The Size, Shape, Density, and Albedo of Ceres from Its Occultation of BD +80471
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This manuscript totals 36 pages, including 6 figures and 7 tables.

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PROPOSED RUNNING HEAD: Occultation by Ceres

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#### Abstract

The occultation of $B D+80471$ by Ceres on 13 November 1984 was observed photoelectrically at 13 sites in Mexico, Florida, and the Caribbean. These observations indicate that Ceres is an oblate spheroid having an equatorial radius of $479.6 \pm 2.4 \mathrm{~km}$ and a polar radius of $453.4 . \pm 4.5 \mathrm{~km}$. The mean density of this minor planet is $2.7 \mathrm{gm} / \mathrm{cm}^{3} \pm 5 \%$, and its visual geometric albedo is 0.070 . While the surface appears globally to be in hydrostatic equilibrium, firm evidence of real limb irregularities is seen in the data.


## INTRODUCTION

It has long been known that Ceres is the largest asteroid. However, knowledge of the precise size of this object has been quite uncertain with even modern diameter determinations disagreeing substantially. For example, Brown et al. (1982), using infrared radiometry, found the diameter of Ceres to be $953 \pm 50 \mathrm{~km}$. Nearly contemporaneous Very Large Array observations at 2 and 6 cm were interpreted by Johnston et al. (1982) to indicate a diameter of $818 \pm$ 82 km.

Precise information about the size and shape of Ceres would be valuable for a variety of reasons. Ceres is one of three minor planets whose mass is accurately known (Schubart and Matson, 1979). Given an accurate value of the diameter, the mean density could be computed with sufficiently small uncertainty to be indicative of Ceres' composition. The shape of the asteroid may provide clues to internal structure, while an accurate diameter would extend by nearly a factor of two the size range over which the radiometric technique and other indirect methods of size determination can be tested.

In recent years the dimensions of several minor planets have been measured by observation of occultations of stars. Such observations when made with an appropriately distributed network of telescopes typically yield diameters with uncertainties of only $1 \%$ or $2 \%$ (see, e.g., Millis et al., 1981). Application of this technique to Ceres,


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however, had been thwarted because conventional occultation searches based on star catalogs identified few attractive occultations observable in accessible parts of the world (Taylor, 1981). In 1983 a specially designed photographic search at last turned up an occultation of the star $B D+80471$ by Ceres occurring on 13 November 1984 (Millis et al., 1984). This paper discusses photoelectric observations of that occultation made at 13 sites in Mexico, the United States, and the Caribbean. Preliminary discussions of various small subsets of these data have been published by Hubbard et al. (1985), Millis et al. (1985), and Oswalt et al. (1986).


## OBSERVATIONS

The final predicted ground track for the 13 November occultation based on a plate taken with the 0.5 -meter Carnegie double astrograph at Lick Observatory is shown in Figure 1. Prior to the occultation, the cross-track uncertainty in the predicted position of the ground track was believed to be no more than one-fourth the width of the track. In fact, the observed track was displaced northward relative to the predicted track by nearly one-third its width.

The coordinates and altitudes of the observing sites are listed in Table I. Also given are the names of the observers and limited information about the instrumentation used for the observations. In
most cases, the observations were made with portable telescopes and photometric equipment specifically designed for occultation work. Only at Zacatecas, Mexico; Melbourne, Florida; and South Miami, Florida, was the occultation observed at established observatory sites.

Photometric parameters for Ceres and $B D+80471$ are listed in Table II along with the expected depths of the occultation lightcurve for different passbands and the expected signal-to-noise ratios for different observing sites. The magnitude and color indices of Ceres are from the TRIAD file (Bowell et al., 1979), while those for the star were measured with the 1.8 -meter Perkins telescope at Lowell Observatory. The signal-to-noise ratios were computed using the relationship given by Millis and Elliot (1979). It is readily evident from Table II that the change in brightness at immersion was too small to be detected by visual or video techniques. However, with photoelectric equipment, the signal-to-noise ratio for the event, while small, was sufficiently high to permit the duration of the occultation to be determined with an uncertainty of no more than $1 \%$ (see Millis and Elliot, 1979). A typical lightcurve of this occultation is shown in Figure 2.

The observed times of immersion and emersion are listed in. columns 2 and 3 of Table III. The quoted uncertainties in these times are estimates by the individual observers. Because of differences in instrumentation, observing conditions, etc., the data are rather
heterogeneous, but each observer has made an effort to establish the uncertainties in the data as objectively as possible. All observations have been given equal weight in the analysis that follows.

ANALYSIS
We have analyzed the observations of the 13 November occultation using standard methods which have been described in detail elsewhere (e.g., Wasserman et al., 1979). Each observed time of immersion or emersion can be combined with the coordinates and altitude of the corresponding site, the asteroid's ephemeris; and the coordinates of the occulted star to define the position of a point on the apparent limb of Ceres. The points derived from all of the observations in Table III are plotted in Figure 3. A circle has been fitted to the data by least squares.

Two serious discrepancies are apparent in Figure 3. The point corresponding to the emersion timing at South Miami (point "A" in the figure) deviates from other points near the same chord by an amount greater than can be explained by any reasonable topography. Large negative excursions in the recorded signal, subsequently found to be due to electronic problems, are present in the South Miami data for an interval of several seconds around the expected emersion time. We believe that the reappearance of the star occurred while the
photometric signal was degraded and that the time originally reported by the observers simply corresponds to the cessation of the electronic difficulties. In any case, one is clearly justified in discarding the emersion timing from South Miami.

The other significant disagreement in the total data set concerns the emersion times recorded at Providenciales (point " $B^{\prime \prime}$ in Figure 3) in the Caicos Islands and near Chamela along the western coast of Mexico (point "C" in Figure 3). Both sites are near the southern edge of the occultation ground track; as a consequence, these observations are particularly important in distinguishing between a circular and elliptical limb profile. Emersion in the Chamela observations is well defined, and the interpretation of those data seems unequivocal. The Providenciales observers, on the other hand, reported two "flashes" prior to their originally identified time of emersion. During the second and more prominent of these, the signal returned to the preoccultation level for approximately 2 seconds. If we identify egress with the onset of this feature, then the Providenciales observations are in good agreement with those from Chamela. Since there is no other obvious way to resolve the discrepancy between the two data sets, we have adopted arbitrarily this identification in the analysis that follows. The alternative would be to discard the Providenciales emersion timing, but that approach would affect the results of the following analysis by amounts that are small compared
with the quoted uncertainties.
In Figure 4 a circle has been fitted to the final data set by least squares. The resulting value for the mean diameter of Ceres is listed in Table IV together with corrections to the right ascension and declination of the asteroid indicated by the least-squares solution. (By convention, we have assumed that all position error resides in the asteroid's ephemeris.) The quoted errors reflect only the uncertainty in fitting a circle to the observed data points and do not include the uncertainty in the observations themselves. Table $V$ contains the ephemeris of Ceres that was used in this analysis. It is based on orbital elements given in the 1975 Leningrad Ephemeris of Minor Planets. The radial differences between the observed points and the fitted circle are listed in columns 4 and 5 of Table III. Qualitatively the circular limb profile fits the data reasonably well, but the southernmost and northernmost chords deviate from the circle in, perhaps, a nonrandom way.

Figure 5 shows the same data as in Figure 4 but fitted with an elliptical limb profile rather than a circle. Also indicated in this figure are constraints placed on the solution by the negative observations by J. F. Barral at Tonantzintla Observatory and M. Frueh at McDonald Observatory. The semimajor and semiminor axes of the fitted ellipse, the position angle of the semiminor axis, and the adjustments to the right ascension and declination of Ceres that


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result from the least-squares solution are given in Table IV. The radial residuals are listed in columns 6 and 7 of Table III. Note that the rms residual per degree of freedom (see Table III) is significantly less in the case of the elliptical fit than for the case of a circular limb profile. Consequently, the inclusion of the additional free parameters inherent in the elliptical solution appears to be justified, and we adopt that solution as best representing the apparent limb of Ceres at the time of the occultation. As in the case of the circular solution, the quoted errors for the elliptical solution are a measure of the uncertainty in the least-squares fit without regard for the errors inherent in the observations themselves.


## DISCUSSION

The occultation observations give the profile of the asteroid in the plane of the sky. However, in order to compare the occultation results with previous diameter measures and to use the occultation data to derive physical parameters of interest, we must assess the three-dimensional figure of Ceres. Tedesco et al. (1983), on the basis of extensive photoelectric photometry of this asteroid, concluded that Ceres is approximately spheroidal and that the obliquity of its pole is small. These conclusions follow from the fact that Ceres displays the same low-amplitude rotational lightcurve regardless of ecliptic
longitude. Indeed, in 1984 about one week after the occultation, one of us (Piironen) recorded the lightcurve shown in Figure 6. For all practical purposes, this lightcurve is identical to those published by Tedesco et al. (1983) from the 1975-76 apparition. The arrow in Figure 6 marks the rotational phase at which the 13 November occultation occurred. Ceres at that phase was at an intermediate brightness, indicating that the effective diameter derived from the occultation observations will be close to the average for all rotational phases. In fact, the total rotational brightness variation can be explained by a variation in apparent effective diameter with rotational phase of less than $2 \%$. It may be less because all or part of the 9 -hour-period brightness variation could be produced by differences in albedo across the surface of Ceres.
The assertion that the obliquity of the rotational pole of Ceres is small can be further tested by reference to the occultation observations. Dermott (1979) has noted that the rotational period of Ceres is sufficiently short that an appreciable equatorial bulge should be present. If the rotational pole is perpendicular to Ceres' orbit, then the declination of the sub-Earth point on the asteroid was $2: 7$ at the time of the occultation. Consequently, we viewed the object essentially equator-on, and the minor axis of the ellipse fitted to the occultation data in Figure 5 should have very nearly the same orientation as the rotational pole of Ceres. Assuming zero obliquity
for the pole, its position angle at the time of occultation would have been $335^{\circ} 2$. The least-squares fit to the data gave $331.5 \pm 6.2$ (see Table IV). This remarkable agreement lends credence not only to the assertion of a small obliquity, but also to the validity of the elliptical limb profile.

One can further ask whether the degree of oblateness observed is in line with expectations. It is first necessary, however, to compute the mean density of Ceres. Schubart (1974) has determined the mass of Ceres to be $(5.9 \pm 0.3) \times 10^{-10} \mathrm{M}_{\odot}$ based on the gravitational interaction of this asteroid with Pallas. Assuming Ceres to be an oblate spheroid whose polar and equatorial radii are respectively equal to the semiminor and semimajor axes of the fitted ellipse in Figure 5, the mean density of Ceres is found to be $2.7 \mathrm{gm} / \mathrm{cm}^{3}$, with an uncertainty of $5 \%$. The uncertainty in density is due almost entirely to the uncertainty in the asteroid's mass. This value for Ceres' density is identical to within the quoted uncertainties with•the density of Pallas (Millis and Elliot, 1979), the only other minor planet whose mass and diameter are sufficiently well determined to permit computation of a reliable density. It appears that Ceres, like Pallas, is primarily rocky in composition.

Dermott (1979) has noted that, for a rotationally distorted body whose surface is in hydrostatic equilibrium, the difference between the length of the equatorial radius, $a$, and the polar radius, $c$, is
given by

$$
\begin{equation*}
\mathrm{a}-\mathrm{c}=\frac{15}{16} \mathrm{HB} \mathrm{~B} . \tag{1}
\end{equation*}
$$

$B$ is effectively the mean radius of the body and for a homogeneous object $H=1$. The quantity $\beta$ is given by

$$
\begin{equation*}
\beta=\Omega^{2} / \pi G\langle\rho\rangle, \tag{2}
\end{equation*}
$$

where $\Omega$ is the asteroid's rotational angular velocity, $G$ is the gravitational constant, and $\langle\rho\rangle$ is the mean density of the body. Taking Ceres' rotational period of $0.37812 \pm 0.00004$ day from Tedesco et al. (1983) and the density and mean radius from the present paper, we find the predicted difference between the equatorial radius and the polar radius to be $29 \pm 1 \mathrm{~km}$. The observed difference from this paper is $26 \pm 5 \mathrm{~km}$, which is consistent with the assumption that Ceres is basically homogeneous (i.e., it is not strongly differentiated) and that its surface has achieved, at least on a global scale, a state of hydrostatic equilibrium. The uncertainty in the measured a-c, however, is too large to prove that $H=1$.

On a local scale the residuals listed in Table III indicate definite departures from the mean equilibrium figure of up to about 10 km . To produce the observed maximum residuals through observational error would require that the reported times of immersion and emersion be in error by on order of 1 second or that the observing site locations be incorrect by as much as 10 km or a combination of the
above. We believe that site locations in all cases are known to better than 1 km unless large systematic errors are present in modern topographic maps. The timing uncertainty in our observations is in most cases 0.1 second or less, although in a few instances it may be as much as 0.5 second. In any case, we are convinced that real irregularities in the limb profile of Ceres having a scale of a few kilometers are seen in the occultation data. Limb features of similar magnitude have been detected in other well-observed asteroid occultations (e.g., Millis et al., 1981). Dermott has stated that a C-type asteroid like Ceres if composed of carbonaceous chondritic material would not be expected to have the structural strength to support topography of greater than about 1 km over a long period of time. The implications of the observed limb irregularities, therefore, deserve further exploration.

It is of interest to compare the occultation diameter of Ceres (i.e., the effective diameter or the diameter of a circle having the same cross-sectional area as the actual elliptical profile) with previous estimates. The various earlier determinations are given in Table VI. Those values are to be compared with $932.6 \pm 5.2 \mathrm{~km}$ derived from the occultation observations. Note that only the determinations based on infrared radiometry are within their quoted 1 uncertainty of the correct value.

Based on Ceres' absolute $V$ magnitude of $3^{m} .61$ from Tedesco et al. (1983) and the occultation diameter, the visual geometric albedo of Ceres is found to be 0.070. In this regard Ceres is typical of other dark, C-type asteroids.

## SUMMARY

The more important physical characteristics of Ceres derived from the observations of the occultation of $B D+80471$ are summarized in Table VII. The apparent profile of Ceres seen at the time of the occultation is consistent with that of a homogeneous object of low obliquity whose surface is generally in hydrostatic equilibrium. Irregularities having a vertical scale of a few kilometers were seen on the limb of this minor planet.

## ACKNOWLEDGMENTS

The reader no doubt has surmised that the effort devoted to identifying, predicting, and observing the 13 November 1984 occultation by Ceres was substantial. We were assisted in this research project by many individuals whose names do not appear in the author list. Space is not available to acknowledge all of these contributions individually, but they are nonetheless appreciated.

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TABLE I. Observing Sites

|  | Site No. | Site Name | W Longitude | Latitude | Altitude (meters) | Observers | Telescope (meters) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | Melbourne, Florida | $5^{\text {h }} 22^{\text {m }} 32$ S 71 | +28.03'30:7 | 12 | A. Smith <br> D. Kornbluh <br> K. Izor | 0.41 |
|  |  | Culiacan, Mexico | 70935.3 | +24 4810 | 147 | W. Hubbard <br> R. Goff | 0.36 |
|  | 3 | Chicalahua, Mexico | 70723.38 | +24 1128.02 | 380 | L. Wasserman <br> R. Nye | 0.36 |
|  | 4 | Burns Lake, Florida | 52455.4 | +25 5342 | 3 | M. Mooney <br> S. Ireland <br> D. Leibow | 0.28 |
| $\bigcirc$ | 5 | Oasis, Florida | 52488.35 | +25 5228.4 | 0 | G. Schneider | 0.36 |
|  | 6 | South Miami, Florida | 52112.13 | +25 4237.0 | 3 | D. Parker <br> W. Douglass <br> J. Beish | 0.36 |
| 品 | 7 | Mazatlan, Mexico | $7 \quad 541.5$ | +23 1155.0 | 79 | H. Reitsema <br> B. Marcialis | 0.36 |
|  | 8 | No Name Key, Florida | 52516.45 | +24 4149.8 | 0 | J. Klavetter <br> H. Povenmire <br> L. Reed | 0.36 |
|  | 9 | Zacatecas, Mexico | 6509.7 | +22 4356.0 | 2714 | B. Zellner | 0.50 |
|  |  | San Blas, Mexico | 718.5 | +21 3252 | 1 | R. Millis <br> W. Osborn | 0.36 |
| z | 11 | Great Exuma, Bahamas | 536.00 | +23 3025.2 | 0 | K. Meech <br> E. Strother | 0.36 |
|  | 12 | Chamela, Mexico | 7009.77 | +19 278.57 | 40 | M. A'Hearn <br> R. Schnurr <br> S. Jones | 0.36 |
|  | 13 | Providenciales, Caicos Islands | 44902.4 | +214635 | 3 | T. Oswalt <br> J. Rafert | 0.36 |

## TABLE II

## Photometric Parameters

| . | V | B-V | U-B |
| :---: | :---: | :---: | :---: |
| Ceres | 7.13 | 0.72 | 0.42 |
| $+8 \cdot 471$ | 10.31 | 0.50 | 0.05 |
|  | V | $\underline{U}$ |  |
| $\Delta m_{\text {immersion }}$ | $0 \cdot 0.057$ | 0.096 |  |
| Predicted signal-to-noise ${ }^{3}$ | $18^{1}$ | $44^{2}$ |  |
| Required signal-to-noise ${ }^{3}$ | 4 | 4 |  |

${ }^{1}$ Assuming a $35-\mathrm{cm}$ aperture telescope and a sea level site (i.e., a "worst-case" situation).
${ }^{2}$ Assuming a $35-\mathrm{cm}$ aperture telescope and an observing site at 2000 meters altitude (i.e., a best-case situation).
${ }^{3}$ Quoted signal-to-noise ratios are for 1 -second integrations. See Millis and Elliot (1979).

TABLE III
Observed Times of Immersion and Emersion

| Site Name | Immersion U.T. |  |  | $\begin{aligned} & \text { Emersion } \\ & \text { (U.T.) } \end{aligned}$ |  |  | Residuals (km) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Circle Fit | Ellipse Fit |  |
|  |  |  |  | Imm. | Em. | Imm. | Em. |
| Melbourne, Florida |  | $\mathrm{h}_{42}$ | $2^{m} 6501 \pm 0.15$ |  |  |  |  | ${ }_{43}$ | $3^{m}{ }^{\text {s }} 855 \pm 0.15$ | -9.26 | $-5.43$ | 8.13 | -9.50 |
| Culiacan, Mexico |  |  | $513.5 \pm 0.3$ |  |  |  | 4 | 46 | 20.3+0.5 | $-4.42$ | 7.83 | 5.19 | 1.53 |
| Chicalahua, Mexico |  |  | $510.6 \pm 0.1$ | 4 | 46 | $20.1 \pm 0.1$ | -15.14 | 17.13 | -10.42 | 11.32 |
| Burns Lake, Florida |  |  | $205.44 \pm 0.13$ |  |  | - | -4.61 | - | -3.25 | - |
| Oasis, Florida |  |  | $203.82 \pm 0.04$ | 4 |  | $14.46 \pm 0.04$ | -2.50 | 10.09 | -1.38 | 5.21 |
| South Miami, Florida |  |  | $158.5 \pm 0.1$ | $(4$ |  | 13.8 $\pm 0.1$ )* | -4.16 | - | -4.44 | - |
| Mazatlan, Mexico |  |  | $507.6 \pm 0.5$ | 4 |  | $17.9 \pm 0.3$ | 6.30 | -8.67 | 3.73 | -11.79 |
| No Name Key, Florida |  |  | $206.53 \pm 0.04$ |  |  | - | 6.43 | - | 0.48 | - |
| Zacatecas, Mexico |  |  | $443.0 \pm 0.1$ | 4 |  | $52.4 \pm 0.2$ | 11.62 | 1.41 | 4.46 | 1.84 |
| San Blas, Mexico |  |  | $506.9 \pm 0.1$ | 4 |  | $09.9 \pm 0.1$ | 3.93 | -5.88 | -6. 21 | -0.98 |
| Great Exuma, Bahamas |  |  | $128.99 \pm 0.01$ | 4 | 42 | $30.27 \pm 0.014$ | 13.34 | -3.24 | 5.21 | 2.85 |
| Chamela, Mexico |  |  | $526.20 \dagger$ | 4 |  | $52.48 \dagger$ | 6.12 | -12.47 | 3.34 | -1.53 |
| Providenciales, Caicos Islands | 4 |  | $120.19 \pm 0.11$ | 4 |  | $42.0 \pm 0.2$ | 6.82 | -17.30 | 4.63 | -7.06 |
|  | (4 $4150.90 \pm 0.18$ )* |  |  |  |  |  |  |  |  |  |
| rms residual per degree of freedom (km) |  |  |  |  |  |  | 9.84 |  | 6.74 |  |

*Originally reported time of emersion. This time has been rejected, as discussed in the text. TThe uncertainty in the duration of the occultation at Chamela is $\pm 0.1$ second. However, the uncertainty in the absolute timing at this site may be as much as $\pm 1.0$ second.

TABLE IV
Results of Least-Squares Fits to the Data

Circular limb profile:

| Mean radius | $471.6 \pm 2.2 \mathrm{~km}$ |
| :--- | :--- |
| Right Ascension correction | $-0.00890 \pm 0.00012 \mathrm{sec}$ |
| Declination correction | $0.181 \pm 0.003$ |

Elliptical limb profile:

| Semimajor axis | $479.6 \pm 2.4 \mathrm{~km}$ |
| :--- | :--- |
| Semiminor axis | $453.4 \pm 4.5 \mathrm{~km}$ |
| Position angle of north pole of | $331.5 \pm 6: 2$ |
| semiminor axis |  |
| Right Ascension correction | $-0.00884 \pm 0.00008 \mathrm{sec}$ |
| Declination correction | $0.1 .76 \pm 0 . .003$ |
| Equivalent radius $(\sqrt{\mathrm{ab}})$ | $466.3 \pm 2.6 \mathrm{~km}$ |

## TABLE V

Astrometric, Geocentric Ephemeris of Ceres


## TABLE VI

| Technique | Diameter (km) | Reference |
| :---: | :---: | :---: |
| Filar micrometer | $781 \pm 87$ | Barnard (1895) |
| Lunar occultation | $1200 \pm 250$ | Dunham et al. (1974) |
| Polarimetry | $1016 \pm 50$ | Morrison and Zellner (1979) |
| Radio | $818 \pm 82$ | Johnston et al. (1982) |
| Infrared | $953 \pm 50$ | Brown et al. (1982) |
| Infrared | $962 \pm 30$ | Lebofsky et al. (1984) and Lebofsky (1985). |

## TABLE VII

## Ceres Occultation Results

| Equatorial radius | $479.6 \pm 2.4 \mathrm{~km}$ |
| :--- | :---: |
| Polar radius | $453.4 \pm 4.5 \mathrm{~km}$ |
| Effective diameter | $932.6 \pm 5.2 \mathrm{~km}$ |
| Mean density | $2.7 \pm 0.14 \mathrm{gm} / \mathrm{cm}^{3}$ |
| Visual geometric albedo | 0.070 |

FIGURE CAPTIONS


#### Abstract

Figure 1. The predicted ground track of the 13 November 1984 occultation of $B D+80471$ by Ceres.


Figure 2. The lightcurve of the occultation by Ceres observed at San Blas, Mexico (Site 10, Table I). The observations were made through a B filter of the UBV system. Arrows mark the times of immersion and emersion.

Figure 3. A least-squares fit of a circular limb profile to the originally reported data set. Discrepant points are marked $A, B$, and C. These points are discussed in detail in the text.

Figure 4. A circular limb profile fitted by least squares to the adopted data set. Parameters of the solution are given in Table IV.

Figure 5. An elliptical limb profile fitted by least squares to the adopted data set. Parameters of the solution are given in Table IV.
Figure 6. The composite lightcurve of Ceres observed at LowellObservatory on 18 and 20 November 1984. The arrow indicates the phaseof the lightcurve at which the occultation occurred. Observations from20 November are marked with dots ( ) , 18 November with squares ( $\square$ ),and the first. eight points of 18 November shifted one period ( $0^{d_{37}} 812$ )
with diamonds (>)



Figure 2.


Figure ${ }^{3}$


Figure 4.


Figure 5.


Figure 6.

