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Effect of Aspect Ratio on Sidewall Boundary-Layer Influence in Two-Dimensional Airfoil Testing

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SUMMARY

The effect of the sidewall boundary-layers in airfoil testing in two-dimensional wind tunnels is investigated. The non-linear crossflow velocity variation induced because of the changes in the sidewall boundary-layer thickness is represented by the flow between a wavy wall and a straight wall. Using this flow model, a correction for the sidewall boundary-layer effects is derived in terms of the undisturbed sidewall boundary-layer properties, the test Mach number and the airfoil aspect ratio. Application of the proposed correction to available experimental data showed good correlation for the shock location and pressure distribution on airfoils.

INTRODUCTION

A simplified analysis in the form of a modified Prandtl-Glauret rule to account for the attached sidewall boundary-layer effects was proposed by Barnwell (Ref. 1). This was later extended to transonic speeds using the von Karman similarity parameter by Sewall (Ref. 2). The Barnwell-Sewall correction for the test Mach number has been found to be quite effective in giving good agreement between the measurements and the predictions of the Grumfoil computer code for various airfoils tested in the Langley 0.3-m Transonic Cryogenic Tunnel (Ref. 3-5). From these studies, it appears that the change in the sidewall boundary-layer characteristics due to the airfoil pressure field can be a significant source of blockage correction, particularly at transonic speeds.

The Barnwell-Sewall correction has been derived under certain assumptions of simplified boundary-layer treatment and linear variation of the crossflow velocity across the width of the tunnel. These assumptions imply

that the airfoil chord is sufficiently large so that the effect of the side-wall boundary-layers can be considered to be quasi- one dimensional. Barnwell has shown recently (Ref. 6) that the linear crossflow assumption is justified provided $(4\delta*/b)(b/c)^2$ is small. Hence, this assumption is likely to become less accurate when the width of the tunnel is much larger than the airfoil chord. (i.e., for high aspect ratio models).

Since the inapplicability of the Barnwell's correction to large aspect ratio models is mainly due to the linear crossflow assumption, it appears that the validity of this correction can be improved if a more realistic assumption for the crossflow velocity variation is made. In the present report, this has been attempted by considering the compressible flow between a straight wall and a wavy wall. For this problem, the ratio of the crossflow velocity at any point in the flow to that at the wall is only a function of the distance from the wavy wall. It is assumed that the crossflow velocity variation along the airfoil span with sidewall boundary-layer effects can be represented by this wavy wall flow model. Using this approach, a modification to the Barnwell-Sewall correction is proposed to account for the airfoil aspect ratio.

NOMENCLATURE

b	Semi-width of the tunnel
c	Airfoil chord
c_ℓ	Lift coefficient
C_n	Normal force coefficient
C_p	Pressure coefficient
H	Shape factor of the sidewall boundary-layer
k	Constant (See Eq. 16)

k_1	Constant (See Eq. 5)
k_2	Constant ($=2\pi\beta\ b/\ell$)
ℓ	Wavelength of the wavy wall
ℓ_s	Length of source distribution
M	Mach number
n	Coordinate normal to the wavy wall (Fig. 2)
U	Velocity
u	Perturbation velocity in the x-direction
v	Perturbation velocity in the y-direction
w	Perturbation velocity in the z-direction
w_0	Crossflow velocity at the sidewall
x	Streamwise coordinate
x_s	Shock location on airfoil surface
y	Normal coordinate
z	Spanwise coordinates
δ^*	Sidewall boundary-layer displacement thickness
ϵ	Amplitude of the wavy wall
β	Compressibility factor
ϕ	Velocity potential for the 2-D wavy wall flow (Fig. 2)
ϕ_w	Velocity potential corresponding to wind tunnel flow

Subscripts

c	Corrected values
e	Conditions at the edge of boundary-layer
exp	Experimental values
∞	Free stream condition

ANALYSIS

For the steady subsonic flow over an airfoil in a nominally two-dimensional wind tunnel of width 2b (Fig. 1), the development of the boundary-layer on the sidewalls introduces a spanwise velocity across the width of the tunnel. This spanwise velocity is maximum at the sidewall, and zero at the mid-plane because of the symmetry. In general, the flow in the tunnel tends to become three-dimensional and the corresponding small perturbation equation for the flow in the tunnel is

$$(1-M_\infty^2)\ \phi_{w,xx} + \phi_{w,yy} + \phi_{w,zz} = 0 \qquad (1)$$

The corresponding boundary condition for the spanwise velocity is imposed at the sidewall ($z = \pm b$)

$$\frac{\partial \phi_w}{\partial z}\bigg|_{z=\pm b} = \mp\ U_e\ \frac{\partial \delta^*}{\partial x} \qquad (2)$$

where U_e is the velocity at the edge of the boundary-layer. Following Barnwell, the rate of boundary-layer growth on the sidewalls can be approximated by

$$\frac{\partial \delta^*}{\partial x} = -\ \frac{\delta^*}{U_e} \left(2 + \frac{1}{H} - M_e^2\right) \frac{\partial U_e}{\partial x} \qquad (3)$$

In arriving at equation (3), it is assumed that the sidewall boundary-layer can be approximated by a flat plate boundary-layer with its equivalent

length much longer than the airfoil chord c, so that the change in the side-wall boundary-layer thickness is predominantly due to model induced chord-wise pressure gradients. In airfoil tests, the interest is often confined to pressure measurements over the midspan region of the airfoil. The influence in this region due to sidewall boundary-layers is an integrated effect of what is happening at the airfoil/sidewall junction and will be relatively insensitive to the details of the boundary-layer development at the sidewall. Hence, instead of solving the complicated problem of three-dimensional boundary-layer development at the airfoil/sidewall junction, Barnwell combined equations (2) and (3), and assumed a linear variation of the spanwise velocity between the sidewall and the mid-span. This assumption of linear variation implies that the change in the streamtube area is gradual so that the sidewall boundary-layer effect can be treated one-dimensionally (Ref. 8).

In the present treatment, the effective shape of the sidewall is represented by a wavy wall of amplitude ϵ and wave length ℓ. It can be argued that the values of ϵ and ℓ will be related in some way to the sidewall boundary-layer thickness and the airfoil chord, respectively. While it may be difficult to identify a priori the exact nature of dependence, it is hoped that this wavy wall representation will at least provide an insight into the variation of sidewall boundary-layer effects with changes in ϵ and ℓ, which are in effect equivalent to changing the sidewall boundary-layer thickness and the airfoil aspect ratio.

For the two-dimensional wavy wall model shown in Figure 2, the perturbation velocity potential can be written as (Ref. 9)

$$\phi = k_1 e^{-k_2 n/b} \left[1 + e^{2k_2(n/b-1)} \right] \sin \frac{2\pi x}{\ell} \tag{4}$$

where
$$k_1 = \frac{U_\infty}{\beta} \frac{\varepsilon}{1 - e^{-2k_2}} \tag{5}$$

and
$$k_2 = \frac{2\pi \beta b}{\ell} \tag{6}$$

By differentiating equation (4), the normal velocity variation can be written as

$$\frac{\partial \phi}{\partial n} = \frac{k_1 k_2}{b} \left[-e^{-k_2 n/b} + e^{-2k_2} \cdot e^{k_2 n/b} \right] \sin \frac{2\pi x}{\ell} \tag{7}$$

At the wavy wall (n=0), the normal velocity is given by

$$\left(\frac{\partial \phi}{\partial n} \right)_{n=0} = \frac{k_1 k_2}{b} \left(e^{-2k_2} - 1 \right) \sin \frac{2\pi x}{\ell} \tag{8}$$

At the straight wall, the normal velocity is zero. Therefore,

$$\left(\frac{\partial \phi}{\partial n} \right)_{n=b} = 0 \tag{9}$$

Using equations (7) and (8), the ratio of the normal velocity at any point to that at the wavy wall can be written as

$$\frac{\partial\phi/\partial n}{(\partial\phi/\partial n)_{n=0}} = \frac{w}{w_o} = \frac{e^{-k_2 n/b} - e^{-2k_2}e^{k_2 n/b}}{1 - e^{-2k_2}} \tag{10a}$$

or
$$\frac{w}{w_o} = \frac{\text{Sinh}\{k_2(1 - n/b)\}}{\text{Sinh } k_2} \tag{10b}$$

Writing $z = b - n$, equation (10b) can be written as

$$\frac{(\partial\phi/\partial z)}{(\partial\phi/\partial z)_{z=b}} = \frac{\text{Sinh}(k_2 z/b)}{\text{Sinh } k_2} \tag{11}$$

Equation (11) gives a relation for the variation of the spanwise velocity which reduces to a linear relationship for small values of the parameter k_2. This situation occurs either when the Mach number approaches unity or the wave length (alternatively, the airfoil chord) is large compared to the width of the tunnel.

The relation derived in equation (11) can be used in an empirical manner to represent sidewall boundary-layer effects by using the value of the spanwise velocity induced due to boundary-layer at the sidewall. Combining equations (2) and (11),

$$\frac{\partial\phi_w}{\partial z} = - U_e \frac{\partial\delta^*}{\partial x} \frac{\text{Sinh}(k_2 z/b)}{\text{Sinh } k_2} \tag{12}$$

$$= \delta^*\left(2 + \frac{1}{H} - M_e^2\right)\frac{\text{Sinh}(k_2 z/b)}{\text{Sinh } k_2} \phi_{w,xx} \tag{13}$$

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Differentiating equation (13) with respect to z, it follows

$$\frac{\partial^2 \phi_w}{\partial z^2} = \frac{k_2 \delta^*}{b} \left(2 + \frac{1}{H} - M_e^2\right) \frac{\text{Cosh } (k_2 z/b)}{\text{Sinh } k_2} \frac{\partial^2 \phi_w}{\partial x^2} \tag{14}$$

Combining equations (1) and (14), the flow in the wind tunnel with sidewall boundary-layers can be approximated by

$$(1 - M_\infty^2 + k) \phi_{w,xx} + \phi_{w,yy} = 0 \tag{15}$$

where

$$k = \frac{\delta^*}{b} \left(2 + \frac{1}{H} - M_e^2\right) \left[k_2 \frac{\text{Cosh } (k_2 z/b)}{\text{Sinh } k_2}\right] \tag{16}$$

At the median section (z=0), equation (16) reduces to

$$k = \frac{\delta^*}{b} \left(2 + \frac{1}{H} - M_e^2\right) \left(\frac{k_2}{\text{Sinh } k_2}\right) \tag{17}$$

The factor $k_2/\text{Sinh } k_2$ depends on the test Mach number and the airfoil aspect ratio. The form of the equation (15) is the same as that originally proposed by Barnwell except that in the present case the definition of the term k is different as given by equation (16). It has been shown in Reference (8), that the small disturbance equation (15) representing the sidewall effects can be interpreted as causing changes in both the test Mach number and the airfoil thickness. The modification to account for the transonic effects are given in References (2) and (8). Hence, the correction to the test Mach number and forces can be done in a similar

manner using the value of k defined by equation (17) which also accounts for the airfoil aspect ratio.

The corresponding expressions for the corrected Mach number (M_c) and the corrected pressure coefficient $(C_{p,c})$ are given by

$$\frac{1 - M_\infty^2 + k}{M_\infty^{4/3}} = \frac{1 - M_c^2}{M_c^{4/3}} \tag{18}$$

$$C_{p,c} = \left(\frac{M_\infty^2}{M_c^2}\right)^{1/3} C_p \tag{19}$$

RESULTS AND DISCUSSION

The present analysis allows for the nonlinear variation of the spanwise velocity as compared to Barnwell's assumption of linear variation which is strictly correct for narrow tunnels or low aspect ratio models. This is demonstrated in Figure 3 by plotting the ratio of the spanwise velocity (w) at any spanwise station to that at the wall (w_0), for values of $k_2 = 0, 2,$ and 5. Except for small values of k_2 ($= 2\pi\beta b/\ell$), the variation tends to become non-linear. This non-linear variation introduces non-uniform side-wall boundary-layer effects across the span of the airfoil. The magnitude of the sidewall boundary-layer effect is given by the gradient of the span-wise velocity (Figure 4). For $k_2 = 0$, corresponding to Barnwell's assumption of linear variation of the spanwise velocity, the gradient is uniform across the width of the tunnel. With increasing k_2, the gradient increases near the wall but reduces rapidly towards midspan to the value given by $k_2/\text{Sinh}(k_2)$. The variation of the gradient at the mid-span is shown in Figure 5. This shows that near the mid-span the effect of the wavy

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wall is to reduce the crossflow velocity gradient with decrease in the wave length (or increasing aspect ratio of the airfoil).

It must be noted that the present method is based on two-dimensional considerations and it represents conservative values of the aspect ratio correction factor. It is likely that the three dimensional nature of the flow at the airfoil/sidewall junction will further alleviate the effects near the midspan.

When applying the present aspect ratio correction, it is necessary to define what constitutes a typical length scale ℓ in terms of the airfoil chord c. This is examined by comparing the present results with some of the experimental data. Initially, the shock position correlation on a supercritical airfoil tested in the ONERA tunnel is attempted. The measured shock positions for two different sidewall boundary-layer thicknesses of $\delta^*/b = 0.023$ and .054 are shown in Figure 6a. The effect of applying the Barnwell-Sewall correction without accounting for the aspect ratio effects is shown in Figure 6b. It may be noticed that this method tends to over correct and the correlation is not entirely satisfactory. The effect of incorporating the aspect ratio correction is shown in Figures 6c and 6d, assuming $\ell = 2c$ and $\ell = c$, respectively. For this case, assuming the wave length to be equal to the airfoil chord appears to give better correlation.

Assuming $\ell = c$, the normal coefficient measurements (Ref. 2) on a supercritical airfoil in the Langley 6"x19" tunnel have been correlated (Figures 7a, b, and c). For this case, considering the scatter in the experimental data, the aspect ratio correction does not seem to significantly influence the correlation.

It may be noted that the aspect ratio of the models in the Onera tests

and the Langley tests were respectively 0.73 and 1. These tests do not represent a wide range of aspect ratios and it is difficult to generalize from these limited comparisons what is the best value for the representative length scale in terms of the airfoil chord.

Recently, pressure distribution measurements on two different chords of the Cast-10 airfoils (Ref. 4) were made in the Langley 0.3-m Transonic Cryogenic Tunnel over a wide range of Reynolds and Mach numbers. The aspect ratio of the models were 1.33 and 2.66, respectively. These test results demonstrated that on the higher aspect ratio model, the application of the Barnwell-Sewall method often over estimated the sidewall boundary layer effects. As mentioned in the introduction, it has been the experience with several airfoil tests in the TCT, the Barnwell-Sewall sidewall boundary-layer correction was adequate for the 6" chord (or aspect ratio of 1.33) model normally employed. The effect of applying the sidewall boundary-layer corrections with and without the aspect ratio effect for the pressure distribution on the higher aspect ratio (=2.66) model is shown in Figures 8a and 8b. For applying the aspect ratio correction, it has been assumed that $\ell = 2c$. This assumption is based on the fact that the effect of the airfoil on the sidewall boundary-layer is distributed over a distance of about twice the chord of the airfoil. From Figures 8a and 8b, it may be seen that the aspect ratio correction certainly improves the agreement between the measurements and the calculated pressure distribution using the Grumfoil code. For this aspect ratio, with the assumption of $\ell = 2c$, the correction to the test Mach number is negligible.

While there can be certain ambiguity about the extent of the aspect ratio correction required, it appears that the proposed correction will at

least provide a conservative estimate of the reduction in the sidewall boundary-layer effects on higher aspect ratio models. It must be noted that the present corrections account for only the blockage effects. Detailed measurements on the Cast-7 airfoil (Ref. 10) over a wide range of aspect ratios suggest that the downwash effects can be significant. However, considering the uncertainties in angle of attack in two-dimensional airfoil testing, the present correction is useful when making theoretical calculations of the pressure distribution with prescribed lift coefficient.

CONCLUSIONS

1. A correction for the sidewall boundary-layer effect in airfoil testing has been proposed taking into account the aspect ratio of the model.

2. The correction proposed, based on the flow between a wavy wall and a straight wall, shows significant reduction in sidewall boundary-layer effects with increasing aspect ratio of the model.

3. Comparison with the experimental data on shock location and pressure distribution on airfoils using the present correction gave good correlation.

4. In the limit of vanishing aspect ratio, the present correction reduces to the Barnwell-Sewall method.

ACKNOWLEDGEMENTS

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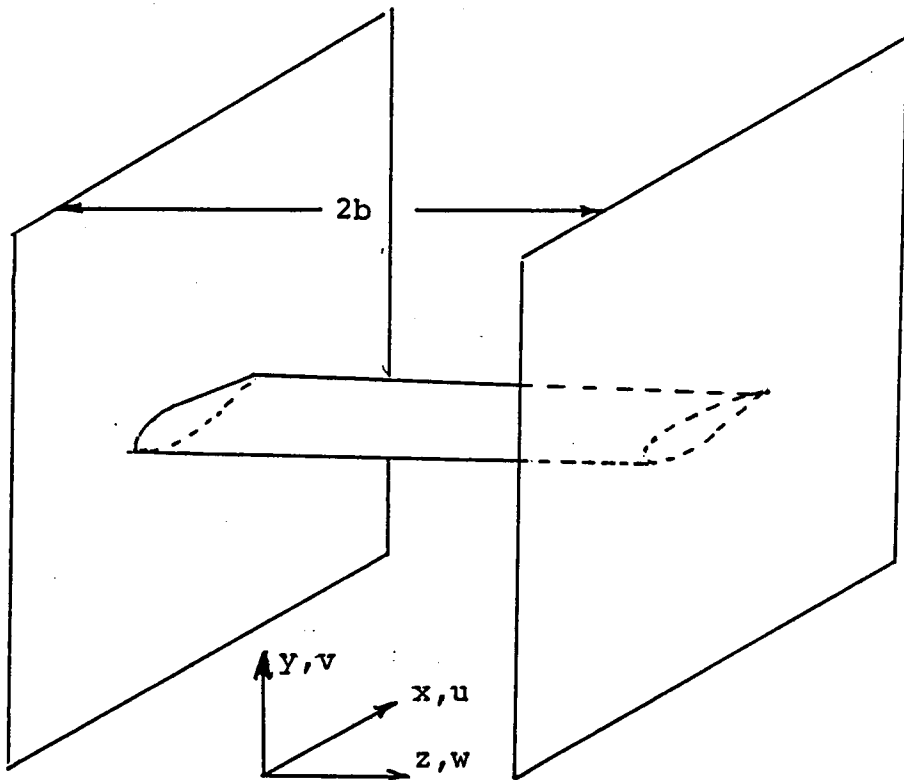   Tests. AIAA Journal, Vol. 22, February 1984, pp. 297-299.

Figure 1: Airfoil model in a two-dimensional
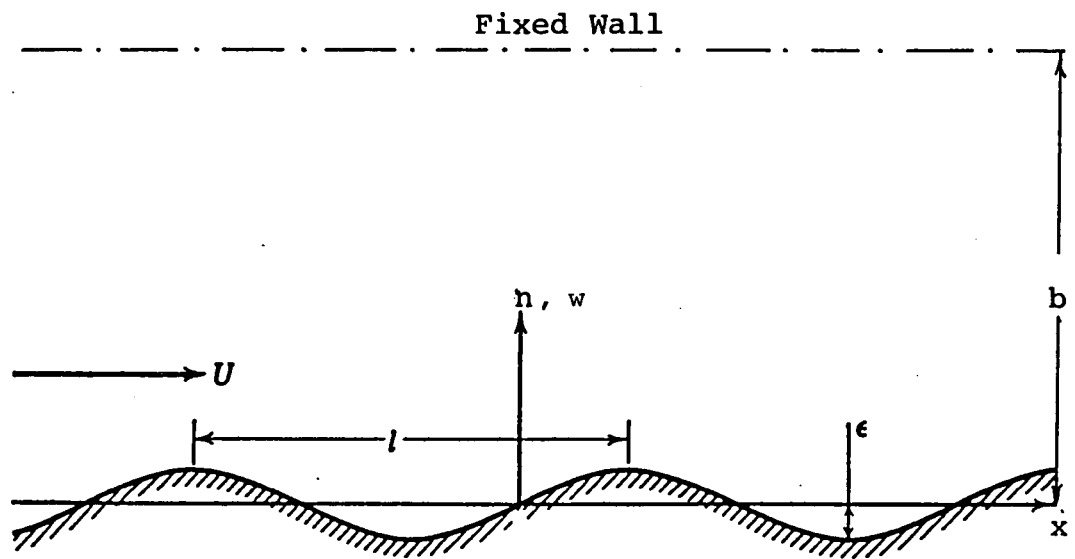wind tunnel and the coordinate
system.

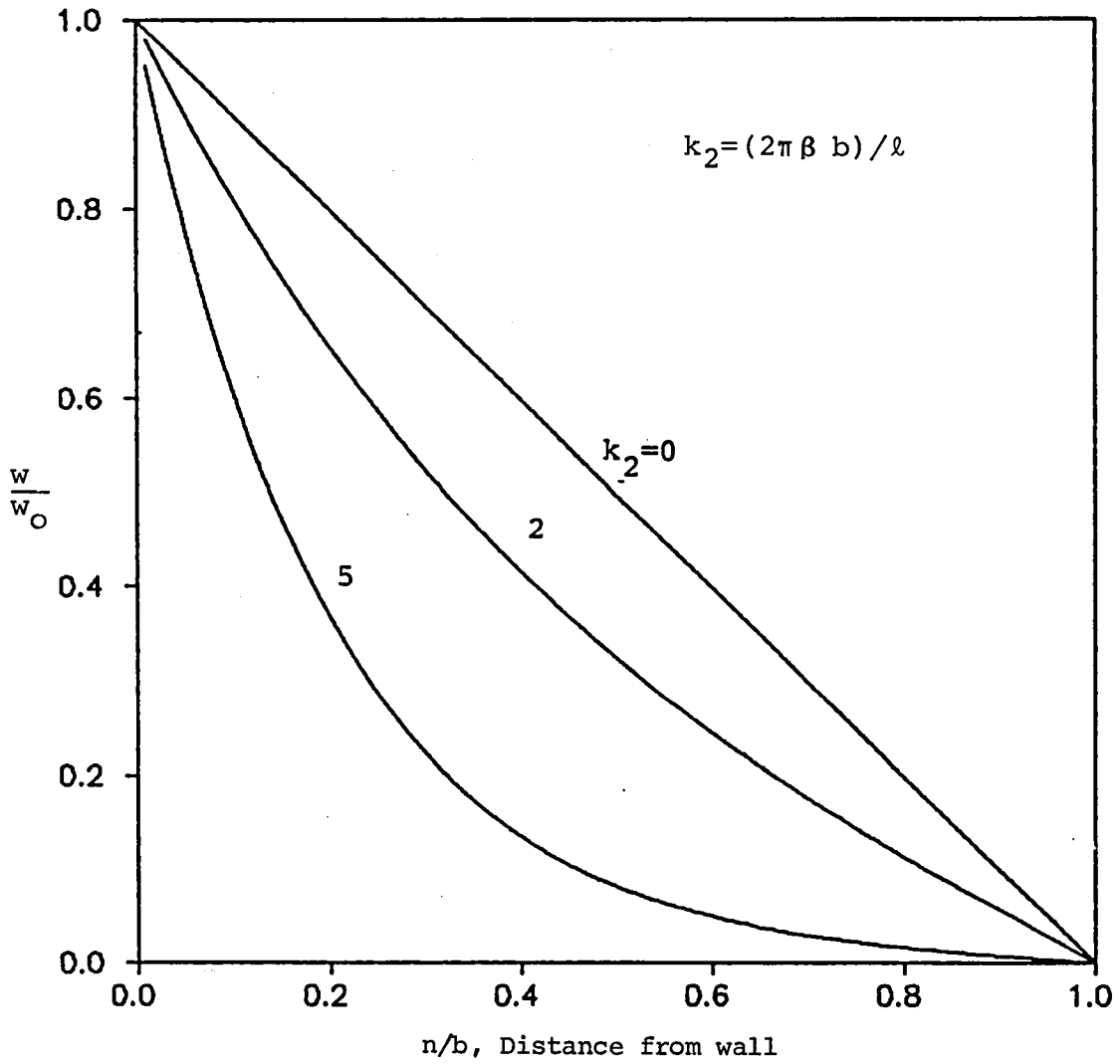Figure 2: Flow between a wavy wall and fixed wall.

Figure 3:   Variation of the normal velocity across
            the width of the tunnel for different k_2.
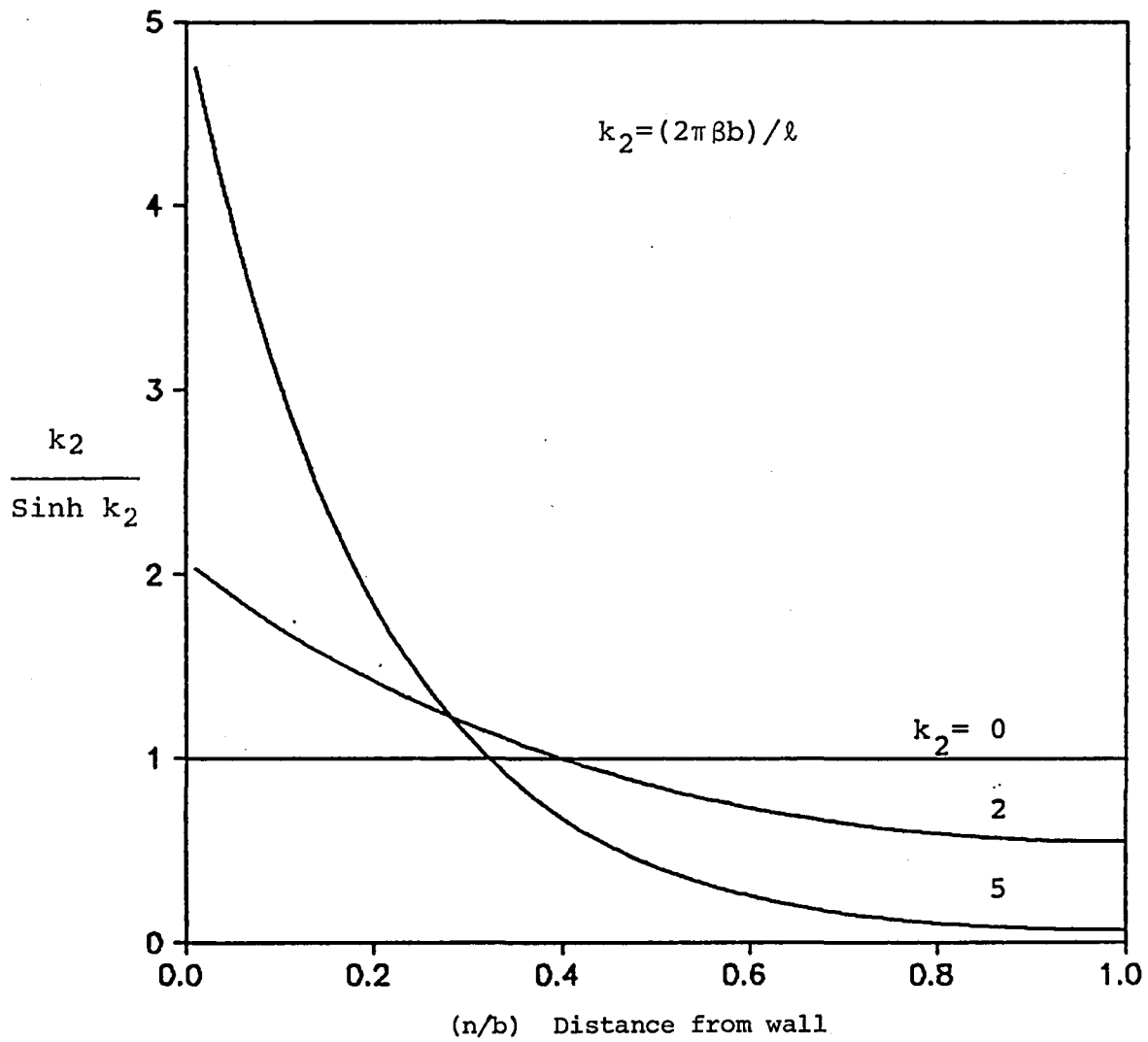
Figure 4: Variation of the normal velocity gradient across the width of the tunnel for different k_2.
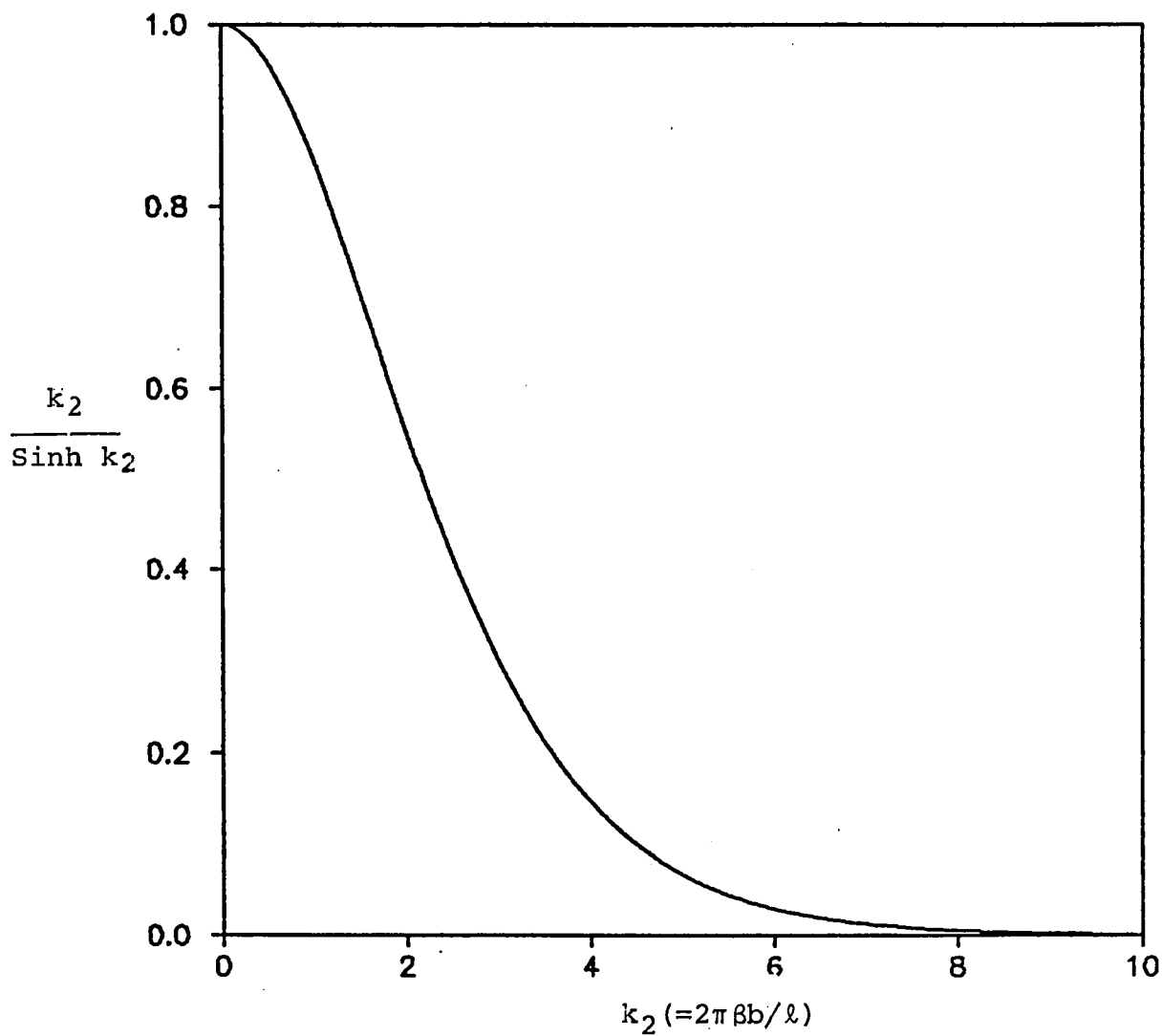
Figure 5: Variation of the normal velocity gradient (i.e., Aspect ratio correction factor in the median plane).
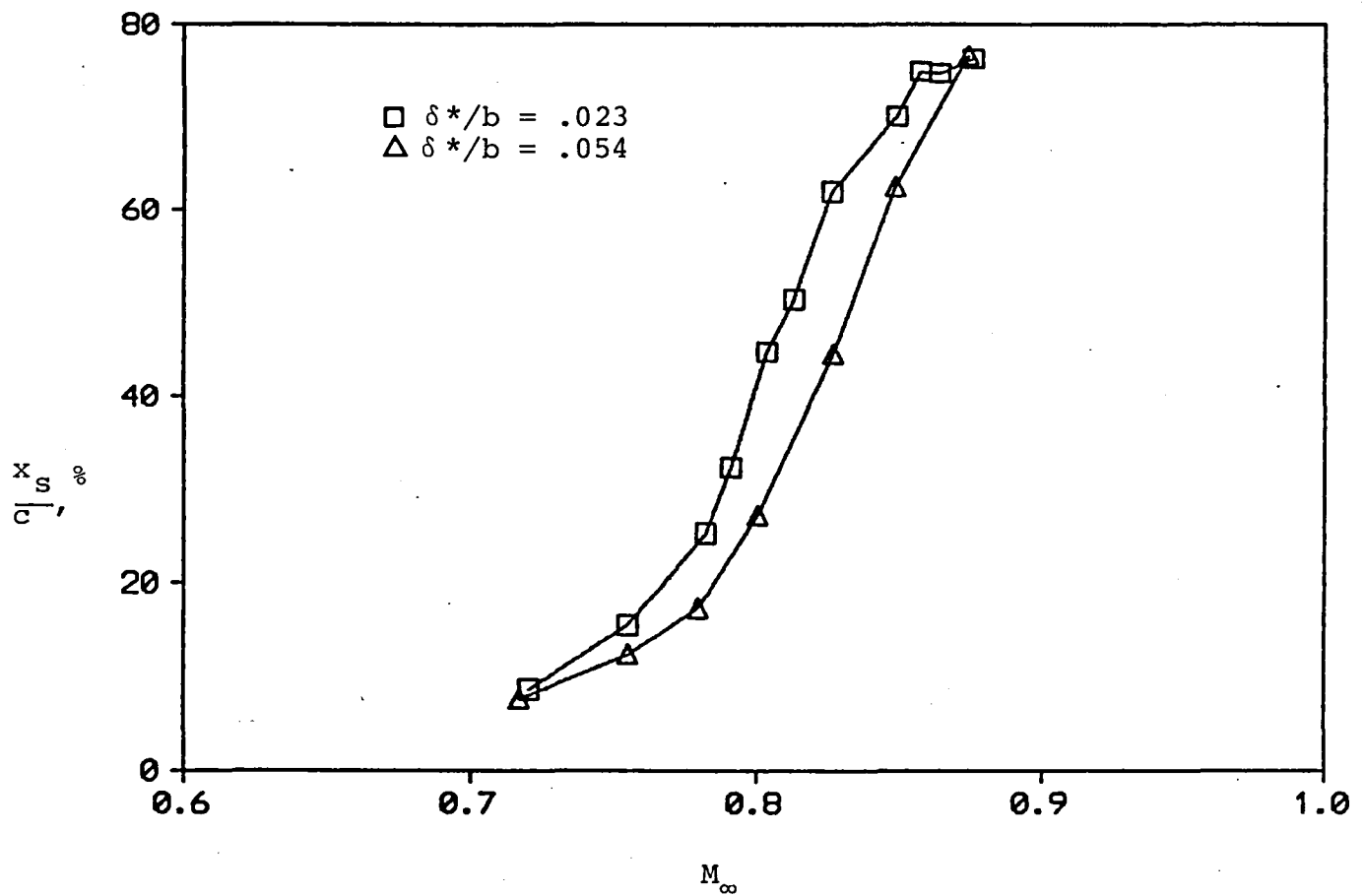
Figure 6 a: Measured shock locations on a super-
critical airfoil with different side-
wall boundary-layer thicknesses.
(Data from Reference 7)
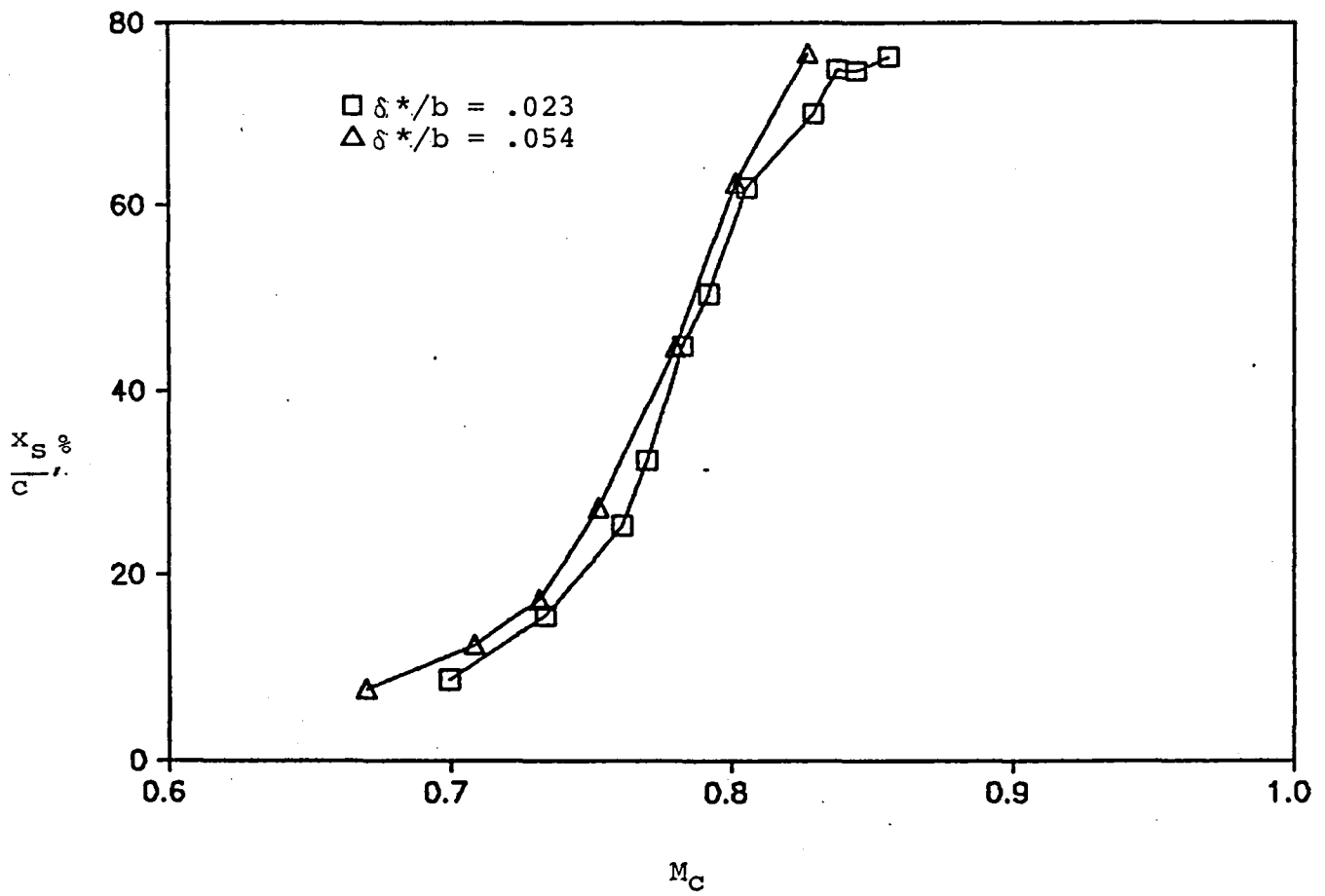
Figure 6b: Correlation of shock location using
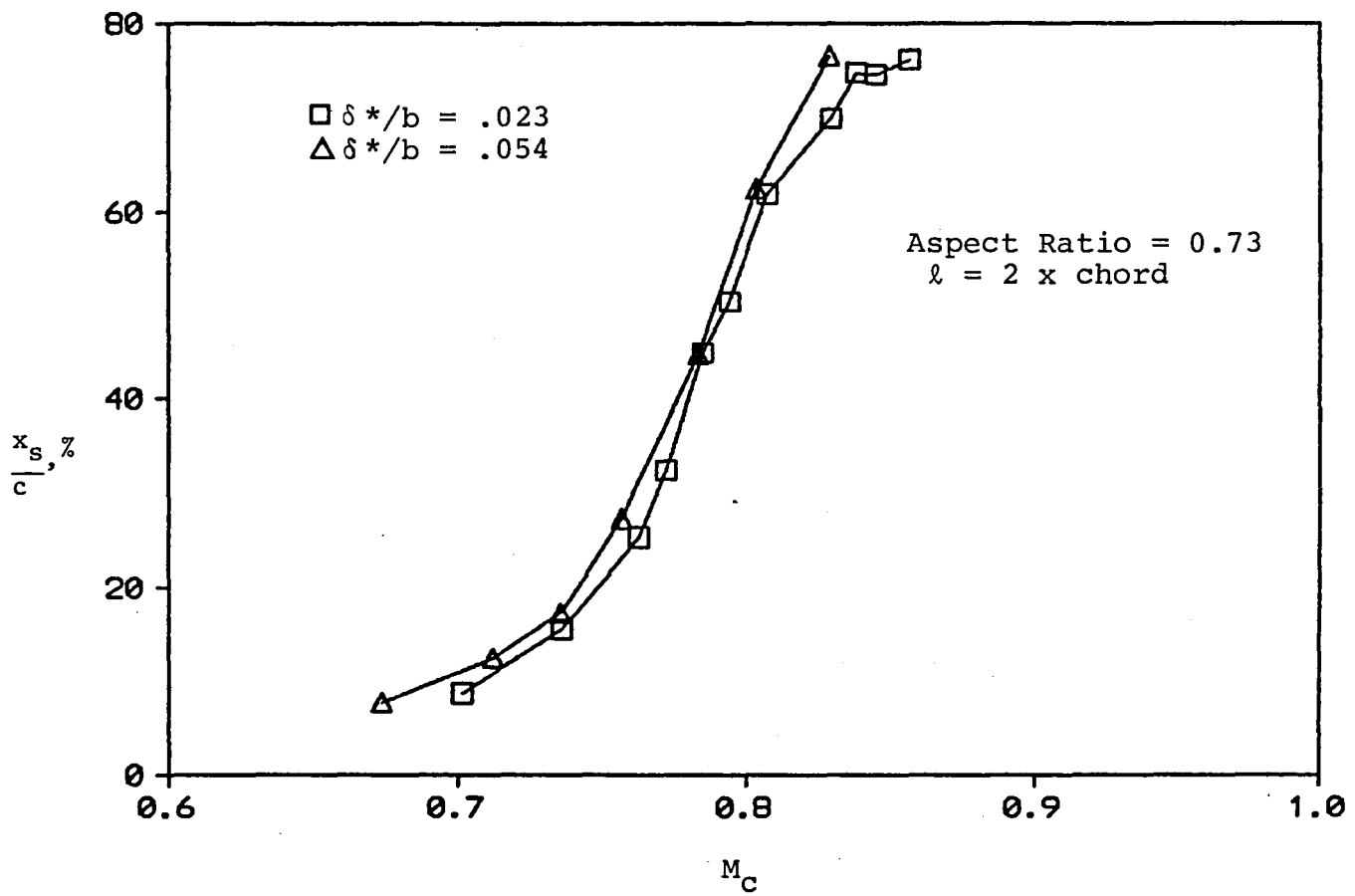   Barnwell-Sewall sidewall boundary-
             layer correction.

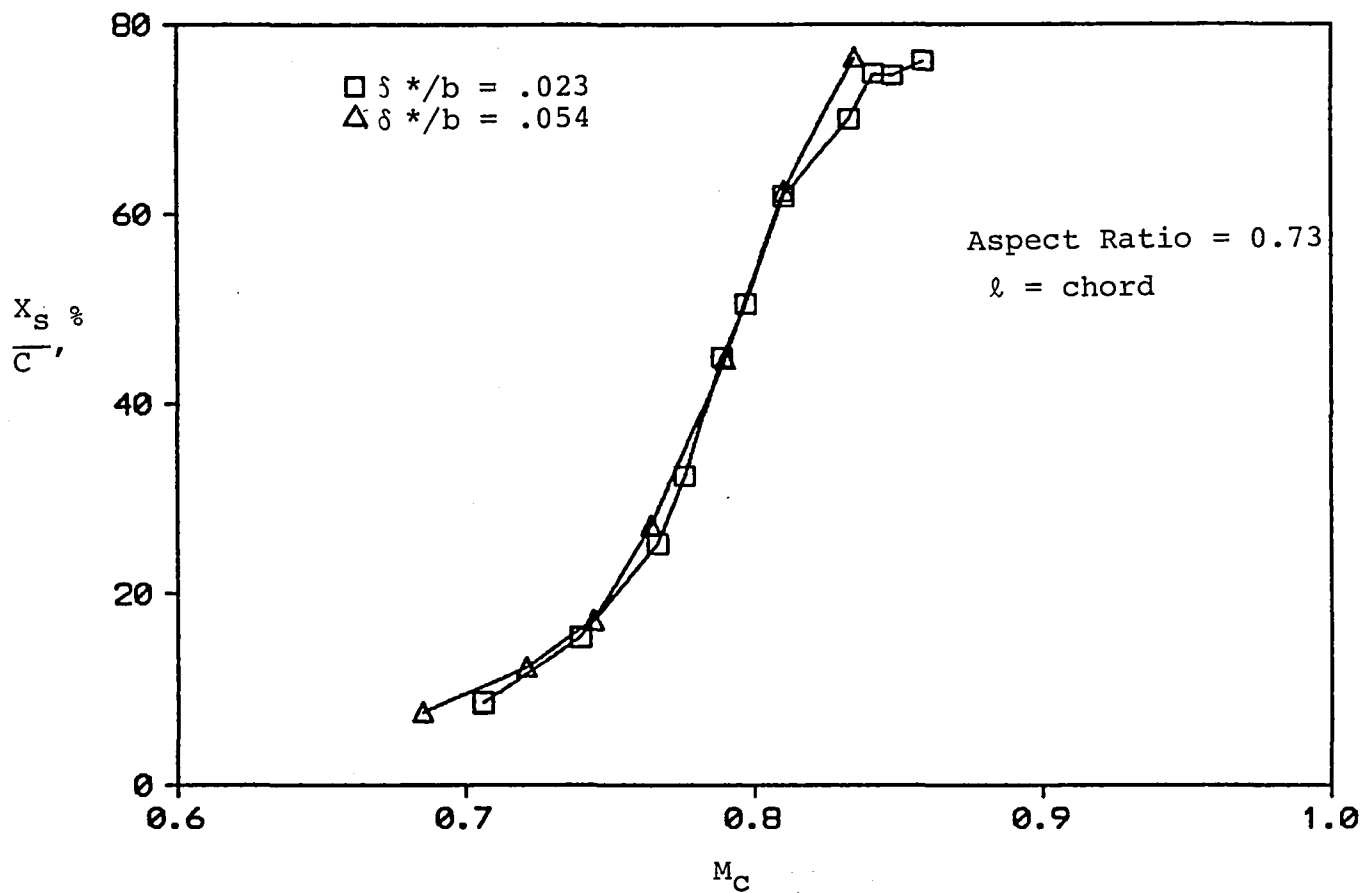Figure 6c: Correlation of shock location using
            the present method.

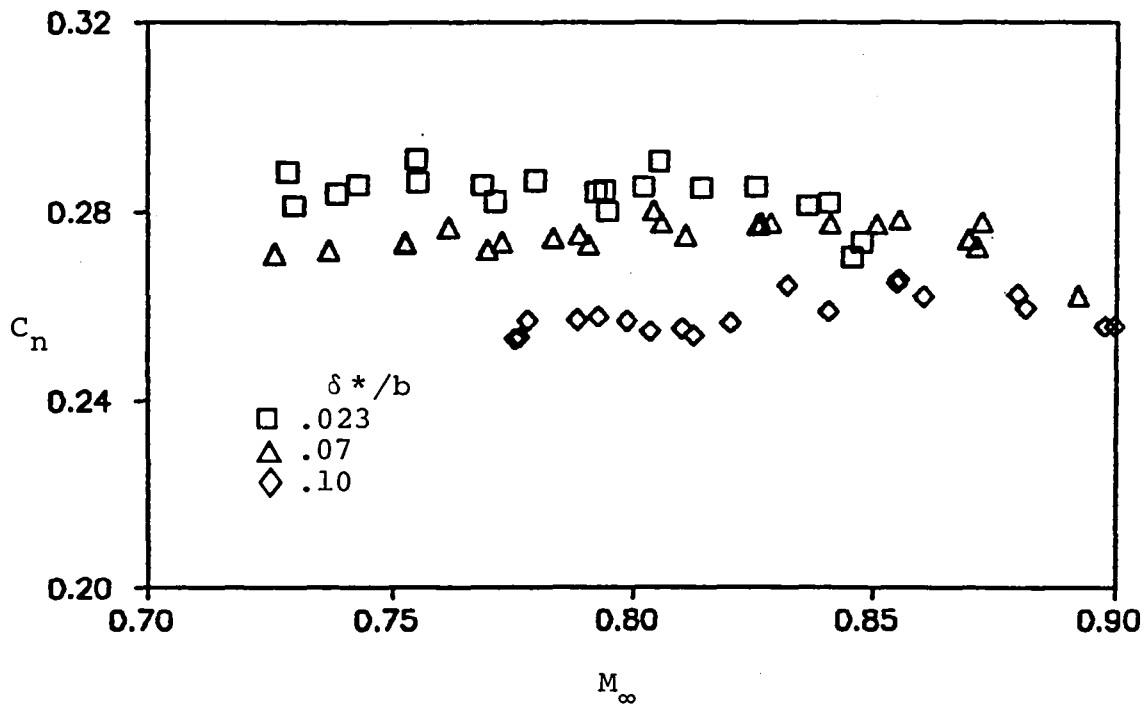Figure 6d: Correlation of shock location using
the present method.

0.32

0.28

C_n

0.24

0.20

0.70   0.75   0.80   0.85   0.90

M_∞

$\delta * /b$
□ .023
△ .07
◇ .10

Figure 7a: Measured normal force data on a super-
critical airfoil in the Langley 6"x19"
with different sidewall boundary-layer
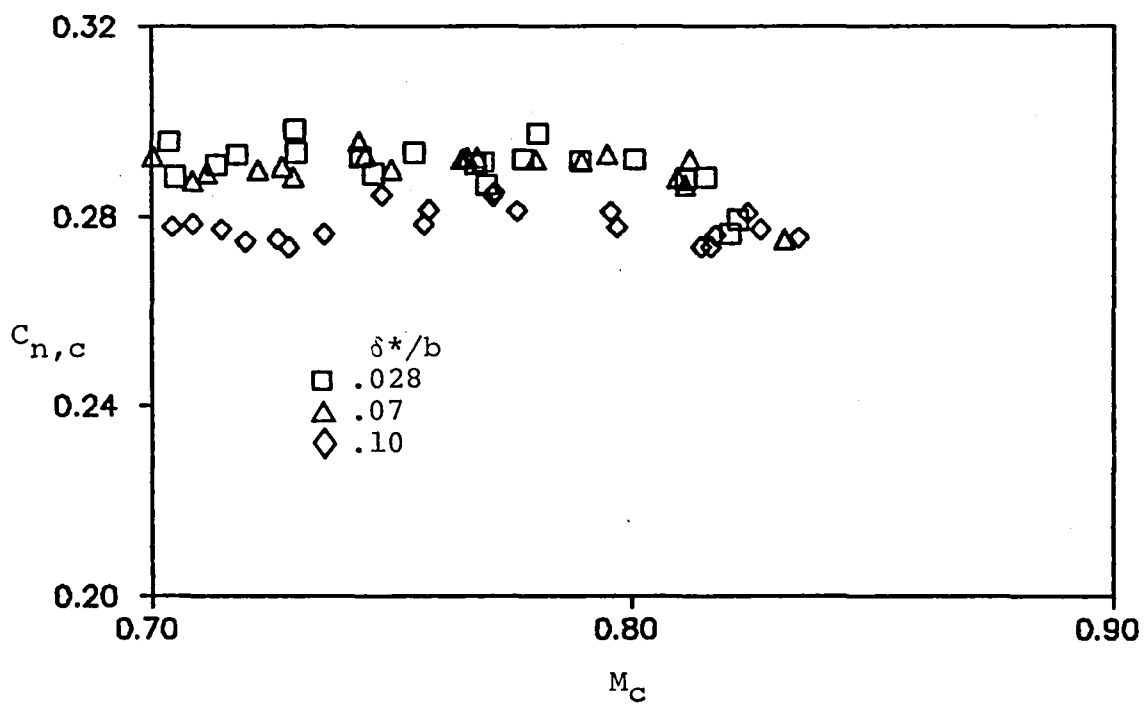thicknesses (from Ref. 2)

Fig 7b: Correlation of normal force coefficient
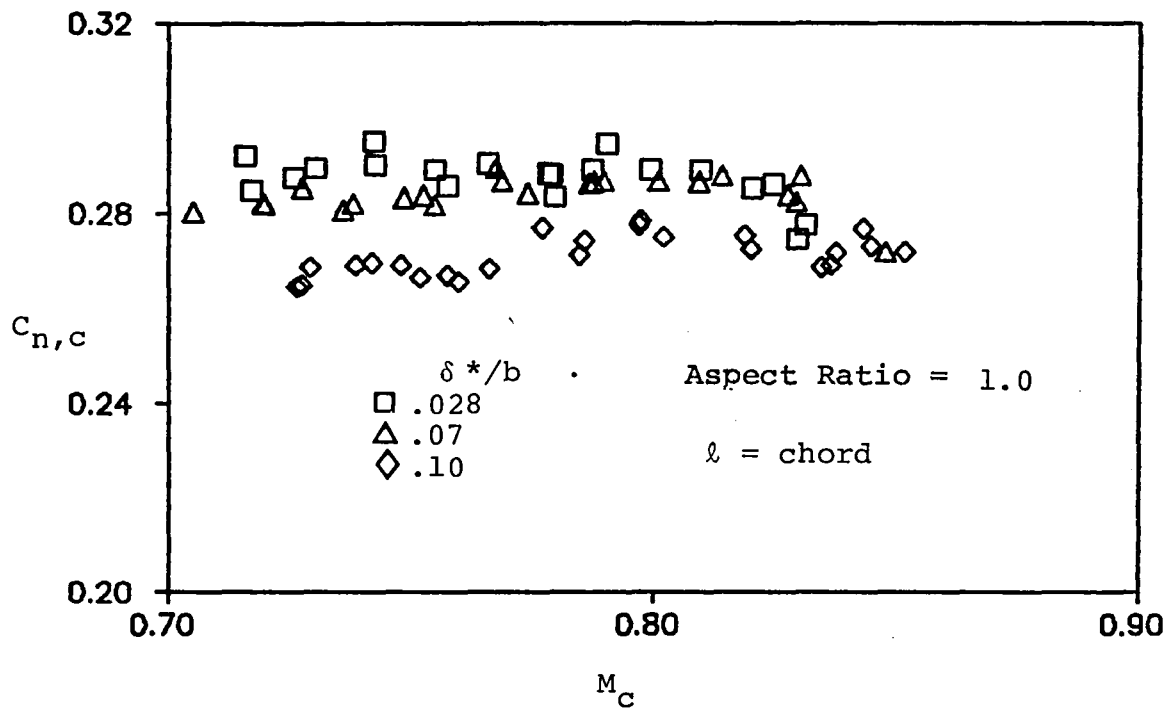   using Barnwell-Sewall method.

Figure 7c: Correlation of normal force coefficient
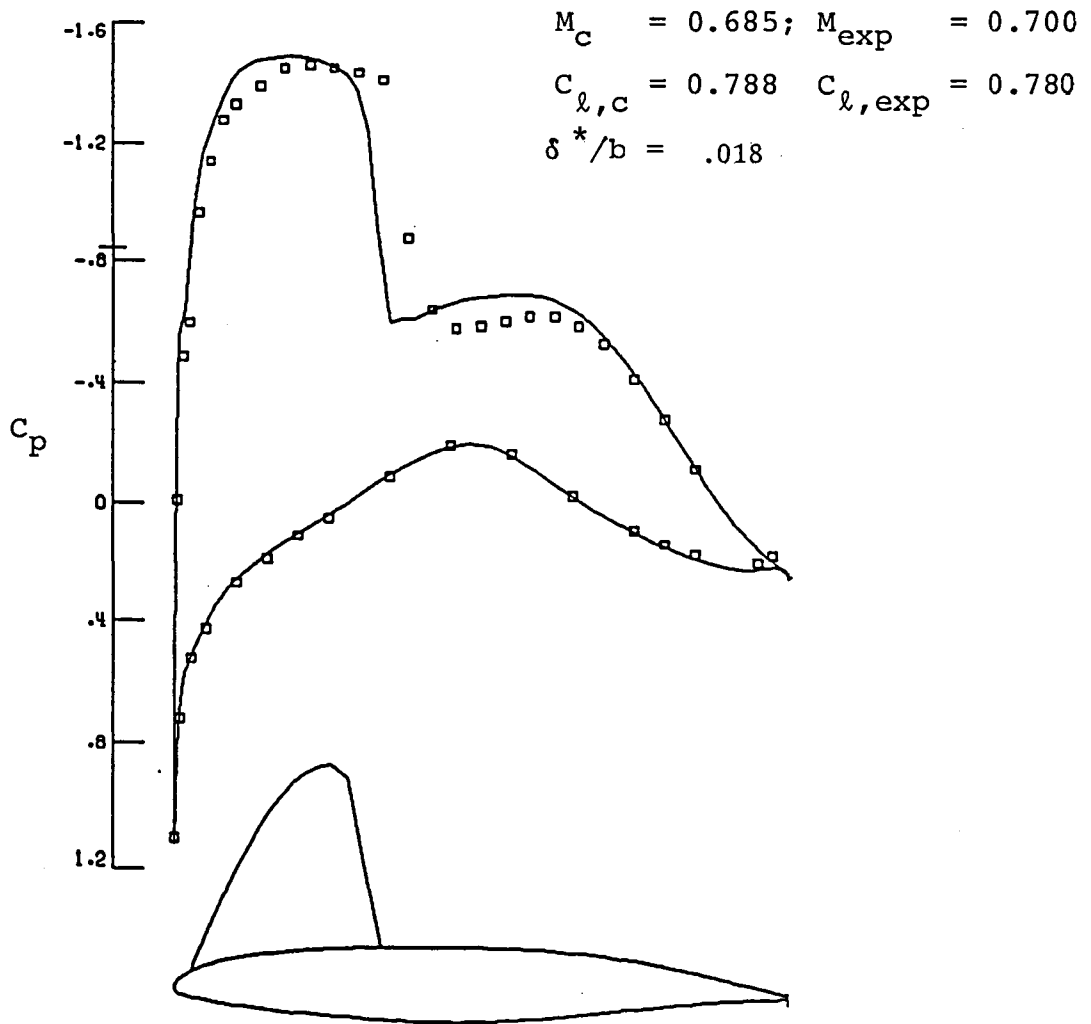using the present method.

$$M_c = 0.685; \quad M_{exp} = 0.700$$
$$C_{\ell,c} = 0.788 \quad C_{\ell,exp} = 0.780$$
$$\delta^*/b = .018$$

Figure 8a: Comparison of pressure distribution on Cast-10 airfoil with Grumfoil code predictions using Barnwell-Sewall correction (Ref. 2).
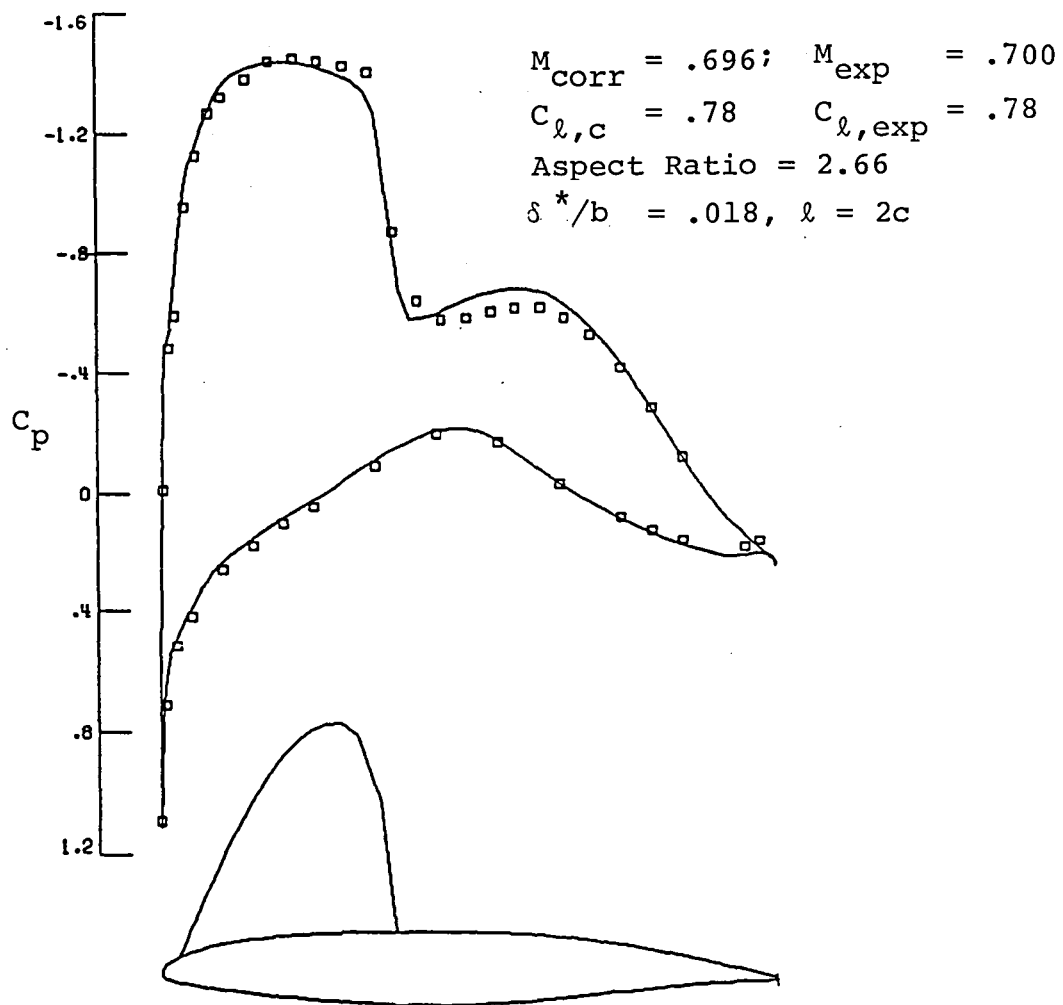
Figure 8 b: Comparison of pressure distribution
on Cast-10 airfoil with Grumfoil
code predictions using the present
correction.

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15. Supplementary Notes

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16. Abstract

The effect of sidewall boundary layers in airfoil testing in two-dimensional wind tunnels is investigated. The non-linear crossflow velocity variation induced because of the changes in the sidewall boundary-layer thickness is represented by the flow between a wavy wall and a straight wall. Using this flow model, a correction for the sidewall boundary-layer effects is derived in terms of the undisturbed sidewall boundary-layer properties, the test Mach number and the airfoil aspect ratio. Application of the proposed correction to available experimental data showed good correlation for the shock location and pressure distribution on airfoils.

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