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SENSITIVITY ANALYSIS AND OPTIMIZATION OF NODAL POINT PLACEMENT FOR VIBRATION REDUCTION

JOCELYN I. PRITCHARD, HOWARD M. ADELMAN, AND RAPHAEL T. HAFTKA

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National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

SUMMARY

A method is developed for sensitivity analysis and optimization of nodal point locations in connection with vibration reduction. A straightforward derivation of the expression for the derivative of nodal locations is given, and the role of the derivative in assessing design trends is demonstrated. An optimization process is developed which uses added lumped masses on the structure as design variables to move the node to a preselected location - for example where low response amplitude is required or to a point which makes the mode shape nearly orthogonal to the force distribution, thereby minimizing the generalized force. The optimization formulation leads to values for added masses that adjust a nodal location while minimizing the total amount of added mass required to do so. As an example, the node of the second mode of a cantilever box beam is relocated to coincide with the centroid of a prescribed force distribution, thereby reducing the generalized force substantially without adding excessive mass. A comparison with an optimization formulation that directly minimizes the generalized force indicates that nodal placement gives essentially a minimum generalized force when the node is appropriately placed.

INTRODUCTION

The current trend in engineering design of aircraft and spacecraft is to incorporate in an integrated manner, various design requirements and to do so at an early stage in the design process (refs. 1, 2). Incorporation of vibration design requirements is one example of this. The conventional approach of meeting vibration requirements has been to "fix" a design for vibration, sometimes after a serious problem has been detected. Technology advances are leading to more complicated aircraft and spacecraft with higher speed and performance requirements and, therefore, it is more important to include vibration requirements early in the design process.

In helicopter rotor blade and fuselage design, stringent requirements on ride comfort, stability, fatigue life of structural components, and stable locations for electronic equipment and weapons lead to design constraints on vibration levels (refs. 3-5). Some of the methods previously used to control structural vibration in rotor blades include pendulum absorbers (ref. 6), active isolation devices (ref. 7), additional damping (refs. 5, 8), vibration absorbers which create "anti-resonances" (refs. 9, 10), and tuning masses to place frequencies away from driving frequencies (refs. 5, 11-14). Efforts to incorporate the above concepts for vibration reduction in systematic optimization techniques are described in references 10, 15-19. References 20, 21 contains surveys of applications of optimization methods for vibration control of helicopters.

Recently, the concept of "modal shaping" has been proposed as a method to reduce structural vibration, especially in helicopters (refs. 3, 4). In this method, vibration modes of rotor blades are altered through structural modification to make them nearly orthogonal to the air load distribution - thus reducing the generalized (modal) force. This paper deals with the concept of nodal point placement which is related to modal shaping and consists of modifying the mass distribution of a structure to place the node of a mode at a desirable location. Typical candidates for nodal point placement are locations where low response amplitude is required such as pilot or passenger seats, locations of sensitive electronic equipment, weapon platforms or engine mounts. Nodal point placement also has the potential for reducing overall response by placing a node at a strategic location of a force distribution to reduce the generalized force.

The objectives of this paper are to develop and demonstrate the concept of nodal point placement and develop a mathematical optimization procedure based on this concept to reduce vibration. An important ingredient in the optimization procedure is the derivative of the nodal point location with respect to a design variable. This derivative quantifies the sensitivity of a nodal location to a change in a design variable and is referred to as a sensitivity derivative. The sensitivity derivative of the nodal location is derived in this paper. The equation involves the derivative of the vibration mode with respect to the design variable and the slope of the mode shape at the nodal point and is easily implemented in a vibration analysis program using available or easily-computed quantities. Analytical results are presented for the sensitivity derivatives for a beam model of a rotor blade and compared with finite differences for an independent check. The sensitivity derivatives have been employed in an optimization procedure for placing a node at a specified location by varying the sizes of lumped masses while minimizing the sum of these masses. Optimization results are shown for placement of a node at a prescribed location on the beam model.

SENSITIVITY DERIVATIVE OF NODAL POINT LOCATION

The modal deflection normal to the length of a one-dimensional structure is denoted $u(x,v)$ and represented by the solid line in figure 1. The deflection and the nodal point location denoted by $x_{np}(v)$ are both functions of a design variable, v, and when the design variable is perturbed, the deflection shape changes to the shape shown by the dashed line. The derivative of the nodal location with respect to a design variable gives information on how design changes affect nodal point locations and thus vibration response. As will be seen later in this paper, the sensitivity derivative is an important ingredient in optimization of nodal locations.

The formulation of the derivative of the nodal location is based on expanding the perturbed mode in a Taylor series about the nominal nodal point. Neglecting the higher order terms, we have the equation:

$$u(x_{np} + dx_{np}, v + dv) = u(x_{np}, v) + \left.\frac{\partial u}{\partial x}\right|_{x_{np},v} dx_{np} + \left.\frac{\partial u}{\partial v}\right|_{x_{np},v} dv \tag{1}$$

The term on the left side of the equation and the first term on the right are displacements of the nodal points of the perturbed and nominal mode shapes, respectively which are zero. Since $x_{np} = x_{np}(v)$, it follows that

$dx_{np} = \frac{dx_{np}}{dv} dv$. Therefore, from (1)

$$\left.\frac{\partial u}{\partial x}\right|_{x_{np},v} dx_{np} + \left.\frac{\partial u}{\partial v}\right|_{x_{np},v} dv = \left(\left.\frac{\partial u}{\partial x}\right|_{x_{np},v} \frac{dx_{np}}{dv} + \left.\frac{\partial u}{\partial v}\right|_{x_{np},v}\right) dv = 0 \tag{2}$$

Noting that dv is arbitrary and solving for dx_{np}/dv leads to the formula for the nodal point derivative*

$$\frac{dx_{np}}{dv} = -\left[\frac{\partial u/\partial v}{\partial u/\partial x}\right]_{x_{np},v} \tag{3}$$

*Equation (3) is also applicable to nodal line movement in two-dimensional structures. If a nodal line is parallel to the y-axis of a plate for example, then equation (3) gives the change in the x-location with respect to change in a design variable. For a nodal line parallel to the x-axis, equation (3) applies; provided x is replaced by y in the equation. For the general case where the nodal line is not parallel to the x or y direction, equation (3) gives the derivative of the location of a nodal line in a direction normal to the line.

The two ingredients in the formula are $\partial u/\partial v$, the derivative of the mode shape at the nodal point and $\partial u/\partial x$, the slope of the mode shape at the nodal point. The value of $\partial u/\partial x$ is obtained from the nominal mode shape; and the value of $\partial u/\partial v$ is obtained by Nelson's method (ref. 22) which will be described in the next section.

IMPLEMENTATION OF SENSITIVITY ANALYSIS

GENERAL APPROACH

The calculation of derivatives of nodal point locations has been implemented in a general purpose finite element program (ref. 23). In a finite element analysis, the components of the vibration eigenvector are generally available only at the grid points of the model. Linear interpolation is used to locate the node between grid points. Once the node location is found, interpolation is used to obtain the slope of the mode shape $\partial u/\partial x$ at the node and the mode shape derivative $\partial u/\partial v$ at the node. It is noted that the modal deflection u is a subset of the eigenvector ϕ and therefore the derivative $\partial u/\partial v$ is a subset of the eigenvector derivative $\partial \phi/\partial v$.

NELSON'S METHOD FOR EIGENVECTOR DERIVATIVES

A free-vibration problem with no damping, is governed by the following eigenvalue equation

$$(K - \lambda M)\phi = 0 \tag{4}$$

In equation (4) K is the stiffness matrix, M is the mass matrix, ϕ is the eigenvector, and λ is the eigenvalue equal to the square of the frequency. The eigenvector is normalized such that the generalized mass is unity

$$\phi^T M\phi = 1 \tag{5}$$

By taking the derivative of equation (4) with respect to a design variable v the following equation emerges:

$$(K - \lambda M) \frac{d\phi}{dv} = \frac{d\lambda}{dv} M\phi - \frac{dK}{dv} \phi + \lambda \frac{dM}{dv} \phi \tag{6}$$

Because this equation is singular a direct solution for $\frac{d\phi}{dv}$ is not possible. However, the general solution to equation (6) is expressible in the following form:

$$\frac{d\phi}{dv} = q + c\phi \tag{7}$$

where q is a particular solution found by setting one component of the eigenvector derivative equal to zero and deleting the corresponding row and column from equation (6) and solving for the remaining components. The constant c is found by taking the derivative of the normalization condition in equation (5) and substituting equation (7) into the resulting expression.

$$2\phi^T M \frac{d\phi}{dv} = - \phi^T \frac{dM}{dv} \phi \tag{8}$$

$$c = - \phi^T Mq - \frac{1}{2} \phi^T \frac{dM}{dv} \phi \tag{9}$$

EXAMPLE PROBLEM

The example problem used for the sensitivity analysis study is a cantilever beam representation of a rotor blade developed in reference 14 and shown in figure 2. The beam in figure 2(a) is 193 inches (4.9 m) long and is modeled by 10 finite elements of equal length. The model contains both structural mass and lumped (non-structural) masses. The beam has a box cross section as shown in figure 2(b) and the material properties and cross sectional dimensions are summarized in table 1. There are eight lumped masses at various locations along the length of the beam. The values of the masses are the design variables in this study and their values and locations are shown in table 1. The values of the masses were generated in reference 14 in an optimization procedure to minimize mass subject to frequency constraints and serve as nominal values for the current sensitivity study. Sensitivity studies are performed in which the derivative of the nodal location for the second mode is computed with respect to the lumped masses.

RESULTS OF SENSITIVITY ANALYSIS

Derivatives of the nodal point location for the second mode were calculated using equation (3). For an independent check on the implementation of equation (3), the derivatives were also calculated by finite differences. The finite difference derivatives in contrast to the analytical derivatives (equation 3) require a precise determination of the nodal location. The reason for this is that the quantities in equation (3), $\partial u/\partial v$ and $\partial u/\partial x$, vary slowly in the vicinity of the node. Conversely the finite difference method subtracts the nominal and perturbed node locations to calculate the derivative and even small errors in these values can lead to large errors in the derivatives. The finite difference calculations begin with an eigenvalue analysis for the nominal design variables. From examination of the eigenvector, the element containing the node is identified. The displacements and slopes at the end points of this element are extracted from the eigenvector and used to define a third order polynomial. The root of the polynomial that lies in the element is the nodal location. Next, the design variable is perturbed (by 0.1 percent) and the process is repeated to find the perturbed nodal location. A forward finite difference formula is then used to calculate the derivative.

The sensitivity results are shown in table 2. The two methods generally agreed within about two percent. Examination of table 2 shows both positive and negative values of the derivatives. A positive value indicates that an increase in the mass moves the nodal point to the right of the nominal location and a negative value indicates that an increase in mass moves the node to the left. The derivatives in table 2 show that increases in the masses at grid points 10 and 11 are the most effective ways (per unit mass) to move the node to the right. Similarly, decreases in the masses at grid points 10 and 11 or increases in the masses at grid points 6 and 7 have the largest effects (per unit mass) in moving the node to the left.

OPTIMIZATION FORMULATION

In this section, we will show how node locations are adjusted using mathematical optimization. The optimization problem is to place a node at a desired location by varying the magnitudes of lumped masses while minimizing the total lumped mass. CONMIN, a general-purpose optimization program, (ref. 24) is utilized as the optimizer. The formulation of the problem consists of defining an objective function (the quantity to be minimized); the constraints (limitations on the behavior of the model); and the design variables (the parameters of the model to be changed in order to find the optimum design). The optimizer requires derivatives of both the objective function and the constraints. The formulation for this problem is as follows:

The objective function, f, is the sum of the lumped masses , i.e.

$$f = \sum_{i=1}^{N} M_i \tag{10}$$

The constraint, g, which must be negative or zero for an acceptable design, expresses the requirement that the nodal point x_{np} be placed within a distance δ from a desired location x_o that is,

$$g = |x_{np} - x_o| - \delta \leq 0 \tag{11}$$

The design variables consist of the sizes of the lumped masses. Constraints on the largest and smallest acceptable values of the design variables are required by the optimizer. These values are arbitrarily set. The derivatives of the objective function with respect to the design variables are

$$\frac{\partial f}{\partial v_i} = \frac{\partial f}{\partial M_i} = 1.0 \qquad i = 1,2,\ldots,N \tag{12}$$

and the derivatives of the constraints are equal to positive or negative values of the nodal point sensitivity derivatives i.e.

$$\frac{\partial g}{\partial v_i} = \pm \frac{\partial x_{np}}{\partial v_i} \tag{13}$$

calculated from equation (3).

OPTIMIZATION PROCEDURE

The sequence of operations in the optimization procedure is illustrated in figure 3. The overall procedure consists of two nested loops. Each pass through the outer loop is referred to as a cycle which involves a full analysis and a sensitivity calculation. The first computation is to generate the structural model of the beam, excluding the values of lumped masses. As the first step in the outer loop the lumped masses (the current design variables) are inserted into the model. Next, the vibration analysis is performed and the nodal location and the slope of the mode shape at the nodal point are found by interpolation of grid-point eigenvector displacements. The sensitivity analysis block includes calculating the vibration mode shape derivatives by Nelson's method and calculating the nodal point derivative from equation (3). The inner loop is contained in the optimizer block which consists of the optimization program of reference 24 and an approximate analysis for calculating the objective function and the constraints (see ref. 25). The approximate equations are

$$f = f_o + \sum_i \frac{\partial f}{\partial v_i} \Delta v_i \tag{14}$$

$$g = g_o + \sum_i \frac{\partial g}{\partial v_i} \Delta v_i \tag{15}$$

These equations give the change in the objective function from f_o to f and the change in a constraint from g_o to g corresponding to a change in design variables Δv_i. To assure that the linear approximations in eqs. 14 and 15 are valid, the size of Δv_i is limited to ten percent of v_i. Use of these approximations saves computational time and effort in the inner loop where many evaluations of the objective function and constraints are required. Development of these and other techniques and demonstration of their benefits are described in reference 26. Once the inner loop iterations have converged the next cycle of the outer loop begins using the current design variables as the new values of the lumped masses. These masses are then inserted in the structural model and the process continues until convergence of the outer loop is achieved.

NODAL PLACEMENT OPTIMIZATION

The model used in the optimization procedure is shown in figure 4 and is the same beam structure described in figure 2 and table 1. The node for the second mode is to be placed within δ = 1.0 inch (.0254 m) of x_o = 164 inches (4.16 m). The location x_o is chosen because it is the centroid* of a representative air load distribution (fig. 5) given in reference 3 for a rotor blade. The design variables are the masses at joints 9, 10, and 11 having initial values of 5.21 lbm (2.36 kg), 6.55 lbm (2.97 kg), and 6.60 lbm (2.99 kg) (from ref. 14) - a total of 18.36 pounds (8.32 kg), and the initial location of the node is 154.7 in (3.929 m). The upper and lower bounds on the design variables are arbitrarily set at 50. lbm (23 kg) and 0.5 lbm (.23 kg), respectively.

The optimization procedure converged to the final design shown in table 3, in which the masses were 0.5 lbm** (.23 kg), 3.70 (1.68 kg), and 20.25 lbm (9.19 kg) - a total of 24.45 lbm (11.10 kg), and the nodal point is located at 163 inches (4.140 m). The optimization history is shown in figure 6. The optimizer initially adds mass to bring the nodal point to within one inch of the desired location (fig. 6a). For the remainder of the cycles, as shown in figure 6b, the optimizer concentrates on minimizing the total mass by shifting mass among the three locations. Basically, mass was shifted from the two inboard locations to the tip where mass is most effective in moving the nodal point. For example, the mass at grid point 9 is reduced from 5.21 lbs (2.36 kg) to 0.5 lbs (.23 kg) while the tip mass is increased

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*As shown in Appendix B, placing the nodal point for the second mode of a beam at the centroid of the force distribution results in a near-minimum value of the corresponding generalized force.

**Lower bound

from 6.6 lbs (2.99 kg) to 20.25 lbs (9.19 kg). Excessive addition of mass is avoided (only 6 additional pounds were needed) because of the effectiveness of relocating mass to the tip.

EFFECT OF NODAL POINT PLACEMENT ON GENERALIZED FORCE

One of the potential applications of nodal point placement is the reduction of overall vibration response by generalized force minimization. In this section, the generalized force from a design based on nodal point placement is compared with the true minimum obtained by a method which will now be described.

FORMULATION OF GENERALIZED FORCE MINIMIZATION

In this formulation the objective function is the generalized force given by

$$f = \phi^T F \tag{16}$$

where ϕ is the eigenvector and F is a vector of the distributed force. The design variables are the same as those in the previous optimization example; i.e., lumped masses. In order for the comparison of designs to be valid, a constraint is imposed that the sum of the masses used as design variables be less than or equal to $M^* = 24.45$ lbm (11.10 kg) - the mass that was required in the nodal point placement optimization. Therefore, the constraint is

$$g = \sum_{i=1}^{N} M_i - M^* \leq 0 \tag{17}$$

The derivative of the objective function is

$$\frac{\partial f}{\partial v_i} = F^T \frac{\partial \phi}{\partial v_i} \tag{18}$$

where the eigenvector derivative $\partial\phi/\partial v_i$ is obtained by Nelson's method. The derivative of the constraint is given by

$$\frac{\partial g}{\partial v_i} = \frac{\partial g}{\partial M_i} = 1.0 \qquad i = 1,2,\ldots,N \qquad (19)$$

This optimization formulation was used to minimize the generalized force for the steady state air load distribution in figure 5.

COMPARISON OF DESIGNS

The results of this study are summarized in table 4, in which the design variables, total mass, generalized force, and nodal point locations are shown for three designs: the initial design, the final design from node placement, and the final design from the direct minimization of the generalized force. The node placement procedure is very effective in minimizing the generalized force - giving 10.8 lbf (48.04 N), compared to 10.0 lbf (44.48 N) from the direct method when both were started at a design with a generalized force of 20.8 lbf (92.52 N). The direct minimization procedure, while not dealing directly with the nodal location nevertheless places the node essentially at the same point as the node placement design: 163.8 inches (4.161 m) vs. 163.0 inches (4.140 m).

CONCLUDING REMARKS

This paper has described sensitivity analysis and optimization methods for adjusting mode shape nodal point locations with application to vibration reduction. The paper begins with a derivation of an expression for the derivative of the nodal location with respect to a design variable. Sensitivity analyses were performed on a demonstration problem which consisted of a box beam model of a helicopter rotor blade. In these analyses, the derivatives of the nodal location for the second mode with respect to the magnitudes of lumped masses on the beam were calculated. It was shown that these derivatives gave useful information on the effect of the masses on the node location and indicated which masses were most effective in moving the nodal point. Next, the paper described an optimization procedure to place a node at a prescribed location by adjusting the magnitudes of lumped masses while minimizing the sum of these masses. A general-purpose optimization program was used and the nodal point derivatives were a key ingredient in the procedure. This optimization procedure was then used in an example where the nodal point for the second mode of a cantilever beam model of a rotor blade was placed at a location close to the centroid of a force distribution. This location was chosen as a result of a numerical study (described in an appendix) where it was shown that this choice for the nodal location gave a minimum generalized force. We were successful in moving the node to the desired location requiring only six pounds of lumped mass on a 193-inch (4.90 m) beam that weighed 117 pounds (53.1 kg).

Finally, to evaluate the potential for nodal placement to reduce vibration, the generalized force for the second mode was calculated and compared to the minimum generalized force obtained in this paper by a separate optimization procedure. It was found that the nodal placement procedure gave a generalized force which was very close to the minimum. The results in this paper suggest that adjusting the mode shapes of structures by relocating nodal points has potential for reducing both overall and local response levels in vibrating structures.

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APPENDIX A: NOMENCLATURE

b	width of box cross section
c	constant used in Nelson's method (eq. 9)
d	side wall thickness of box cross section (fig. 2)
E	Young's modulus
F	force vector
f	objective function
g	constraint function
h	height of box cross section
I	identity matrix
K	stiffness matrix
L	length of beam
M	mass matrix
M_i	lumped mass equal to ith design variable
M^*	sum of design variables (mass)
N	number of design variables
q	particular solution in Nelson's method
t	upper and lower wall thickness of box cross section (fig. 2)
u	modal deflection
v, v_i	design variable
x	coordinate along one-dimensional structure
x_{np}	nodal point location
x_o	desired nodal point location
δ	allowable distance from desired nodal point location
λ	eigenvalue, square of frequency
ρ	weight density
ϕ	eigenvector
T (superscript)	transpose of matrix

APPENDIX B - EFFECTIVENESS OF LOCATING NODE AT CENTROID OF FORCE DISTRIBUTION TO REDUCE GENERALIZED FORCE

At an early stage in this work, a numerical study was performed to investigate the effect on the generalized force of placing the nodal point of the second mode shape of a simply-supported beam at the centroid of a force distribution. Given the force distribution (fig. 7(a)) with its centroid at 52 percent of the length of the beam, the generalized force $\phi^T F$ was calculated for 11 arbitrary shape functions (fig. 7(b)) having different nodal locations x_{np} which varied between 25 percent and 75 percent of the beam length. As shown in figure 8, the smallest generalized force in fact occurs when the node is placed beween 50 and 55 percent of the beam length (essentially at the centroid of the force distribution). While this is not a proof, it does show that the centroid of the force distribution is a good choice for the location of a nodal point to obtain a low value of generalized force.

TABLE 1. DETAILS OF FINITE-ELEMENT MODEL OF ROTOR BLADE
(see figure 2)

Material Properties and Cross-Sectional Dimensions

Element. No.	E	ρ	b	h	t	d
1	0.490×10^7 (psi)	.07 (lb/in³)	3.75 (in)	2.5 (in)	.8 (in)	.1 (in)
	3.378×10^{10} (Pa)	1938.00 (kg/m³)	.0953 (m)	.064 (m)	.0203 (m)	.00254 (m)
2-10	0.585×10^7 (psi)	.07 (lb/in³)	3.75 (in)	2.5 (in)	.8 (in)	.1 (in)
	4.033×10^{10} (Pa)	1938.00 (kg/m³)	.0953 (m)	.064 (m)	.0203 (m)	.00254 (m)

Values of Lumped Masses at Grid Points

Grid Pt. No.	3	4	6	7	8	9	10	11
Mass lbm (kg)	3.04 (1.38)	1.67 (0.757)	6.40 (2.90)	7.46 (3.38)	10.75 (4.88)	5.21 (2.36)	6.55 (2.97)	6.60 (2.99)

TABLE 2. COMPARISON OF ANALYTICAL AND FINITE DIFFERENCE DERIVATIVES OF NODAL POINT LOCATIONS FOR SECOND MODE OF CANTILEVER BEAM OF FIGURE 2

Mass at Grid Point –	$\frac{dx_{np}}{dv}$ Analytical*		Finite Difference*	
3	- 0.0278	(- 0.156)	- 0.0277	(- 0.155)
4	- 0.0881	(- 0.493)	- 0.0880	(- 0.493)
6	- 0.231	(- 1.29)	- 0.230	(- 1.29)
7	- 0.237	(- 1.33)	- 0.236	(- 1.32)
8	- 0.166	(- 0.930)	- 0.165	(- 0.924)
9	- 0.00380	(- 0.0213)	- 0.00361	(- 0.0202)
10	0.309	(1.73)	0.309	(1.73)
11	0.828	(4.64)	0.826	(4.63)

*in/lbm (cm/kg)

TABLE 3. INITIAL VS. FINAL DESIGNS FOR NODAL POINT OPTIMIZATION (see figure 4)
x_o = 164.0 in. (4.166 m); δ = 1.0 in. (.0254 m)

		Initial Design	Final Design
M_1	lbm at Grid Point 9	5.21	0.50
	(kg)	(2.36)	(0.23)
M_2	lbm at Grid Point 10	6.55	3.70
	(kg)	(2.97)	(1.68)
M_3	lbm at Grid Point 11	6.60	20.25
	(kg)	(2.99)	(9.19)
M^*	lbm	18.36	24.45
	(kg)	(8.32)	(11.10)
Node Location x_{np}	(in.)	154.7	163.0
	(m)	(3.929)	(4.140)

TABLE 4. FINAL DESIGNS, NODE LOCATIONS AND MODAL FORCES FROM NODE PLACEMENT VS. DIRECT FORCE MINIMIZATION - SECOND MODE OF CANTILEVER BEAM (see figure 2)

PARAMETER	DESIGN		
	Initial	Final From Node Placement	Final From Direct Minimization
M_1 lbm	5.21	0.50	0.50
(kg)	(2.36)	(0.23)	(0.23)
M_2 lbm	6.55	3.70	1.75
(kg)	(2.97)	(1.68)	(0.79)
M_3 lbm	6.60	20.25	22.20
(kg)	(2.99)	(9.19)	(10.07)
M^* lbm	18.36	24.45	24.45
(kg)	(8.32)	(11.10)	(11.10)
Generalized Force lbf	20.8	10.8	10.0
(N)	(92.52)	(48.04)	(44.48)
Node location x_{np} in.	154.7	163.0	163.8
(m)	(3.929)	(4.140)	(4.161)

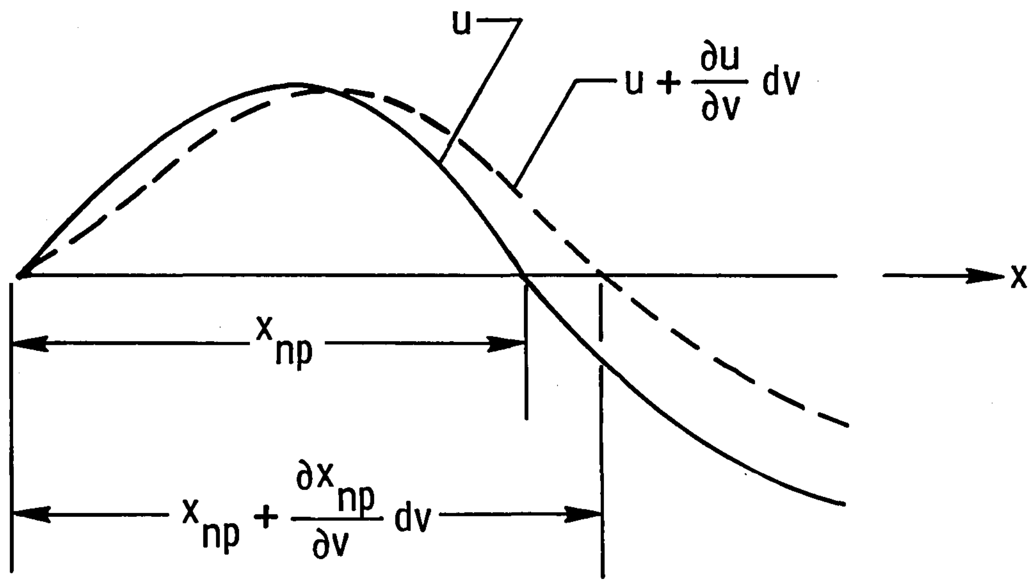
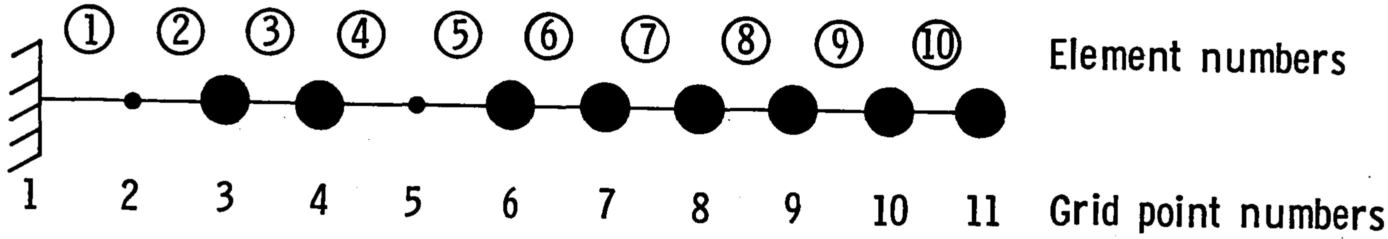
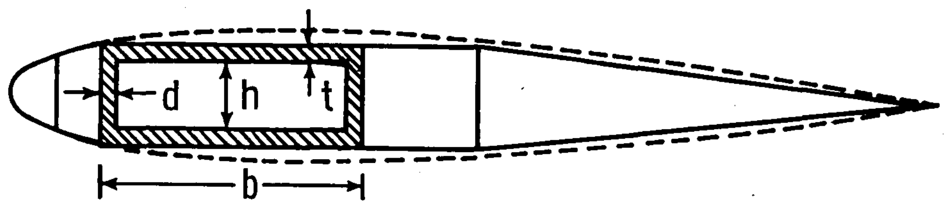


Figure 1.- Nominal and perturbed mode shapes and nodal points for one-dimensional structure.



(a) Finite element model of rotor blade
(10 equal elements of length 19.3 in. (.490 m))



(b) Cross-sectional detail of rotor blade showing dimensions of box beam

Figure 2.- Rotor blade model.

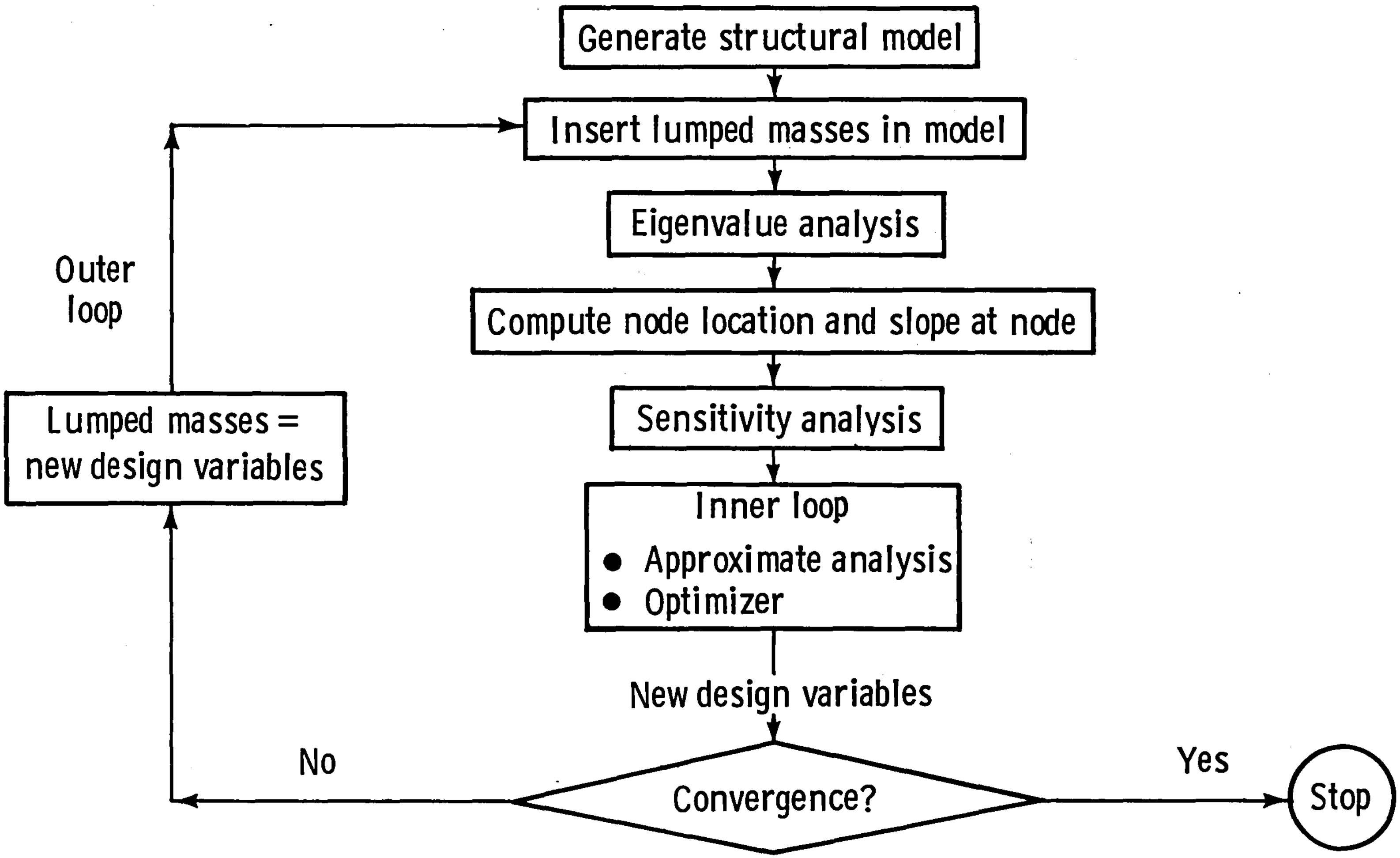
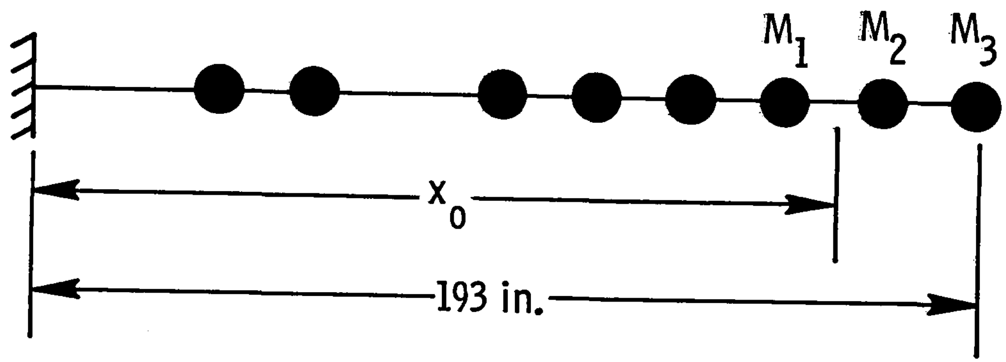


Figure 3.- Optimization procedure for nodal placement.



- Desired node location: $x_0 = 164.0$ in. (4.166 m)
- Allowable distance: $\delta = 1.0$ in. (.0254 m)
- Design variables: M_1 M_2 M_3
- Upper bounds on design variables: 50 lb (23 kg)
- Lower bounds on design variables: 0.5 lb (.23 kg)

Figure 4.- Optimization problem specifications.

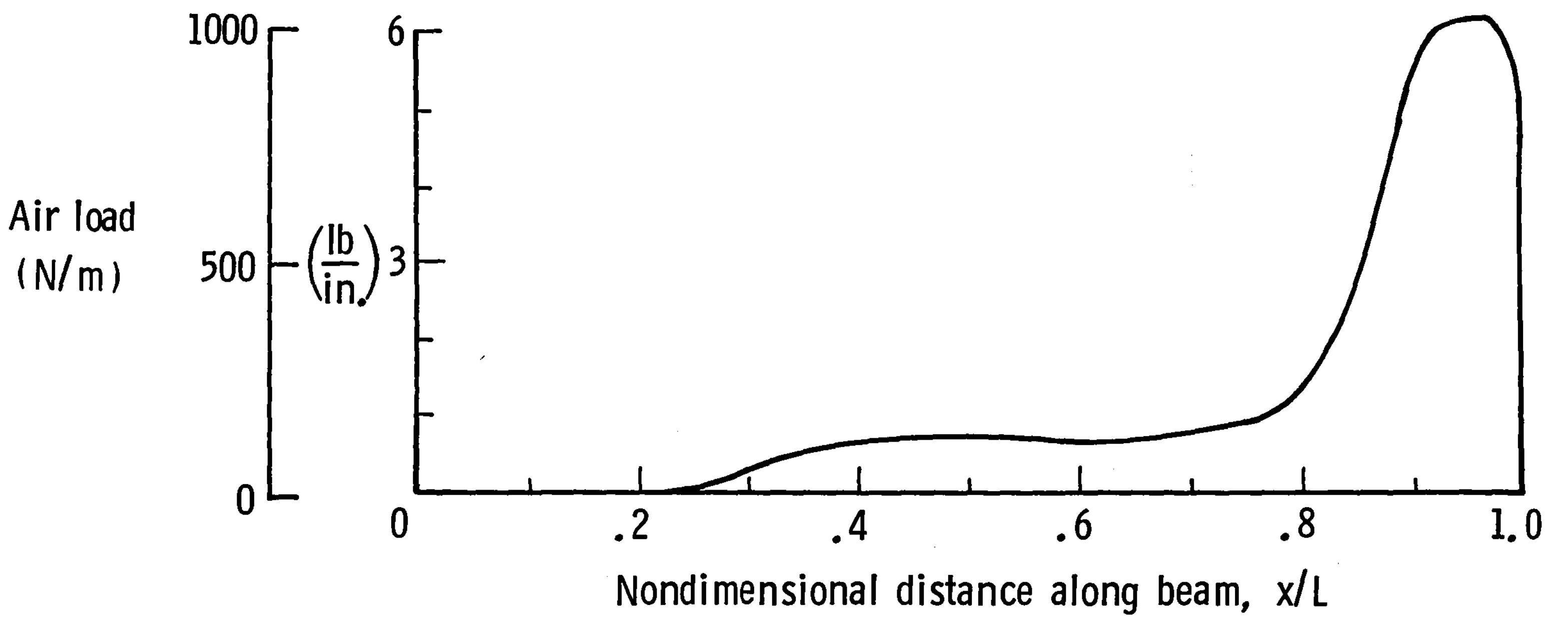
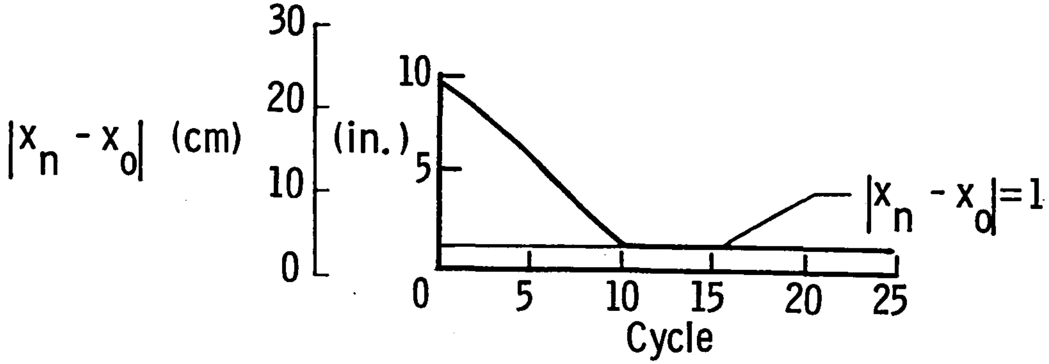
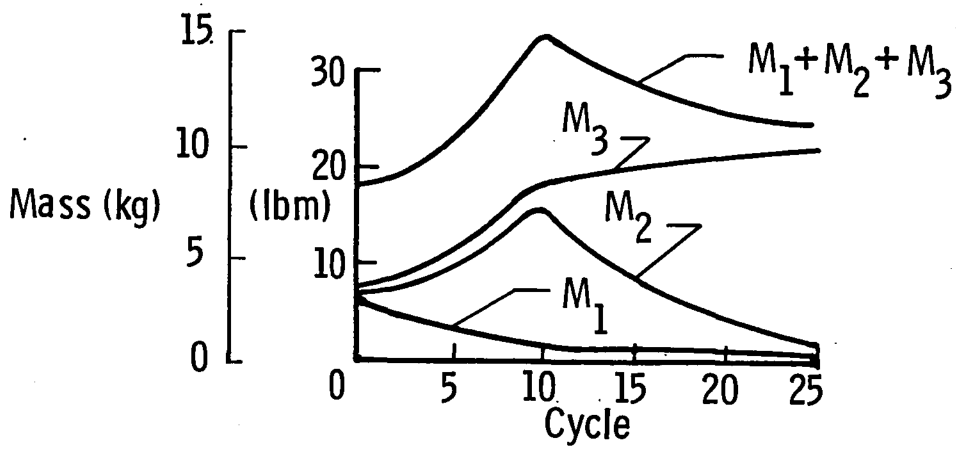


Figure 5.- Sketch of air load distribution for generalized force minimization study.

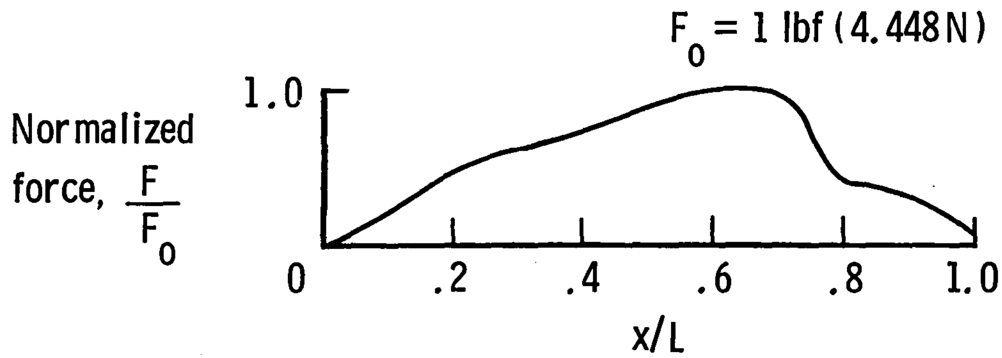


(a) Convergence of node location

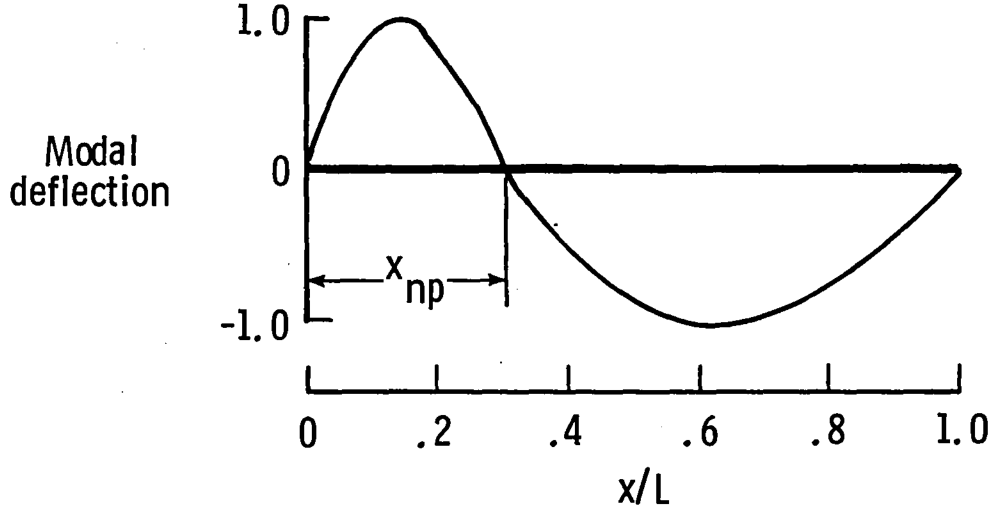


(b) Convergence of weight

Figure 6.- Convergence of optimization procedure for nodal placement.



(a) Force distribution, F
(centroid at x/L = .52)



(b) Typical mode shape, ϕ having nodal location, x_{np}

Figure 7.- Mode shape and force distribution used to study the effect of nodal location on generalized force (see Appendix B).

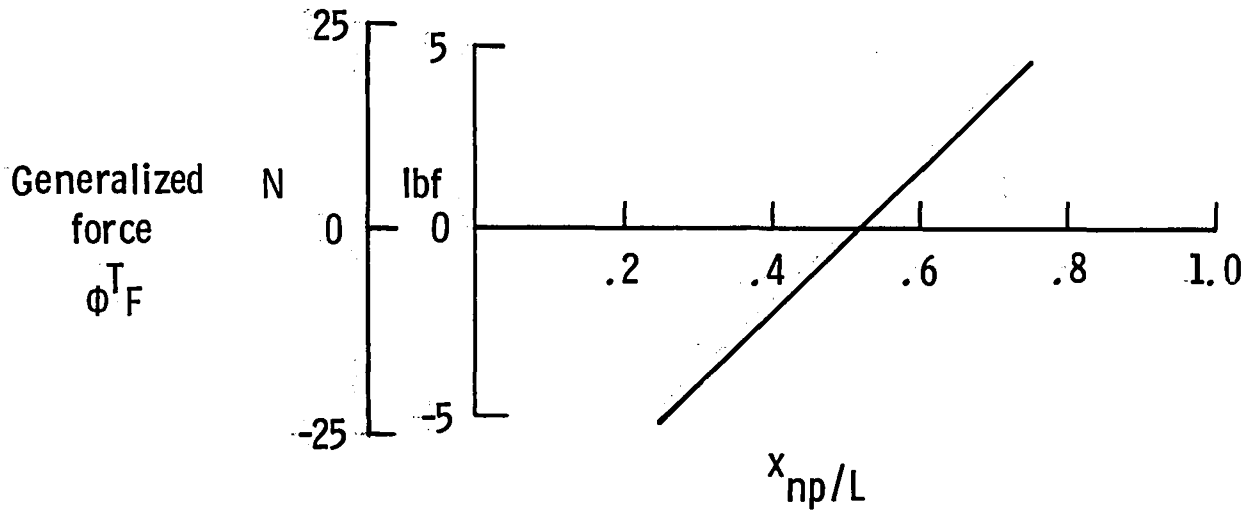


Figure 8.- Generalized force vs. nodal location (x_{np}) for force and typical mode shape of figure 7.

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Jocelyn I. Pritchard, Howard M. Adelman, and Raphael T. Haftka

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Jocelyn I. Pritchard and Howard M. Adelman, NASA Langley Research Center, Hampton, VA.
Raphael T. Haftka, Virginia Polytechnic Institute and State University, Blacksburg, VA.

16. Abstract

A method is developed for sensitivity analysis and optimization of nodal point locations in connection with vibration reduction. A straight-forward derivation of the expression for the derivative of nodal locations is given, and the role of the derivative in assessing design trends is demonstrated. An optimization process is developed which uses added lumped masses on the structure as design variables to move the node to a preselected location - for example, where low response amplitude is required or to a point which makes the mode shape nearly orthogonal to the force distribution, thereby minimizing the generalized force. The optimization formulation leads to values for added masses that adjust a nodal location while minimizing the total amount of added mass required to do so. As an example, the node of the second mode of a cantilever box beam is relocated to coincide with the centroid of a prescribed force distribution, thereby reducing the generalized force substantially without adding excessive mass. A comparison with an optimization formulation that directly minimizes the generalized force indicates that nodal placement gives essentially a minimum generalized force when the node is appropriately placed.

17. Key Words (Suggested by Author(s))

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