NASA Technical Memorandum 86242

Continuous Prediction of Spartan Visibility From Orbiter Over Modeled Free-Flight Mission

Joseph C. King

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Joseph C. King
Goddard Space Flight Center
Greenbelt, Maryland

National Aeronautics
and Space Administration

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INTRODUCTION

Orbiter operations involving Spartan and other free-flying objects need the ability to see and detect such objects optically. This ability depends primarily on the brightness of the object relative to other sources present. The present analysis develops a general mathematical model by which realistic estimates of brightness and visible range can be made, based on a simplified model of the free-flyer as an optical reflector, its attitude program, and the predicted ephemerides of the Orbiter and the sun.

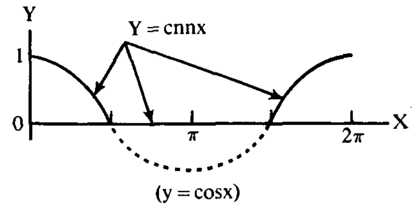
Because of the uncertainties in predicting actual Spartan-Orbiter separation distances, the visibility prediction problem is formulated here to compute maximum working range from the other variables. Working range is defined as that separation distance within which visual or optical tracking is operationally feasible, e.g., within which the brightness is sufficient for reliable detection by the Orbiter star trackers (third stellar magnitude).

The Spartan reflector model is illuminated both by direct solar radiation and radiation reflected by the earth (albedo), producing a model source of defined directional intensity. Its intensity in the Orbiter direction (along orbit) yields the desired maximum range directly. The required geometric and photometric calculations involve a number of angles in space, which are readily computed from the basic directions defining their sides. The time-dependent directions are determined by straightforward calculation from fundamental relationships and constants.

In practical application, a suitable series of computed range predictions, covering all or part of a Spartan free flight, can be plotted against time and compared to a set of actual range predictions or envelopes. The results permit determination of the intervals of usefulness of optical tracking for the mission, either as a primary or auxiliary mode. They may also suggest consideration of remedial measures, such as improving Spartan reflective properties or limiting the separation distance of Spartan from Orbiter.

NOMENCLATURE

A	azimuth of coordinate plane. A_{13} is azimuth at axis 1 of plane of axes 1 and 3.
C	range-intensity correlation constant
C_m	stellar magnitude definition constant
cnn	non-negative cosine
DF	fraction of day from 0^h UT (universal time)
DN	day number (of year)
d	distance (range), Orbiter to Spartan, meters
E	illumination, lux (lumens/m^2)
$\hat{e}$	earth center to earth albedo element unit vector
ecl	eclipse function: $\text{ecl}\,\breve{\phi}_{S(\odot)} = \left\{\begin{matrix}1\\0\end{matrix}\right\}$ when $\cos\breve{\phi}_{S(\odot)} \left\{\begin{matrix}\geq\\<\end{matrix}\right\} -\dfrac{\sqrt{r_S^2 - r_\oplus^2}}{r_S}$
FFET	free-flight elapsed time (Spartan)
$\hat{f}$	Spartan face normal (unit vector)
h	Spartan altitude, meters
i	orbit inclination, °
I	luminous intensity, candelas (lumens/steradian)
JD	Julian date (day number counting from Jan. 1, 4713 B.C.)
JD_{NY}	Julian date of last Jan. 0
L	longitude of sun
$\mathcal{L}$	secular component of longitude of sun
M	mean anomaly of sun
MET	mission elapsed time
N	orbital revolution number (Spartan released N = 1)
$\hat{R}_{ES}$	earth albedo element to Spartan unit vector
r	distance, earth albedo element to Spartan, meters
$r_\oplus$	radius of earth, 6378.14 km (equatorial)
S	area, m^2
S_{AU}	surface area of 1 AU radius sphere, m^2
$\hat{s}$	Spartan position unit vector
T	time in Julian centuries from the new standard epoch J2000.0 = 2000 Jan. 1.5 = JD245 1545.0 (Ref. 4)
t	time since last ascending node passage, sec
$-\hat{v}$	Spartan negative velocity unit vector
α	right ascension angle on celestial sphere, °
γ	off-nadir angle of element E (from Spartan), °
δ	declination angle on celestial sphere, °



ϵ	obliquity of ecliptic (angle to equator), °
ζ	sunlight reflection angle from earth albedo element, °
η	sunlight incidence angle on Spartan face, °
θ	Spartan angular displacement from ascending node
λ	orbital longitude, measured from ascending node of orbit, °
μ	sunlight reflection angle from Spartan face, °
ρ	reflectance (ρ_A = 0.30; ρ_i's given Figure 1)
σ	albedo flux incidence angle on Spartan face, °
τ	orbit period, sec
Φ	total luminous flux of sun, 3.79 x 10^{28} lumens
$\breve{\phi}$	sunlight incidence angle on earth albedo element, °
$\breve{\phi}_{S(\odot)}$	sun-earth-Spartan angle ("solar colatitude" of Spartan)
ψ	geocentric angle between albedo integration element and Spartan radius
Ω	right ascension of ascending node of orbit, °
ω	azimuth of albedo integration element, measured CW from Spartan meridian
☊	ascending node
$\hat{\odot}$	sun (from earth) unit vector

<u>Subscripts</u>

A	albedo
a	dummy angle symbol
D	direct (solar illumination)
d	deployment (Spartan)
E	end of mission
ES	albedo integration element to Spartan line
e	earth center to earth albedo element line
f	Spartan face normal
i	index identifying Spartan faces (i = 1, 2, 3, . . .6)
j	index identifying circular boundaries in earth albedo zone integration (j = 0, 1, 2, . . .7)
k	index identifying sectoral boundaries in earth albedo zone integration (k = 0, 1, 2, . . .17)
n	rotation serial number
R	recovery
S	Spartan
v	along velocity vector
w	dummy axis index
1, 2, 3	floating axis identifiers
3m	third stellar magnitude
$\odot$	sun
$\oplus$	earth

Superscripts

$\bullet$	derivative wrt time
$\hat{}$	unit vector
$\breve{}$	complement of (angle). $\check{\alpha} = 90° - \alpha$
$\tilde{}$	supplement of (angle). $\tilde{\alpha} = 180° - \alpha$
$\frown$	conjugate (explement) of (angle). $\overset{\frown}{\alpha} = 360° - \alpha$

CONTINUOUS PREDICTION OF SPARTAN VISIBILITY FROM ORBITER OVER MODELED FREE-FLIGHT MISSION

SUMMARY

For a Spartan free-flight mission specified by:

1) Spartan attitude program,
2) Spartan optical reflective properties,
3) Orbit elements,
4) Time phasing of attitude, orbital, and solar positions,

compute and plot vs. time the estimated range at which the apparent brightness of Spartan is equal to some minimum useful level (e.g., third stellar magnitude for Orbiter star trackers). Considering only direct solar illumination of Spartan, the third magnitude range d_{3mD} is derived from the luminous intensity I_D of Spartan by

$$d_{3mD} = C\, I_D^{1/2} \tag{1}$$

where C = 2484.37 m/lumen$^{1/2}$ (see CONSTANTS)

$$I_D = \frac{E_{\odot(\oplus)}}{\pi} \sum_{i=1}^{6} \rho_i\, S_i \,\mathrm{cnn}\, \eta_i \,\mathrm{cnn}\, \mu_i \,\mathrm{ecl}\, \check{\phi}_{s(\odot)} \tag{2}$$

Including the additional illumination of Spartan by sunlight reflected from the earth (albedo), the third magnitude range becomes

$$d_{3m} = C(I_D + I_A)^{1/2} = C\, I^{1/2} \tag{3}$$

$$\text{where } I_A = \frac{2E_{\odot(\oplus)}\, \rho_A}{\pi^2} \sum_{i=1}^{6} \sum_{j=o}^{7} \sum_{k=o}^{17} \mathrm{cnn}\check{\phi}_{jk}\, S_j \cos\zeta_j \;\mathrm{cnn}\sigma_{ijk}\, \rho_i S_i \mathrm{cnn}\mu_i\, r_j^{-2} \tag{4}$$

The above symbols are identified under NOMENCLATURE. Derivations and values are given in the discussion which follows.

CONSTANTS

Range-Intensity Correlation Constant

Equations (1) and (3) relate range (distance d_{3m}) for third magnitude brightness (illumination E_{3m}) to the luminous intensity of the "source" (Spartan). They are derived from the stellar magnitude definition

$$10^{0.4m}\, E = C_m \tag{5}$$

and the geometric relationship between intensity and illumination

$$E = I/d^2 \tag{6}$$

C_m is a constant (arbitrary but established) which can be evaluated from a known reference, such as the sun's luminous flux $\Phi = 3.79 \times 10^{28}$ lumens and designated magnitude $m = -26.8$ at earth. Substituting (5) in (6)

$$d_{3m} = (10^{0.4m}/C_m)^{1/2}\, I^{1/2} = C\, I^{1/2}$$

where $C = 10^{0.2(m - m_{\odot})} (S_{AU}/\Phi_{\odot})^{1/2} = 2484.37$ m/lumen$^{1/2}$

S_{AU} is the surface area of the sun-centered spherical surface through the earth over which the sun's total luminous flux is considered distributed in computing the flux density (illumination $E_{\odot(\oplus)}$) at the earth (1 AU = 1.496×10^{11} m. away).

Spartan Reflector Model

For the purposes of this analysis, the optical reflective properties of Spartan 1 are approximated by the Spartan Reflector Model shown in Figure 1. The figure lists the approximate face areas S_i and diffuse reflectance values ρ_i for use in Equations (2) and (4).

Earth Albedo

The average earth albedo value $\rho_A = 0.30$ is adopted for use in this analysis. This value, taken from Reference 1, appears to be conservative. The illumination of Spartan by earth albedo flux is estimated via the approximate integration scheme shown in Figure 2, which defines the area element S_j.

ANGLES

The six generic angles which appear in the two intensity equations ((2) and (4)) are angles between 11 directions in space: the Spartan position vector $\hat{s}$, the negative velocity vector $(-\hat{v})$, the sun (from earth center) $\hat{\odot}$, the earth albedo element (from earth center) $\hat{e}$, the albedo element-to-Spartan line $\hat{R}_{ES}$, and the six Spartan face normals $\hat{f}_i$. These directions are all derived in the next section and expressed in equatorial coordinates (right ascension α and declination δ on the celestial sphere). The required angles between the individual pairs are then determined using a single general relationship (spherical law of cosines):

$$\cos a = \cos\breve{\delta}_1 \cos\breve{\delta}_2 + \sin\breve{\delta}_1 \sin\breve{\delta}_2 \cos(\alpha_2 - \alpha_1) \quad (7)$$

Table 1 lists the six generic ray angles, specifies their defining directions (sides), and tabulates their coordinates for entry into Equation (7) to compute the size of the particular angle defined.

Table 1. Ray Angle Determination

Angle	Initial Side (1)			Terminal Side (2)		
	Description	Direction Coordinates		Description	Direction Coordinates	
a		α_1	δ_1		α_2	δ_2
η_i	Spartan face normal, $\hat{f}_i$	α_{fi}	δ_{fi}	Sun line, $\hat{\odot}$	$\alpha_{\odot}$	$\delta_{\odot}$
μ_i	Spartan face normal, $\hat{f}_i$	α_{fi}	δ_{fi}	Negative velocity vector, $-\hat{v}$	α_{-v}	δ_{-v}
$\breve{\phi}_{jk}$	Albedo element normal, $\hat{e}$	α_e	δ_e	Sun line, $\hat{\odot}$	$\alpha_{\odot}$	$\delta_{\odot}$
$\phi_{s(\odot)}$	Spartan position vector, $\hat{s}$	α_s	α_s	Sun line, $\hat{\odot}$	$\alpha_{\odot}$	$\delta_{\odot}$
ζ_j	Albedo element normal, $\hat{e}$	α_e	δ_e	Element-to-Spartan line, $\hat{R}_{ES}$	α_{ES}	δ_{ES}
σ_{ijk}	Spartan face normal, $\hat{f}_i$	α_{fi}	δ_{fi}	Element-to-Spartan line, $\hat{R}_{ES}$	α_{ES}	δ_{ES}

$$\cos a = \cos\breve{\delta}_1 \cos\breve{\delta}_2 + \sin\breve{\delta}_1 \sin\breve{\delta}_2 \cos(\alpha_2 - \alpha_1) \quad (7)$$

Face	Normal	Area, S_i	Reflectance (diffuse), ρ_i
1	X	1.301 m^2	0.05
2	-X	1.301 m^2	0.82
3	Y	2.694 m^2	↓
4	-Y	2.694 m^2	↓
5	Z	2.357 m^2	↓
6	-Z	2.357 m^2	↓

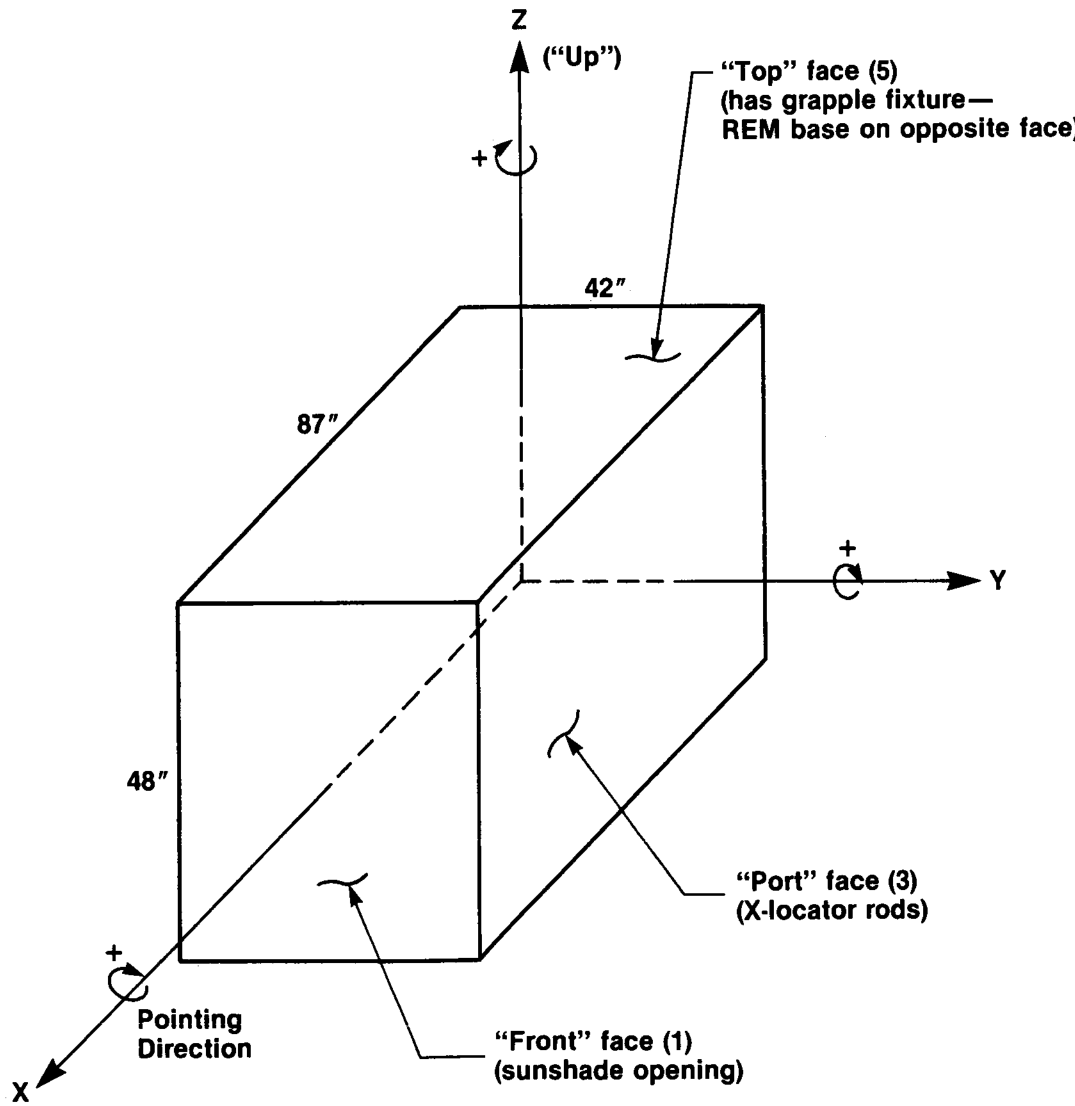
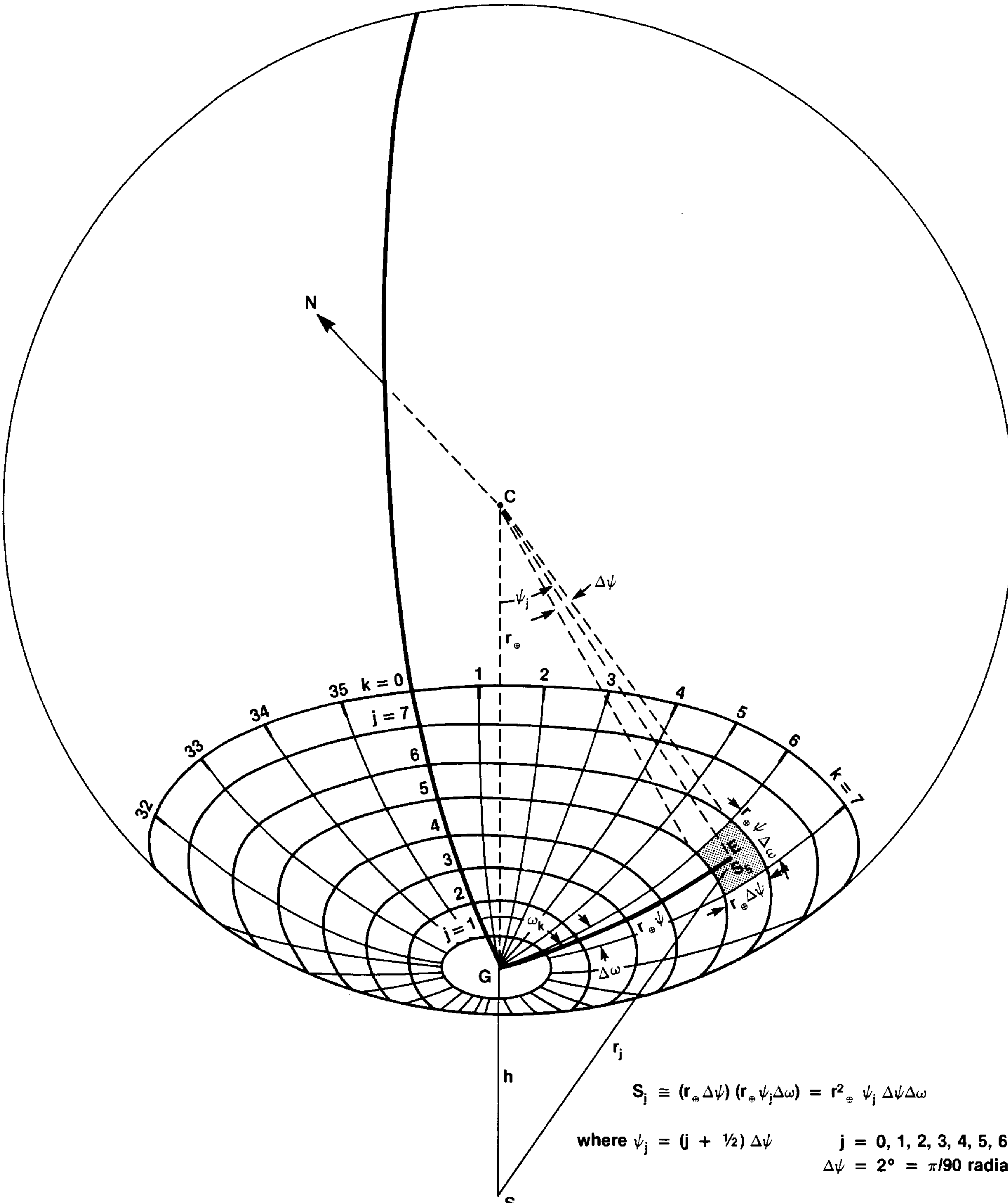


Figure 1. Spartan Reflector Model with "Pointing" Reference Frame



$$S_j \cong (r_\oplus \Delta\psi)\,(r_\oplus \psi_j \Delta\omega) = r^2_\oplus\ \psi_j\ \Delta\psi\Delta\omega$$

where $\psi_j = (j + ½)\ \Delta\psi$ $\qquad j = 0, 1, 2, 3, 4, 5, 6, 7$

$\Delta\psi = 2° = \pi/90$ radians

$\omega_k = (k + ½)\Delta\omega$ $\qquad k = 0, 1, 2, 3. . .17$

$\Delta\omega = 10° = \pi/18$ radians

Figure 2. Approximate Integration of Earth Albedo Illumination of Spartan

DIRECTIONS

Sun ($\hat{\odot}$)

The coordinates of the sun at any instant can be computed using formulas from Reference 2:

$$\alpha_{\odot} = \arctan(\cos\epsilon \tan L) \tag{8}$$

$$\delta_{\odot} = \arcsin(\sin\epsilon \sin L) \tag{9}$$

where $\epsilon = 23.441°$
$L = \mathcal{L} + 1.°916 \sin M + 0.°020 \sin 2M$
where $\mathcal{L} = 280.°460 + 36000.°772\ T$
$M = 357.°528 + 35999.050\ T$
where $T = (JD - 2451545.0)/36525$

The Julian date JD of a given instant can be determined in various ways. A convenient way to accommodate a range of launch dates is to express JD as the sum:

Julian Date of preceding January 0	JD_{NY}	=	______________
Day number (of year)	DN	=	______________
Fraction of day from 0^h UT	DF	=	______________
Julian Date (sum of above three)	JD	=	______________

Spartan Position Vector ($\hat{s}$)

Referring to Figure 3, the Spartan equatorial coordinates α_s and $\breve{\delta}_s$ are found by solving the right spherical triangle ☊GL to obtain

$$\tan\lambda_s = \cos i \tan\theta \tag{10}$$

$$\cos\breve{\delta}_s = \sin i \sin\theta \tag{11}$$

The desired Spartan coordinate α_s is obtained by adding the right ascension of the orbit ascending node Ω to the Spartan orbital longitude λ_s

$$\alpha_s = \Omega_o + [(N-1)\tau + t]\dot{\Omega} + \arctan(\cos i \tan\theta) \tag{12}$$

where Ω_o = right ascension of ascending node immediately preceding Spartan release
N = orbital revolution number (Spartan released during revolution $N = 1$)
τ = orbit period
t = time since last ascending node passage
$\dot{\Omega}$ = nodal precession rate
$\theta = \dot{\theta}t = 360°t/\tau$

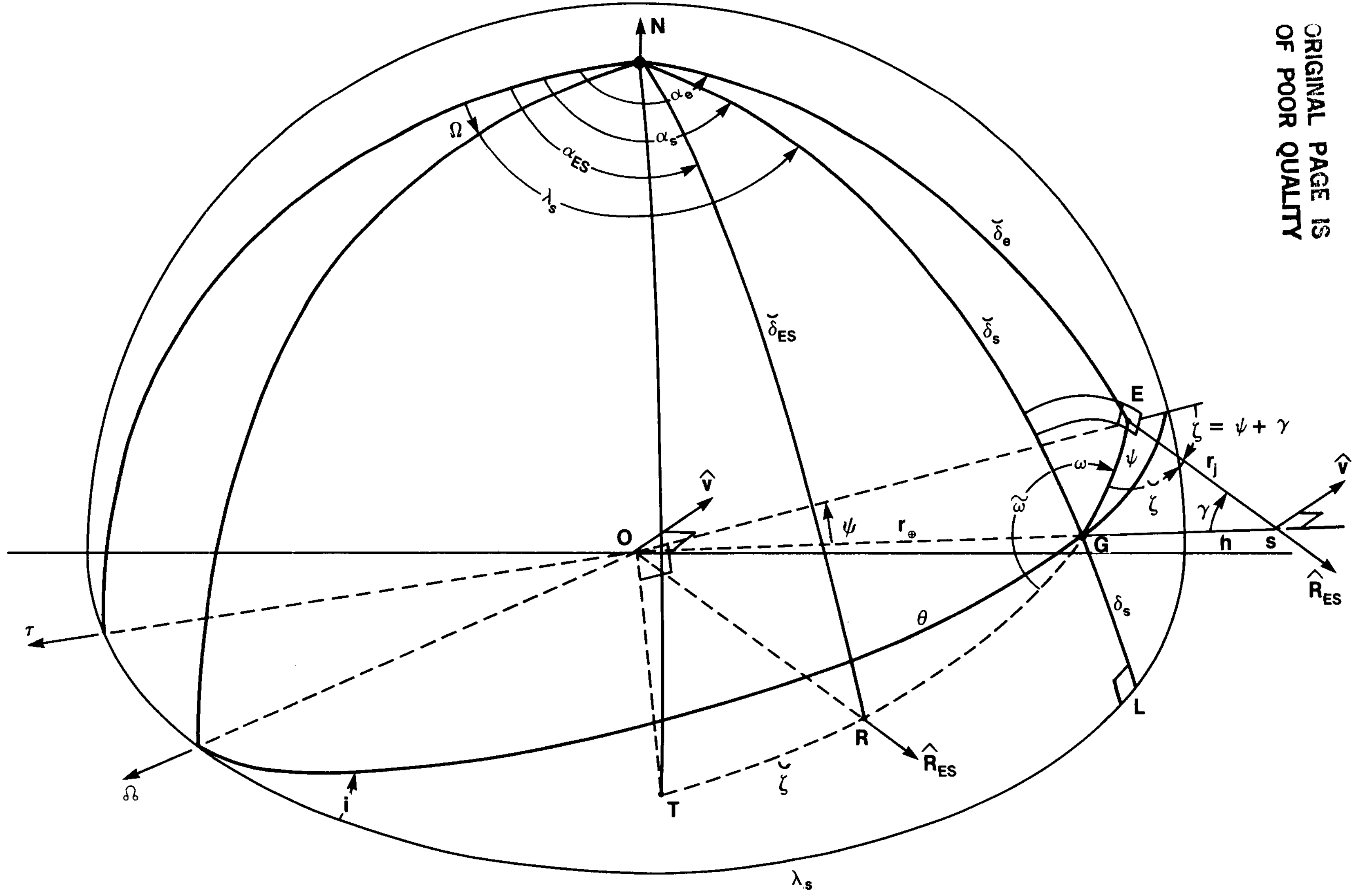


Figure 3. Geometric Relationships Determining Directions

Negative Velocity Vector Direction ($-\hat{v}$)

The Spartan velocity vector direction at a given moment coincides with that of the radius vector 90° ahead of Spartan. Substituting $\theta_v = \theta + 90°$ into Equations (10) and (11)

$$\tan \lambda_v = -(\cos i)/\tan \theta \tag{13}$$

$$\cos\breve{\delta}_v = \sin i \cos \theta \tag{14}$$

Again the orbital longitude λ_v is converted to right ascension by adding Ω, the right ascension of the node

$$\alpha_v = \Omega_o + [(N-1)\tau + t]\,\dot{\Omega} + \arctan (\cos i)/\tan \theta \tag{15}$$

Albedo Element Direction ($\hat{e}$)

Referring to Figure 3, the coordinates α_e and $\breve{\delta}_e$ of the albedo element E are found by solving the oblique spherical triangle GNE yielding

$$\cos \breve{\delta}_e = \cos\breve{\delta}_s \cos \psi + \sin\breve{\delta}_s \sin\psi \cos\omega \tag{16}$$

$$\sin (\alpha_e - \alpha_s) = (\sin\psi \sin\omega)/ \sin\breve{\delta}_e \tag{17}$$

ψ and ω are explicitly defined and used to specify the grid of elements for the summation approximating the total albedo illumination of Spartan (see Figure 2).

Element-to-Spartan Direction ($\hat{R}_{ES}$)

The element-to-Spartan line direction $\hat{R}_{ES}$ coordinates can be obtained from Figure 3 by producing a line from the earth center parallel to $\hat{R}_{ES}$. If the radius OE is swung 90° clockwise along the great circle of EG to T, it coincides with the radius OT, which is parallel to the surface tangent at E. If the radius is swung back through the elevation angle $\breve{\zeta}$ to R, it coincides with the desired parallel to $\hat{R}_{ES}$ through O. The desired coordinates are those of R, α_{ES} and $\breve{\delta}_{ES}$, found by solving the oblique spherical triangle RNG

$$\cos \breve{\delta}_{ES} = \cos\breve{\delta}_S \cos(90° - \psi - \breve{\zeta}) + \sin\breve{\delta}_S \sin(90° - \psi - \breve{\zeta}) \cos \varpi \tag{18}$$

$$\sin (\alpha_S - \alpha_{ES}) = \sin\omega \cos(\psi + \breve{\zeta})/ \sin\breve{\delta}_{ES} \tag{19}$$

The zenith angle ζ, as well as the distance r_j, can be found by solving the oblique plane triangle EOS

$$r_j = [r_\oplus^2 + (r_\oplus + h)^2 - 2r_\oplus (r_\oplus + h) \cos \psi]^{1/2} \tag{20}$$

$$\sin\zeta = (r_\oplus + h)(\sin\psi)/r_j \tag{21}$$

Spartan Face Normals ($\hat{f}_i$)

The time-varying orientation in space of the Spartan face normals is determined by the Spartan attitude program, which is preplanned based on observing and operating requirements. The attitude program is typically specified by an initial orientation (direction of one axis and rotational position of Spartan about it), followed by a timed sequence of rotations about selected axes. The specification format is illustrated by Table 2, which is the basic attitude program planned for Spartan 1.

While a plan like that of Table 2 completely determines the Spartan attitude program, it does not give explicitly the axis (face normal) directions needed here for photometric calculations. These can be computed using Equations (22) through (27), following the cyclic substitution routine illustrated in Table 3.

Table 2. Attitude Program for Spartan 1 (4-23-84)

Time			Rotation				Orientation After Rotation			
Julian Date, JD	Rev. No., N	Time since Node, t	Type No.	Serial No. η	Axis*	Displacement, ΔA	Pointing* Axis: Rt. Asc. α	Pointing* Axis: Decl. δ	Up* Axis: Azim.** A	
(days)		(sec)				(deg)	(deg)	(deg)	(deg)	
(To be determined. See Figure 6)	3	1997	0	0	—	0.0	20.662	45.101	265.559 ← Initial Orient.	Acquisition and Calibration circuit
		2085	1	1	Roll	-28.547	•	•	• →	
		2217	2	2	Ptich	-175.866	•	•	• →	
		3458	3	3	Roll	15.889	•	•	• →	
		3541	4	4	Pitch	-88.899	↓	↓	↓	
		4977	5	5	Roll	15.388	three orientation coordinates for all six axis directions (±X, ±Y, ±Z, the face normals) to be computed using algorithm shown schematically in Table 3.			
		5045	6	6	Pitch	-59.593				
		5117	7	7	Roll	-23.464				
		5329	8	8	Pitch	-38.016				
	4	55	9	9	Roll	32.478				
		136	10	10	Pitch	-67.477				
		2181	11	11	Roll	0.276				
		2266	12	12	Pitch	-108.397				
		3472	13	13	Roll	12.347				
		3552	14	14	Pitch	-86.899				
		4976	15	15	Roll	15.091				
		5045	16	16	Pitch	-61.523				
		5118	17	17	Roll	-22.860				
Do 10 times	4, 5,. . .13	5329	18	18, 28,. . .108	Pitch	-38.016				Science Circuit 1
	5, 6,. . .14	55	19	19, 29,. . .109	Roll	32.478				
		136	20	20, 30,. . .110	Pitch	-67.477				
		2181	21	21, 31,. . .111	Roll	0.276				
		2266	22	22, 32,. . .112	Pitch	-108.399				
		3473	23	23, 33,. . .113	Roll	15.804				
		3561	24	24, 34,. . .114	Pitch	-99.619				
		4989	25	25, 35,. . .115	Roll	15.284				
		5052	26	26, 36,. . .116	Pitch	-48.344				
		5118	27	27, 37,. . .117	Roll	-22.852				
Do 3 times, but		5327	28	118, 124, 130	Pitch	-38.016				Science Circuit #2
	15, 18, 21	55	29	119, 125, 131	Roll	32.478				
		136	30	120, 126, 132	Pitch	-67.477				
	16, 19, 22									
Skip on 3rd pass	17, 20	2214	31	121, 127	Pitch	67.477				
		2294	32	122, 128	Roll	-32.478				
		4151	33	123, 129	Pitch	38.016				
↑	23	2188	34	133	Roll	14.662				Attit. Maint. Circ.
		2266	35	134	Pitch	-22.366				
		2303	36	135	Roll	50.000				

Ticks are asc. node crossings

*Roll and Pitch rotations are about the X (Pointing) and Y axes respectively, positive clockwise when looking toward the origin along the positive axis. See Figure 1.

**Azimuth angles are measured in the YZ plane from north to the Z axis, positive clockwise when looking toward the origin along the positive X axis.

Table 3. Completion of Body Axes Coordinates Matrix via Cyclic Algorithm

J. Date (days)	Time Rev. No. N	Time since Node (sec) t	Rotation Ser. No. η	Rotation Axis (Program Index)	Dis*placement (°) ΔA	Coordinates of Axes After Rotation (°): Rt. Asc. α_x	(±) Roll Decl. δ_x	Azim. A_x	Rt. Asc. α_y	(±) Pitch Decl. δ_y	Azim. A_y	Rt. Asc. α_z	(±) Yaw Decl. δ_z	Azim. A_z	(cylic repeat of prev. columns): Rt. Asc. α_x	(±) Roll Decl. δ_x	Azim. A_x	Rt. Asc. α_y	(±) Pitch Decl. δ_y	Azim. A_y
			0																	
			1																	
			2																	
			3																	
	Axes Coordinates Algorithm																			
	Rotation Axis Selection Indices								Known			Calculated (Eqs. (22) thru (31))								
	Yaw ↓			Pitch ↓			Roll ↓		α_1 ↑	δ_1 ↑	A_1 ↑	α_2 ↓	δ_2 ↓	A_2 ↓	α_3 ↓	δ_3 ↓	A_3 ↓			
			8	Pitch																
			9																	
			10																	
				Roll ↓																
			15	Roll																
			16																	
			17																	
			18																	
			19	Yaw																
			20	↓																
			21	Yaw																
			22																	
			⋮																	

*Add the present displacement ΔA_n to the previous azimuth A_{n-1} of the current rotation axis to get the present azimuth A_n.

$$\cos\breve{\delta}_2 = \sin A_{13} \cos\breve{\delta}_1 \quad (22)$$

$$\cos A_{21} = \frac{\sin\breve{\delta}_1}{\sin\breve{\delta}_2} \quad (23)$$

$$\cos \lambda_2 = -\sin A_{13} \cos A_{21} \quad (24)$$

$$\cos \breve{\delta}_3 = \cos A_{13} \sin\breve{\delta}_1 \quad (25)$$

$$\sin A_{32} = -\frac{\cos\breve{\delta}_1}{\sin\delta_3} \quad (26)$$

$$\cos \lambda_3 = \cos A_{13} \sin A_{32} \quad (27)$$

The geometric basis of Equations (22) through (27) is illustrated in Figure 4. The main sketch depicts the overall geometric relationships and defines the pertinent variables. The two small sketches are details of quadrantal spherical triangles which appear on the main sketch. Their solutions via their polar triangles yield Equations (22) through (27).

The additional relationships needed for formal completion of Table 3 are straightforward:

$$\alpha_2 = \alpha_1 + \lambda_2 \quad (28)$$

$$\alpha_3 = \alpha_1 + \lambda_3 \quad (29)$$

$$\left.\begin{aligned} \alpha_{-w} &= \alpha_w \pm 180^\circ \quad (30) \\ \delta_{-w} &= -\delta_w \quad (31) \end{aligned}\right\} \quad w = X, Y, Z$$

BODY REFERENCE AXES

The body reference axis system used in this analysis (Figure 1) is chosen for convenience, clarity, and compatibility with conventional usage. It is referred to as the ''Pointing'' system because the positive X axis is in the experiment pointing direction. The Z axis is conventionally ''up'' from the REM base in the Orbiter. The Y axis is to the left or port (along X-locator rods) when facing in the pointing direction, yielding a right-handed system. Rotations are considered positive which appear clockwise when viewed toward the origin.

The Pointing system is shown in Figure 5 in relation to physical features of Spartan, and also in relation to the ''ACS'' system and two standard Orbiter reference systems. Note that all directions in the ACS system are opposite to those of the Pointing System. Positive rotations have the same physical direction in both systems because opposite look directions in defining clockwise rotation offset the opposite axis directions.

COMPUTING AND PLOTTING SCHEME

Generating the required range vs. time plots is a stepwise process requiring an arbitrary number of discrete range calculations, depending on the length of the mission, the particular Spartan attitude program, etc. Selecting a suitable calculating and plotting procedure may require iteration in some cases, but for a

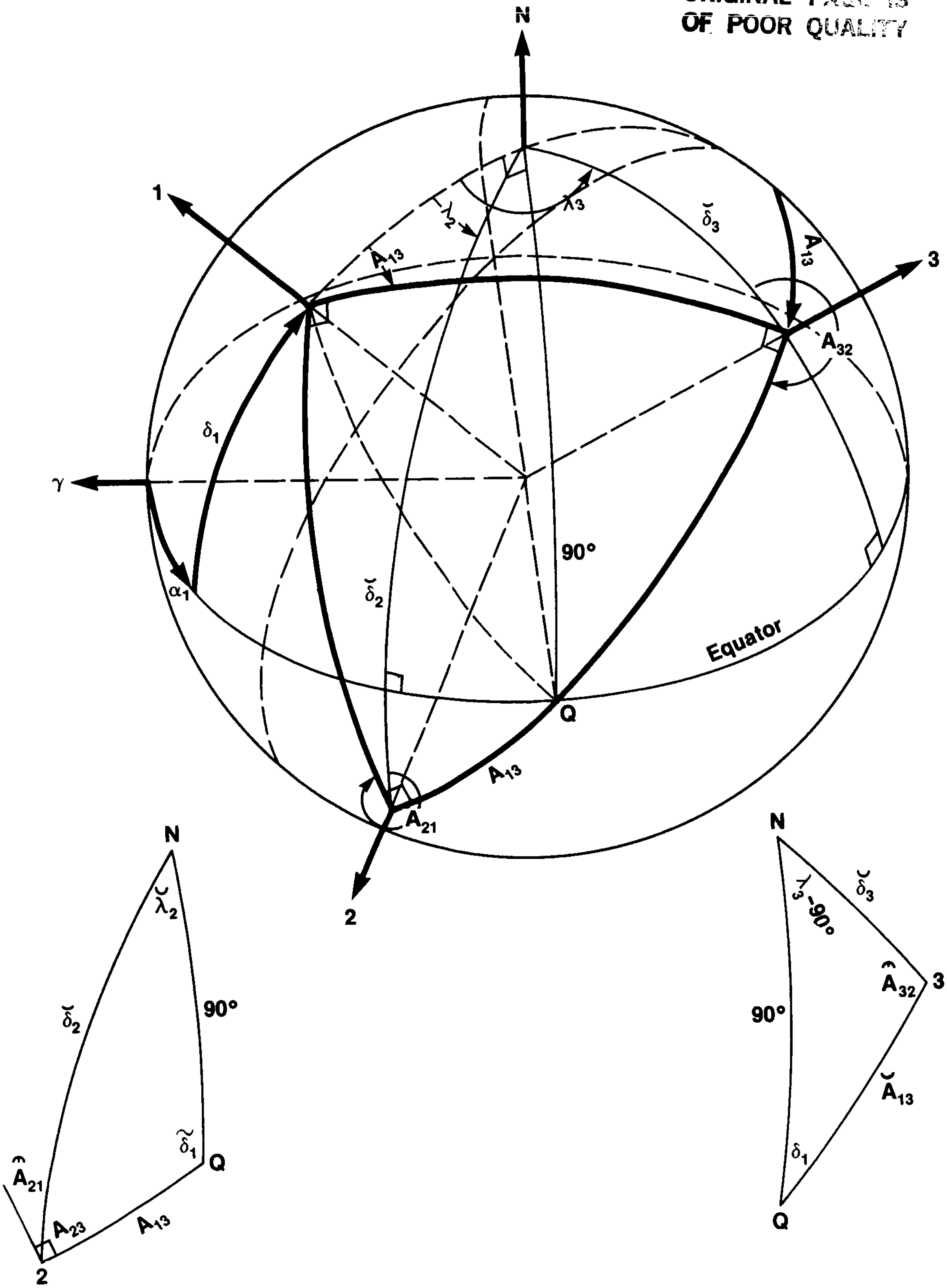


Figure 4. Relationships Between Equatorial (α, δ)/Azimuth (A) Orientation Coordinates of Three Orthogonal Axes

practical attitude program in the form of Table 2, the simple procedure illustrated in Figure 6 will be a convenient tentative choice.

Figure 6 also illustrates the time phasing relationships between the Spartan attitude program, which is phased primarily to the nodal passages ($☊_0$, $☊_1$, $☊_2$, . . .), STS mission events, and the basic Julian Date time reference system. As noted in Table 2, the Julian Date phasing is conveniently left open in preflight modeling, primarily to accommodate likely variations in launch date and time.

	Reference Axis Systems			
Face Identifying Features	**Pointing**	**ACS***	**Body AXES****	**Orbiter Structural Body****
Pointing (front)	**X**	**(-X)**	**X**	**-X**
Antipointing (rear)	**-X**	**Roll (X)**	**-X**	**X**
X-Locator Rods (port)	**Y**	**(-Y)**	**-Y**	**-Y**
SOB Side (starboard)	**-Y**	**Pitch (Y)**	**Y**	**Y**
Grapple Fixture (up)	**Z**	**(-Z)**	**-Z**	**Z**
REM (down)	**-Z**	**Yaw (Z)**	**Z**	**-Z**

***The ACS axes are left-handed if taken in the roll, pitch, yaw order. The others are right-handed.**
****See Reference 3.**

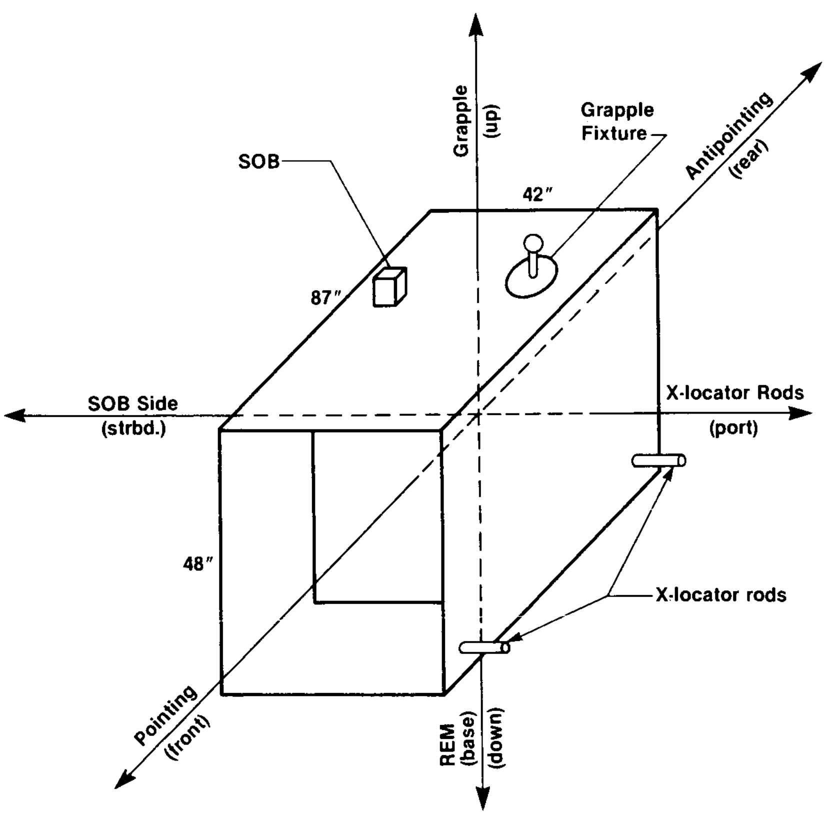


Figure 5. Spartan Body Reference Axes

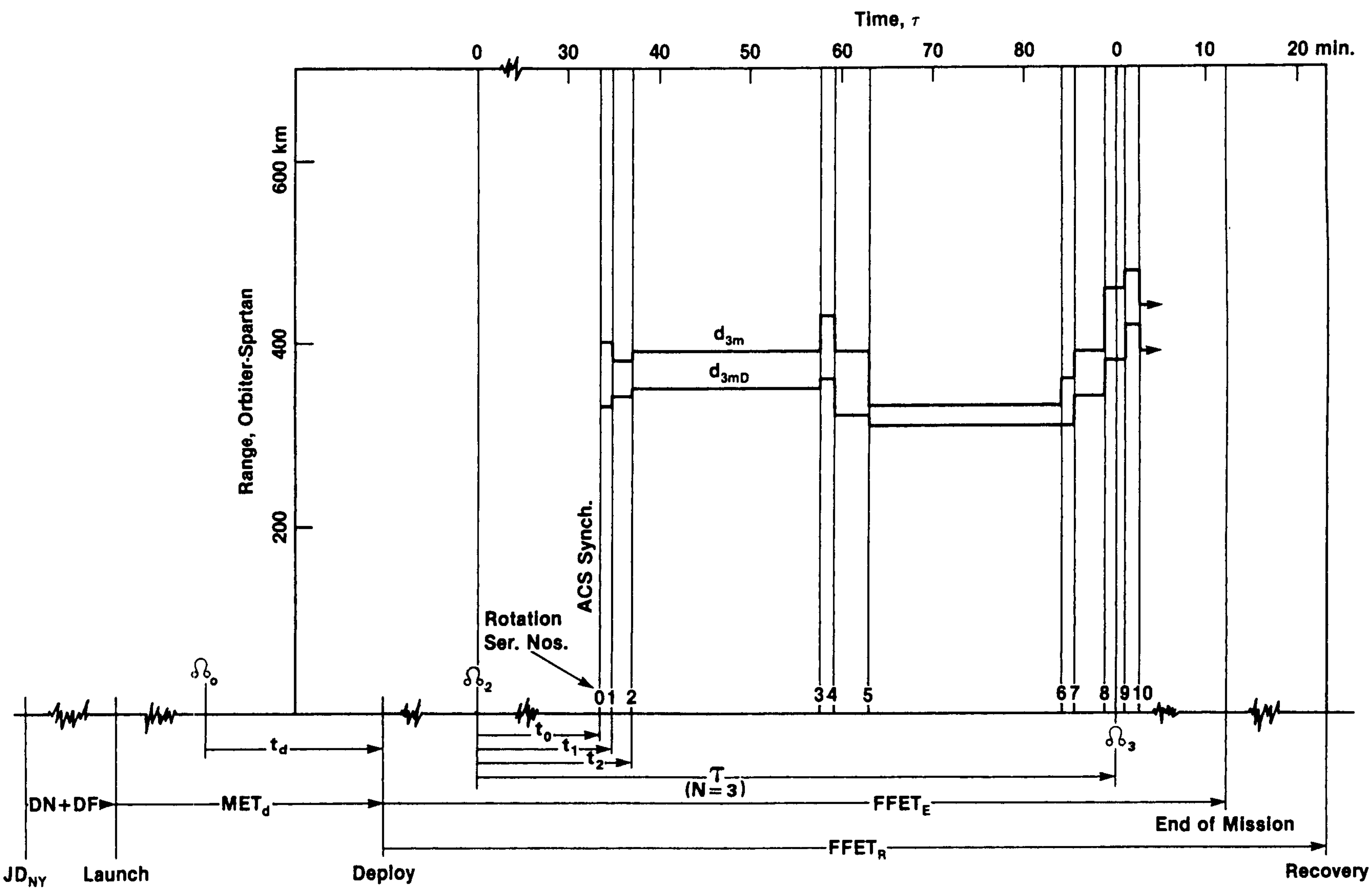


Figure 6. STS/Spartan Events Time Line with Trial Visibility Range Plot Format

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16. Abstract

Orbital operations in the neighborhood of other satellites or free-flying objects need the ability to see and detect such objects optically. This ability depends primarily on the brightness of the object relative to other sources present. The present analysis and computational procedure provides a means for predicting the visual brightness of a satellite when viewed from a nearby satellite in the same orbit. It is designed specifically for estimating the brightness of Spartan free-flyers from the STS Orbiters which release and later retrieve them, but the basic methods are applicable to other satellite-to-satellite visibility prediction problems. The Spartan reflector model defined herein is illuminated both by direct solar radiation and by the earth (albedo), producing a model source of defined directional intensity. The intensity in the Orbiter direction (along orbit) yields the desired maximum range directly. The required geometric and photometric calculations involve a number of angles in space, which are readily computed from the basic directions defining their sides. The time-dependent directions are determined by straight-forward calculation from fundamental relationships and constants.

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