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Parameterized Materials and Dynamic Response Characterizations in Unidirectional Composites

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SUMMARY

The values of phase velocities of ultrasonic waves in transversely isotropic media are presented in terms of a) the fiber volume fraction of a unidirectional fiberglass epoxy composite with constant matrix properties and b) the ratio between extensional moduli in the longitudinal and transverse directions of the composite when the properties of the fibers are changed, at a constant fiber volume fraction.

The model of a homogeneous transversely isotropic medium is adopted to describe the relations between elastic properties and velocities. The displacements due to an oscillatory point source in an infinite medium are used as one measure of comparison of the behavior of the unidirectional composite according to the variations of the parameters, as described above.

Values of phase velocities , elastic moduli, Poisson's ratios and displacements due to a point source can be read from the parameterized plots for a known fiber volume fraction or a known ratio between extensional moduli of the composite . Alternatively fiber volume fraction and the ratio between extensional moduli can be inferred from the plots when the values of the phase velocities are known; for example from experimental measurements. Thus, such parameterized curves may be useful in nondestructive mechanical property and material degradation characterizations.

INTRODUCTION

In composites manufactured from continuous fibers, anisotropy often arises due to fiber orientation and the fact that the fiber stiffness is usually much larger than the matrix stiffness. The homogeneous elastic properties of such composites are usually represented by a complete set of independent elastic constants which are expressed either as a stiffness matrix or a compliance matrix which is defined by the stress-strain relationships.

When a transversely isotropic model is adopted for unidirectional composites, a complete set of elastic constants consists of five independent elements . These elastic constants can be determined a) experimentally either directly by mechanical testing or by phase velocity measurements of stress waves propagating through the material [1-4] or b) Theoretically by the rule of mixtures, which combines the elastic properties of the components according to the the fiber volume fraction of the final composite [5].

In a unidirectional composite, if either the fiber volume fraction or the fiber properties at a constant fiber volume fraction are changed, the resulting changes of the elastic constants of the stiffness matrix can be quantified. Further, given the elastic constants, it is possible to predict the characteristics of stress waves travelling through the medium, such as phase velocities and displacements .

The assumption of a nondispersive medium is adopted in the present study. In terms of stress wave propagation, this means

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that waves of different frequencies, propagating along a given direction in the medium, travel with the same velocity . It has been shown that the phase velocities can be expressed as functions of the angle O (that is, the angle between the normal to the front of the propagating wave and the fiber direction) and the elastic constants when a nondispersive model is adopted [6].

In this work the elastic moduli, elastic constants of the stiffness matrix, phase velocities of longitudinal and shear stress waves, and displacements due to an oscillatory point source are related to two basic parameters of an unidirectional composite . The two independent parameters under consideration are the fiber volume fraction for a unidirectional fiberglass epoxy composite and the fiber properties for a unidirectional composite with a constant fiber volume fraction of 0.6 where the the matrix is epoxy .

UNIDIRECTIONAL FIBERGLASS EPOXY COMPOSITE BEHAVIOR FOR VARIATIONS OF FIBER VOLUME FRACTION

Elastic Properties for Variable Fiber Volume Fraction

The values of the elastic constants of the stiffness matrix are generated for a unidirectional fiberglass epoxy composite with different fiber volume fractions. The values assumed for the properties of the components are [7]

Extensional modulus of the glass fiber: E_f= 72.40 GN/m^2

Extensional modulus of the epoxy matrix: E_m= 5.59 GN/m^2

Poisson ratio of the glass fiber: 0.22

Poisson ratio of the epoxy matrix: 0.40

Density of the glass fiber: ρ_f = 2540 kg/m^3

Density of the epoxy matrix: ρ_m = 1250 kg/m^3

The fiber volume fraction is varied from zero (isotropic epoxy material) to 1 (isotropic E glass fiber material). The elastic moduli and the elastic constants of the stiffness matrix corresponding to the various combinations of fiber and matrix are calculated using the rule of mixtures [5]. The properties are referred to a cartesian coordinate system xyz, in which z coincides with the fiber direction,(see Fig.1), and the Love [8] notation is adopted for the elastic constants of the stiffness matrix.

The resulting values for the elastic moduli (E_z, E_x,G_{xz} and G_{xy}), elastic constants of the stiffness matrix (C_{11}, C_{12}, C_{13}, C_{33} and C_{44}) and Poisson's ratios (ν_{xz}, ν_{zx} and ν_{xy}) are shown

in Figs. 2, 3 and 4, respectively, all plotted versus the fiber volume fraction, V_f.

Phase Velocities for Variable Fiber Volume Fraction

Using the expressions determined previously for the phase velocities in transversely isotropic media [6], the values of the velocities of SH,SV and P plane waves with normals at 0°, 30°, 45°, 60° and 90° with respect to the fiber direction (z) are calculated. The notations SH, SV and P refer to the modes of the propagating waves [6]. SH stands for a shear wave with a polarization direction that is allways parallel to the xy plane (Fig.1) , SV stands for a shear wave with a polarization direction that is allways in the same plane of propagation , and P stands for a longitudinal wave with a polarization direction that is allways in the same plane of propagation [6] . The resulting phase velocities are shown in Figs. 5 , 6 and 7 for SH, SV and P waves, respectively .

Displacements due to Oscillatory Point Source for Variable Fiber Volume Fraction

The displacements due to an oscillatory point source in infinite medium for points located along the principal directions are also calculated. The theoretical formulation and solution of this problem were previously analyzed . Expressions for the displacements were determined in terms of the elastic constants of the the stiffness matrix and the position of the point of

interest [9].

In the present work it is assumed that the medium is subjected to an oscillatory point force of unit amplitude (1 N) at a frequency of 1 MHz which is located at the origin of the coordinate system. In the adopted coordinate system the oscillatory load is applied in the x direction, see Fig.8 . The displacements are denoted by u, v and w corresponding to the directions defined by the coordinate axes x, y and z, respectively.

The displacements are calculated for points located along the coordinate axes at 2 m distances from the origin. For the coordinate system adopted, these points are defined by A(2,0,0), B(0,2,0) and C(0,0,2) for the x, y and z directions . (Refer to Fig.1) . The contributions of the modes SH, SV and P are calculated separately . The displacements are shown in Figs. 9, 10 and 11 for the points A, B and C, respectively. Observe that displacements other than u, for points along the coordinate axes, are zero or are of a lower order of magnitude than u . The displacements v and w are zero for points in the xz and xy planes, respectively, by reasons of loading symmetry. The displacements v and w for points in yz and xz planes, respectively, are of a lower order of magnitude ;thus they are neglected in the asymptotic solution of the displacements due to a point source [9].

Directions of Maximum Displacements for SV Mode of Propagation

Depending on the specific values of the elastic constants of the stiffness matrix, some composites produce a special pattern of propagation of the SV waves. The special pattern of the wave surface corresponding to this mode, denoted by W(SV) [6], is characterized by an overlapping behavior of the wave front (having two cuspidal edges) which occurs at certain angles with respect to the fiber direction . As a consequence of the geometric pattern of the W(SV) front, the displacements for points in the medium located along the limiting directions of the wave front cuspidal edges correspond to maxima [9].

The limiting directions corresponding to the cuspidal edges of the SV mode front were determined for various fiber volume fractions . These are the directions of maxima for displacements. The directions are shown in Fig.12, plotted versus fiber volume fraction V_f. Observe that the maximum positions first appear at a fiber volume fraction of 0.07 and disappear for fiber volume fractions larger than 0.84 . So the range defining the existence of overlap in the W(SV) wave surface goes from $V_f = 0.07$ to $V_f = 0.84$. For fiber volume fractions smaller than 0.07 or larger than 0.84, the composite approaches the isotropic behavior of the epoxy matrix or the glass fiber material, respectively, so the directions of maximum displacements are coincident with the direction z.

UNIDIRECTIONAL COMPOSITE BEHAVIOR FOR VARIATION OF FIBER PROPERTIES

Elastic Properties for Variable Fiber Properties

The values of the elastic properties are generated for unidirectional composites with epoxy as the matrix and fibers with different extensional moduli at the constant fiber volume fraction of 0.60. The values assumed for the properties of the components are:

Extensional modulus of the fiber: 5.0 GN/m^2 = E_f = 500 GN/m^2

Extensional modulus of the epoxy matrix: E_m = 5.59 GN/m^2

Poisson ratio of the fiber: 0.22

Poisson ratio of the epoxy matrix: 0.40

The density of the composite is taken as 1850 kg/m^3 for the range of fiber properties.

The elastic moduli, elastic constants of the stiffness matrix and Poisson's ratios corresponding to the various combinations resulting from the variations of the fiber properties are calculated using the rule of mixtures [2]. The elastic constants of the stiffness matrix (C_{11}, C_{12}, C_{13}, C_{33} and C_{44}) and Poisson ratios (ν_{xz}, ν_{zx} and ν_{xy}) are shown in Figs. 13, and 14, respectively, plotted versus the ratio between the extensional moduli of the composite in the longitudinal and transverse directions, E_z/E_x. The values adopted for the properties of the fiber are assumed, but the resulting properties of the composite cover the range of properties of

actual composites.

Phase Velocities for Variable Fiber Properties

The expressions for phase velocities of waves in a transversely isotropic medium are used for these calculations [3]. Again, the values of the phase velocities are calculated for the directions of $0°$, $30°$, $45°$, $60°$ and $90°$. The resulting values are shown in Figs. 15, 16 and 17 , for SH, SV and P waves, respectively. The notation for the wave modes is in accordance with the previous sections.

Displacements due to Oscillatory Point Source for Variable Fiber Properties

The displacements for the points A(2,0,0), B(0,2,0) and C(0,0,2) are again calculated for the same loading conditions as described earlier. The resulting values for the displacements are shown in Figs 18, 19 and 20 for the points A, B and C. Observe that displacements other than u, for points along the coordinate axes, are zero or of a lower order of magnitude than u. The displacements v and w are zero for points in the xz and xy planes, respectively, by reasons of loading symmetry. The displacements v and w for points in yz and xz planes, respectively, are of a lower order of magnitude; thus they are neglected in the asymptotic solution of the displacements due to a point source [9].

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Directions of Maximum Displacements for SV Mode of Propagation

Following the same procedures used for the variable fiber volume fraction calculations, the positions of maxima for the displacements corresponding to the SV mode are computed for variations of the fiber elastic properties. For most of the range of the fiber moduli considered, there are directions of maxima. It was observed that the directions of maxima first appear at a ratio of $E_z/E_x = 2.27$, (see Fig.21) and continue beyond the upper limit of E_z/E_x plotted.

CONCLUSIONS

The graphs of phase velocities of SH and P waves show that the values of velocities increase a) with decreasing angle of the normal of the propagating wave with respect to the z axis (that is, the fiber direction); b) with increasing fiber volume fraction, (Figs. 5 and 7), for the entire V_f range, except for values of $V_f < 0.2$; and c) with increasing ratio E_z/E_x of the composite (See Figs. 15 and 17.) .

As opposed to the SH and P modes, the SV mode velocities increase with increasing angle of the normal to the wave front with respect to the z axis, until 60°, and then decreases until 90°, where the values of velocity are again the same as for the 0° direction (see Figs. 6 and 16). Equal velocities for 0° and 90° is a result of the transversely isotropic model adopted to describe the composite.

The displacements for the points A, B and C show a decrease a) with increasing V_f (Figs. 9, 10 and 11; except for P waves in the region defined by $V_f < 0.2$); b) with increasing E_z/E_x, as shown in Figs. 18, 19 and 20.

The typical decreasing of all displacements versus increasing fiber volume fraction is in agreement with the corresponding increases in the elastic constants of the stiffness matrix ,Figs. 3 and 13. There is an exception, however, in the behavior of the u displacement at point A. Observe that in Fig.3 the properties C_{11}, C_{12} and C_{13} decrease for increasing V_f, from $V_f=0$ to $V_f=0.2$ which justifies the pattern of u displacements for the point A in the same range of

variation of V_f, see Fig. 9. In the case of the variation of E_z/E_x, all elastic constants of the stiffness matrix, see Fig.13, increase for the entire range analyzed so that the corresponding u displacements , see Figs. 18, 19 and 20 , also increase in all the E_z/E_x range.

The positions of maxima for the displacements corresponding to SV provide a basis for determining if and where the overlapping phenomenon of the W(SV) wave surface occurs, for known values of V_f or E_z/E_x. If the phenomenon of the cuspidal overlapping is capable of being detected experimentally, it would then be possible to estimate the values of V_f and E_z/E_x from Figs.4 ,5 ,6 and 15, 16, 17 for velocity measurements and from Figs. 9, 10, 11 and 18, 19, 20 for displacement measurements.

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Fiber direction

$z \equiv 3$

$C(0,0,2)$

$B(0,2,0)$

$y \equiv 2$

$A(2,0,0)$

$x \equiv 1$
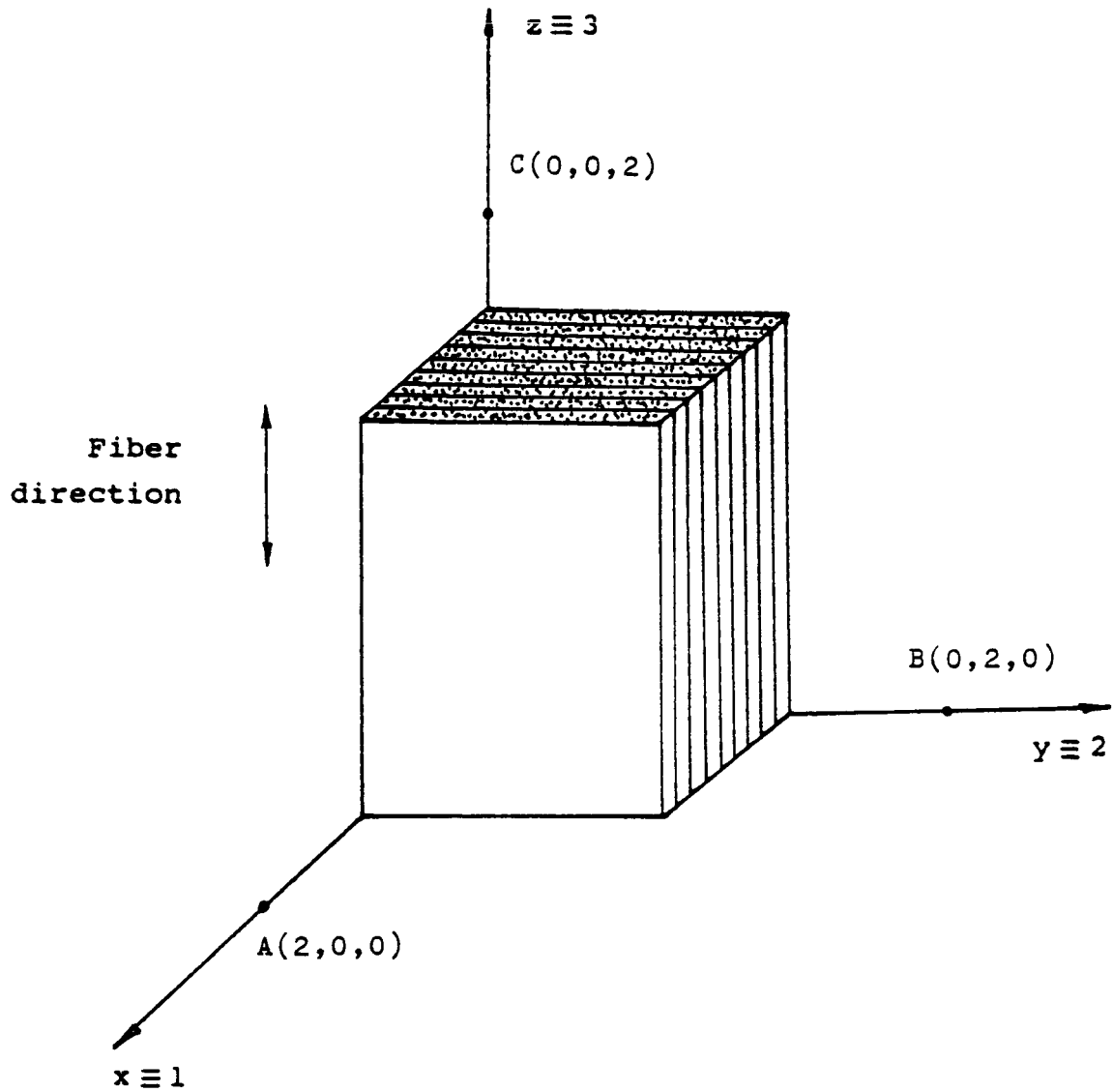
Fig. 1   Position of material axes with respect to coordinate axes for transversely isotropic medium.
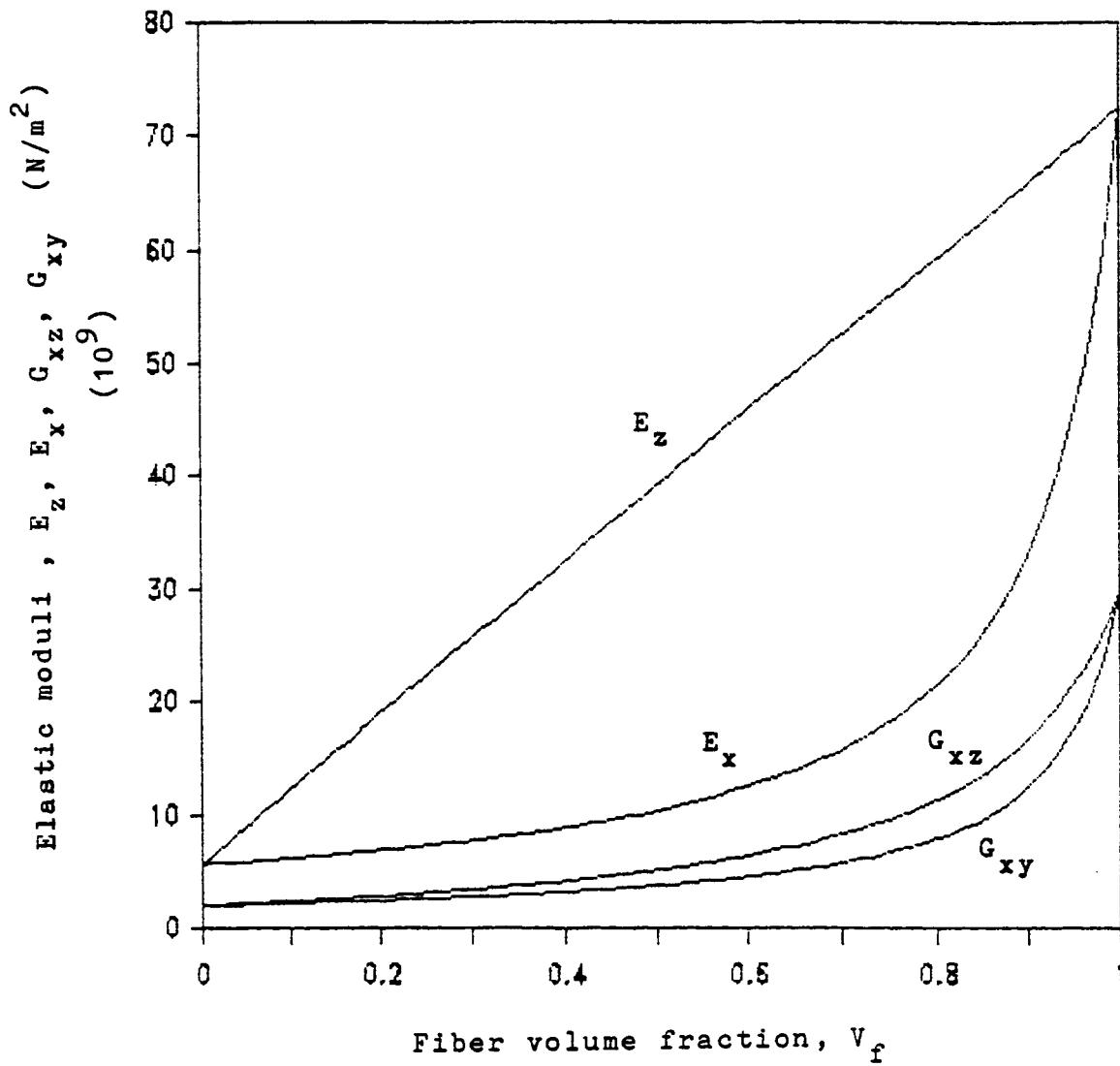
Fig. 2 Variation   of elastic   moduli   in   unidirectional
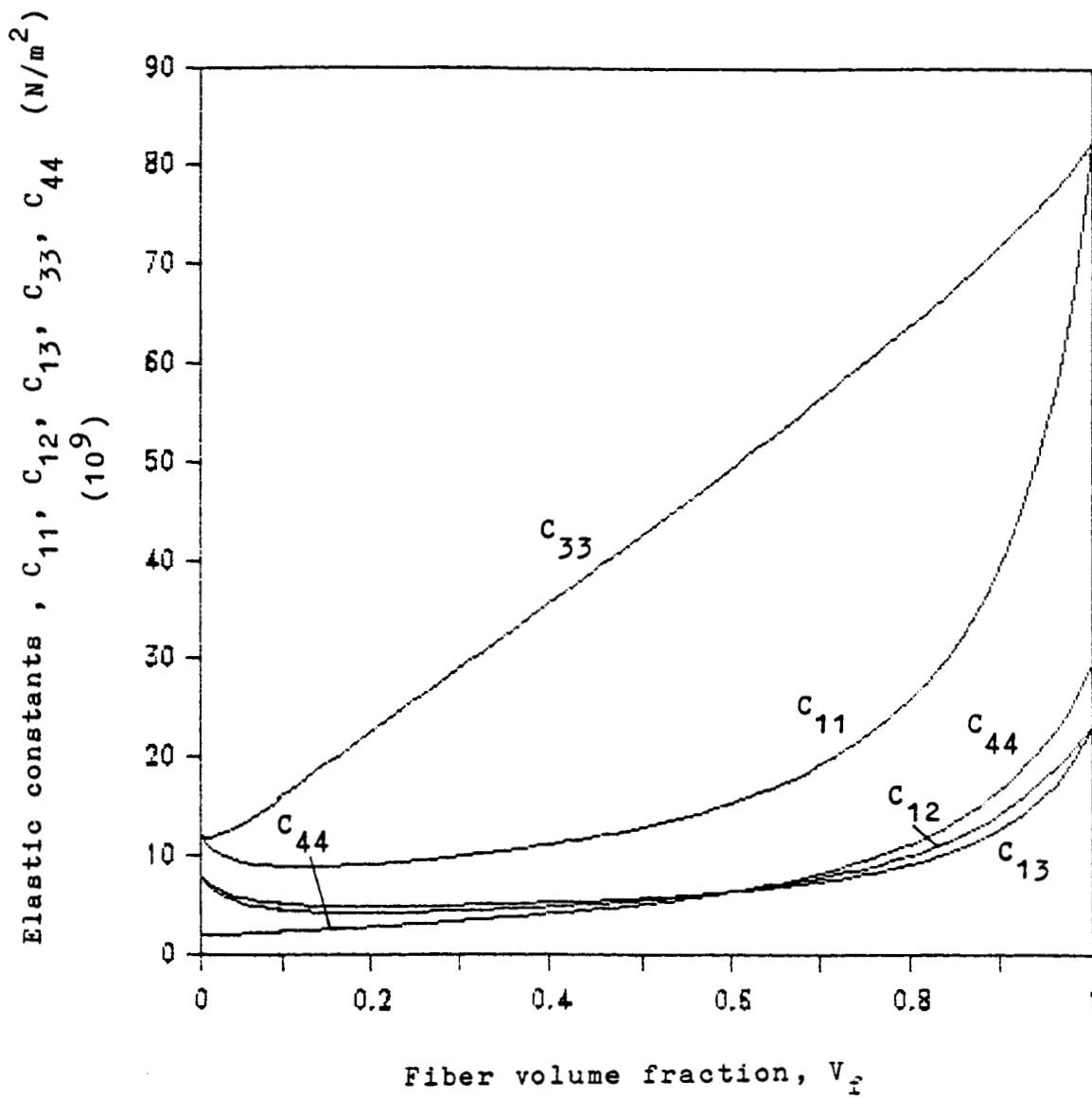fiberglass   epoxy composite versus   fiber   volume
fraction.

Fig. 3 Variation of the elastic constants of stiffness matrix in unidirectional fiberglass epoxy composite versus fiber volume fraction.
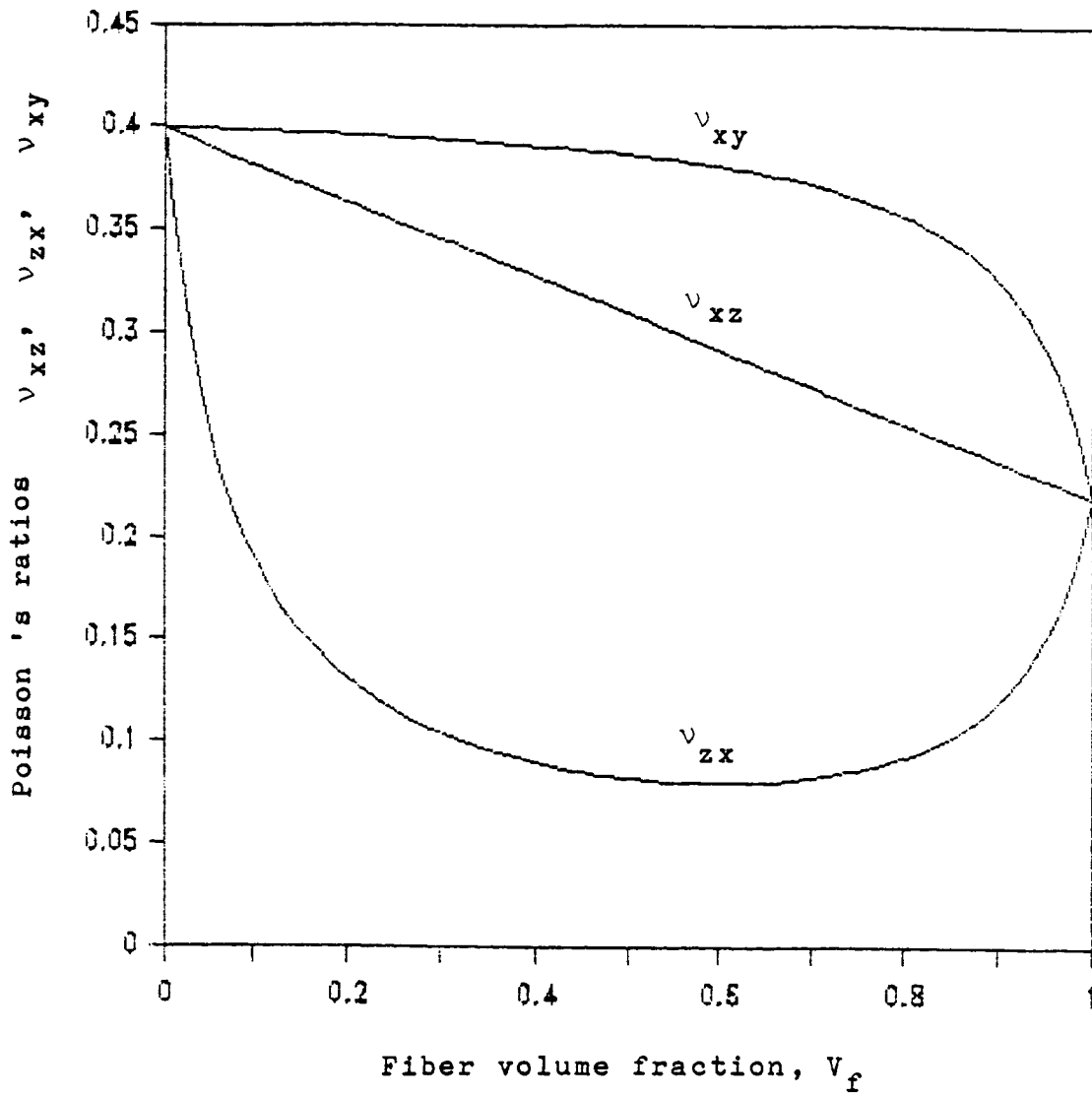
Fig. 4   Variation   of   Poisson's ratios   in   unidirectional
fiberglass epoxy   composite   versus   fiber   volume
fraction. (First   index   indicates   deformation
direction; second index indicates load direction).
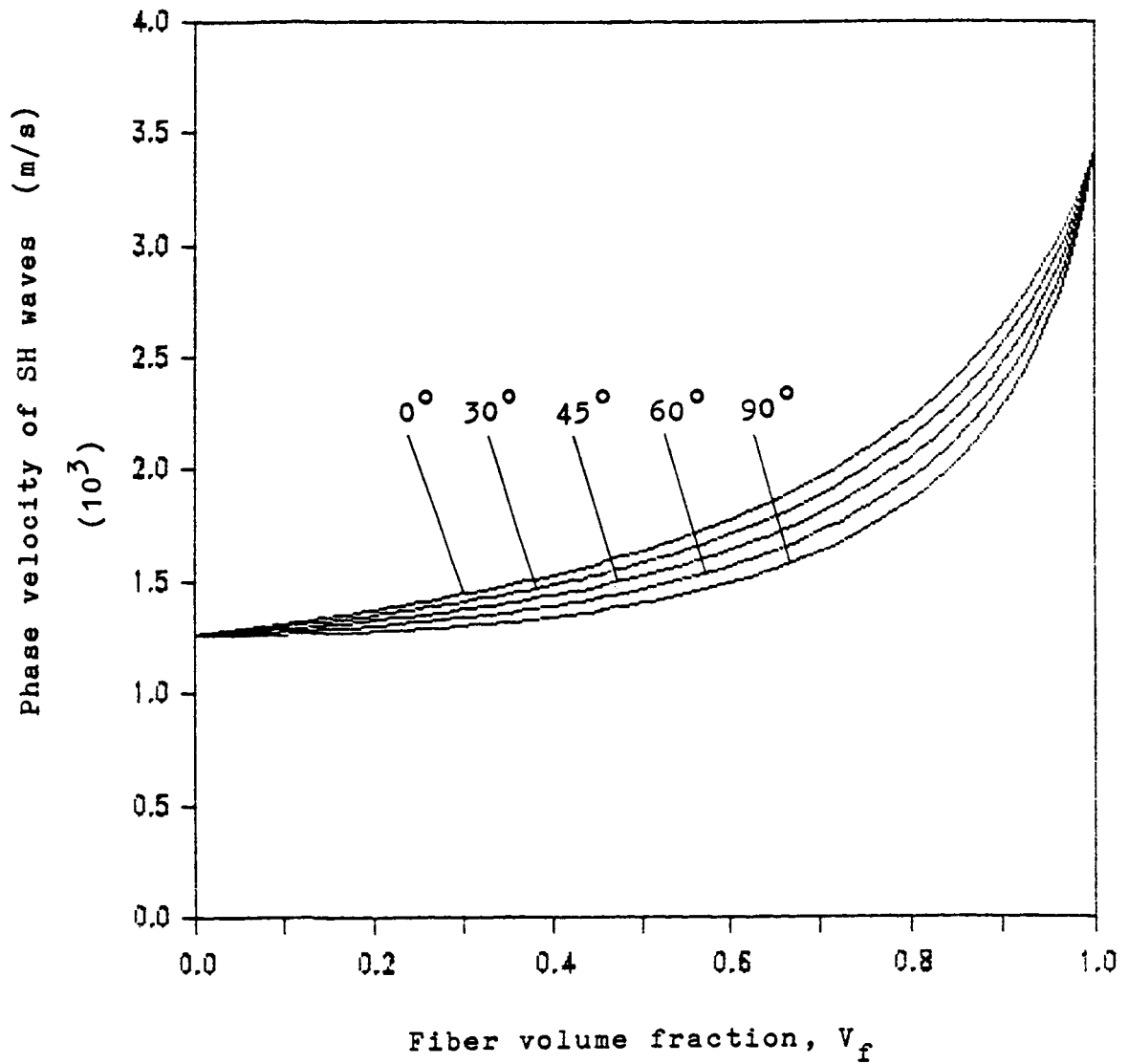
Fig. 5 Variation of phase velocity of SH waves (with
normals $0°$, $30°$, $45°$, $60°$ and $90°$ with
respect to fiber direction) in unidirectional
fiberglass epoxy composite versus fiber volume
fraction.
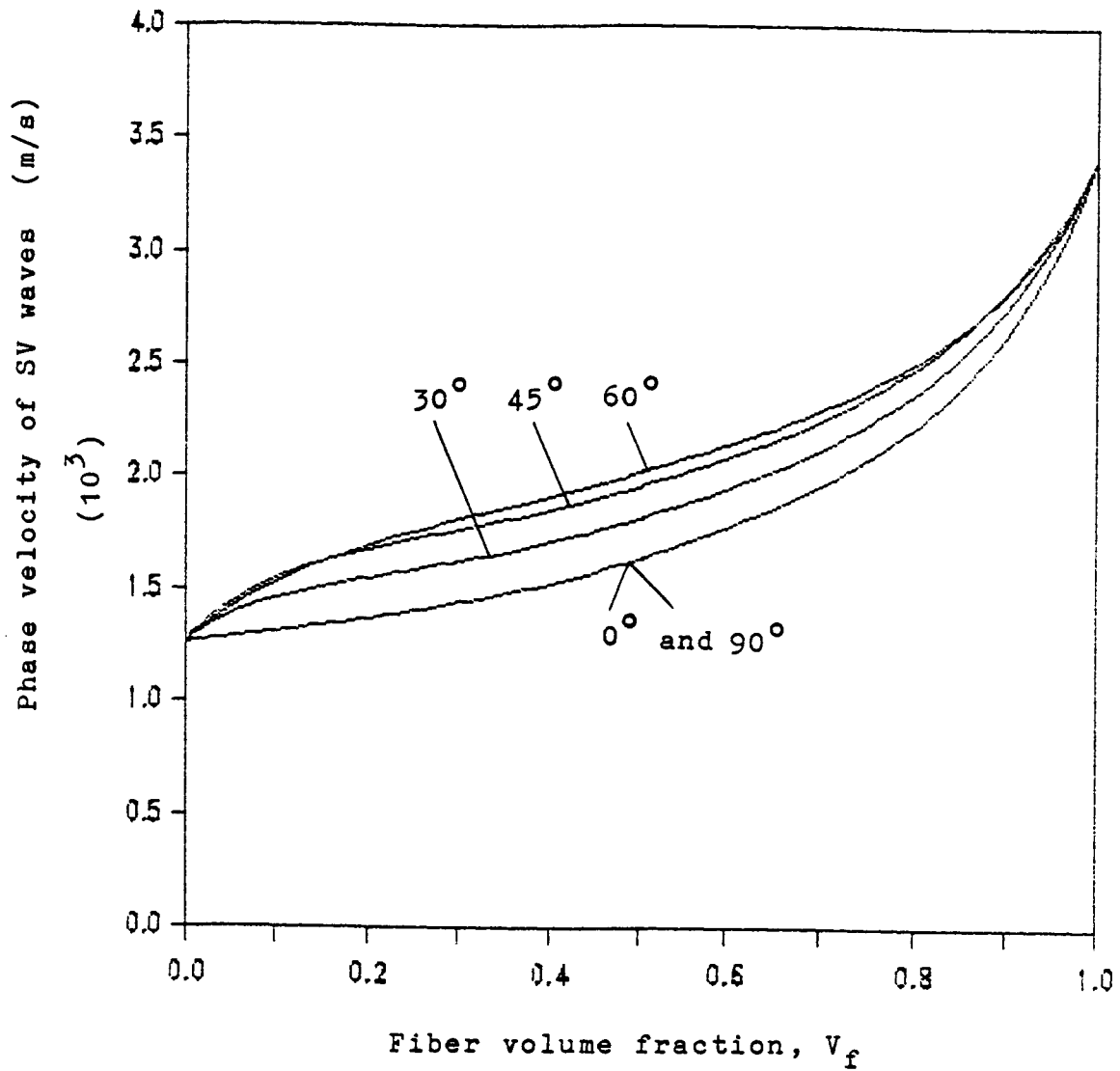
Fig. 6 Variation of phase velocity of SV waves (with normals $0°$, $30°$, $45°$, $60°$ and $90°$ with respect to fiber direction) in unidirectional fiberglass epoxy composite versus fiber volume fraction.
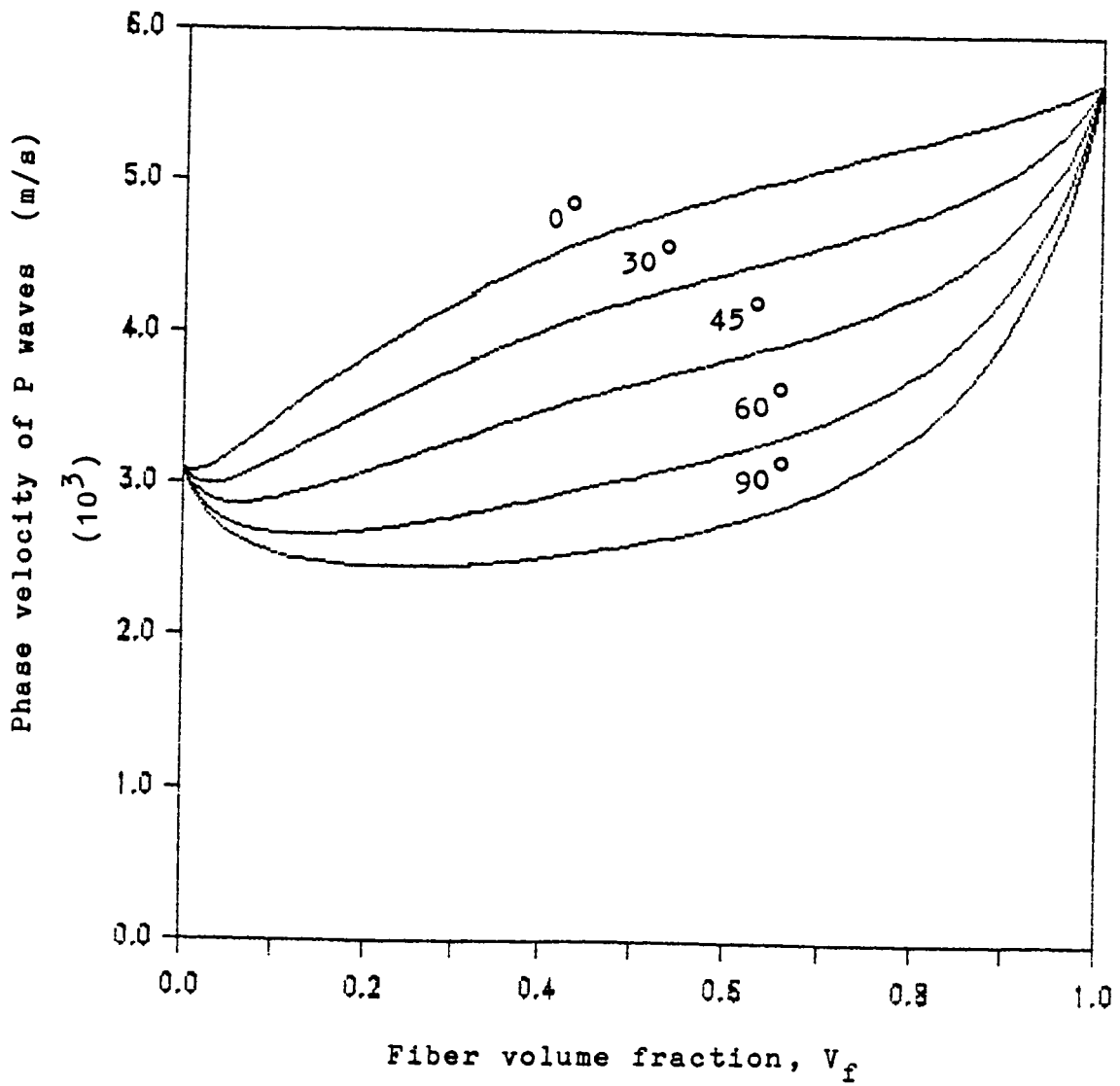
Fig. 7 Variation of phase velocity of P waves (with normals $0°$, $30°$, $45°$, $60°$ and $90°$ with respect to fiber direction) in unidirectional fiberglass epoxy composite versus fiber volume fraction.
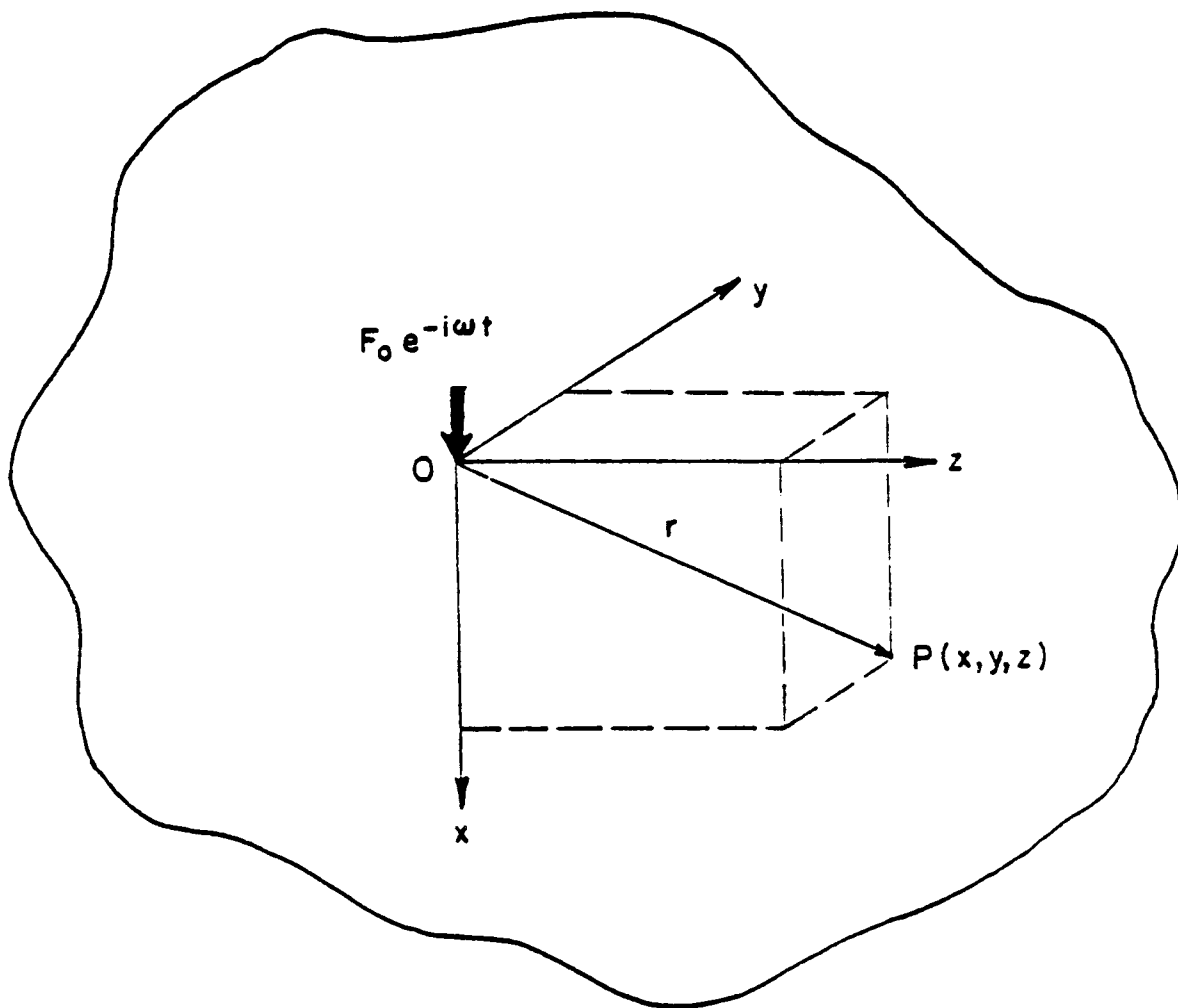
Fig. 8 Schematic illustrating sinusoidal point load
exciting an infinite transversely isotropic
medium, where xy is isotropic plane in cartesian
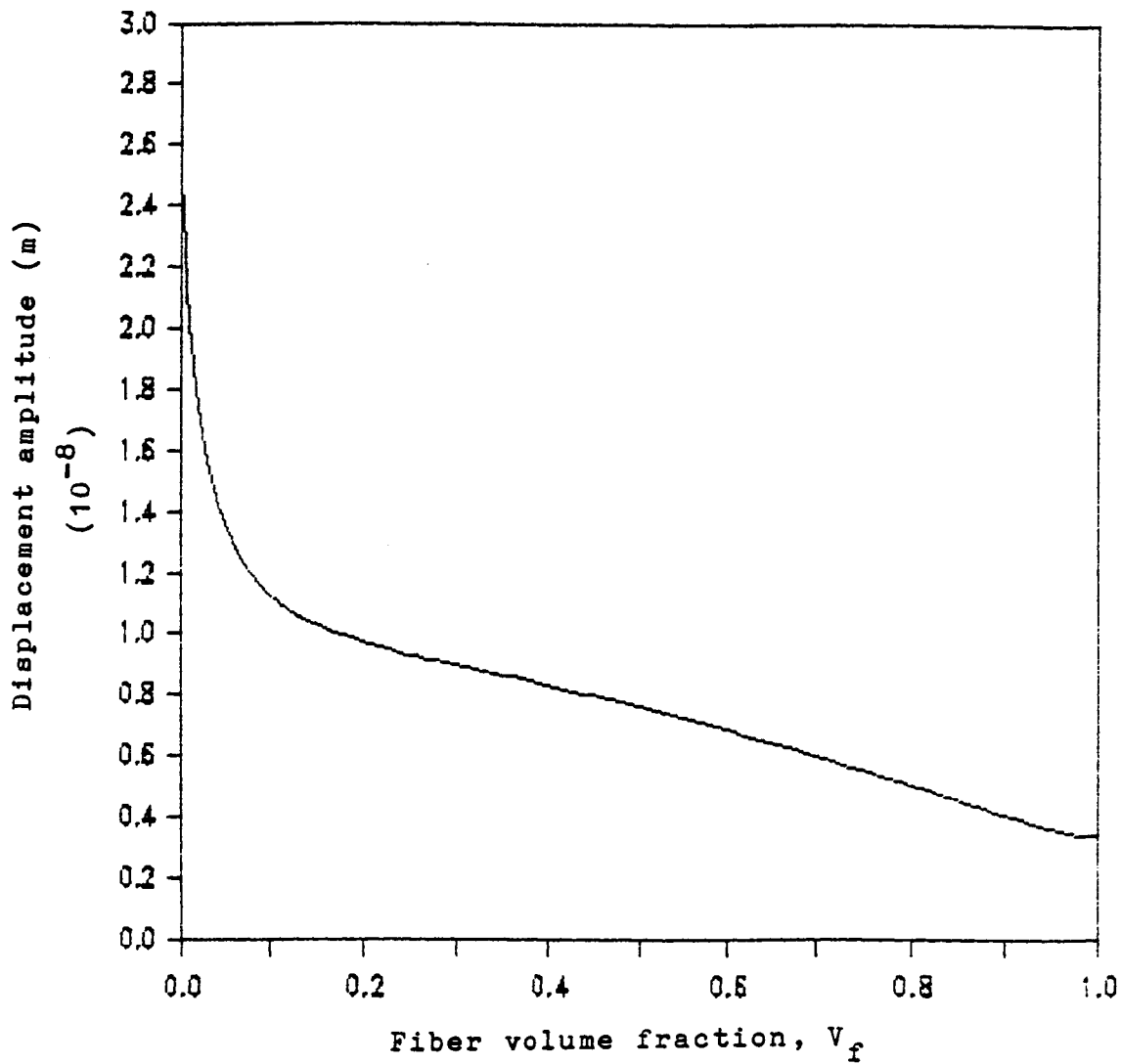coordinate system defined by (x,y,z).
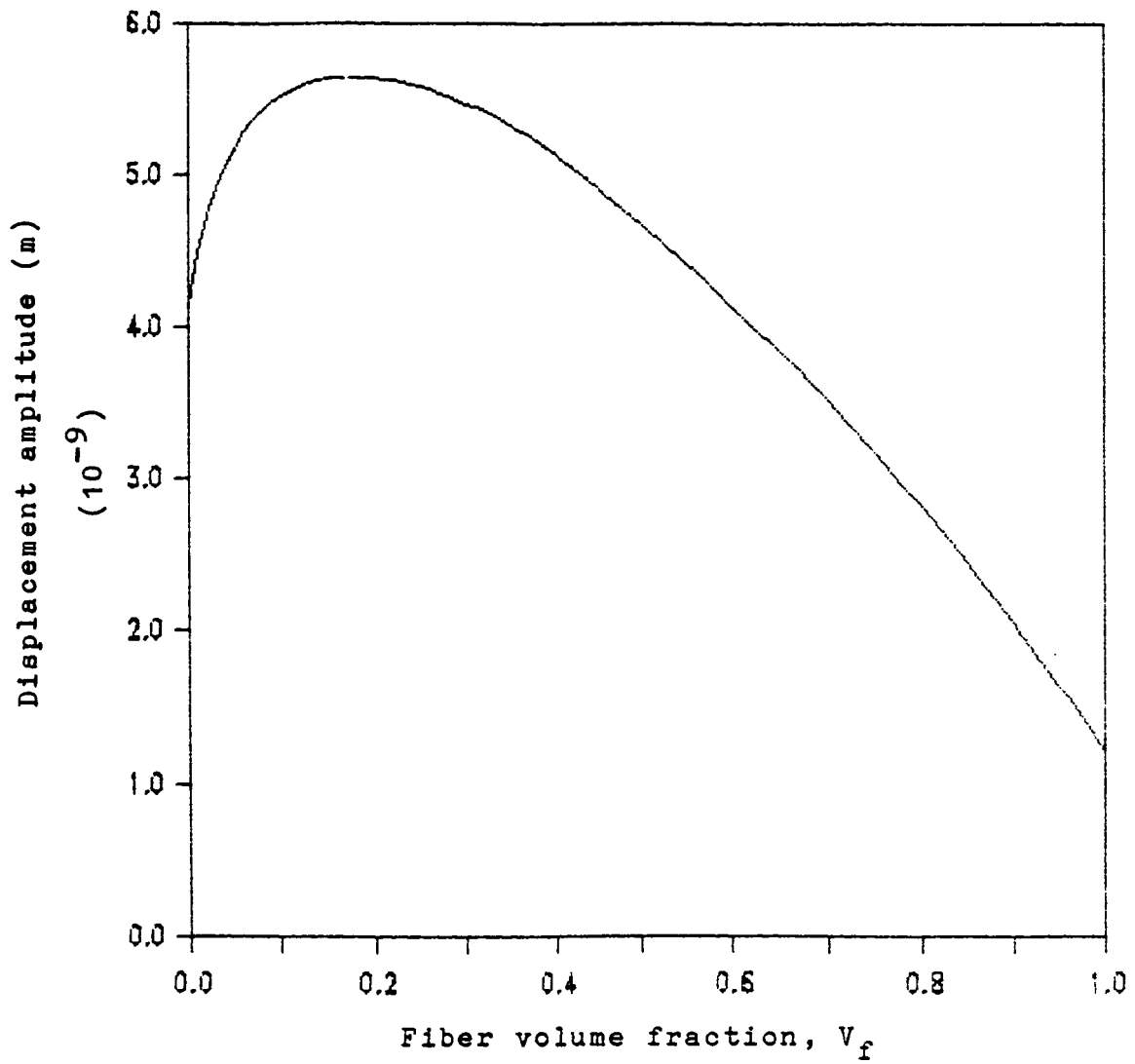
Fig. 9   Variation of amplitude of u displacement   due to   P
wave   at   A(2,0,0)   in   unidirectional   fiberglass
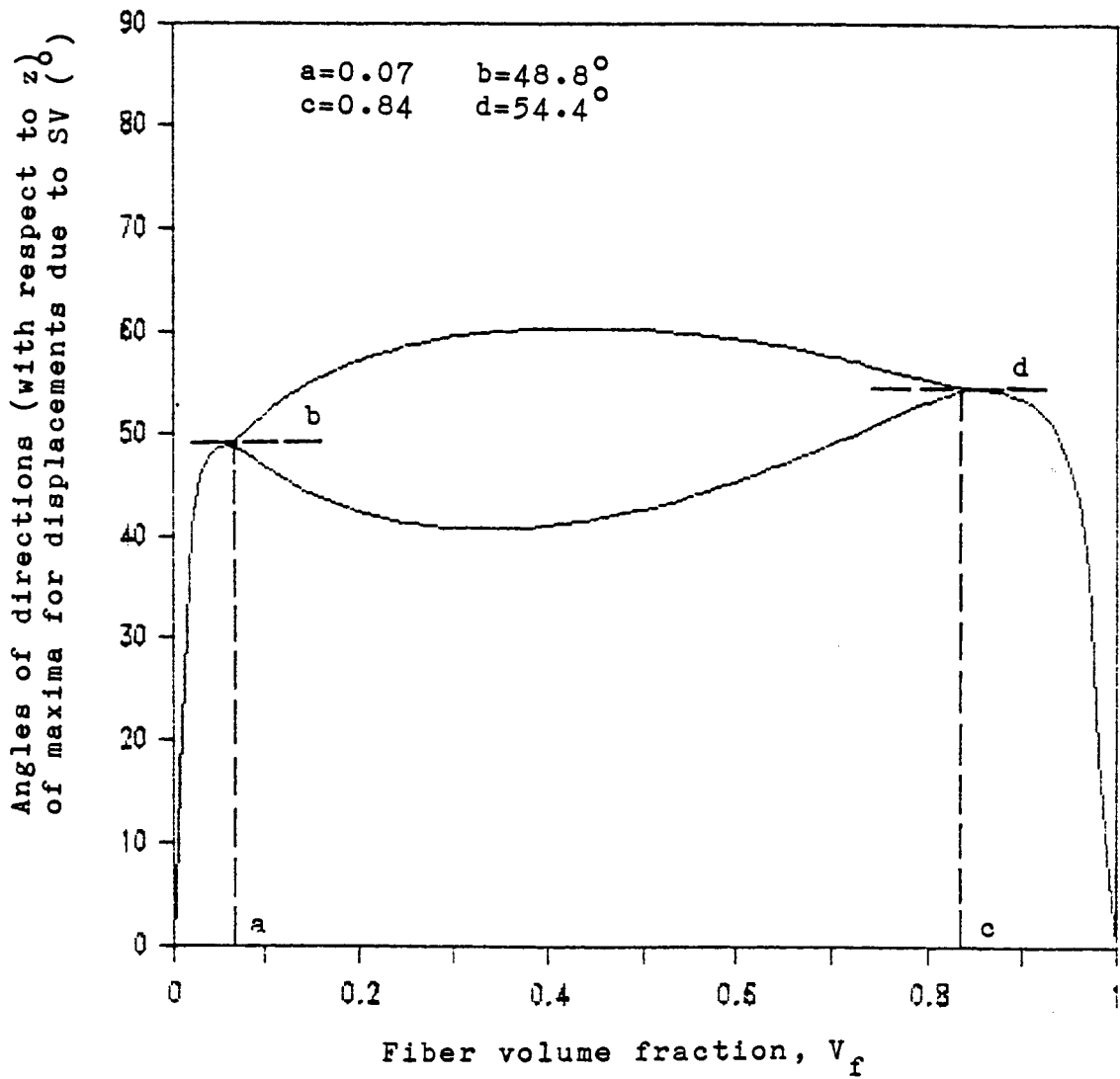epoxy composite versus fiber volume fraction.
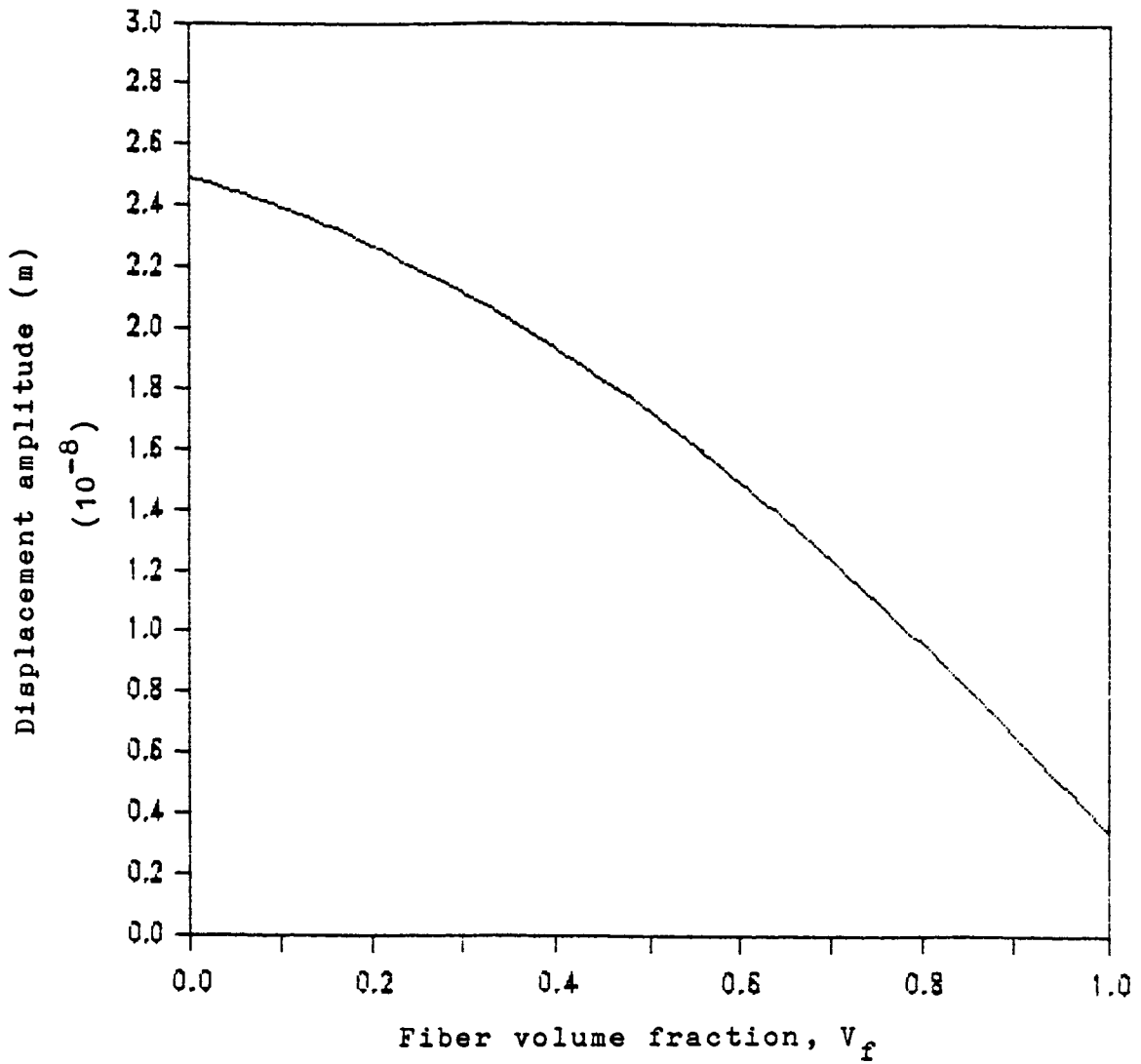
Fig.10   Variation of amplitude of u displacement   due   to
          SH   wave at B(0,2,0) in   unidirectional   fiberglass
          epoxy composite versus fiber volume fraction.

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Fig.11 Variation of amplitude of u displacement due to SV wave at C(0,0,2) in unidirectional fiberglass epoxy composite versus fiber volume fraction.

Fig.12   Direction (with respect to z axis) of maximum
          displacements due to SV wave in unidirectional
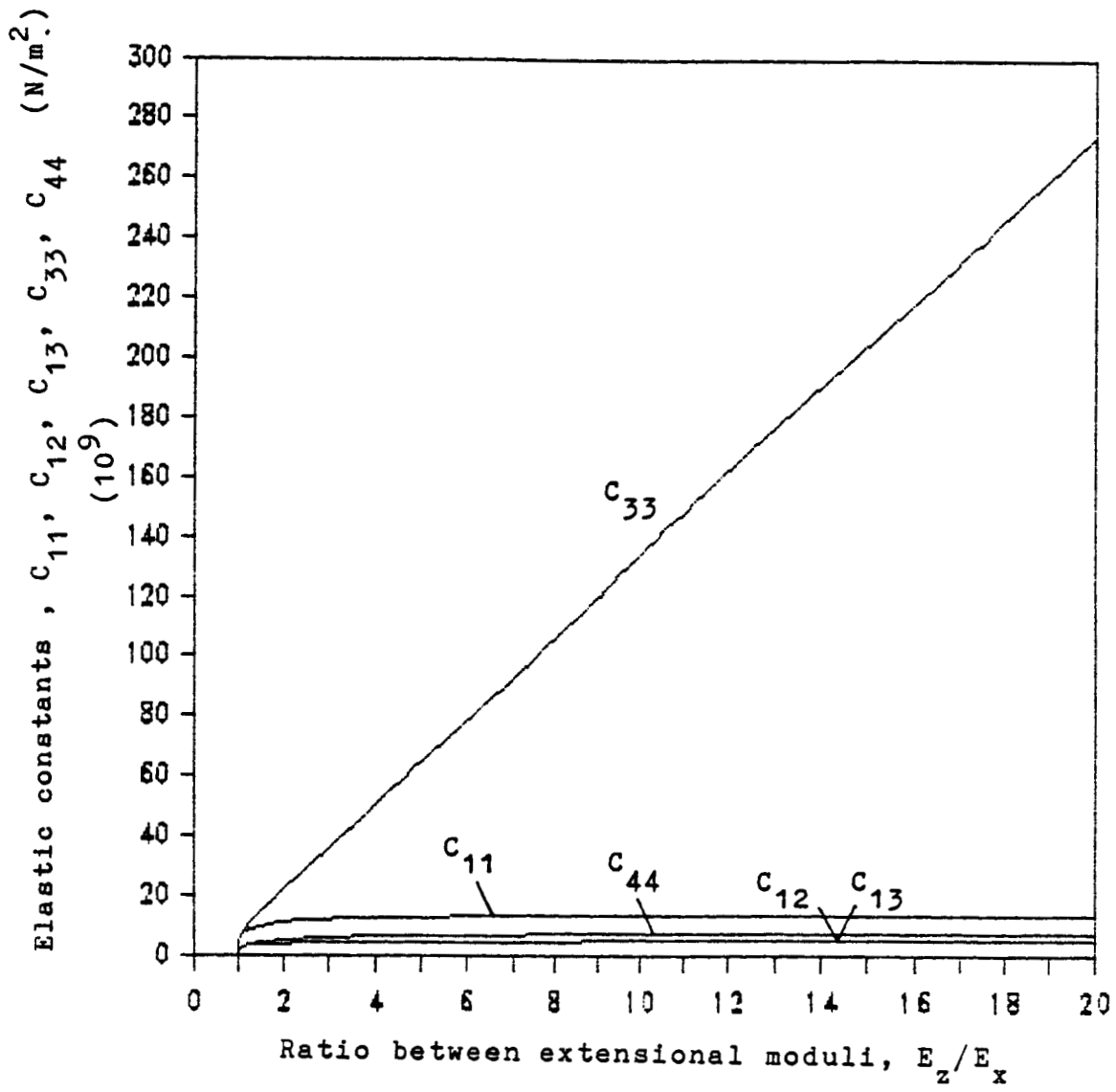   fiberglass epoxy composite versus fiber volume
          fraction.

Fig.13   Variation of elastic constants of stiffness   matrix
          in   unidirectional   composite   versus   ratio   of
          extensional   moduli,   at   constant   fiber   volume
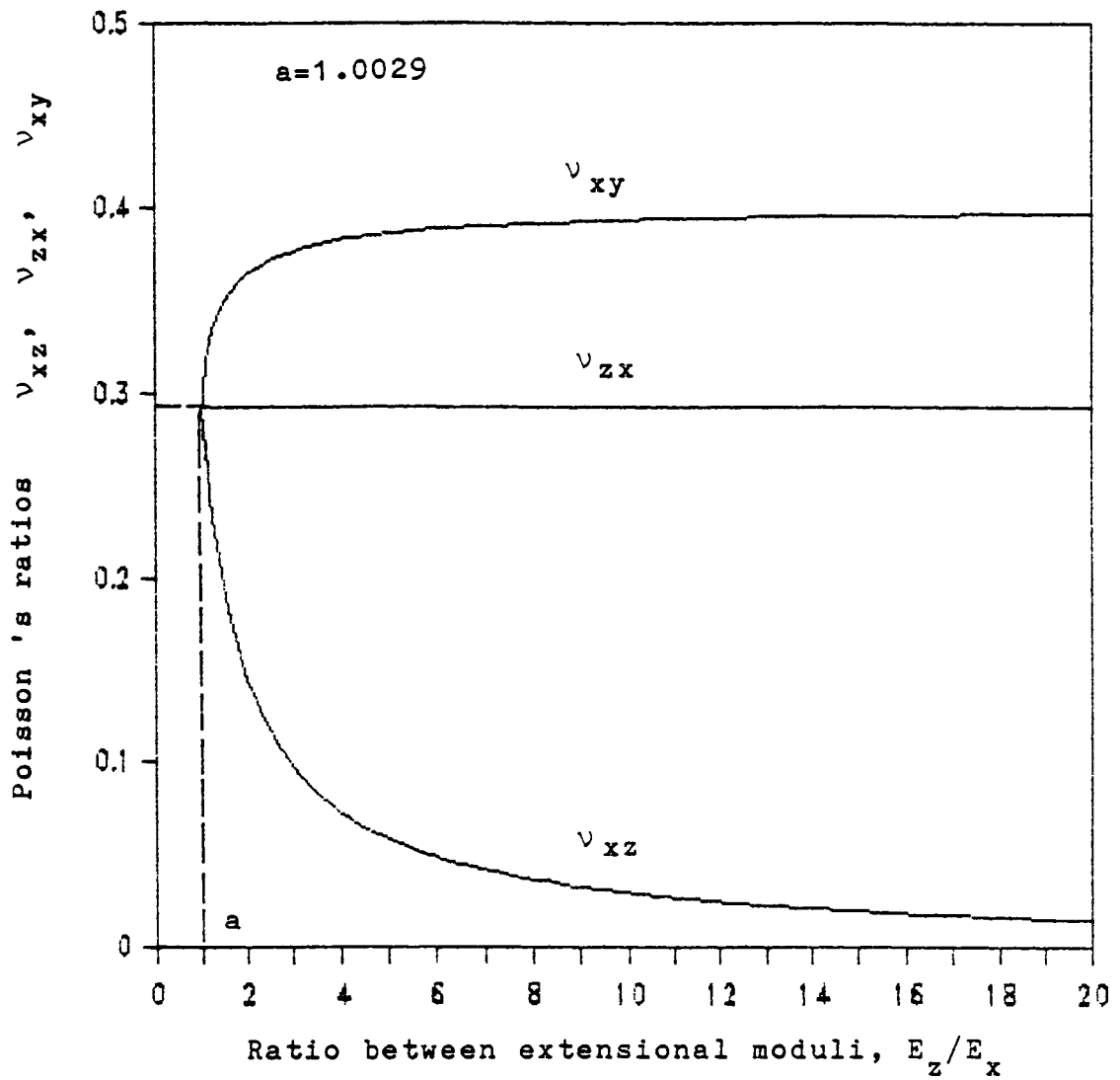   fraction of 0.6.

Fig.14   Variation   of Poisson's   ratios   in   unidirectional
         composite   versus ratio of extensional   moduli,   at
         constant   fiber volume fraction of 0.6   .(First
         index indicates deformation direction; second index
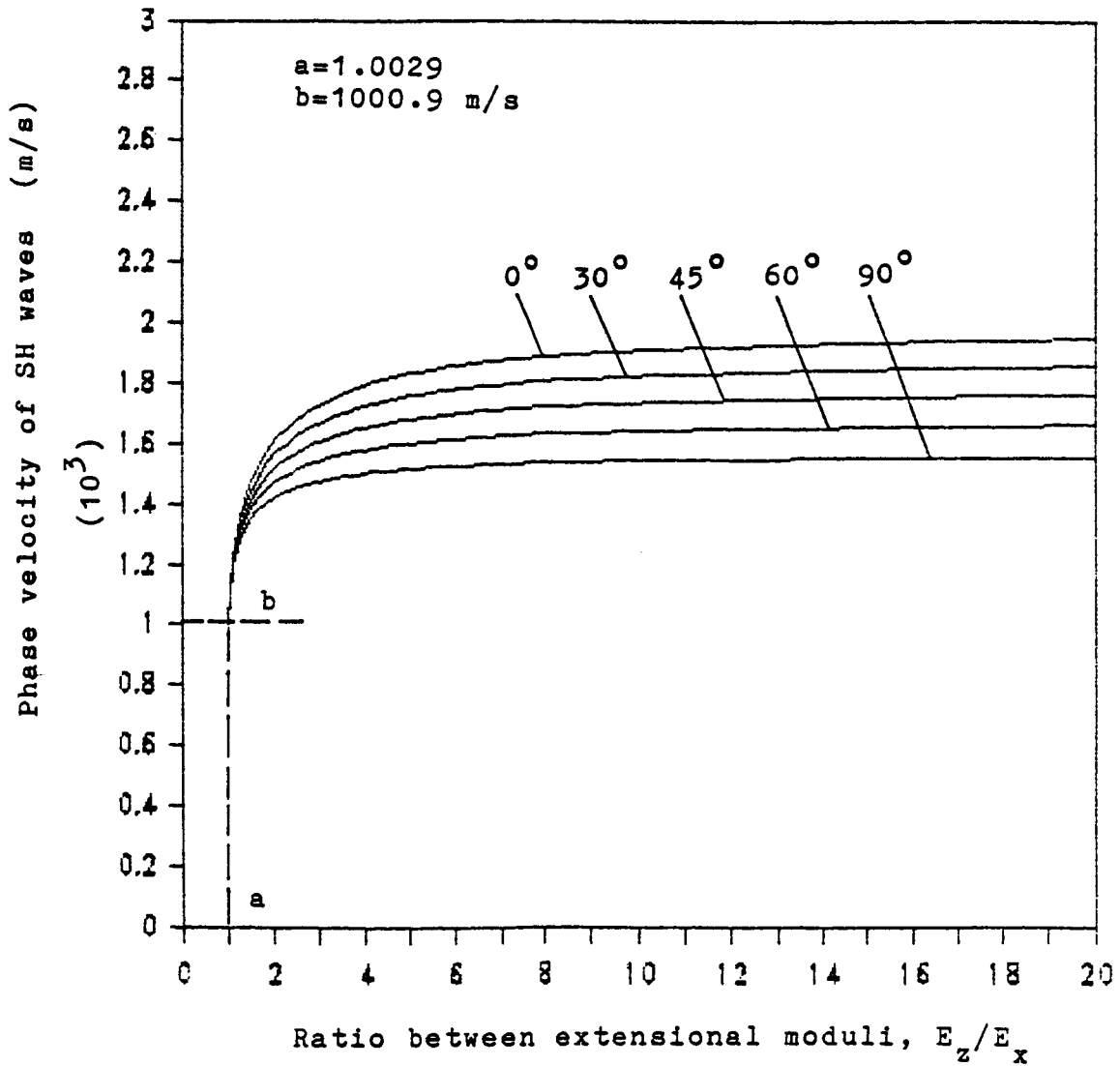   indicates load direction.)

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Fig.15   Variation of phase velocity of SH waves (with
normal $0°$, $30°$, $45°$, $60°$ and $90°$ with respect
to fiber direction) in unidirectional composite
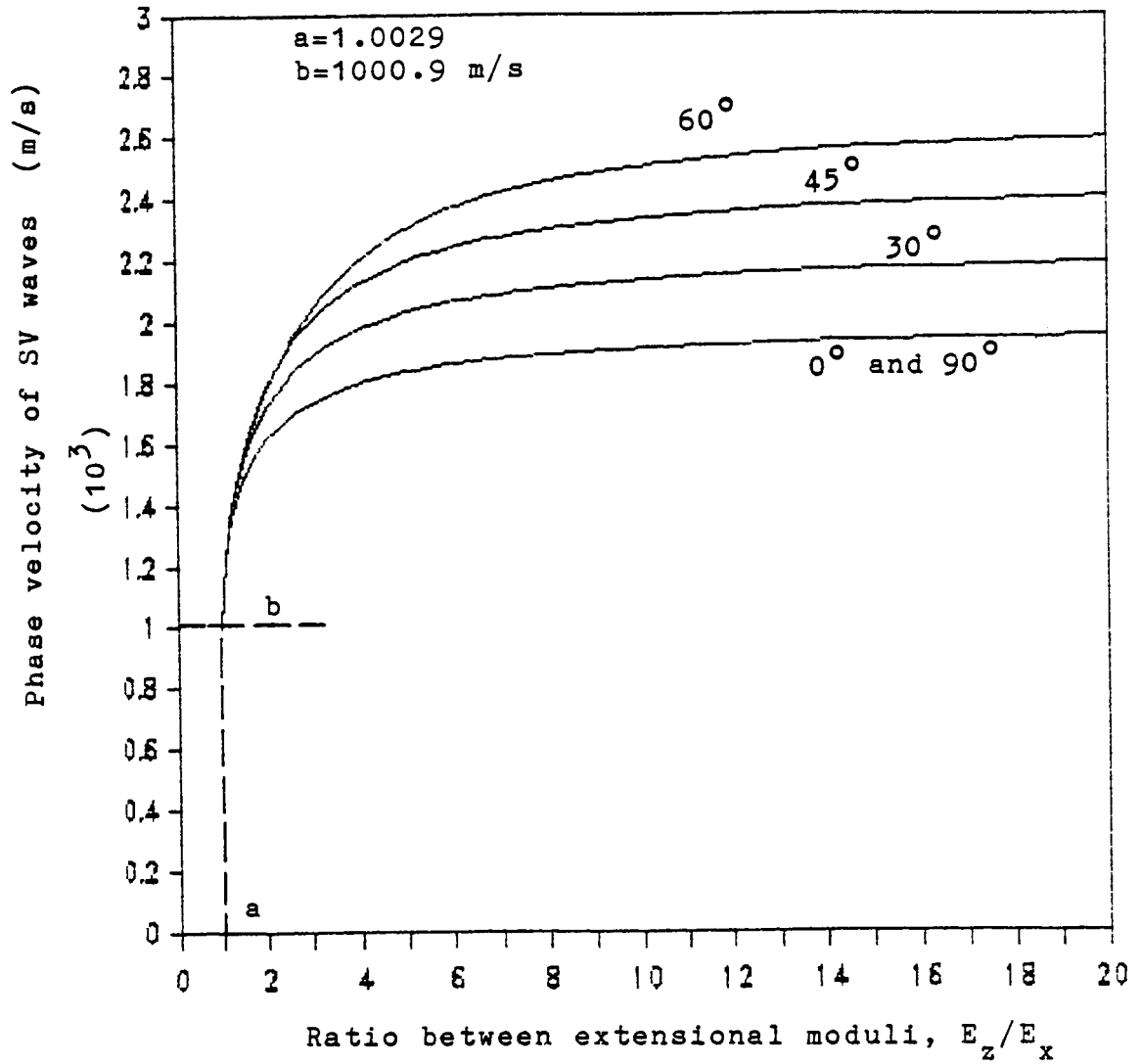versus ratio of extensional moduli, at constant
fiber volume fraction of 0.6 .

Fig.16   Variation of phase velocity of SV waves (with normal 0°, 30°, 45°, 60° and 90° with respect to fiber direction) in unidirectional composite versus ratio of extensional moduli, at constant fiber volume fraction of 0.6 .
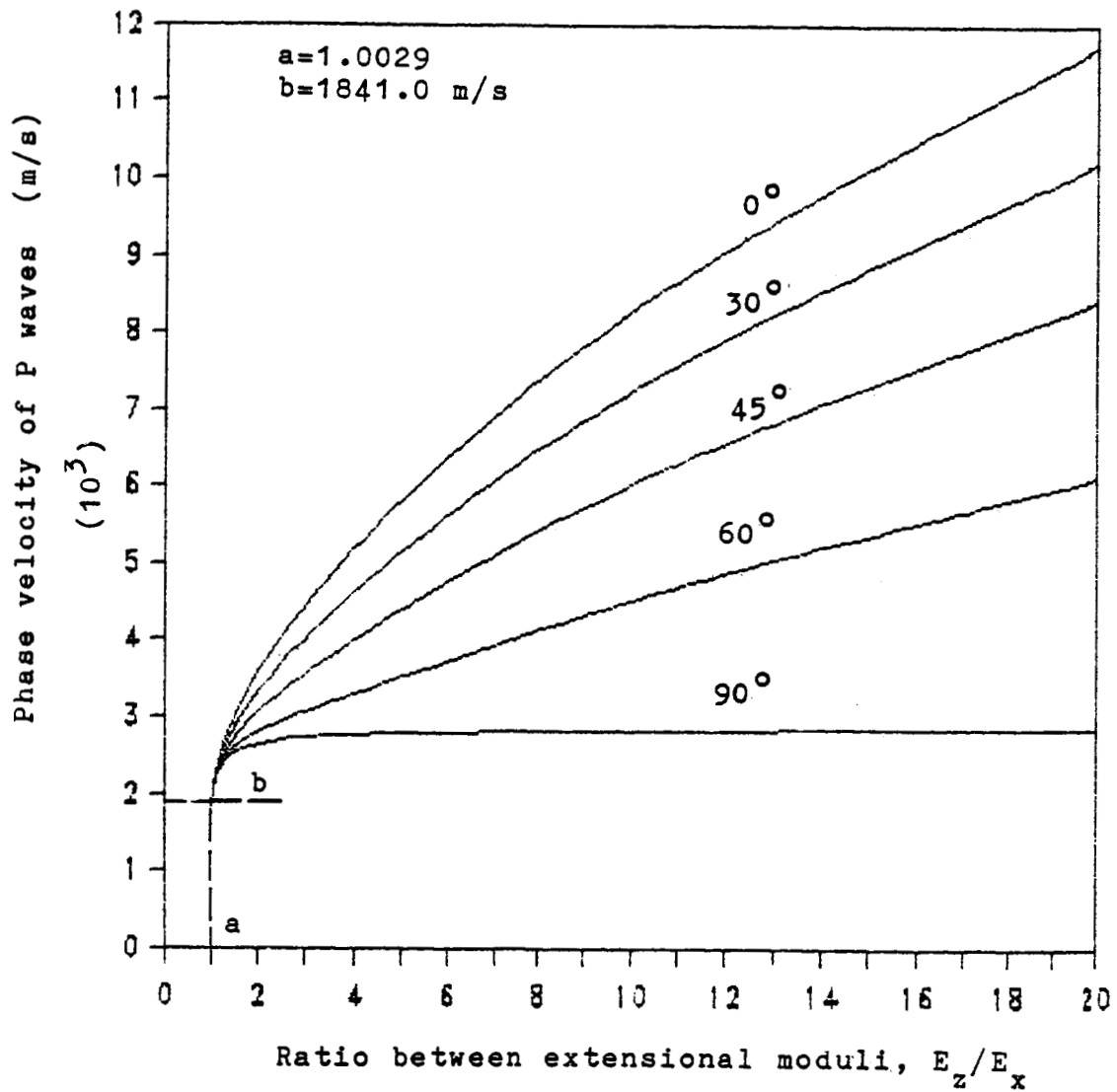
Fig. 17 Variation of phase velocity of P waves (with
normal 0°, 30°, 45°, 60° and 90° with respect
to fiber direction) in unidirectional composite
versus ratio of extensional moduli, at constant
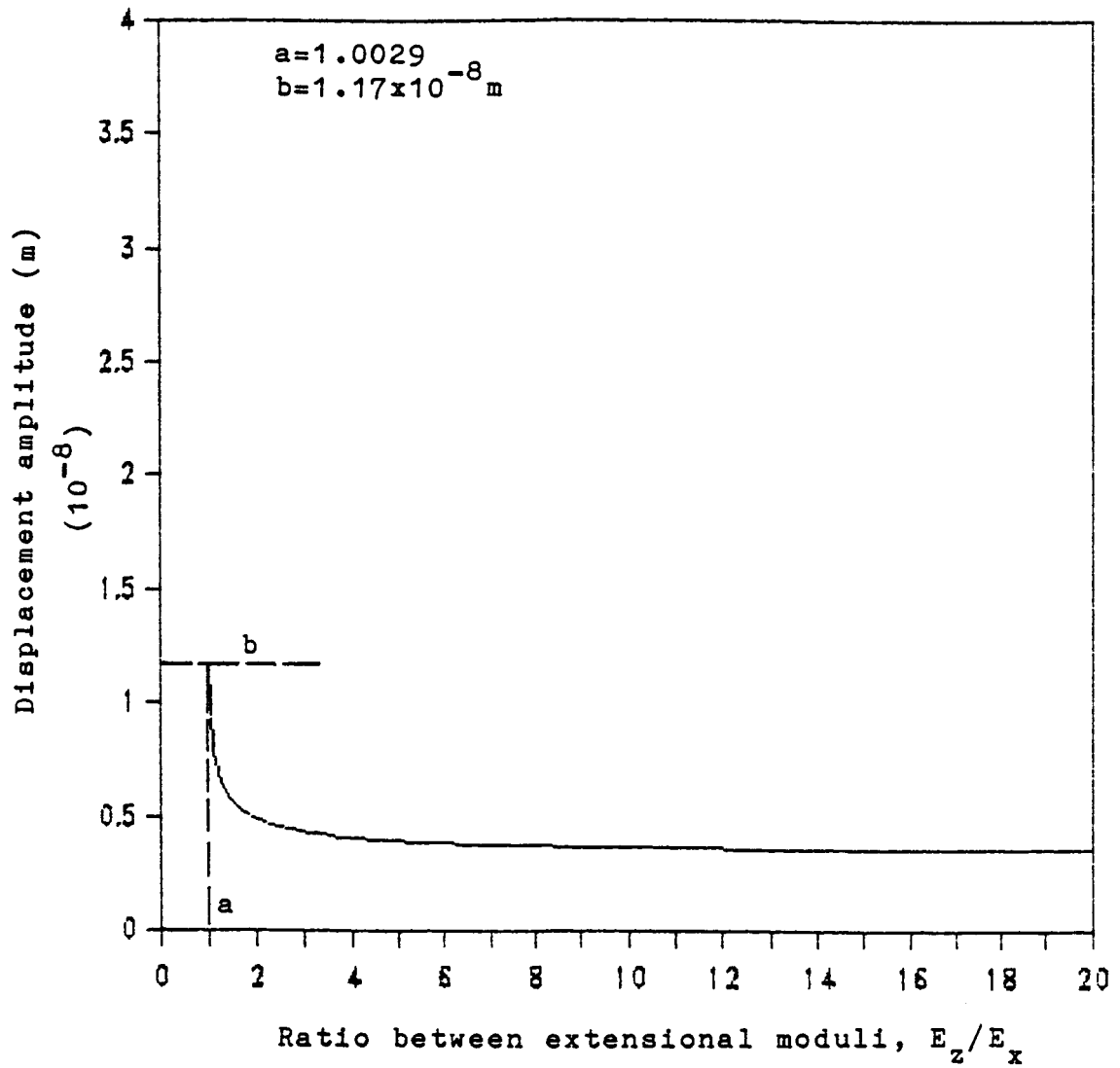fiber volume fraction of 0.6 .

Fig. 18 Variation of amplitude of u displacement due to P wave at A(2,0,0) in unidirectional composite versus ratio of extensional moduli, at constant fiber volume fraction of 0.6 .

Fig. 19 Variation of amplitude of u displacement due to SH wave at B(0,2,0) in unidirectional composite versus ratio of extensional moduli, at constant fiber volume fraction of 0.6 .
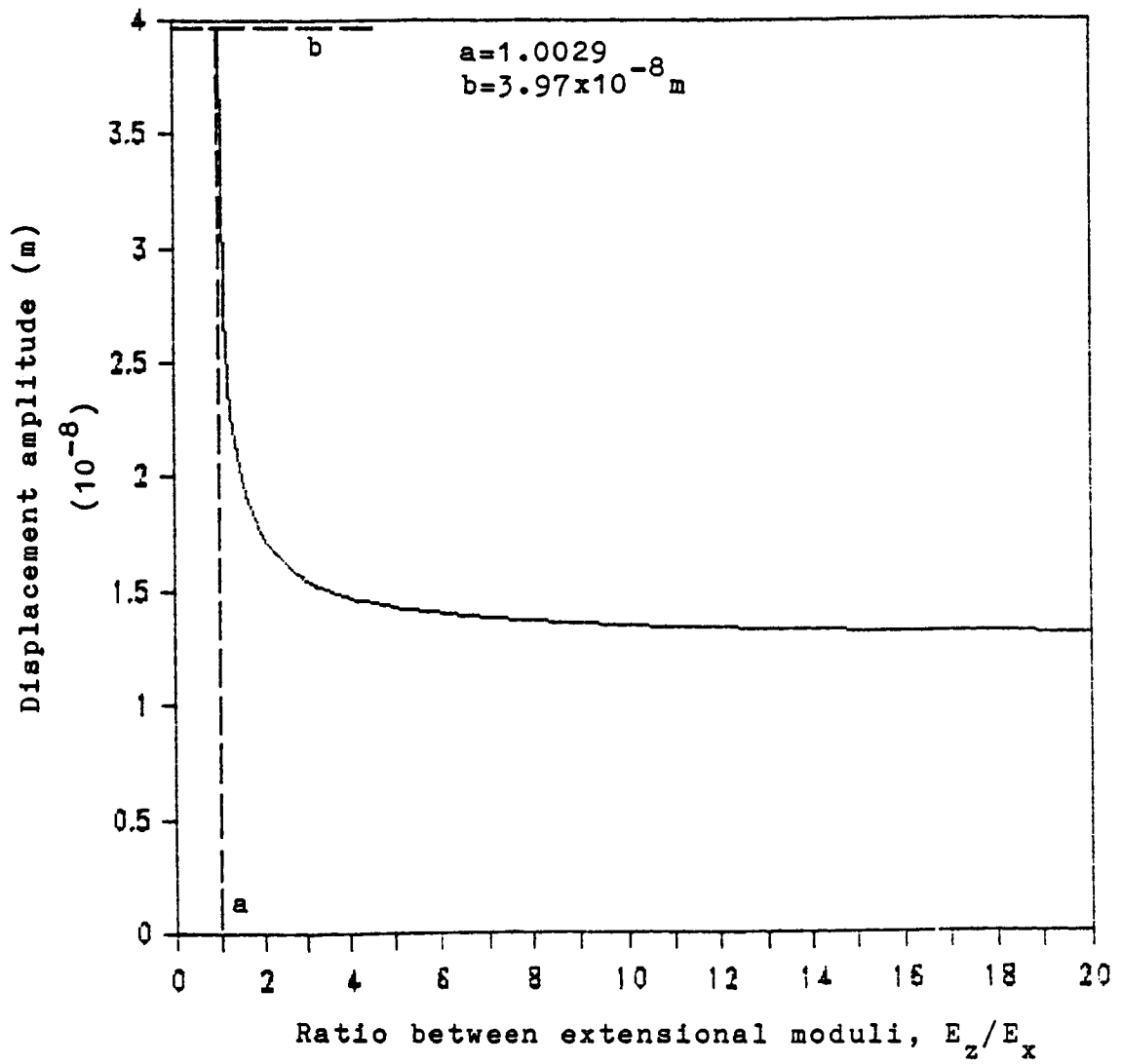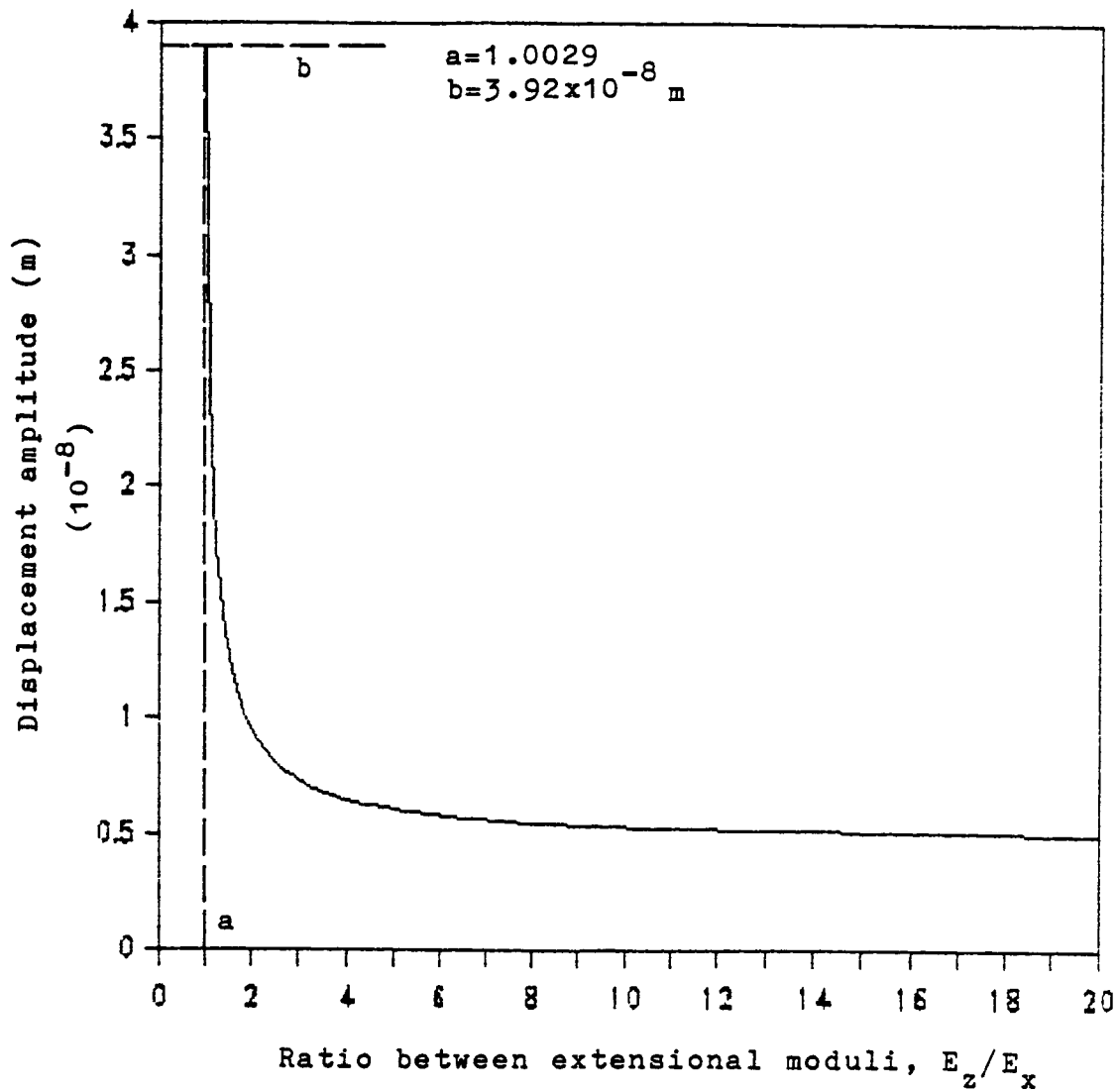
Fig. 20   Variation of amplitude of u displacement due to SV
           wave at C(0,0,2) in unidirectional composite versus
           ratio   of   extensional moduli,   at   constant   fiber
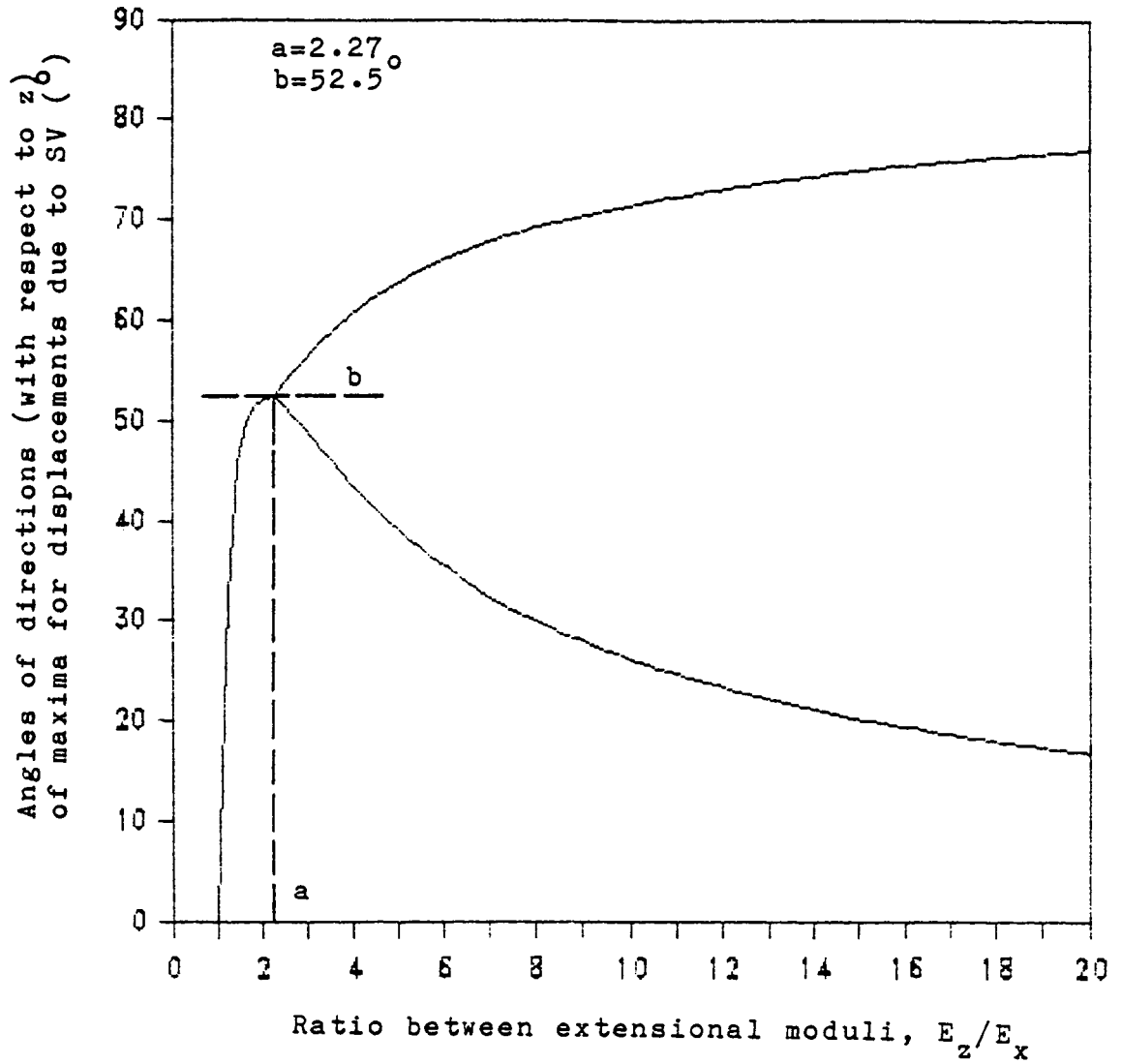   volume fraction of 0.6 .

Fig.21   Direction (with respect to z axis) of maximum
          displacements due to SV wave in unidirectional
          composite versus ratio of extensional moduli, at
          constant fiber volume fraction of 0.6 .

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15. Supplementary Notes

Project Manager, Alex Vary, Structures Division, NASA Lewis Research Center.

16. Abstract

The values of phase velocities of ultrasonic waves in transversely isotropic media are presented in terms of the fiber volume fraction of a unidirectional fiberglass epoxy composite with constant matrix properties and the ratio between extensional moduli in the longitudinal and transverse directions of the composite when the properties of the fibers are changed, at a constant fiber volume fraction. The model of a homogeneous transversely isotropic medium is adopted to describe the relations between elastic properties and velocities. The displacements due to an oscillatory point source in an infinite medium are used as one measure of comparison of the behavior of the unidirectional composite according to the variations of the parameters, as described above. Values of phase velocities, elastic moduli, Poisson's ratios and displacements due to a point source can be read from the parameterized plots for a known fiber volume fraction or a known ratio between extensional moduli of the composite. Alternatively fiber volume fraction and the ratio between extensional moduli can be inferred from the plots when the values of the phase velocities are known; for example from experimental measurements. Thus, such parameterized curves may be useful in nondestructive mechanical property and material degradation characterizations.

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