobservability of guide stars that may be caused by such factors as excessive star brightness errors, non-optimal selection of the brightness threshold of the STR, or the use of non-existing guide stars.

In this report, three automatic target acquisition methods will be presented that are all based on the use of an imaging star tracker. Although these methods have general applicability, they are designed for implementation on SIRTF. The three methods are distinguished by the number of guidestars that are required per target, the amount of computational capability necessary, and the time required for the complete acquisition process. The simplest method is based on using a guide star which is close ( $10^{\circ}$ or less for SIRTF) to the target and is unique by virtue of its brightness. The process consists of slewing to the area of
the sky where the guide star is expected, searching until it enters the FOV, calibrating the attitude about two axes, followed by a slew to the target where the tracking stars are acquired. This method is called the "Lone Star Method." Having to select and store only one guide star, in addition to the tracking stars, is one advantage of the method. Another important advantage is that the search for the bright star can be carried out without stopping and thus the search pattern can be carried out rather quickly.

The "Few Star Method" has the advantage of typically requiring only three guide stars per target, which are to be located inside the FOV when it is centered on the target. Actually, in most cases these guide stars would also be used as tracking stars that have to be selected anyway. This method carries with it a time penalty arising from the need to search an area that is sufficiently large to include the guide stars. This search can, depending upon the pointing error, be quite lengthy because the dim guide stars require a search pattern which stops at each new addition to the search area.

The "Many Star Method" eliminates the lost time spent searching for the guide $\operatorname{star}(\mathrm{s})$ by storing all stars that may be encountered after the initial (but inaccurate) slew to the target. This enables the recognition scheme to determine the exact attitude at the conclusion of the slew and then to step directly to the desired target without. a search. Depending on the attitude uncertainty, this method can require the selection and storage of a large number of guide stars per target.

## 2. STAR PATTERN UNIQUENESS

Successful target acquisition requires the unambiguous identification of a small group of "recognition stars" located inside a larger region of the sky that will be referred to as the "uniqueness area." Regardless of the acquisition method, the recognition stars are defined to be those guide stars that are observed by the STR. A necessary condition for the unambiguous recognition is that the pattern defined by the recognition stars is unique within the uniqueness area.

For the Many Star Method (MSM), this implies that the pattern of those guide stars that happen to be located inside the STR FOV, is unique among all the patterns that can be formed by the guidestars that were selected within the pre-defined uniqueness area. For the Few Star Method (FSM), it means that the pattern of the few guide stars has to be unique among all the patterns that can be formed by the stars observed within the search area. In the case of the Lone Star Method (LSM), uniqueness means that the single guide star is unique to the area where a search is carried out to find it.

If the brightness and position of the stars could be both known and measured to an extremely high accuracy, a pattern consisting of just two stars would have a high probability of being unique within the entire sky. In reality, the position of stars is known rather precisely, but the uncertainty in brightness can be quite large. For stars up to a visual magnitude $m_{v}=15$, as in the case of SIRTF and the Space Telescope, the standard deviation in the position and
brightness is expected to be respectively between .5 and 1 arcsec. and .3 magnitude (Werner, 1983).

The onboard recognition algorithm (the "recognizer"), can use the geometry of star patterns, the brightness of the stars in the pattern, and knowledge about the maximum attitude error about the boresight as information for the identification of the recognition stars. The recognizer has to allow for sufficient tolerance in relative star positions and brightness to account not only for the uncertainty in knowledge of star position and brightness, but also for the anticipated STR measurement errors of those quantities. If these tolerances have been selected to be too close, the danger exists that the recognition star pattern will not be accepted by the recognizer. On the other hand, tolerances that are too large will reduce the uniqueness probability of the recognition star pattern. If it is known that the error about the boresight is limited to a certain value, as will normally be the case for small pointing errors, the recognizer can use this knowledge and impose a rotational constraint on the star pattern.

Intuitively, one can see that the uniqueness probability of a recognition star pattern increases with the number of stars in the pattern. This is due to the fact that the number of independent parameters necessary to define the pattern does increase. The number of parameters necessary will be called the "Uniqueness Dimension" of the pattern. The uniqueness dimension has geometric and brightness contributions. If it is a priori known that the rotation about the boresight is limited, then there is an additional rotational contribution.

In Table 1, the uniqueness dimension and the elements that comprise it are shown as a function of the number of recognition stars in the pattern. For two stars, the geometric contribution consists simply of the angular distance between the stars or its equivalent. By adding a sign convention, e.g., positive is going from star 1 to star 2 , two stars do define a three axis coordinate system in which two coordinates specify the relative direction of additional stars. Hence, the geometric contribution for a pattern consisting of three stars is made up of a sign convention, one coordinate to specify the direction of star 2 and two coordinates to define the direction of star 3.

Table 1. Uniqueness Dimension

| No. of <br> Stars | Position <br> Sign <br> Coord. | Brightn. | Rotation | Uniqueness <br> Dimension |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 2 | 0 | 1 | 2 | 1 | 4 |
| 3 | 1 | 3 | 3 | 1 | 4 |
| 4 | 1 | 5 | 4 | 1 | 11 |
| k | 1 | $2 k-3^{\circ}$ | k | 1 | $3 k-1 *$ |

```
* for k> 1
** for k > 2
```

Since, in general, the position of the stars is much better known that the brightness, the weight of the position coordinates is much higher than that of the brightness values. However, for patterns comprising less than 4 stars, the bright-
ness and rotation contributions can be quite important due to the small number of position coordinates.

When there are $k$ recognition stars among a total of $n$ guide stars, or $n$ observed stars, as in the case of the FSM, the probability of the pattern being unique may be approximated by:

$$
\begin{equation*}
p_{u}=\left(1-p_{s}(k)\right)^{n k} \tag{1}
\end{equation*}
$$

Here, $p_{s}(k)$ is the probability of a chance match of the pattern of recognition stars with a pattern of guide stars for the case where the pattern of recognition stars is translated in the sky to a position where a randomly chosen star of the pattern coincides with a randomly chosen guide star, and the pattern is allowed to assume any orientation permitted by the rotational constraint of the recognizer. If the uniqueness area is large relative to the STR FOV, the chance match probability may be approximated by the following equations:

$$
\begin{align*}
& p_{s}(k)=\left(n_{g} a_{t g}(k)\right)^{k-1}  \tag{2a}\\
& p_{s}(k)=p_{b}^{k}\left(n_{g} a_{t g}(k)\right)^{k-1}  \tag{2b}\\
& p_{\theta}(k)=p_{b}^{k}\left(n_{g} a_{t g r}(k)\right)^{k-1} \tag{2c}
\end{align*}
$$

respectively, for the cases where the recognizer uses geometry only, geometry plus brightness, and geometry, brightness and rotation as constraints. In equations (2), $n_{g}$ is the mean number of guide stars per STR FOV, $a_{t g}(k)$ is the mean value of the areas in the sky that are created by the position tolerance used by
the recognizer ("tolerance areas") in units of STR FOV area, $a_{\text {tgr }}(k)$ is the equivalent if a rotational constraint is imposed, and $p_{b}$ the probability that the brightness of a randomly chosen recognition star matches the brightness of a randomly chosen guide star. The brightness match probability may be approximated by:

$$
\begin{equation*}
p_{b}=1-10^{-\varepsilon t w_{m}} \tag{3}
\end{equation*}
$$

where $w_{m}$ is the tolerance used by the recognizer for the brightness in terms of magnitude, and $s l$ is the slope of the $10 \log$ value of integrated star density (stars per degree) versus star magnitude. When there are only 2 recognition stars and no rotational constraint is imposed, equations (1) and (2) must be modified to account for the perfect correlation of the two tolerance areas involved.

Using SIRTF parameters, the uniqueness probability has been calculated as a function of the size of the uniqueness area and the number of recognition stars in the pattern. The result is shown in Fig. 2. A guide star density of 6.7 stars per FOV has been used, which gives a $99 \%$ probability of having at least 2 guide stars within the FOV assuming a Poisson distribution. The tolerance on angular distance between stars was set at 5 arcsec while the brightness tolerance was 1 magnitude. A worst case slope of the star density versus magnitude, equivalent to $s l=.5$, was assumed which corresponds to pointing towards the galactic center.

For small values of the uniqueness area, the uniqueness probabilities, as
FIGURE 2
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UNIQUENESS AREA (STAR TRACKER FOV) $\rightarrow$
shown in Fig. 2 are actually too low, due to the fact that the "edge effect" of the uniqueness area has not been accounted for. This implies that a star tolerance area is always assumed to be located inside the uniqueness area, something which is not always true, especially not for guide stars that are located near the edge of the uniqueness area.

Even though the probability of a (chance) brightness match is .68 , using the brightness constraint does help as is shown in Fig. 2. When, in addition to the geometric and brightness constraints, a rotational constraint is imposed by the recognizer, the uniqueness probability can improve markedly. This is shown in Table 2, where for a uniqueness area 25 times the STR FOV area, the uniqueness probabilities are given for the three recognizer imposed constraint situations. A rotational constraint of 30 arcmin was used.

Table 2. Uniqueness Probability for SIRTF

| Number of <br> Recogn. <br> Stars | Constraints imposed by Recognizer <br> Geometry <br> only |  <br> brightn. | Geom., brightn. <br> \&rotat. |
| :---: | :---: | :---: | :---: |
|  | .000 | .000 | .793 |
| 2 | .113 | .499 | .999 |
| 3 | .982 | .996 | .999 |
| 4 |  |  |  |

One advantage of the Lone Star Method (LSM) is that the guide star may be selected using a currently available computer accessible star catalog, such as the SAO catalog. This catalog is complete to a visual magnitude of 8 . Uniqueness of
the guide star means that, within the area that may be searched for it, the lone recognition star needs to be unique only by virtue of its brightness. Using the catalog, it is necessary to verify this uniqueness when selecting the lone star.

The probability of finding a unique bright star depends on the size of the uniqueness area and the required brightness margin between the recognition star and other stars in the uniqueness area. In addition, it depends on the rate of change of the integrated star density as function of the star magnitude. The brightness or magnitude margin $(\Delta m)$ is necessary to enable the brightness threshold to be set low enough to insure that the lone star will be observable and high enough to prevent other stars in the uniqueness area to exceed it.

Using SIRTF parameters, the required value of $\Delta m$ is shown in Fig. 3 as a function of the uniqueness probability for three different values of the standard deviation of the star magnitude uncertainty. The uniqueness area was taken to be 90 times the size of the STR FOV, and a star density was assumed that can be expected when pointing toward the galactic center. Because the rate of change of the star density is highest in this region, it represents the worst case uniqueness situation. However, since the star density in this region is also maximal, it turns out that the probability of finding a suitable lone star in a given area around the target is about the same as near the galactic poles where the density is minimal.
 LONE

## 3. ACQUISITION METHODS

In this section, the three acquisition methods will be described in more detail. The situation for the MSM is illustrated in Fig. 4. Because of a post-slew pointing error, the STR FOV is not centered on the target; however, it is located within the pre-defined uniqueness area. In this area a sufficient number of guide stars ("+" in Fig. 4) has been selected to insure a high enough probability to have the desired number of guidestars within the STR FOV, regardless of where the FOV "lands." In the example in Fig. 4, the pointing uncertainty is assumed to be one-dimensional, and the STR FOV is circular. Consequently, the uniqueness area is circular with a radius equal to the sum of the pointing error tolerance $r_{e}$ and the FOV radius $r_{f}$.

The stars that are observed by the STR ("o" in Fig. 4) are also shown in figure. Because of the uncertainty of star brightness, the brightness or magnitude threshold of the STR has to be set-sufficiently conservatively to insure that even the faintest guide star will be observable. As a result, the observed stars are, in general, a superset of the guide stars that are located inside the FOV. The incompleteness of the catalog from which the guide stars have been selected is another reason for observing additional stars. Due to either catalog unreliability or large brightness errors, not all of the guide stars inside the FOV may be observable, as illustrated in Fig. 4. In the example of Fig. 4, there are 2 recognition stars.


A successful recognition requires that at least the following four conditions be satisfied:

1. At least 2 recognition stars need to be present.
2. At least 2 of the recognition stars need to have errors in both position and brightness that are less than the tolerance used by the recognizer.
3. If a rotational constraint is used, then the attitude error about the boresight needs to be less than the tolerance used by the recognizer.
4. If a pattern consisting of $k$ recognition stars does meet the aforementioned requirements, then chance matches of star patterns comprising $k$ or more observed stars will not be possible.

In an idealized situation, where the star catalog is both $100 \%$ complete and reliable, and it is possible to choose the brightness threshold such that only the guide stars will exceed it, and, in addition, the tolerances used by the recognizer always exceed the errors, then above set of conditions reduces to: (1) there should be at least two guide stars in the STR FOV, and (2) the pattern of guide stars inside the FOV should be unique within the uniqueness area.

This idealization is useful because it allows for an easy determination of the upper boundary of the MSM recognition performance. Using this idealization and assuming a perfect recognizer as well, the success probability of recognition is given by:

$$
\begin{equation*}
p_{r b}=\sum_{i=2}^{n_{l}} p_{p}(k) * p_{u}(k)+p_{u}\left(n_{l}\right)\left(1-\sum_{i=0}^{i=n_{l}} p_{p}(k)\right) \tag{4}
\end{equation*}
$$

where $p_{p}(k)$ is the probability of having $k$ guide stars within the FOV,
$p_{u}(k)$ is the probability that the pattern of these $k$ recognition stars is unique, and $n_{l}$ is the maximum number of observed stars that can be processed by the STR. A preliminary value of $n_{l}$ for the SIRTF STR is 20 . If the uniqueness area is large relative to the STR FOV, $p_{p}(k)$, which is a binomial distribution, may be approximated by a Poisson distribution.

Applying the same parameters that were used in Section 2 to determine the pattern uniqueness, assuming a Poisson distribution for $p_{p}(k)$, and using equation (1) for $p_{u}(k)$, the recognition success probabilities were calculated. For uniqueness areas of respectively $10,100,1000$, and 10000 times the FOV area, the success probabilities were found to be $.93, .89, .85$, and .79 when geometry only was used, and $.95, .90, .89$, and .82 when brightness was used by the recognizer in addition. Because of the neglect of the edge effect of the uniqueness area, and the assumption of a Poissonean distribution of the number of recognition stars, these numbers are too conservative when the uniqueness area is small. For a uniqueness area of 25 , the success probability, which is .91 when geometry only is used, climbs to .93 when brightness is used in addition, and equals .98 when adding a 30 arcmin rotational constraint.

A robust, high-performance recognition algorithm has been designed for the MSM. It includes the following steps:

1. Formation of all possible pairs of guide stars having a mutual distance less than the STR FOV diameter and greater than a minimum distance dictated by the STR spatial resolution.
2. Formation of all pairs of observed stars having a mutual angular distance in excess of the above-mentioned minimum.
3. Application of the following criteria to determine which group of guide star pairs match each pair of observed stars:

- Matching pairs have a difference in star distance of less than the distance tolerance used by the recognizer.
- Matching pairs have a difference in orientation about the boresight of less than the rotation tolerance used by the recognizer.
- Matching pairs have stars that do not differ more in brightness than allowed by the recognizer (magnitude tolerance).

4. Generation of a "confirmation matrix" using the information of the matching pairs. The rows of this matrix correspond to the guide stars ( $g s(1)-g s(n)$ ), and the columns correspond to the observed stars (os (1)-os ( $m$ )). The value of each element of the matrix represents the number of times the associated observed and guide stars were found to match. For example, when an observed star pair consisting of os (i) and os $(j)$ matches a guide star pair consisting of $g s(k)$ and $g s(l)$, then this is a "confirmation" that the observed star $i$ is the same as guide star $k$, and observed star $j$ the same as guide star $l$. This confirmation is quantized by incrementing the values of matrix elements $k, i$ and $l, j$ by 1 .
5. Elimination of all possible matches of guide and observed stars, as indicated by a nonzero value in the confirmation matrix, that are likely to be spurious. This is done by determining the largest matrix element and setting to zero all matrix elements that are less than an a priori percentage of the maximum confirmation value.
6. Determination of the pointing error that is associated with each of the remaining matches.
7. Assumption that, if the majority of those derived pointing errors do agree, taking into account the effect of a rotational error, the stars that belong to that majority are the recognition stars.

When the pointing errors are relatively small, as may be expected for

SIRTF, this method works very well. However, for larger errors, the probability of possible matches with more than one group of guide stars will increase. In addition, the number of guide star pairs can become excessive, as it increases roughly with the third power of the characteristic dimension of the uncertainty area, thus requiring a large onboard computational capability.

The former problem can be solved by modifying step 4 , where the increment of the matrix element is not just 1 , but depends on the perceived quality of the match that is based on the distance difference, the rotational angle, the differences in brightness, and the inferred pointing error. Modification of steps 6 and 7 to allow for more than one match and addition of a step to determine the most likely match is another, more complicated, solution. When the number of guide stars becomes too large to handle, the uniqueness area needs to be divided into a number of smaller areas that, for a circular FOV, have overlaps equal to the FOV diameter.

The technique of using star or feature pairs as an element in the recognition process is frequently applied (Strikwerda and Junkins, 1981; Wong and Lai, 1981; Bolles and Cain, 1982). Based on the fact that the FOV has the highest probability of landing in the center region of the uniqueness area, it is possible to reduce the number of guide stars without affecting the success probability. The way to do this is by increasing the guide star density in the center and decreasing the density toward the edge of the uniqueness area. Increasing the effective FOV by stepping the FOV a few times to observe a larger area is another way to
reduce the number of required guide stars at the cost of increased acquisition time.

The necessity to select and store a large number of guide stars per target is a disadvantage of the MSM. This problem can be avoided by using the FSM, where only a few guide stars close to the target need to be selected, as is shown in Fig. 5. The idea of the FSM is to effectively increase the STR FOV until it includes the guide stars. The "hexagon stepping method," which is shown in Fig. 5, is a good way to increase the effective FOV because it minimizes overlap. After each step, the number of stored observed stars is incremented, and a star recognition procedure is executed.

The recognition method is the same as for the MSM, with the roles of the observed and guide stars reversed. However, for equal sizes of the search area and the uniqueness area, the number of FSM observed stars will be greater than the number of MSM guide stars. This is because of the magnitude margin required for the brightness threshold. Also, the attitude error that is built up when stepping causes the position errors of the observed stars to be greater than those of the MSM guide stars, which will adversely affect the recognition performance. In addition, special precautions are required to avoid double images in overlap regions.

The time required for the search is the major disadvantage of the FSM. The number of steps required to cover an area that consists of $i$ layers of hexagons, arranged around the "start hexagon" is equal to $6 \sum_{k=1}^{i} k$, where $i$ is the number

of layers. In Fig. 6, the number of search steps is shown as function of the radius of the circle that can be inscribed in the search area. Assuming SIRTF actuators, which can provide an angular acceleration of $7 \times 10^{-4} \mathrm{rad} / \mathrm{sec}^{2}$ (Werner and Lorell, 1982), a negligible settling time, and a required integration time of 1 second, the anticipated search time for SIRTF is also shown in Fig. 6.

The main disadvantages of the MSM and the FSM, respectively, are the large number of guide stars required and the large time penalty incurred. The LSM, which is sketched in Fig. 7, requires, in addition to the tracking stars, only one bright star per target and has a much smaller time penalty associated with it than the FSM. The process consists of slewing to the area of the sky where the guide star is expected, searching until it enters the FOV, calibrating the attitude about two axes, and then following with a slew to the target where the tracking stars are acquired.

An efficient search pattern, for the case where the pointing uncertainty is one-dimensional, is generated by controlling the spacecraft such that the boresight describes a sequence of semicircles that are alternately centered on the initial position or on a point at a distance equal to the FOV radius from the start location. The speed at which this search can be carried out is limited by the torqueing capability of the control system, the requirement of sufficient signal generation by the lone star, and the desire to have the lone star within the FOV during at least one STR sampling interval.

LONE STAR METHOD


* TRACKING STARS
PREVIOUS TARGET

For SIRTF, a scan speed of 10 arcmin per second is found to be a very conservative maximum, if lone stars up to $m_{v}=7$ are considered. Since a sampling interval of less than .1 seconds will be easily feasible, the image will travel less than 1 arcmin between samples, thus allowing a search pattern without the need for any substantial overlap. Assuming actuators capable of providing angular accelerations of up to $7 \times 10^{-4} \mathrm{rad} / \mathrm{sec}^{2}$, the time required to search to a certain distance from the starting point is shown in Fig. 8, for SIRTF. Up to the sixth semicircle, the scan speed is constrained by the torque limitations of the actuators.

If a quasi-spiral search is used, as described above, and a post-slew pointing error up to $r_{e}$ is considered, it follows that the lone star needs to be unique within a circular area (the "uniqueness area") with a radius equal to $2\left(r_{e}+r_{f}\right)$. Given the integrated star density versus $m_{v}$ and the necessary magnitude margin $\Delta m$, and assuming that only stars with a magnitude up to $8-\Delta m$ are considered as lone star candidates, the required size of the area of the sky around the target to insure a given probability of containing a lone star candidate can be determined as a function of the radius of the uniqueness area.

This "lone star area" is shown in Fig. 9 as a function of the radius of the uniqueness area, for three values of $\Delta m$. The relations are shown for a confidence level of $99 \%$ at two very different sky locations, the galactic polar regions and the region of the galactic center. For small uniqueness areas, the higher density of bright stars at the galactic center outweighs the disadvantage of



a lower probability of being unique, and as a result, it is easier to find lone star candidates at the this region than at the polar regions. For large uniqueness areas, this is no longer true.

The criterion for uniqueness on which Fig. 9 is based does account for the magnitude margin $\Delta m$, but it neglects the value of the magnitude uncertainty, which, as shown in Fig. 3, is also an important parameter. Considering the distribution of lone star candidates associated with having a $99 \%$ probability of having at least one lone star candidate, it may be seen that a uniqueness probability of, e.g., .9 for each of the candidates is sufficient to keep the probability of a unique lone star at $98.5 \%$.

As an example, consider the situation where observations of the galactic center region are required, and it is known that the standard deviation of the brightness is . 5 magnitude, while the uniqueness area is less than 4.4 square degrees. Then from Fig. 3, to obtain a .8 probability of uniqueness for each lone star candidate requires a magnitude margin of 2. Next, from Fig. 9, using a uniqueness area radius of $71 \operatorname{arcmin}$, a lone star area of 200 square degrees is required for a $98.5 \%$ probability of finding a suitable lone star. For a circular area, this means that there is a $\mathbf{9 8 . 5 \%}$ probability for the slew angle to be less than $8^{\circ}$. For SIRTF, the slew time is equal to $10 \sqrt{\alpha}$, where $\alpha$ is the slew angle in degrees. Therefore, a 28 second slew time would be required for an $8^{\circ}$ distance between the lone star and the target.

Not being capable of calibrating the attitude about the boresight can be a serious disadvantage of the LSM. If it is necessary to arrive at the target with a pointing error less than, e.g., $10 \%$ of the FOV diameter, then in the case of SIRTF this would require the error about the boresight to be less than 8.5 arc$\min$, if the lone star is $10^{\circ}$ from the target. If the product of the error about the boresight and the angle between the target and the lone star is too large, then either a second bright star could be used or, after the slew to the target, a search for the tracking stars could be carried out as for the FSM.

The possibility that, due to the distance between the target and the lone star, the lone star may not have entered its observation window, while the target has, is an additional problem associated with the method.

## 4. MONTE CARLO SIMULATIONS

A computer program was created to enable evaluation of the performance of various star pattern recognition algorithms by means of Monte Carlo simulation. Since this type of simulation is relatively straightforward, it allows accurate determination of the recognition performance as a function of the many parameters defining the sky, the star catalog, the STR, and the recognition strategy. In addition, the simulator is an excellent tool for the verification of subsequent analytical methods.

While the simulator has been designed for the evaluation of the MSM, the FSM is sufficiently similar to the MSM to allow deduction of its performance from the MSM results. The LSM is simple enough to permit prediction of its performance by means of analysis.

The required amount of computation time, about 5 minutes on an IBM 3033 for 1000 pointings, is a drawback of analysis by simulation. This time pertains when the uniqueness area is limited to 15 times the STR FOV area, which is expected to be typical for SIRTF, and it will increase rapidly as the uniqueness area is expanded.

The star brightness, as given by the star catalog and as perceived by the STR, will be expressed in terms of "catalog magnitude" $m_{c}$ and "instrument magnitude" $\boldsymbol{m}_{\boldsymbol{i}}$ respectively. In the case of the Space Telescope (ST) star cata$\log$, the catalog magnitude will be equal to the $V$ or visual magnitude for stars north of $\delta=+3^{\circ}$, and equal to the $J$ magnitude for stars south of that
declination (Werner, 1983).

The input parameters that are to be specified for the simulation process are discussed briefly in the following.

The integrated star density is assumed to be an exponential function of $m_{c}$. This is a reasonable simplification which, for $m_{v}$ less than 15 , may be made over a range of some 5 magnitudes. This is supported by Fig. 10, where the minimum, mean, and maximum densities are shown. These densities were derived from the Bahcall and Soneira Galaxy Model (Bahcall and Soneira, 1980) and information from Allen (1976). Hence, specification of the star density function over the magnitude interval of interest requires the input of only two parameters.

The completeness, reliability, limiting magnitude, and magnitude uncertainty are star catalog properties that need to be specified. The completeness is defined as the fraction of real stars up to the limiting magnitude that are included in the catalog, while reliability specifies the fraction of stars in the cata$\log$ corresponding to real stars. The total positional uncertainty of the stars, which contains both catalog and STR contributions, is an important input parameter. A normal distribution is assumed for the magnitude and position errors. The star catalog being prepared for the ST mission, which is likely to be used for SIRTF, is expected to be 90 to $95 \%$ complete at 15 th magnitude, to have star position errors ranging from .5 to $1 \operatorname{arcsec}(1 \sigma)$, and to have magnitude errors of $.3(1 \sigma)$ (Werner, 1983). Compared to the error contributed by the

catalog, the positional error due to the STR is expected to be negligible.
The STR parameters include the size of the FOV, the maximum number of stars that can be processed simultaneously, the dynamic range, and the minimum angular distance between stars that, in view of the spatial resolution and image point spread function, can be handled. In addition, the error in the instrument magnitude, which is assumed to have a normal distribution, has to be defined. This error is due to both an error in the assumed STR sensitivity and the difference in spectral response between the STR and the filter on which the cata$\log$ magnitude is based; therefore, it will be a function of the color distribution of the stars. For a given region of the sky, this color distribution can be determined by using the Bahcall and Soneira Galaxy model.

To define the post-slew attitude error, both the pointing uncertainty, which is assumed to be equal about both axes, and the error about the boresight need to be given. These attitude errors are assumed to have normal distributions.

The group of parameters discussed next are all associated with the recognition strategy. Based upon the pointing uncertainty, the size of the uniqueness area around the target needs to be defined. In addition, the density of "catalog stars" in the uniqueness area has to be specified. This can be done so that, while taking into account both the catalog reliability and the size of the uniqueness area relative to the FOV, a high enough probability of having an adequate number of recognition stars inside the STR FOV can be obtained. It should be noted that the magnitude limit of the star catalog constitutes an upper boundary
for the density. The STR brightness threshold is to be set such that even the faintest guide star selected from the uniqueness area has a fair chance of being observable. This parameter is set relative to the magnitude of the faintest guide star that will be generated by the simulator.

The tolerances used by the recognizer for the error in angular distance between star pairs, the attitude error about the boresight, and for the error in the instrument magnitude, are additional input parameters. Here, the error of the instrument magnitude is the difference between the predicted and the measured instrument magnitude. In addition, the confirmation value threshold, that was discussed in Section 3, needs to be set.

Currently, the simulator assumes a random spatial distribution of the stars according to a uniform distribution. Stars that are too close to each other are simply not selected as guide stars for those cases where the difference in brightness would be small enough to cause an unacceptable position error. Since about $70 \%$ of the stars belong to binary or multiple star systems (Allen, 1976; Herczeg, 1982), it is felt that more research is desirable to determine what impact this has on the probability of finding suitable guide stars.

In addition to generating the success rate of recognitions, the simulator also provides enough information about failed cases to reveal their causes. This does facilitate the improvement of the recognizer.

## 5. SIMULATION RESULTS \& COMPARISON OF ACQUISITION

## METHODS

The simulator was used to predict the MSM recognition performance for the range of pointing errors expected for SIRTF. To maximize performance, the recognition algorithm incorporated constraints on the errors in star position, star magnitude, and in angle about the boresight. For the results presented here, the values of these constraints were selected such that the probability of the errors exceeding them was less than $\mathbf{1 \%}$. In addition, the uniqueness area was sized to have a $99 \%$ probability of fully containing the STR FOV.

For standard deviations of 1 arcsec, 1 magnitude, $2.5 \operatorname{arcmin}$, and $10 \operatorname{arcmin}$, applying to star position, star magnitude, pointing, and rotation error respectively, and using a guide star density high enough to ensure a $99 \%$ probability of having at least 3 guide stars in the FOV, 984 out of 1000 recognitions were successful. The recognizer was capable of correctly dispositioning the failures as such. All failures were caused by the presence of a bright guide star that, due to a limitation of the STR dynamic range to 5 magnitudes ( $=$ intensity range of 100), made it impossible to observe any other guide stars in the FOV. Because these bright stars have a very high probability of being unique, failures caused by them can be eliminated by modifying the recognizer so that it can also handle a single unique bright star.

The effect of catalog completeness was studied by assuming a completeness of only $50 \%$, a very pessimistic value. To obtain an equal number of guide
stars, this low level of completeness would require a catalog one magnitude deeper, as well as a STR capable of observing stars that are 1 magnitude fainter. Also, the number of observed stars would double on the average. The recognition performance dropped to 935 successes out of 1000 cases. Of the failures, 19 were caused by dynamic range problems, while a large part of the remaining failures were caused by the rejection of observed star pairs that were closer than the 1.5 arcmin limit used. Again, most of the failures can be avoided by modification of the recognizer but at the cost of increased complexity.

Selection of only the brightest observed stars as recognition star candidates is an effective way to reduce the onboard computational load. This method will work well when the catalog is both highly complete and reliable. Using only the 3 brightest observed stars (out of an average 7.18 ), it turned out that 972 out of 1000 recognition attempts were successful when the catalog was $100 \%$ complete and reliable. The limited dynamic range of the STR caused 17 of the failures. When only the 2 brightest observed stars were used, the success rate dropped to $93.5 \%$. A further drop to 91.7 and 89.7 occurred when both the pointing and attitude error about the boresight were increased by a factor of 2 and 3 respectively. This drop is in line with the associated reduction in pattern uniqueness. The recognizer incorrectly designated two of the 103 failures that occurred with the largest pointing error as successes.

In Table 3, the three acquisition methods are compared in terms of required number of guidestars, acquisition time, and post-calibration slew angle for three
different pointing uncertainties. With a $99 \%$ requirement for the FOV to remain inside the uniqueness area, these uncertainties translate into areas of $7.5,20$, and 38.5 times the FOV area of the SIRTF STR in the case of the MSM, and into areas with radii of 41,67 , and 93 arcmin in the case of the LSM. It is assumed that the recognizer used for the FSM and MSM imposes position, magnitude, and rotation constraints.

Table 3. Comparison of Acquisition Methods

| Method | ```Pointing uncertainty ( \(1 \sigma\) ) (arcmin)``` | Req'd number of guide st. | Acquisition time (sec) | Post Calibr. slew angle (degree) |
| :---: | :---: | :---: | :---: | :---: |
| Lone ${ }^{3}$ star (LSM) | 5 | 1 | 48 | 4.7 |
|  | 10 | 1 | 78 | 7.7 |
|  | 15 | 1 | 116 | 12.0 |
| Few star (FSM) | 5 | 3 | 105 | < 2 |
|  | 10 | 3 | 215 | < 2 |
|  | 15 | 3 | 364 | <.2 |
|  | 5 | 54 | 4 | $<.2$ |
|  | 10 | 134 | 6 | <. 4 |
|  | 15 | 258 | 8 | <. 6 |

1. $99 \%$ probability to complete acquisition within stated time.
2. $99 \%$ probability of slew angle to be less than stated.
3. Brightness margin of 2 magnitudes.
4. Guide star density consistent with $99 \%$ probability of having at least 2 guide stars inside FOV.

The LSM; which needs only one bright guide star, becomes rather impractical if the pointing error is large. This is because the resulting large distance of the lone star to the target, combined with the error about the boresight, makes it less likely for the FOV to arrive at the target with a sufficiently small error to allow acquisition of the tracking stars. Adding an FSM type search for them would correct this problem, but it would further increase the acquisition time. In addition, it becomes more likely that, due to viewing constraints, a lone star cannot be used efficiently with its target under all conditions.

Due to the very high uniqueness probability, no more than 3 guide stars are required for the FSM. However, for large pointing errors the search time becomes prohibitive. The negligible acquisition time makes the MSM very attractive. Obviously, the large number of guide stars that need to be selected and stored per target is the main disadvantage of the method. However, in view of the relatively small attitude errors that are expected, this is considered to be the most attractive acquisition method for SIRTF. Rapid advances in computer technology will make the method even more attractive in the future, even in the presence of large pointing errors.

The number of guide stars required for the MSM can be reduced significantly, at the cost of an increased acquisition time, by stepping the STR FOV a number of times to increase the effective FOV. This is illustrated in Fig. 11 where the guide star reduction factor is shown as function of the radius of the uniqueness area for up to 6 steps. For SIRTF, each step would cost

approximately 6 seconds. The uniqueness area radii that correspond with the three pointing errors of Table 3 are 2.7, 4.5, and 6.2 times the FOV radius.

## 6. CONCLUSIONS

Three methods of automatically acquiring a desired target have been developed and analyzed. All of the methods are capable of the acquisition, but there are varying degrees of complexity, onboard computation time and memory required to perform the task, and slew accuracy requirements.

The Lone Star Method is considerably simpler than the other two and requires a minimal memory size. However, it may require as much as 1 or 2 minutes more time for the acquisition than the Many Star Method and places much higher constraints on the slew accuracy requirements, especially about the boresight.

The Few Star Method retains the low memory requirements of the Lone Star Method and is the superior method in terms of the slew accuracy; however, its disadvantage is a potentially large time required ( $2-6 \mathrm{~min}$ ) to search for the desired star pattern.

The Many Star Method virtually eliminates any time penalty for searching or slewing inefficiently, but requires a substantial number of star parameters (coordinates and brightness) in memory. The actual number is highly dependent on the slew accuracies of the pointing system. Table 3 indicates that a pointing uncertainty of 15 arcmin will require the parameters of 260 stars in memory for each target.

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