~~31p.~~
25P

CONTROLLER DESIGN
via
STRUCTURAL REDUCED MODELING BY FETM

October 13, 1986

IN-30247

A Semi Annual Report
(Grant No. NAG-1-622)

Prepared for
NASA Langley Research Center,
Hampton, Virginia 23665

Submitted by

Drexel University
Department of Mechanical Engineering and Mechanics
Philadelphia, Pennsylvania 19104

(NASA-CR-179838) CONTROLLER DESIGN VIA STRUCTURAL REDUCED MODELING BY FETM Semiannual Report (Drexel Univ.) 25 p CSCL 20K
N87-10408
Unclas
G3/39 44334

Principal Investigator:

Ajmal Yousuff
Assistant Professor
(215) 895-1868

TABLE OF CONTENTS

page

Abstract 4

I. Introduction 5

II. Reduced Modeling by FETM Method 7

2.1 Review of FETM Method 8

2.2 The Modified FETM Method 11

2.3 Example: Simply Supported Beam 17

III. Controller Design by Modified FETM Method 20

IV. Conclusions and Recommendations for Future Research 21

References 24

Appendix

Foreward

This semi annual report was prepared by the Department of Mechanical Engineering and Mechanics, Drexel University, Philadelphia, Pennsylvania 19104 under NASA Grant NAG-1-622. The work was performed under the direction of Dr. Jer-Nan Juang of the Structural Dynamics Branch of NASA Langley Research Center, Hampton, Virginia 23665.

The technical work was conducted by Dr. Ajmal Yousuff, Principal Investigator and the numerical work was performed by Mr. Mike Konstantinidis, graduate research assistant. This report covers the theoretical research performed during January 01, 1986 - June 30, 1986, and the numerical work generated during January 01, 1986 - September 30, 1986.

The investigator of this study wishes to thank Dr. Jer-Nan Juang for his guidance and support of this research, and his colleagues Dr. Leon Bahar and Dr. Tein-Min Tan for several discussions and assistance.

ABSTRACT

The Finite Element - Transfer Matrix (FETM) method has been developed to reduce the computations involved in analysis of structures. This widely accepted method, however, has certain limitations, and does not directly produce reduced models for control design. To overcome these, a modification of FETM method has been developed. The modified FETM method easily produces reduced models that are tailored toward subsequent control design. Other features of this method are its ability to (i) extract open loop frequencies and mode shapes with less computations, (ii) overcome limitations of the original FETM method, and (iii) simplify the design procedures for output feedback, constrained compensation, and decentralized control.

This semi annual report presents the development of the modified FETM, and through an example, illustrates its applicability to an output feedback and a decentralized control design.

I. INTRODUCTION

The objectives of this research are to develop an algorithm to produce reduced models of structures and to integrate a controller synthesis scheme in this algorithm. This semi annual report describes the status of the research. Major part of the research has focused on the development of the reduced modeling procedure. This is based on the Finite Element - Transfer Matrix method. The second objective, namely, the controller synthesis procedure, has been dealt with only preliminarily. Numerical investigations have been carried out to study the applicability of the reduced modeling procedure in control design.

The Finite Element - Transfer Matrix (FETM) method has been developed to overcome the problem of large size matrices that are encountered while modeling flexible structures within acceptable accuracy. The main advantage with the FETM method is that it yields a reduced set of equations by operating at (finite-) elemental level. The advantages offered by the FETM method are not fully utilized in the area of control design - this is since the FETM method has been developed primarily for open loop structural analysis. The method developed in this research modifies the FETM method to include control issues. The essential feature of the modified FETM method is its ability to model the entire structure in terms of any selected degrees of freedom. The choice of these dof is to be determined from the control objectives. Other features of this method are its ability to (i) produce reduced order models for control design, (ii) extract open loop frequencies and mode shapes with less computations, (iii) overcome the limitations of the original FETM method, and (iv) simplify the design procedures for output feedback, constrained compensation, and decentralized control.

With this perspective, this report is organized as follows: In Section II, we briefly review the original FETM method, point out its limitations, and present the modifications developed in this research. We will also offer a numerical example to illustrate the procedure. In Section III, we present our preliminary analysis of this procedure's applicability to control design and use an example to illustrate the suggested method. We will show an output feedback design and a decentralized control design. Section IV concludes this report by summarizing our findings, and raising issues that need to be addressed in future research. The findings of this research have been presented at two conferences and copies of these papers are included in Appendix.

II. REDUCED MODELING BY FETM METHOD

The structural analysts, owing to the ever persistant problem of large size matrices, have developed the FETM method. In the standard finite element analysis, the stiffness and mass matrices are computed for the structure, and the natural frequencies are then determined from an eigenvalue problem. The FETM method does not directly solve the eigenvalue problem. Instead, a transfer matrix, which depends on natural frequencies, is calculated which relates the displacements and forces at right boundary to those at left boundary of the structure. The correct natural frequencies are then extracted by iteration when the boundary conditions on left and right edges of the structure are specified. One of the advantages of the FETM method is that it yields a reduced set of equations by operating on the structure at the (finite-) elemental level. Another attraction of this method is that it offers a straightforward means of substructuring.

The advantages offered by the FETM method are not fully utilized in the area of control design. The FETM method can be modified to incorporate the mission (control) objectives at the modeling phase. This is achieved by first identifying the dof based on the sensor/actuator locations and mission objectives, and then by calculating a transfer matrix that relates these dofs to the rest of the dofs (i.e, instead of relating the displacements and forces at the boundaries; see below for details). This method is particularly suitable for decentralized control, output feedback, and constrained compensation designs. The modified FETM method eliminates the limitations of the original method, while still offering considerable computational ease.

With a brief mathematical review of the FETM method, we present our modifications in what follows. A numerical example is included herein to illustrate the procedure.

2.1 Review of FETM Method*

The transfer matrix method is generally associated with the concept of stiffness. Pestel and Lechie[1] demonstrated that, for one-dimensional structural members such as beams, the transfer matrix could be derived from the stiffness matrix. Dokainish[2] was the first to suggest a combined FETM method. He recognized that the transfer matrix method could be extended to two-dimensional structures by deriving the transfer matrix for a plate strip from the corresponding stiffness matrix. Since the publication of Dokainish's paper in 1972, several authors (notably, [3-7]) have proposed refinements and extensions of the FETM method. All of these authors derive the transfer matrix from the stiffness matrix in much the same fashion as Dokainish. This derivation is summarized below.

If a dynamic stiffness matrix S is defined as $S = K - \omega^2 M$ where K is the stiffness matrix, M the mass matrix, and ω a natural frequency of the structure, then the equations of motion may be written as

$$Sq = f. \tag{2.1}$$

Here q are the nodal displacements, and f are the nodal forces. Considering now the ith substructure within the structure, eqn(2.1) may be partitioned as

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}_i \begin{bmatrix} u_L \\ u_I \\ u_R \end{bmatrix}_i = \begin{bmatrix} f_L \\ f_I \\ f_R \end{bmatrix}_i . \tag{2.2}$$

The subscripts L, I, and R refer to quantities on the left boundary, in the interior, and on the right boundary of the substructure.

---

* Due to appropriateness, we reproduce Section 2 of [9] with minor modifications in the first half of this section.

For free vibration, there are no forces on the interior; i.e., $f_I = 0$, (valid for open loop analysis, and not for closed loop analysis which may involve active force actuators located at interior nodes). This information is used to eliminate u_I from eqn(2.2) which produces the condensed set of equations

$$\begin{bmatrix} S_{LL} & S_{LR} \\ S_{RL} & S_{RR} \end{bmatrix}_i \begin{bmatrix} u_L \\ u_R \end{bmatrix}_i = \begin{bmatrix} f_L \\ f_R \end{bmatrix}_i \tag{2.3}$$

where S_{LL}, S_{LR}, S_{RL}, and S_{RR} involve the submatrices of the dynamic stiffness matrix S in eqn(2.2). The transfer matrix for the ith substructure T_i is then found by manipulating eqn(2.3) to yield

$$\begin{bmatrix} -S_{LR}^{-1}S_{LL} & S_{LR}^{-1} \\ S_{RL}-S_{RR}S_{LR}^{-1}S_{LL} & S_{RR}S_{LR}^{-1} \end{bmatrix}_i \begin{bmatrix} u_L \\ f_L \end{bmatrix}_i = \begin{bmatrix} u_R \\ f_R \end{bmatrix}_i \tag{2.4a}$$

or, simply,

$$T_i X_{L_i} = X_{R_i} \tag{2.4b}$$

For the adjacent (i+1)th substructure, equilibrium and continuity of displacements require that

$$X_{L_{i+1}} = X_{R_i} \tag{2.5}$$

Equations (2.5) and (2.4b) are combined to give

$$X_{R_{i+1}} = T_{i+1} T_i X_{L_i} \ . \tag{2.6}$$

Extending eqn(2.6) to N substructures results in

$$X_N = T_N T_{N-1} \cdots\cdots T_1 X_0 , \tag{2.7}$$

where X_N and X_0 are the "state" vectors on the extreme right and left boundaries of the entire structure.

The derivation of the transfer matrix from the dynamic stiffness matrix requires the inversion of the submatrix S_{LR} as shown in eqn(2.4a). In a strict sense, this inversion is possible if and only if S_{LR} is a square nonsingular matrix. However, S_{LR} is a square matrix only if there is an equal number of nodes (actual dof) on the right and left boundaries of the substructure. Moreover, if the structure includes any rigid body modes and the dynamic stiffness matrix is evaluated at the rigid body (zero) frequency, then S_{LR} may be singular even if it is square, and even eqn(2.3) may be unobtainable. For simplicity however, we will focus only on flexible modes. Thus, the submatrix S_{LR} cannot be inverted in general. Therefore, the above formulation of combined FETM method is only applicable to models which have the same number of nodes on all substructure boundaries.

Recognizing that rectangular transfer matrices could occur in practice, Pestel[8] proposed using a left-inverse of S_{LR}. This is possible only if the number of nodes on the left boundary of a substructure is equal to or greater than the number on the right boundary. This restricts Pestel's approach to structural models in which the number of nodes on the substructure boundaries decreases monotonically from one exterior boundary to the other. As suggested by Degen, *et.al.*,[9], a further extension of Pestel's procedure for rectangular transfer matrices would be to use a generalized inverse, rather than a left-inverse of S_{LR}. This however introduces additional approximations into the derivation of the transfer matrix from the stiffness matrix. More recent developments to overcome these limitations has been reported by Degen, *et.el.*,[9], employing a mixed finite element formulation. In [9], the energy expression is obtained as a Reissner functional, which allows the governing equations to be obtained in mixed form, that is, as a combination of stresses and displacements. This formulation permits the boundaries of a particular substructure to have different number of dof "within certain limits"; the limits being that the number of the nodes on the right boundary is not too large relative to the number on the left boundary. Hence, the limitations of the original FETM method [1-7], though reduced in [8,9], are not completely eliminated.

2.2 Modification of the FETM Method

All the above methods [1-9] have been developed for open loop (free vibration) analysis. A control engineer, however, is more concerned with the closed loop design and analysis, and requires an appropriate reduced model of the structure. None of the above methods yield reduced models conveniently. Moreover, in order to control the structure, actuators may be located at interior nodes of the structure, invalidating the assumption $f_I = 0$ used in deriving eqn(2.3). Thus, in order to address the issues of unequal number of dof, development of appropriate reduced models, and non-zero (feedback) applied forces at interior nodes, suitable modifications of the FETM method are needed. These modifications, developed during the current investigation, are now reported.

Frequency Determination: First, we present the modified FETM method to calculate the natural frequencies. Consider the frequency determination by the conventional finite element method: one solves the following equation which is obtained after enforcing boundary conditions, to obtain the frequency.

$$|\mathbf{K} - \omega^2 \mathbf{M}| = 0 \; ; \; \mathbf{K},\mathbf{M} \in \mathbf{R}^{(n,n)} \tag{2.8a}$$

Partition eqn(2.8a) as

$$\begin{vmatrix} K_{11}-\omega^2 M_{11} & K_{12}-\omega^2 M_{12} \\ K_{12}^T-\omega^2 M_{12}^T & M_{22}-\omega^2 M_{22} \end{vmatrix} = 0, \tag{2.8b}$$

where K_{11}, M_{11} are (n_1,n_1); K_{22}, M_{22} are $(n_2,n_2;\ n_2=n-n_1)$, and K_{12}, M_{12} are (n_1,n_2), with n_1 being quite arbitrary for now. (Later, we will use the control objectives to determine n_1). Then, assuming that $[K_{11}-\omega^2 M_{11}]$ is nonsingular, eqn(2.8b) can be written as

$$|(K_{22}-\omega^2 M_{22}) - (K_{12}^T-\omega^2 M_{12}^T)(K_{11}-\omega^2 M_{11})^{-1}(K_{12}-\omega^2 M_{12})| = 0, \tag{2.9}$$

and the frequency can be extracted from this equation.

Reduced Model from Assembled Equations: We will now relate the development (2.8a)-(2.9) to the analysis of the homogenoeus system

$$M\ddot{q}(t) + Kq(t) = 0, \quad q \; \varepsilon \; \mathbf{R}^n. \tag{2.10}$$

As usual, assuming a harmonic solution, we get

$$[K - \omega^2 M]q(t) = 0, \tag{2.11a}$$

which in partitioned form is written as

$$[K_{11}-\omega^2 M_{11}]q_1(t) + [K_{12}-\omega^2 M_{12}]q_2(t) = 0 \;, \; q_1 \; \varepsilon \; \mathbf{R}^{n_1} \tag{2.11b}$$

$$[K_{12}-\omega^2 M_{12}]^T q_1(t) + [K_{22}-\omega^2 M_{22}]q_2(t) = 0 \;, \; q_2 \; \varepsilon \; \mathbf{R}^{n_2} \tag{2.11c}$$

Using eqn(2.11b) we obtain

$$q_1(t) = -[K_{11}-\omega^2 M_{11}]^{-1}[K_{12}-\omega^2 M_{12}]q_2(t), \tag{2.12}$$

establishing a relationship between $q_1(t)$ and $q_2(t)$. Hence, eqn(2.11c) yields

$$[M_{22}+M_{12}^T T_{12}(\omega)]\ddot{q}_2(t) + [K_{22}+K_{12}^T T_{12}(\omega)]q_2(t) = 0, \tag{2.13a}$$

where

$$T_{12}(\omega) \triangleq -[K_{11}-\omega^2 M_{11}]^{-1}[K_{12}-\omega^2 M_{12}]. \tag{2.13b}$$

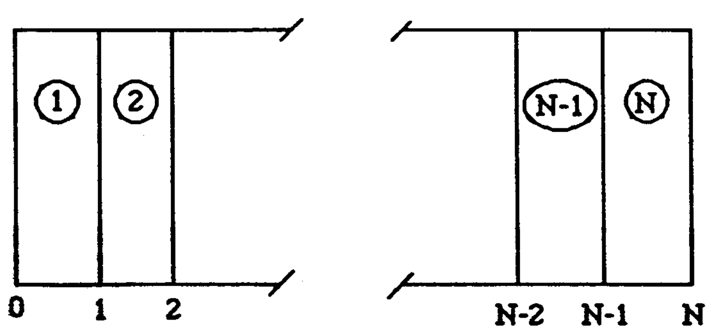
Note that the 'reduced model' (2.13a) of dimension n_2 still represents the original 'large model' of dimension n, with the dof q_1 absorbed/transferred into q_2. The frequency must satisfy

$$|\,[K_{22}+K_{12}^T T_{12}(\omega)] - \omega^2 [M_{22}+M_{12}^T T_{12}(\omega)]\,| = 0, \tag{2.14}$$

which in view of eqn(2.13b) is the same as eqn(2.9). Therefore, it is possible to extract the frequencies (and the mode shapes using eqn(2.12)) of the original system (2.10) from the reduced model (2.13a), by transferring the q_1-information into q_2.

It is in order now to make the following observations. The system representation (2.10) is obtained after enforcing the boundary conditions, and represents a homogeneous system. Eqn(2.10) represents the 'assembled' set of equations, and hence is of large order, i.e., n may be quite large. Since n is large, either n_1 and/or n_2 may still be large - if n_1 is large, then the matrix being inverted in eqn(2.12) is large; if n_2 is large, then the eigenvalue problem (2.13a) still involves large matrices. This approach (as presented thus far) operates only on the assembled equations, and not at an elemental level as proposed by Dokainish. Therefore, we would like to develop a method that operates at the elemental level - and hence, less calculations - which would yield the same reduced model (2.13).

Reduced Model from Elemental Equations: We will follow the method similar to that of Dokainish with a few modifications. We will consider a free(no applied forces) plate vibration



problem for illustration. Consider Strip1. The strip itself may contain a number of elements, whose equations of motion may be assembled together to yield the following equation of motion for Strip1.

$$\begin{bmatrix} M_{ll,1} & M_{lr,1} \\ M_{rl,1} & M_{rr,1} \end{bmatrix} \begin{bmatrix} q_{l,1} \\ q_{r,1} \end{bmatrix} + \begin{bmatrix} K_{ll,1} & K_{lr,1} \\ K_{rl,1} & K_{rr,1} \end{bmatrix} \begin{bmatrix} q_{l,1} \\ q_{r,1} \end{bmatrix} = \begin{bmatrix} 0 \\ F_{r,1} \end{bmatrix} \tag{2.15}$$

where the n_1 dimensional variables $q_{l,1}$ are the free dof at the left section (boundary for Strip1), obtained after enforcing the boundary conditions. (This is different from Dokainish's approach wherein the boundary conditions are not enforced at this step). Since $q_{l,1}$ are the free dof, there are no external forces, leading to '0' at the right hand side of (2.15);

$q_{r,1}$ (n_2 dimensional) are the free dof at the right section. Since (2.15) is the equation of motion for Strip1 by itself, $F_{r,1}$ represents the internal forces at the right section;

top and bottom boundary conditions are also enforced. The top-left and top-right boundary conditions may be quite different (This totally eliminates the limitations of the original FETM method), and n_1 need not be equal to n_2.

In the absence of external forces, the motion of the plate will be harmonic. Thus, similar to the derivation of eqn(2.12), we get

$$q_{l,1}(t) = T_{lr,1}(\omega)q_{r,1}(t) \tag{2.16a}$$

where

$$T_{lr,1}(\omega) = -(K_{ll,1}-\omega^2 M_{ll,1})^{-1}(K_{lr,1}-\omega^2 M_{lr,1}). \tag{2.16b}$$

Note: Eqn(2.16b) requires an inverse of an (n_1,n_1) matrix which is always square. We assume the existense of its inverse.

With the relationship (2.16) available, $q_{l,1}(t)$ can be eliminated from (2.15) to yield

$$\mathbf{M}_{r,1}\ddot{q}_{r,1}(t) + \mathbf{K}_{r,1}q_{r,1}(t) = \mathbf{F}_{r,1}(t) \tag{2.17a}$$

where,

$$\begin{aligned} M_{r,1} &\triangleq M_{rr,1} + M_{lr,1}^T T_{lr,1}(\omega) \\ K_{r,1} &\triangleq K_{rr,1} + K_{lr,1}^T T_{lr,1}(\omega). \end{aligned} \tag{2.17b}$$

Similarly, with Strip2 under consideration, using continiuity conditions ($q_{l,2}=q_{r,1}$) and equiibrium conditions ($F_{l,2}=-F_{r,1}$), and by using (2.17) one gets

$$[M_{ll,2}+M_{r,1}]\ddot{q}_{l,2}(t) + [K_{ll,2}+K_{r,1}]q_{l,2}(t) + M_{lr,2}\ddot{q}_{r,2}(t) + K_{lr,2}q_{r,2}(t) = 0. \tag{2.18}$$

From eqn(2.18) we obtain

$$q_{l,2}(t) = T_{lr,2}(\omega)q_{r,2}(t) \tag{2.19a}$$

where

$$T_{lr,2}(\omega) = -[(K_{ll,2}+K_{r,1})-\omega^2(M_{ll,2}+\overset{M}{\cancel{K}}_{r,1})]^{-1}[K_{lr,2}-\omega^2 M_{lr,2}]. \tag{2.19b}$$

Finally, with eqn(2.19) substituted in the equation of motion of Strip2, we get

$$M_{r,2}\ddot{q}_{r,2}(t) + K_{r,2}q_{r,2}(t) = F_{r,2}(t) \tag{2.20a}$$

where,

$$M_{r,2} \triangleq M_{rr,2} + M_{lr,2}^T T_{lr,2}(\omega) \tag{2.20b}$$

$$K_{r,2} \triangleq K_{rr,2} + K_{lr,2}^T T_{lr,2}(\omega). \tag{2.20c}$$

We pause for a moment now to point out that the same model (2.20) could also be obtained by first assembling the equations of motion of Strips 1 and 2, and then eliminating $q_{l,1}(t)$ and $q_{l,2}(t)$ from it. Hence, eqn(2.20) is a reduced model for Strips 1 and 2 assembled together, but obtained by operating at the elemental level.

One can now proceed similarly and obtain a model, using Strip3, in $q_{r,3}$ coordinates only as

$$M_{r,3}\ddot{q}_{r,3}(t) + K_{r,3}q_{r,3}(t) = F_{r,3}(t). \tag{2.21}$$

Or, one could start from the right boundary (StripN), eliminate $q_{r,N}$ and obtain a model for StripN in terms of $q_{l,N}$ and $F_{l,N}$, and move left to StripN-1,..., and similarly to Strip5. At the end of this, one would have

$$M_{l,5}\ddot{q}_{l,5}(t) + K_{l,5}q_{l,5}(t) = F_{l,5}(t). \tag{2.22}$$

Now consider Strip4:

$$\begin{bmatrix} M_{ll,4} & M_{lr,4} \\ M_{lr,4}^T & M_{rr,4} \end{bmatrix} \begin{bmatrix} \ddot{q}_{l,4} \\ \ddot{q}_{r,4} \end{bmatrix} + \begin{bmatrix} K_{ll,4} & K_{lr,4} \\ K_{lr,4}^T & K_{lr,4} \end{bmatrix} \begin{bmatrix} q_{l,4} \\ q_{r,4} \end{bmatrix} = \begin{bmatrix} F_{l,4} \\ F_{r,4} \end{bmatrix}. \tag{2.23}$$

Assuming that there is no applied force along the right edge of Strip4, we get $F_{r,4} = -F_{l,5}$, and the continuity condition $q_{r,4} = q_{l,5}$. Thus, from eqn(2.22) and eqn(2.23), we get

$$q_{r,4} = T_{rl,4}(\omega)q_{l,4}. \tag{2.24}$$

Now, if there is some external force vector $P_{l,4}$ acting along the left edge of Strip4, then

$$F_{l,4} = P_{l,4} - F_{r,3}. \tag{2.25}$$

Therefore, combining eqns. (2.21), (2.24), (2.25), and (2.23), we finally get

$$M_{l,4}\ddot{q}_{l,4}(t) + K_{l,4}q_{l,4}(t) = P_{l,4}. \tag{2.26}$$

The model (2.26) represents the entire structure, with the boundary conditions taken into account. The dimension of the model is n_4, the number of free degrees of freedom along the left edge of Strip4. If $P_{l,4}$ is zero eqn(2.26) becomes a representation of a homogeneous system.

Note that, when there are no external forces present, we can obtain a reduced model of the structure in terms of any intermediate dof. Hence, if there is external load along the right edge of, say, ith strip, then one gets a model as

$$M_{r,i}\ddot{q}_{r,i}(t) + K_{r,i}q_{r,i}(t) = P_{r,i} \;;\; q_{r,i} \; \varepsilon \; \mathbf{R}^{n_i}. \tag{2.27}$$

Now suppose that only n_{2i} external forces ($P_{2r,i}$), of the possible n_i, are applied at this edge, corresponding to n_{2i} dof, then eqn(2.27) can be written as

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_{1r,i} \\ \ddot{q}_{2r,i} \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} q_{1r,i} \\ q_{2r,i} \end{bmatrix} = \begin{bmatrix} 0 \\ P_{2r,i} \end{bmatrix} \tag{2.28}$$

$$q_{1r,i} \; \varepsilon \; \mathbf{R}^{n_{1i}} \;;\; q_{2r,i} \; \varepsilon \; \mathbf{R}^{n_{2i}} \;;\; n_{1i} + n_{2i} = n_i$$

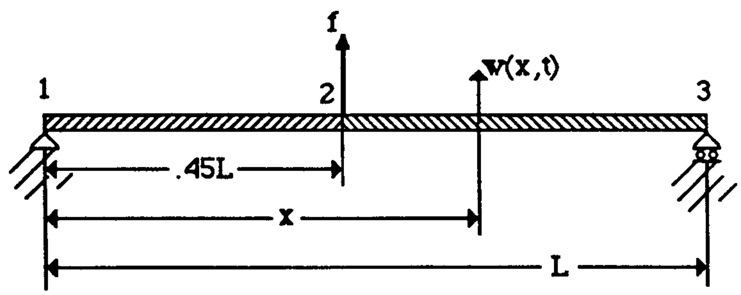
It is then possible to reduce (2.28) further, by eliminating $q_{1r,i}$, to obtain the following model of order n_{2i}.

$$M_R\ddot{q}_R + K_Rq_R = P_{2r,i} \;;\; q_R = q_{2r,i}\,. \tag{2.29}$$

2.3 Example: Simply Supported Beam

In order to present the application of the proposed method and its relation to the FETM method let us consider an output feedback control of a simply supported beam shown below. It is desired to control the linear displacement and linear velocity at point2, by using the linear force actuator (f) located at point2. The available sensors measure the linear position (w_2) and the linear velocity ($\dot{w}_2$) at point2. In this Section, we only develop a reduced model and compute the natural frequencies of this beam, the control design is deferred till next Section. The conventional finite

ρ : mass density per unit area
A : cross-sectional area
E : Modulus of elasticity
I : moment of inertia



Numerical Values: $L = 1$; $\rho A = 420$; $EI = 1$.

element method would yield a model containing 4 dof, namely θ_1, θ_2, w_2, and θ_3. Assuming that this model is too large for subsequent calculations, we wish to obtain a reduced model. We proceed as follows.

For simplicity, divide the beam into two elements as shown. The conventional FETM method (intended for analysis and not for control design) would first develop transfer matrices - assuming some numerical value for the natural frequency - that relate (θ_2,w_2) to (θ_1,w_1), and (θ_3,w_3) to (θ_2,w_2). Then it would obtain an equation relating the dof (θ_3,w_3) at the right boundary to those (θ_1, w_1) at the left boundary. Finally, it would enforce the boundary conditions at the left and right boundaries in this equation and iterate on the assumed frequency to extract the actual frequency. This procedure is accomplished without regard to the control objectives and sensor/actuator locations. Moreover, this procedure does not yield a convenient reduced model for subsequent control design. The following steps illustrate the proposed FETM method.

Step 0: Determination of critical dof.

Since the proposed FETM method is capable of producing reduced models in terms of any dof, the dof appropriate to the control objectives and sensor/actuator locations must be determined. In the example under consideration this dof is w_2, the linear deflection at point2.

Step 1: Element 1.

Write the cubic-beam model for Element 1 with the boundary condition $w_1=0$ enforced. Assume a value for the frequency ω, and eliminate θ_1 dof using the transfer matrix method. This gives a model of Element 1 in terms of (θ_2, w_2).

Step 2: Element 2.

Write the model for Element 2 (with $w_3=0$) and eliminate θ_3 as above, using the same assumed ω. This gives a model of Element 2 in terms of (θ_2, w_2).

Step 3: Reduced Model in terms of w_2.

Combine the models in Steps 1 and 2, to develop a model of the entire beam in terms of (θ_2, w_2). Eliminate θ_2 from this model as above, using the same ω. This gives a model of the beam in terms of only w_2, which is the dof that we wish to control.

Clearly the reduced model is function of the assumed frequency. The model obtained in Step 3 can be claimed to be a reduced model only if the assumed frequency is one of the frequencies of the beam. Hence, Step 4 is required.

Step 4: Extraction of frequencies.

Evaluate the determinant of $[K_R - \omega^2 M_R]$ for the same assumed value of ω. If the determinant is zero, then the assumed frequency is one of the natural frequencies of the system; if not, the assumed frequency should be updated and Steps 1 through 4 repeated until the determinant vanishes. (Currently, a modified secant method has been adopted for this updating scheme, and it works fairly efficiently). For this example, the four fundamental frequencies were computed to be 0.48, 2.15, 5.32, and 10.2 rad/sec.

An interactive program has been developed on IBM PC/XT using FORTRAN which solves for the natural frequencies of an abitrary beam with arbitrary boundary conditions employing the modified FETM. This program can compute the frequencies of a beam with even arbitrary intermediate boundary conditions such as, a hinge at some intermediate point - a situation wherein Dokainish's method is not applicable.

III. CONTROLLER DESIGN BY FETM METHOD

One of the problems encountered in designing a controller for flexible structures is the 'dimension problem', if one chooses to work with ordinary differential equations. The standard finite element method yields models consisting of many degrees of freedom, requiring more computer memory than is available for control design. The conventional FETM method, though uses less number of equations, does not yield models of lower dimensions suitable for control design. The modified FETM method, as explained above, produces reduced models with due consideration to control objectives. One might then use his/her favorite method to design a controller based on these reduced models.

One such approach would be to cast the problem in state space domain. Given the reduced model of eqn(2.29), its state space realization can be obtained as

$$\begin{aligned} \dot{x}_R &= A_R x_R + B_R u \quad ; \quad x_R \triangleq [q_R^T, \dot{q}_R^T]^T \in \mathbf{R}^{n_{2i}} \\ y_R &= C_R x_R \\ z_R &= M_R x_R, \end{aligned} \tag{3.1}$$

where y_R define the variables to be controlled and z_R are the measurements. Note that, for a given control problem, in order for y_R and z_R to be defined properly it is important to choose the dof q_R, and hence x_R, appropriately. Once eqn(3.1) is obtained, it is then possible to employ any of the state space techniques to develop a control policy to produce the required controller as

$$\begin{aligned} \dot{x}_c &= A_c x_c + F_R z_R \; ; \quad x_c \in \mathbf{R}^{n_c} \\ u &= G_R x_c + E_c z_R, \end{aligned} \tag{3.2}$$

where n_c is the order of the desired controller. Depending upon the type of controller sought the parameters $\{A_c, F_R, G_R, E_c\}$ can have different structures; for example, in an output feedback control scheme all parameters except E_c would be zeros.

Having obtained the controller, the evaluation of this controller has to be achieved only with respect to the actual full order model of the system. However, since the full order model is assumed to consist of many degrees of freedom, an efficient means of evaluating the controller is needed. This problem is being investigated jointly by Dr. Leon Bahar of Drexel University and the author of this report, employing a nested procedure for computing matrix exponential. Alternately, one might predict the performance of the controller by using only the reduced model (2.29), if an appropriate procedure could be developed. This issue is also under investigation at Drexel University.

The papers included in Appendix illustrate two controller design methods: (a) an output feedback design, and (b) a decentralized control design. Both these methods are based on the Linear Quadratic Regulator approach.

IV. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

In summary, during the first half of this research support, we have developed a modified FETM method that is capable of producing reduced models of flexible structures to ease the subsequent design of controllers. Unlike the conventional FETM method, the proposed method does not require equal number of dof at each section, i.e., at edges of each strip. Moreover, this procedure does not involve generalized inverses. More significantly, the reduced models produced by this method are tailored toward the control design. The applicability of this method to controller design has been iluustrated through an example.

The progress of this research has raised several issues that need to be addressed:

(1) During the development of the reduced model, the inversion of a matrix involving submatrices of the stiffness and mass matrices is assumed. Precise conditions under which this assumption is valid needs to be established. Currently it is believed that if the assumed natural frequency does not coincide with the frequency of the particular strip under consideration, then the matrix is invertible. This belief has to be formalized.

(2) The iterative numerical scheme employed in the extraction of the natural frequency is a modified secant method. This scheme works quite efficiently for the example considered herein. We believe that the scheme could be accelerated by using the results of the above Issue #1.

(3) Since the reduced model (2.29) depends upon the frequency, each natural frequency yields a different reduced model, and one could conceivably obtain many reduced models. This raises the question of which one (or which few) of the natural frequencies are to be used for the generation of the reduced models? In order to answer this question one needs to determine a priori the 'significant' frequencies, such as those identified by modal cost analysis[10]. This issue has to be resolved for the proposed method to be a feasible controller design scheme.

(4) The controller design based upon the proposed method needs to be explored. Currently, we have used an example to illustrate our thoughts on this design technique. We believe that, since tha reduced model is directly related to the full order conventional finite element model, in terms of

the frequency, the stiffness matrix, and the mass matrix, the performance of the controller with respect to the full model could be predicted. In particular, the stability of the overall closed loop system could be predicted; infact, for the example considered in the first paper of Appendix, one could guarantee stability fo the system for all bandwidth controllers. The stability properties of matrix second order systems would naturally be used here.

(5) An important requirement of any control design is the robustness of the controller. This issue needs to be addressed in future research.

REFERENCES

1. E.C.Pestel, F.A.Leckie, *Matrix Methods in Elastomechanics*, McGraw-Hill, New York (1963).
2. M.A.Dokainish, "A New Approach for Plate Vibrations: Combination of Transfer Matrix and Finite-Element Technique," *J. Eng. Ind. Trans. ASME*, Series B94, 526-530 (1972).
3. G.Chiatti, A.Sestieri, "Analyses of Static and Dynamic Structural Problems by a Combined Finite Element-Transfer Matrix Method," *J. Sound Vib.*, Vol.67, 35-42 (1979).
4. V.H.Mucino, V.Pavelic, "An Exact Condensation Procedure for Chain-like Structures Using a Finite Element-Transfer Matrix Approach," *J. Mech. Design*, V103, 296-303 (1981).
5. T.J.McDonald, K.B.Eversole, "A Combined Finite Element-Transfer Matrix Structural Analysis Method," *J. Sound Vib.*, Vol.51, 157-169 (1977).
6. S.Sankar, S.V.Hoa, "An Extended Transfer Matrix-Finite Element Method for Free Vibration of Plates," *J. Sound Vib.*, Vol.70, 205-211 (1980).
7. M.Ohga, T.Shigematsu, T.Hara, "Structural Analysis by a Combined Finite Element-Transfer Matrix Method," *Comput. Struct.*, Vol.17, 321-326 (1983).
8. E.C.Pestel, "Note: Rectangular Transfer Matrices," *Int. J. Mech. Sci.*, Vol.5, 419-421 (1963).
9. E.Degen, M.Shephard, R.Loewy, "Combined Finite Element-Transfer Matrix Method Based on Mixed Formulation," *Comput. Struct.*, Vol.20, 173-180 (1985).
10. R.E.Skelton, P.C.Hughes, "Modal Cost Analysis for Linear Matrix-Second-Order Systems," *J. Dyn. Sys. Meas. Control*, Vol.102 (1980).

~~ABSTRACT~~

@ ABS The Finite Element - Transfer Matrix (FETM) method has been developed to reduce the computations involved in analysis of structures. This widely accepted method, however, has certain limitations, and does not directly produce reduced models for control design. To overcome these, a modification of FETM method has been developed. The modified FETM method easily produces reduced models that are tailored toward subsequent control design. Other features of this method are its ability to (i) extract open loop frequencies and mode shapes with less computations, (ii) overcome limitations of the original FETM method, and (iii) simplify the design procedures for output feedback, constrained compensation, and decentralized control.

This semi annual report presents the development of the modified FETM, and through an example, illustrates its applicability to an output feedback and a decentralized control design.

@ ABA Author.