535

COSMIC RAY ANISOTROPIES AT HIGH ENERGIES

N. J. Martinic, A. Alarcon, F. Teran
Chacaltaya Cosmic Ray Laboratory
University of La Paz

ABSTRACT
The directional anisotropies of the energetic cosmic ray gas due to the relative motion between the observers frame and the one where the relativistic gas can be assumed isotropic is analyzed. The radiation fluxes formula in the former frame must follow as the Lorentz invariance of $d\underline{p}/E$, where $\underline{p}$, E are the 4-vector momentum-energy components; $d\underline{p}$ is the 3-volume element in the momentum space. The anisotropic flux shows in such a case an amplitude, in a rotating earth, smaller than the experimental measurements from say, EAS-arrays for primary particle energies larger than 1.E(14) eV. Further, it is shown that two consecutive Lorentz transformations among three inertial frames exhibit the violation of $d\underline{p}/E$ invariance between the first and the third systems of reference, due to the Wigner rotation. A discussion of this result in the context of the experimental anisotropic fluxes and its current interpretation is given.

1. Introduction

Using the inertial frames S, S'; the so-called Compton Getting anisotropies can be deduced as follows. (Cf' for instance Gerantos and Martinic, 1977). The Lorentz transformation is written:

$$p_1 = p \sqrt{A}, \qquad E_1 = \gamma p c\ (\zeta - \beta\mu), \qquad \mu_1 = \gamma(\mu - \beta\zeta)/\sqrt{A}; \qquad (1)$$

with $A = 1 + \gamma^2(\beta^2\zeta^2 + \beta^2\mu^2 - 2\beta\zeta\mu)$, γ the Lorentz factor and β the relative speed between the frames, in c-units; $\zeta = E/pc$, and μ (or μ_1) the cosine of the inclination angle of the $\underline{p}$ (or $\underline{p}_1$) momentum with respect to the x-axes of both frames. In Eq. (1) the velocity $\underline{\beta}$ was taken parallel to the x-axes of both systems of reference; E_1(or E) is the total energy of the particle. Notice that $d\underline{p} = p^2 dp\ d\mu 2\pi$, and that it can be checked that $d\underline{p}/E = d\underline{p}_1/E_1$.

$$d\phi_\alpha = f(\underline{p}, E(p), \underline{r}, t)\ p_\alpha d\underline{p}/E \qquad (2)$$

where f is the scalar distribution function in the 8-dimensional space. Further, (see for instance Fisk et al., 1973) it can be shown the $fp^2 = J$, where $J(p, E(p)...)$ is the cosmic ray intensity, i.e. the number of particles per unit of time, solid angle, per unit of surface (normal to the flux direction) and per energy window: E, E+dE. Notice in Eq. (2) that f and $d\underline{p}/E$ are

scalars; and that $d\phi_\alpha$ can be written as $(d\underline{\phi}, d\phi_4)$ and parallel to $(\underline{p}, E)$. The Lorentz transformation can be applied to $d\Phi = |d\underline{\phi}|$:

$$d\Phi_1 = d\Phi \ \sqrt{A} \tag{3}$$

between S and S' frames. Moreover, in polar coordinates, $d\underline{p}/E = p \ dE \ d\mu \ 2\pi$ and, as mentioned, is invariant. Replacing in Eq. (2)

$$f_1 p_1 p_1 dE_1 d\mu_1 = f p \sqrt{A} \ p \ dE \ d\mu \tag{4}$$

and, for $f p^2 = J$

$$J_1 dE_1 d\mu_1 = J\sqrt{A} \ dEd\mu \tag{5}$$

From Eqs. (1) it can be calculated the Jacobian: $dE_1 d\mu_1 = dEd\mu/ \ \sqrt{A}$ and

$$J_1/A = J \tag{6}$$

which can be found in the references as $J_1/p_1^2 = J/p^2$.

From Eq. (6), assuming that J_1 is isotropic one can obtain the cosmic ray anisotropic intensity J, $J = J_1 (p,E(p)) \ \{1+(2+\nu)\beta\zeta\cos\theta + O(\beta^2\zeta^2)\}$, where ν is the exponent of the isotropic power law intensity of the energy. The drawback of expression (6), for energies such that $\zeta=1$ (pc $\simeq$ E), $\nu = O(3)$; and relative speeds of 300 km/s (typical figure for the peculiar velocity of the solar system with respect to distant cosmic ray sources) is that one obtains values of less than 1% for the amplitude of the anisotropy; this value is small compared to experimental measurements for energies larger than 1.E(14) eV. At these energies the amplitude of the anisotropy exhibit a law proportional to $\sqrt{E}$, reaching about 100% for energies of the order of 1.E(20) eV. (cf Linsley, 1983). In consequence, either it should be looked for dynamical sources for the measured anisotropies and discard formula (6), or make a critical appraisal of the conditions of validity of the deductions which led to Eq. (6).

2. The Wigner Effect

Now we use three inertial frames: S, S' and S" which exhibit parallel axes: The S with S' and the S' with S"; one should not be tempted to extrapolate the transitivity property and assume the parallelism among all the former frames. The relative velocity $\underline{\beta}_1$ between S and S' is take parallel to the x-axes of both frames; and $\underline{\beta}_2$ the one between S' and S" has an inclination angle α with respect to the x-axes of the latter frames. Say, S sees S' with the velocity $\underline{\beta}_1$ towards the right side; S' sees S" towards the right and upwards with the speed β_2. Any frame that sees another one to its left shall

see it with reverse sense i.e. the S" sees the S' with a left-downward motion and with speed β_2.

Our aim is to use the Lorentz transformations between S and S" in order to check the invariance of $d\underline{p}/E$ between these frames. It can be expressed that S" shall move with a velocity $\underline{\beta}$ and inclination angles (as seen from S") measured with respect to its x-axis. The ε angle shall be ambigously defined in the S frame due to the Wigner rotation. The latter frame shall rotate an angle $\delta = (\gamma_1-1)\,|\underline{\beta}_1\times\underline{\beta}_2|/\beta^2 - O(\beta_2^2)$ as seen from S"; the axis of rotation is perpendicular to the $\underline{\beta}_1,\underline{\beta}_2$ plane, at least for small β_2. Let us calculate two successive Lorentz transformations: we call $\underline{p}_i$, E_i (i=0, 1, 2) the four-vector momentum-energy in every frame S, S', and S" respectively. The two transformations are (i=0, 1):

$$p_{i+1} = p_i\sqrt{A_i}, \quad E_{i+1} = \gamma_{i+1}\rho_i c(\zeta_i-\beta_{i+1}\mu_i), \quad \mu_{i+1} = \gamma_{i+1}(\mu_i-\beta_{i+1}\zeta_i)/\sqrt{A_i} \qquad (7)$$

In the set of Eq. (7), $A_i = 1+\gamma_{i+1}^2(\beta_{i+1}^2\zeta_i^2+\beta_{i+1}^2\mu_i^2-2\beta_{i+1}\zeta_i\mu_i)$, where $\mu_0 = \cos\theta$ and $\mu_1 = \cos\theta_1$ for i=0; and $\mu_1 = \cos(\theta_1-\alpha)$ and $\mu_2 = \cos(\theta_{2-\alpha})$ for i = 1. In this two sets of transformations θ_1 is the angle measured with respect to the x-axis of its frame, $\zeta_i = E_i/p_i c$. The first equation of Eqs. (7) is the relativistic cosine theroem of vector addition of momenta $\underline{p}_i$ and $\underline{\beta}_{i+1}mc^2$ (m is the rest mass of the particle). Non relativistically, when $\gamma_i = 1$ and $\gamma_i^2\beta_i^2 = 0$, we obtain the cosine formula for vector addition. From the sets of Eqs. (7) one obtains (we drop the 0-subscripts):

$$p_2 = p\sqrt{\{1+\gamma^2(\beta^2\zeta^2+(\underline{\beta}\underline{p}/p)^2-2(\underline{\beta}\underline{p}/p)\zeta)\}} = p\sqrt{A} \qquad (8)$$

and

$$E_2 = \gamma pc\{\zeta-\underline{\beta}\underline{p}/p\}, \qquad (9)$$

where $\gamma = \gamma_1\gamma_2(1+\underline{\beta}_1\underline{\beta}_2)$, and

$$\underline{\beta} = \{\underline{\beta}_2+\underline{\beta}_1/\beta_1\{\gamma_1\beta_1+(\gamma_1-1)\underline{\beta}_2\underline{\beta}_1/\beta_1\}\}/\{\gamma_1(1+\underline{\beta}_1\underline{\beta}_2)\} \qquad (10)$$

that is the relativistic addition of velocities $\underline{\beta}_1$ and $\underline{\beta}_2$. So far the Wigner effect is absent, specially if the angle relations were

$$\cos(\theta_2-\varepsilon) = \gamma\{\cos(\theta-\varepsilon)-\beta\zeta\}/\sqrt{A} = \gamma\{\cos\theta\cos\varepsilon+\sin\theta\sin\varepsilon-\beta\zeta\}/\sqrt{A} \qquad (11)$$

however one obtains

$$\cos(\theta_2-\alpha) = \gamma\{\frac{\cos\alpha+\beta_1\beta_2}{1+\beta_1\beta_2\cos\alpha}\cos\theta + \frac{\gamma_1^{-1}\sin\alpha\sin\theta}{1+\beta_1\beta_2\cos\alpha} - \frac{\beta_2+\beta_1\cos\alpha}{1+\beta_1\beta_2\cos\alpha}\zeta\}/\sqrt{A} \qquad (12)$$

if one writes

$\cos\alpha_1 = (\cos\alpha + \beta_1\beta_2)/(1-\beta_1\beta_2\cos\alpha)$, one recognizes a similar expression as the aberration angle formulae for photon transformation. Using the α_1 angle Eq. (12) transforms into (we call α, α_2)

$$\cos(\theta_2-\alpha_2) = \gamma \left\{\cos\alpha_1\cos\theta + \frac{\sqrt{(1-\beta_1^2)}\ \sin\alpha_1\sin\theta}{\sqrt{(1-\beta_1^2\beta_2^2)}} - \underline{\beta}^\star\underline{\beta}_2/\beta_2\ \zeta\right\}/\sqrt{A} \tag{13}$$

where $\underline{\beta}^\star$ is obtained from Eq. (10) except that the 1, 2 - subscripts of the velocities has been interchanged. It can be seen that $\underline{\beta}$ and $\underline{\beta}^\star$ have the same mudulus but they are not parallel: their difference of inclinations is the Wigner rotation. Notice that Eqs. (11) and (13) are the same equation when $\underline{\beta}_1$ and $\underline{\beta}_2$ are parallel: The Wigner rotation as well as the non-communtativity of the addition of velocities disappear. Although Eqs. (8) and (9) assures the invariance of $E^2-p^2c^2$, between the S and S" frames, one cannot guarantee the invariance of $d\underline{p}/E$: to do that one needs the Eq. (11) in addition of Eqs. (8) and (9).

3. Discussion

In order to investigate the effects of the Wigner rotation we have introduced three inertial frames. The physical picture can be seen as follows: The S; frame is the one where the cosmic ray is isotropic. The S" frame is the observers one, say, the terrestrial EAS detectors. The introduction of the S frame seems to be necessary in order to put up a scenario to allow in S' cosmic ray isotropic fluxes; further, the relative velocity between the former frames can be considered as relativistic i.e. $\beta_1 \simeq 1$. The future (as shall be detected in S" later on) cosmic ray gas, in S can be thought as (may be) non-relativistic and anisotropic. The velocity dispersion in S is a consequence of the acceleration mechanisms of these particles and cannot be pin-pointed to a point source in a given $\underline{\beta}_1$ direction. Besides, at every region of the sky, as seen by S', we have equivalent S frames to guarantee the isotropic fluxes.

We need flux transformations between frames such as the S and S" that takes into account the Wigner effect.

References
Geranios, A., Martinic, J. N., 1977, 15th ICRC, Plovidiv, 11, 192.
Fisk, L. A., Forman, M. A., Axford, W. I., J. Geophys. Res. 78, 995, 1973.
Lindsley, J., 1983, 18th ICRC, Bangalore, Rapporteur paper.