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P-23

(NASA-CR-179934) MIDDLE ATMOSPHERE MODELING  
Final Report (Johns Hopkins Univ.) 23 p  
CSCI 04A

N87-13023

G3/46 Unclass  
44714

Final Report on  
Middle Atmosphere Modeling

by

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December 1, 1984 to November 30, 1985

NAG 5-489

## Brief Summary

The principal research effort accomplished under NASA Grant NAG 5-489 was the research reported in Attachment 1 entitled "Energy Balance Constraints on Gravity Wave Induced Eddy Diffusion in the Mesosphere and Lower Thermosphere." This paper is in press and should appear in the November 1985 issue of Journal of Geophysical Research--Atmospheres.

In this paper we make a strong case for turbulent diffusive transport of constituents and potential temperature at rates significantly less than used by most workers in the field. Recent analysis of microwave derived water vapor mixing ratio profiles in the upper mesosphere by the PI substantiate this conclusion of the paper.

We also argued in this paper that larger diffusion coefficients are permitted for chemically active species than the constraints derived from thermodynamic considerations. Recent work by the PI shows that the large diffusion coefficients used by researchers to transport odd oxygen can be supplied by gravity waves whose amplitudes are near their saturation value.

In addition some preliminary research was carried out in non-LTE radiative transfer. This research will be continued in subsequent NASA grants and used in the analysis of SME data.

# Attachment 1

Energy Balance Constraints on Gravity Wave Induced Eddy Diffusion  
in  
The Mesosphere and Lower Thermosphere

by

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June 1985

*J. G. R. Atmospheres, in press*

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## Abstract

Breaking gravity waves generate and maintain a background level of turbulence which is capable of producing substantial cooling and/or heating in the upper mesosphere and lower thermosphere. The net thermodynamic effect of breaking gravity waves is critically dependent on the eddy Prandtl number ( $P_t$ ) applicable to mesospheric turbulence. When  $P_t \sim 1$ , our calculations of the heat budget for the mesopause region imply that the globally averaged eddy or turbulent diffusion coefficient cannot exceed  $10^6 \text{ cm}^2 \text{ s}^{-1}$ . This upper limit on turbulent diffusion applies to both potential temperature transport and chemically inert tracer transport when radiative damping is negligible. For chemically active species larger diffusion coefficients are permitted, because the effective eddy diffusion coefficient is increased by an additive term  $L/2\gamma^2$ , where  $L$  is the chemical loss rate and  $\gamma$  is the vertical wavenumber.

For  $P_t \sim 4-6$ , the turbulent diffusion of momentum ( $D_M$ ) is sufficiently greater than the turbulent diffusion of heat ( $D_H$ ) that the conversion of gravity wave energy to heat with high efficiency nearly balances the divergence of the downward eddy heat flux in the wave breaking zone. Therefore the heat budget of the mesopause region would no longer provide a powerful and useful constraint on  $D_H$ . If  $P_t$  exceeds 6 with high efficiency for energy conversion to heat, gravity waves would heat the mesosphere throughout the wave breaking region.

## 1. Introduction

Lindzen's (1981) parameterization of turbulence and stress generated by breaking gravity and tidal waves initiated considerable research on constituent transport and the momentum and heat budget of the mesosphere (e.g. Holton 1982, 1983; Dunkerton, 1982; Schoeberl et al., 1983; Apruzese et al., 1984; Fritts and Dunkerton, 1985; Garcia and Solomon, 1985). The identification of a specific mechanism which produces turbulent mixing allows a quantitative description of associated effects even though it may be highly parameterized. On the basis of the research referenced above it would appear that the intensity of the turbulence as represented by the eddy or turbulent diffusion coefficient,  $D(\text{cm}^2\text{s}^{-1})$ , must be greater than  $10^6 \text{ cm}^2\text{s}^{-1}$  from studies of the momentum budget and constituent transport, whereas our previous study of the heat budget (Apruzese et al., 1984) concluded that  $D < 10^6 \text{ cm}^2\text{s}^{-1}$  with a preferred value of  $6 \times 10^5 \text{ cm}^2\text{s}^{-1}$ .

Taken at face value we might deduce that the effective eddy Prandtl number must be greater than one. (The eddy Prandtl number,  $P_t$ , is the momentum diffusion coefficient,  $D_M$ , divided by the thermal diffusion coefficient,  $D_H$ .) In a turbulent atmosphere tracer transport and potential temperature transport should be described by the same coefficient,  $D_H$ , in the limit of a chemically inert species and adiabatic motion, if K theory adequately describes turbulent transport processes (see Kraichnan, 1976, for comments on this simplification). The studies of Allen et al. (1981) and Garcia and Solomon (1985) clearly suggest that  $D_H$  exceed  $10^6 \text{ cm}^2\text{s}^{-1}$  for chemically active species in contrast to the upper limit on  $D_H$  obtained by Apruzese et al. (1984). In the case of Garcia and Solomon (1985) their net  $D_H$  had only a weak dependence on the chemical loss rate and thus should be approximately equal to the heat diffusion coefficient. They assumed  $P_t = 1$  and calculated  $D_M$  with Holton's (1982) modification of Lindzen's (1981) parameterization of breaking gravity waves. As indicated above they obtained  $D_M > 10^6 \text{ cm}^2\text{s}^{-1}$  in the mesopause region, with largest values during solstice.

The principal objective of this report is to re-examine the constraints on turbulence imposed by the heat budget and determine whether there is sufficient theoretical evidence to support the hypothesis that the eddy Prandtl number is greater than one in the mesosphere. The mesopause thermal structure is calculated with turbulent diffusion coefficients commonly used in chemical models and deduced from mean zonal wind deceleration. As seen below, extreme mesopause temperatures of less than 100 K are produced by the large net cooling.

## 2. Model

The model adopted for this study includes the non-LTE version of the Apruzese et al. (1982) two-stream IR algorithm, the physical processes of molecular and turbulent heat conduction, odd oxygen transport, gravity wave dissipation, and NO IR cooling as described in Apruzese et al. (1984). The approximate IR algorithm incorporates exact cooling to space escape probabilities from Fels and Schwarzkopf's (1981) extensive tables on CO<sub>2</sub> transmittances to generate accurate cooling to space rates. Radiative exchange between atmospheric layers is treated approximately; consequently only thermal disturbances with vertical wavelengths greater than 8 km can be treated with sufficient accuracy in the model. None of the results presented below are sufficiently sensitive to our approximate IR cooling and heating rates to warrant a substantially more accurate treatment of IR radiative transfer at this time.

Vertically propagating gravity waves carry energy which is preferentially deposited as heat in the wave breaking zone as a result of the turbulence created. Schoeberl et al. (1983) gave a simple analytic expression for this heating in terms of the efficiency  $\epsilon$  with which gravity wave energy is converted to heat

$$H_g = \frac{N^2 D_M}{2c_p} (1 + P_t^{-1}) \epsilon \quad \text{K s}^{-1} \quad (1)$$

where  $N$  is the buoyancy frequency, and  $c_p$  is the specific heat at constant pressure. Also turbulence generated by breaking gravity waves leads to a downward turbulent heat flux whose divergence (convergence) results in cooling (heating). The usual expression for this process acting on the mean temperature field is

$$C_g = \frac{1}{\rho c_p P_t} \left( \frac{\partial}{\partial z} \left( \rho c_p D_M \left( \frac{\partial T}{\partial z} + \frac{g}{c_p} \right) \right) \right) \quad \text{K s}^{-1} \quad (2)$$

where  $C_g > 0$  for heating,  $\rho$  is mass density,  $T$  is temperature,  $g$  is gravitational acceleration, and  $z$  is altitude.

Given the variety of physical and radiative processes outlined above and discussed in depth in Apruzese et al. (1984) the globally averaged equilibrium temperatures in the mesosphere and lower thermosphere were calculated as a function of the eddy diffusion coefficient,  $D_M$ , and eddy Prandtl number,  $P_t$ . In computing these temperatures it was presumed that the correlation between the turbulent diffusion and temperatures could be represented by the product of their global averages. In other words if  $\langle \rangle$  represents a global average then we assumed

$$\left\langle \frac{\partial}{\partial z} D \frac{\partial T}{\partial z} \right\rangle = \frac{\partial}{\partial z} \langle D \rangle \frac{\partial}{\partial z} \langle T \rangle$$

We have tested this assumption under solstitial conditions using an analytic model of the mesospheric wind and temperature structure. Using Lindzen's formula  $D \propto (\bar{u}-c)^4$  we found that using global averages to represent the heat flux is accurate to better than 1%.

The global heat budget is also affected by the convergence of the downward heat flux associated with the anticorrelation of the zonal mean vertical velocity,  $\bar{w}$ , (a result of gravity wave breaking) and the zonally averaged temperature, i.e. the term  $\frac{1}{\rho} \frac{\partial}{\partial z} \langle \rho \bar{w} \bar{T} \rangle$  in the thermodynamic energy equation. This term is substantially smaller than the net heating term due to gravity waves,  $H_g + C_g$ . For example at the mesopause under solstitial conditions the model of Garcia and Solomon (1985) with a mean meridional circulation driven by gravity wave breaking predicts  $\langle D \rangle = 1.5 \times 10^6 \text{ cm s}^{-1}$  and  $\langle \bar{w} \bar{T} \rangle = 16 \text{ K cm s}^{-1}$ . The  $\bar{w}$  and  $\bar{T}$  fields have an approximate height dependence of  $\rho^{-1/2}$ . The convergence of this heat flux is spread over at least 2 or 3 scale heights and has a magnitude of at most 10% of  $H_g + C_g$ . This is not surprising as  $\bar{w}$  and  $\bar{T}$  have large amplitudes only at polar latitudes; the global average is accordingly small.

### 3. Results

In our initial set of calculations, four representative turbulent diffusion coefficients were constructed and are shown in the top panel of Fig. 1. Model A is our "best fit" profile from Apruzese et al. (1984) (their model 4), although no real effort was expended to generate a truly excellent fit. Model A has an asymptotic value for  $D$  of  $6 \times 10^5 \text{ cm}^2 \text{ s}^{-1}$  at high altitudes and is characterized by very low values throughout the mesosphere.

Model B is an average of the summer and winter diffusion coefficients estimated by Lindzen (1981) for breaking gravity waves. Model C includes, in addition to gravity wave generated turbulent diffusion, Lindzen's estimate of the contribution from the breaking diurnal tide. These two profiles should be regarded as illustrative rather than rigorous globally averaged values of the turbulent diffusion coefficient. Finally Model D is the envelope of the highly structured vertical eddy diffusion coefficient used by Allen et al. (1981) in a



1D model of mesospheric chemistry. Most 1D models of mesospheric chemistry require eddy diffusion coefficients in excess of  $10^6 \text{ cm}^2 \text{ s}^{-1}$  and possibly as high as  $10^7 \text{ cm}^2 \text{ s}^{-1}$ . With a peak value of  $2 \times 10^6 \text{ cm}^2 \text{ s}^{-1}$ , Model D is certainly not an extreme case.

In the middle panel of Fig. 1 the four globally averaged temperature profiles calculated with their respective turbulent diffusion coefficients are compared with the CIRA 1972 temperature profile, which is representative of the observed globally averaged temperature. In these calculations it was assumed that  $P_t = 1$ , i.e.  $D_M = D_H \equiv D$ , and  $\epsilon = 1$ . The assumption of a high efficiency for conversion of gravity wave energy to heat is based on numerical simulations of Fritts and Dunkerton (1984). Note that Model A underestimates the temperature by up to  $20^\circ \text{K}$  between 65 and 75 km, the region where IR cooling is dominated by  $\text{CO}_2$  hot bands and isotopic bands (Dickinson, 1973; Wehrbein and Leovy, 1982). A comparison of our cooling rates with the very recent computations of Dickinson (1985) indicates that our rates are too large in the 63-82 km region. As will be shown below a 40% reduction in the  $\text{CO}_2$  cooling rate eliminates the discrepancy between observed and calculated temperatures. The departure of the Model A calculated temperature profile above 95 km from CIRA 1972 is due to the uncertainty in the  $\text{CO}_2(\nu_2)$  deactivation rate by O collisions as discussed at length in Apruzese et al. (1984). The thermosphere is not of fundamental importance in this study and is not discussed further.

In the lower panel of Fig. 1 the divergence of the turbulent heat flux is illustrated. For Model A below the mesopause the cooling rate does not exceed  $5 \text{ K d}^{-1}$ . With Models B and C which have a peak value of  $D \sim 3 \times 10^6 \text{ cm}^2 \text{ s}^{-1}$ , the respective mesopause temperatures are 65 and 46 K. Note that at the mesopause the value of  $D$  in Model C is significantly larger than Model B's value.

The reason for such cold mesopause temperatures is the extreme divergence of the turbulent heat fluxes. In Model B it peaks at  $\sim 30 \text{ K d}^{-1}$ , whereas in Model C it approaches  $350^\circ\text{K d}^{-1}$ . With solar heating rates of less  $5 \text{ K d}^{-1}$ , the equilibrium temperature must become very cold to minimize IR cooling and maximize convergence of downward heat flux from the thermosphere carried by molecular conduction. Model D also yields a cold mesopause ( $\sim 100 \text{ K}$ ) but up to 65 km is in close agreement with Model A, which suggests that below 65 km the atmospheric heat budget is insensitive to values of  $D$  that do not exceed the Model D profile. Above 80 km the large values of  $D$  in Model D generate a divergence of the turbulent heat flux in excess of  $\sim 63 \text{ K d}^{-1}$ .

To analytically illustrate the strong sensitivity of temperature to turbulent diffusion, we write the net heating due to gravity waves as

$$H_g + C_g = \frac{N^2 D_M}{2c_p} \left[ \epsilon \left( 1 + P_t^{-1} \right) + \frac{7}{P_t} \left( \frac{H}{H_D} - 1 \right) \right] \text{ K s}^{-1} \quad (3)$$

when the small term proportional to  $\frac{\partial^2 T}{\partial z^2}$  is neglected,  $H_D^{-1} = \frac{1}{D_M} \frac{\partial D_M}{\partial z}$ , and  $c_p$  is for a diatomic gas. For typical mesospheric values  $N^2 = \frac{g}{f} \left( \frac{\partial T}{\partial z} + \frac{g}{c_p} \right) = 3.5 \times 10^{-4} \text{ s}^{-2}$ ,  $H = 6 \text{ km}$ ,  $D_M$  in units of  $10^6 \text{ cm}^2 \text{ s}^{-1}$ , with  $P_t = 1$  and  $\epsilon = 1$  as assumed in Fig. 1

$$H_g + C_g = 7.5 \left( 1.4 \frac{H}{H_D} - 1 \right) D_M (10^6 \text{ cm}^2 \text{ s}^{-1}) \text{ K d}^{-1} \quad (4)$$

Thus for constant  $D_M$ ,  $H_g + C_g = -7.5 \text{ K d}^{-1}$  when  $D_M = 10^6 \text{ cm}^2 \text{ s}^{-1}$  or  $-22 \text{ K d}^{-1}$  for  $D_M = 3 \times 10^6 \text{ cm}^2 \text{ s}^{-1}$ , which may be compared against a maximum solar heating rate in this region of  $5 \text{ K d}^{-1}$ . Thus when  $H_g + C_g < -5 \text{ K d}^{-1}$  ( $D_M > 7 \times 10^5 \text{ cm}^2 \text{ s}^{-1}$ ), the mesopause must cool down to decrease IR cooling and increase convergence of the downward thermal conduction heat flux. Note that when  $\frac{H}{H_D} \sim 0.7$ , i.e.  $D_M \propto \exp(z/1.4H)$ , then  $H_g + C_g \sim 0$ . If  $D_M$  increases more rapidly with altitude than

this, gravity waves will produce a net heating of the region. Similarly if  $D_M$  increases more slowly with height than  $\exp(z/1.4H)$  net gravity wave cooling occurs.

Returning to Fig. 1, it is apparent why Model A with an asymptotic value of  $D_M = 6 \times 10^5 \text{ cm}^2\text{s}^{-1}$  is close to the upper limit on  $D_M$ , beyond which disastrous consequences on the mesopause temperature result from gravity wave cooling. Model D has  $D_M$  increasing as  $\exp(z/2H)$  from 50 to 80 km and hence generates net gravity wave cooling over this region. Only when  $D_M > 10^6 \text{ cm}^2\text{s}^{-1}$ , does this cooling rate become excessive and the mesospheric temperature plunges. In Models B and C,  $D_M > 10^6 \text{ cm}^2\text{s}^{-1}$  from 50 to 75 km and  $D_M$  does not increase with height more rapidly than  $\exp(z/2H)$ . The large gravity wave cooling rate in this region drives the mesospheric temperatures below  $100^\circ\text{K}$  above 75 km.

Examination of Eq. (3) reveals that gravity wave cooling is dominated by the last term  $\frac{-7}{P_t}$ . Although over limited regions  $H_D$  can be less than  $1.4H$ , eventually  $D_M$  must level off ( $H_D \gg H$ ) and the last term overwhelms the other terms with net gravity wave cooling. Only in the limit of large  $P_t$  can the contribution of this last term be substantially reduced over the entire mesosphere. For constant  $D_M(H_D = \infty)$  when  $P_t = 6$ ,  $\epsilon = 1$ , then  $H_g + C_g \sim 0$ . With finite  $H_D$ , a lower Prandtl number will yield negligible gravity wave cooling. On the other hand, for large  $P_t (\geq 10)$  gravity wave breaking will only produce net heating.

To test the effect of the Prandtl number dependence numerically, some additional calculations were performed. In these calculations the  $\text{CO}_2$  cooling rates between 63 and 82 km were reduced by 40% to bring them into closer agreement with Dickinson's (1985) more accurate computations and to demonstrate that our conclusions do not depend critically on the IR cooling rates. In addition the  $\text{CO}_2(\nu_2)$  deactivation rate by O collisions was adjusted to

$2 \times 10^{-11} \exp(-800/T) \text{ cm}^3 \text{ s}^{-1}$  (slightly larger than the value adopted by Gordiets et al. (1982)) to bring the calculated temperatures in the thermosphere into agreement with CIRA 1972. For the reader interested in an extended discussion of the issues raised by this adjustment, Section 3.6 of Apruzese et al. (1984) should be consulted.

In Fig. 2 Model 1 contains the above modifications with  $\epsilon = 1$ ,  $P_t = 4$  and a  $D_M$  profile that is  $\sim 3.3$  times the Model A diffusion coefficient in Fig. 1. (The precise expression for  $D_M$  is  $D_M = D_0 D_\infty / (D_0 + D_\infty)$ , where  $D_0 = 3 \times 10^5 \exp [(z-65 \text{ km}/H) \text{ cm}^2 \text{ s}^{-1}]$ ,  $H =$  scale height, and  $D_\infty$  is the asymptotic value at high altitudes given in figure caption.) The asymptotic value of  $D_H$  is  $5 \times 10^5 \text{ cm}^2 \text{ s}^{-1}$  for Model 1, slightly less than for Model A. The calculated mesospheric temperatures in the 65 to 75 km region are now in good agreement with CIRA 1972. Also the lower thermospheric temperatures above 95 km are also in good agreement with CIRA 1972. The principal region of disagreement is between 80 and 95 km where our calculated temperatures exceed CIRA 1972 by up to  $15^\circ \text{ K}$ , and would suggest that enhanced gravity wave cooling can be accommodated by lowering  $P_t$ . If  $P_t = 1$ , then the Model 2 temperature profile is obtained with a mesopause temperature of 140 K. Comparison of Models 1 and 2 illustrates the extreme sensitivity to  $P_t$  that we would expect from our discussion of Eq.(3). It is confined to the region above 80 km where gravity wave cooling exceeds solar heating. In Model 3,  $P_t = 4$  but  $D_M$  is twice its value in Model 1. Above 100 km, the temperature is only lowered by 10 K with this factor of 2 increase in  $D_M$ . Below 100 km the temperature change is insignificant. The weak dependence on  $D_M$  (or  $D_H$ ) when  $P_t = 4$  is due to the large cancellation of gravity wave heating and cooling terms. For example at 90 km,  $H_g + C_g = -2.7 \text{ K d}^{-1}$  for Model 1 and  $-3.4 \text{ K d}^{-1}$  for Model 3, with  $C_g = -6.3$  and  $-10.6 \text{ K d}^{-1}$ , respectively. When compared with

the solar heating rate of  $\sim 11 \text{ K d}^{-1}$ , which is rapidly increasing with height at 90 km, the  $0.7 \text{ K d}^{-1}$  change in  $H_g + C_g$  is insignificant.

#### 4. Discussion and Summary

The present study clearly demonstrates the importance of the Prandtl number for mesospheric turbulence. For large  $P_t$  ( $\sim 6$ ) and  $\epsilon \approx 1$ , gravity wave heating approximately balances gravity wave cooling and the globally averaged atmospheric temperature profile is controlled by radiative processes. For  $P_t > 6$ , gravity wave breaking produces only net heating. In the other limit  $P_t \leq 1$ , gravity wave breaking is generally characterized by very strong cooling but with small heating at the bottom of the breaking zone. In this limit, the cooling is sufficiently strong to impose a powerful constraint on the maximum permissible level of turbulence in the mesosphere without plunging the mesopause temperature down to  $\sim 100 \text{ K}$ . With allowance for uncertainty in our radiative transfer model this upper limit is  $D = 10^6 \text{ cm}^2\text{s}^{-1}$ . Our preferred asymptotic value is  $(5-6) \times 10^5 \text{ cm}^2\text{s}^{-1}$  when  $P_t = 1$ . Below 70 km, the D profile from Model D is an absolute upper limit.

From a close examination of Figs. 5 and 6 in Apruzese et al. (1984), one concludes that deducing a local upper limit to D independently of the total D profile is risky below the mesopause. For example, Model 3 in Apruzese et al. (1984) almost satisfies the upper limit constraint  $D_H < 10^6 \text{ cm}^2\text{s}^{-1}$  ( $D_H = 1.2 \times 10^6 \text{ cm}^2\text{s}^{-1}$  for high altitudes) and is less than the diffusion profile of Model D below 70 km. But the calculated mesopause temperature is  $\sim 130 \text{ K}$ .

As discussed above, more rapid transport of constituents is needed than allowed by the heat transport. Schoeberl et al. (1983) extended the parameterization of linear wave chemical transport by eddy diffusion given in Strobel (1981) to include the effect of breaking gravity waves. Their Eqs. (27) and

(28) may be combined to yield an effective vertical eddy diffusion coefficient of

$$\bar{K}_{zz} = D_H + \frac{L}{2\gamma^2} + \frac{1}{2} D_H \quad (5)$$

where  $\gamma$  is the vertical wave number and  $L$  is the chemical loss rate in units of  $s^{-1}$ . The first term represents diffusion by the background turbulence created by the breaking gravity wave and the second term represents eddy transport by the gravity wave for chemically active ( $L \neq 0$ ) species. Chao and Schoeberl (1984) have demonstrated that the last term, which arises from direct diffusive transport by the breaking gravity wave, should be negligible rather than  $1/2 D_H$  because the perturbation vertical velocity and mixing ratio are almost in quadrature. The eddy transport of heat and constituents by a gravity wave in the breaking region cannot be represented by a linear diffusive process according to their analysis. Schoeberl et al. (1983) showed for chemically active species in the mesosphere and lower thermosphere that the second term in Eq. (5) can equal or exceed the first term when  $D_H$  obeys our upper limit constraint. The authors are not aware of any 1D chemical model that has tested whether  $\bar{K}_{zz}$  with  $D_H < 10^6 \text{ cm}^2 \text{ s}^{-1}$  can adequately account for constituent transport in the mesopause region. If it can be proven that  $P_t = 1$  for mesospheric turbulence, then the  $L/2\gamma^2$  contribution to  $\bar{K}_{zz}$  with  $D_H < 10^6 \text{ cm}^2 \text{ s}^{-1}$  would be essential. Note that expression (5) is valid only when  $k(\bar{u}-c) \gg \gamma^2 D_H$ , and  $L$ , where  $k$  is the horizontal wave number,  $\bar{u}$  is the mean zonal wind, and  $c$  is the zonal phase speed.

The determination of the eddy Prandtl number in the mesosphere is an exceedingly difficult experimental measurement. To date only inferences are available that are not definitive or compelling. Although Justus (1967) determined  $P_t$  to be  $\sim 3$  from photographic tracking of rocket released chemical clouds

and analysis of turbulent wind data in the 90 - 110 km region, there was considerable scatter in his data points. Recently Fritts and Dunkerton (1985) have theoretically examined constituent and heat fluxes driven by localized gravity wave breaking where the breaking zones are small in vertical extent in comparison to the vertical wavelength. In their analysis this localization of turbulence leads to an eddy Prandtl number given by

$$P_t^{-1} = 1 - \frac{2\alpha n}{n+1} + \frac{\alpha^2}{2} \quad (6)$$

where  $\alpha$  is a measure of supersaturation of the breaking gravity wave ( $\alpha = 1$  for saturation),  $n$  is a measure of the localization of turbulence, i.e.  $D = \overline{D} \left( \frac{1 - \cos \phi}{2} \right)^n$  and  $\phi = kx + mz - ct$  is the wave phase with the usual definitions for these quantities. They recommend  $\alpha = 1.3$ ; thus for  $n = 1.5$ ,  $P_t = 3.5$  and  $n = 2$ ,  $P_t \sim 7$ . With  $\alpha = 1$ ,  $n = 1.5$  and  $2$ , the respective values of  $P_t$  are  $3.3$  and  $6$ . With even modest localization of turbulence ( $n = 1$ ),  $P_t \sim 2$  for  $\alpha \sim 1-1.5$ . The analysis of Fritts and Dunkerton (1985) strongly suggests that  $P_t \geq 2$  unless large supersaturation ( $\alpha \geq 2$ ) results in the mesosphere from breaking gravity waves. Then the eddy Prandtl number can be reduced to  $P_t = 1$ . Observations by Theon et al. (1967) and Philbrick et al. (1983) indicate supersaturation by up to  $\alpha \sim 1.3$ , but not  $2$ .

The Fritts and Dunkerton (1985) model is, however, an analytic treatment of convection effects for a single breaking gravity wave discussed by Chao and Schoeberl (1984) and may not accurately represent the breakdown of a gravity wave packet. For example, the turbulent region, initially set up by the folding of the potential temperature surface, will expand in time as stable air is entrained into the turbulent zone. Thus the heat flux will vary in time and a

turbulent zone which best represents the time mean heat flux may be quite different from the zone where breaking initially begins. The actual question of heat transport by breaking gravity waves may not be settled until detailed numerical simulations are performed.

Finally, it is important to recall that the arguments put forth by Chao and Schoeberl (1984) and Fritts and Dunkerton (1985) for heat transport induced by a single breaking wave do not include heat transport produced by stable waves encountering a turbulent zone and/or being radiatively damped. The effect of these processes gives rise to an eddy Prandtl number of only 1. This discussion raises the possibility that  $P_t$  may indeed be variable with an initial value that is large at the onset of wave breaking and as the turbulence develops dissipation of stable waves drives the eddy Prandtl number down to an asymptotic value of 1.

Although this paper has been concerned principally with the globally averaged eddy diffusion coefficient, the constraints on it discussed above are also applicable locally. For example, the maximum values of  $D_M$  occur above the cores of the zonal jets near the stratopause. If  $P_t$  were  $\sim 1$ , then a secondary circulation would have to be set up in the atmosphere to locally balance the strong cooling when  $D_M > 10^6 \text{ cm}^2 \text{ s}^{-1}$ .

**Acknowledgments:** This work was initiated while DFS was at the Naval Research Laboratory. DFS was supported by the Office of Naval Research and NASA's Upper Atmospheric Research Office. At JHU, DFS was supported by NASA Contract NAG 5-489. He was also supported in part by the Naval Research Laboratory at Science Applications International Corporation.



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## Figure Captions

Fig. 1 (Upper panel) Eddy or turbulent diffusion coefficients described in the text as a function of altitude and used to calculate temperature profiles (middle panel) which are compared with CIRA 1972 temperature profile. (Lower panel) the cooling or heating rate associated with the divergence or convergence of the downward eddy heat flux. All calculations with  $P_t = 1$ .

Fig. 2 Calculated temperature profiles with modified  $\text{CO}_2$  cooling rates as a function of altitude compared with CIRA 1972 temperature profile which illustrate sensitivity to  $P_t$ . The respective values of  $P_t$  and asymptotic values of  $D_M$  ( $D_\infty$ ) are: (Model 1)  $[4, 2 \times 10^6]$ , (Model 2)  $[1, 2 \times 10^6]$ , and (Model 3)  $[4, 4 \times 10^6 \text{ cm}^2 \text{ s}^{-1}]$ .

# DIFFUSION COEFFICIENT

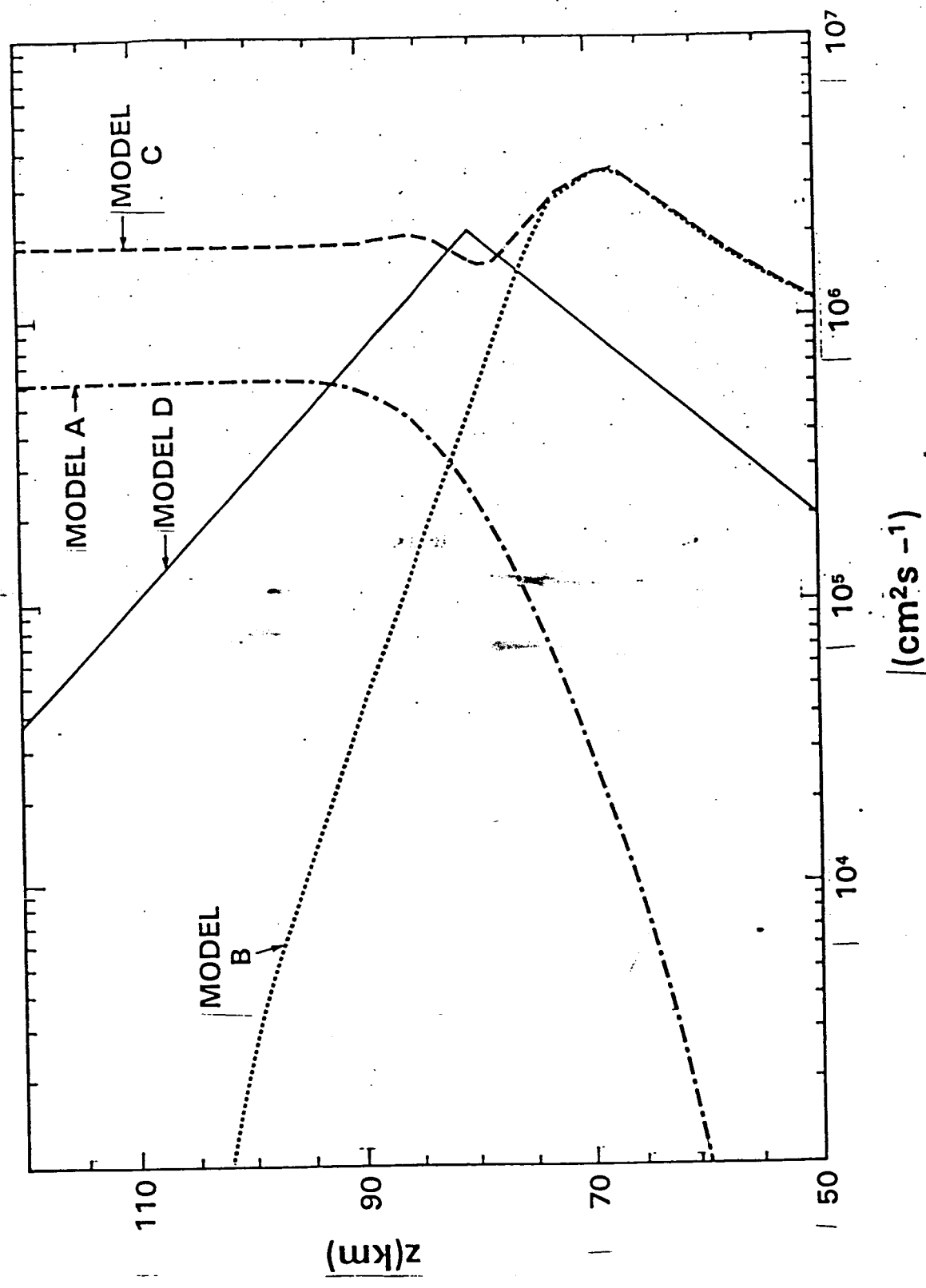


Fig. 1a

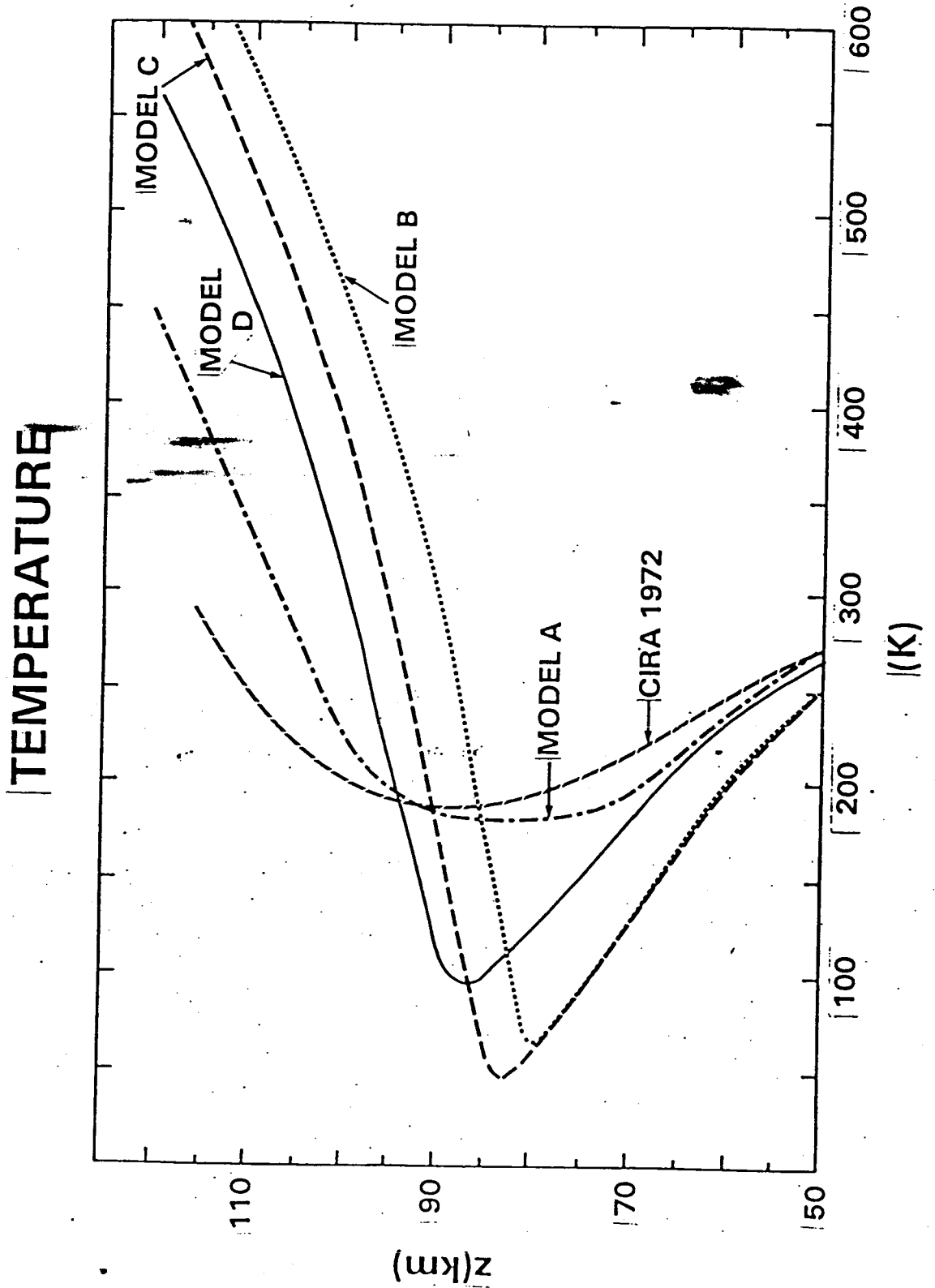


Fig. 1b

# COOLING RATE

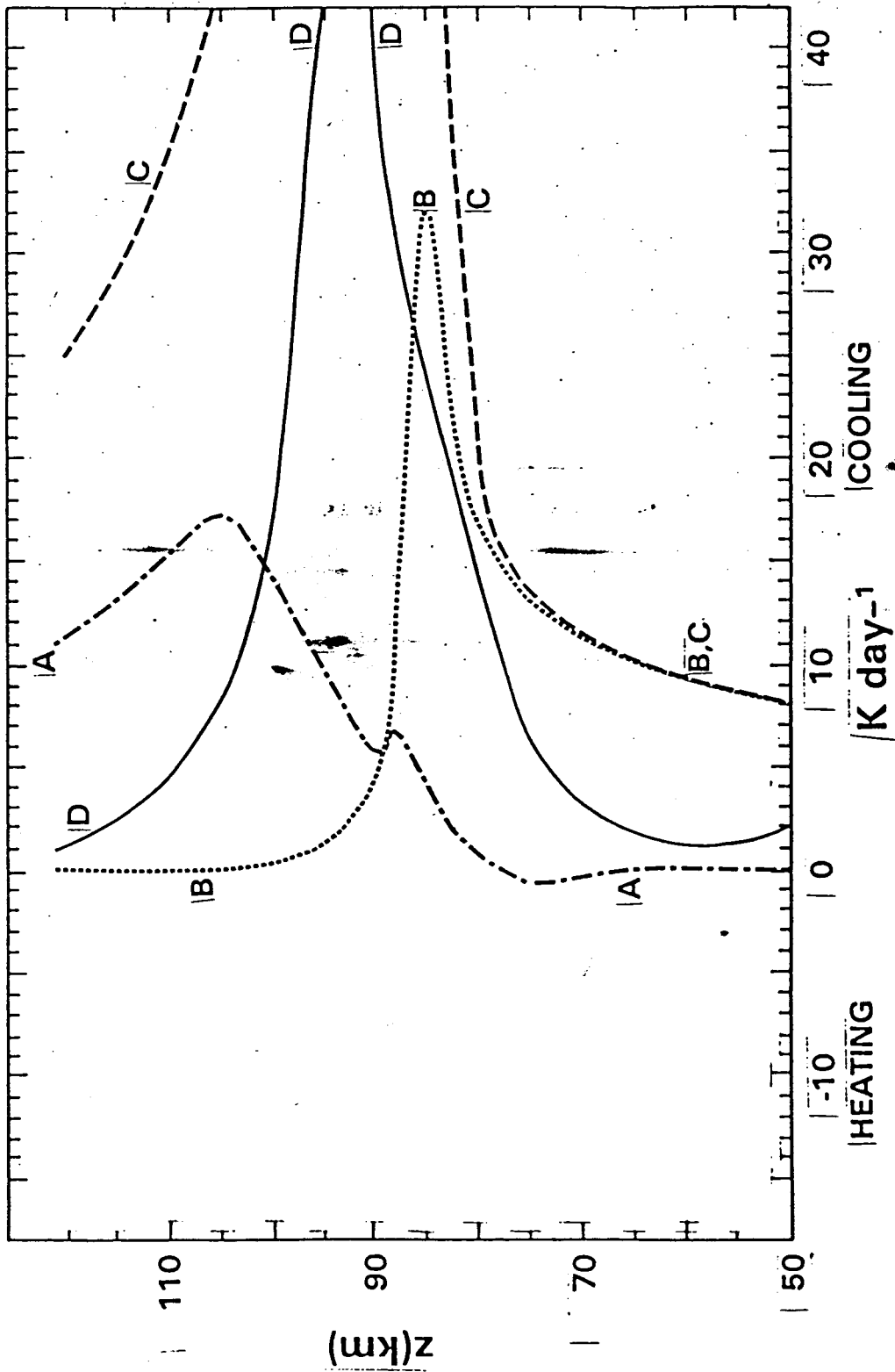


Fig. 1c

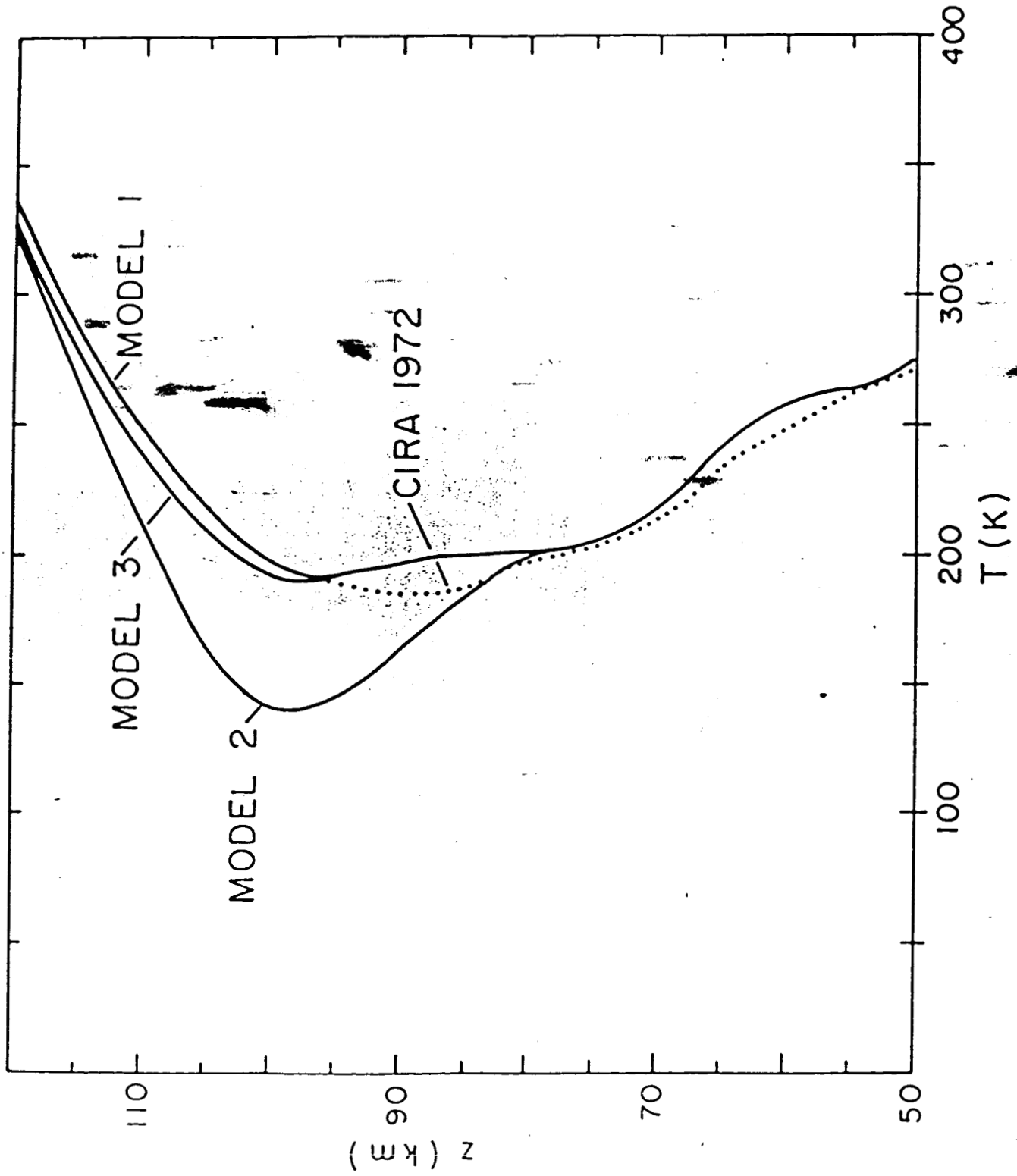


Fig. 2