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SUMMARY

From published research it is known that a quasi-periodic 14 month atmospheric pressure oscillation of a few tenths of a millibar exists in the region of the North and Baltic Seas. At some time in the cycle the associated wind stress has a westerly component that drives a circulation in the North Sea. The results of a dynamical model and comparisons with several North Sea residual circulation studies show that a large sea level gradient results along the Dutch coast. It is this feature that has been referred to as the enhanced pole tide. The dynamic similarity of this pole tide in the North and Baltic Seas to the annual and seasonal wind forced circulations is considered. It is inferred that the large deviations of the pole tide from equilibrium at coastal stations are the result of this sea level set up forced by the 14 month wind stress cycle.

1. INTRODUCTION

The Chandler wobble of the earth's rotation axis has a period near 14 months and sets up the pole tide in the ocean. The amplitude of the wobble determines the equilibrium tide height, or maximum possible tide height if all wobble potential energy were used to raise the sea level without horizontal motion. This value is about a half centimeter in midlatitudes. It is difficult to determine the global shape of the pole tide from observations because of its small amplitude, and since most of the data is taken at coastal stations. Independent data studies over the past several decades (see Lambeck 1980, p. 212-214) have shown that while the pole tide amplitude is usually of the same magnitude as the equilibrium tide, it is anomolously large, up to 3 cm, in the North and Baltic Seas. The salient feature of all these analyses is the steep tide height gradient along the Dutch coast, where the tide height increases northward to five times the equilibrium value (Wunsch, 1974, Figs. 1, 2; reproduced in Lambeck, 1980, Fig. 8.1). The dynamical explanation of this phenomenon is the purpose of this article.

It is important to find an explanation for any significant deviation of the pole tide from equilibrium. One question of geodynamics concerns the possibility of the earth's Chandler wobble energy being damped in the oceans. This could only occur if the pole tide departs significantly from equilibrium over a large part of the ocean basins (Smith and Dahlen, 1981). Theoretical work with Laplace's tidal equations has shown that the pole tide amplitude should be very close to equilibrium for an ocean covered earth (O'Connor and Starr, 1983; Dickman, 1985) and also in bounded ocean basins (Carton and Wahr, 1986; O'Connor, 1986). Although these anomalously large coastal pole tides are not likely to result from anomlously large pole tides over the entire ocean basins, some explanation is desirable.

The observed phase and amplitude of the pole tide compared to the equilibrium tide is referred to as the tidal admittance, and most data studies determine this value for each location. In particular, the phase of the admittance varies greatly at different stations, which is a surprising result for a long period global forcing mechanism. Furthermore, if the observed tide results only from the earth's wobble, then a strong correlation should exist in the time series of monthly pole positions (equilibrium tide values) and the observed tide, which is not the case. Two studies of the secular variations of the pole tide admittance (Naito, 1977; Daillet, 1981) have concluded that the pole tide, especially in the North and Baltic Seas, must be the result of some additional forcing mechanism. Recent work on the analysis of sea level around Britain (Thompson, 1980; Cartwright, 1983) shows that when meteorological effects are removed from time series of sea level data, the the pole tide is found to be 0.6 cm, near its equilibrium value. This indicates that meteorological effects may be present in many pole tide observational studies.

The first investigation into the dynamics of the pole tide in the North Sea was done by Wunsch (1974) who showed that the bottom topography deepening northward would be an important factor in an eastward intensification of the flow regime. The direct forcing of the wobble potential on the North Sea was neglected compared to the deep ocean pole tide forcing at the opening of the sea. This assumption has been found acceptable for all tide models of the North Sea, because of the limited extent of the basin. The solution to the homogeneous equations is then determined by the open boundary condition. A nonequilibrium deep ocean pole tide was specified at the opening, but the resulting solution did not account for the observed tide height gradient along the continental coast. Similar results were obtained in a study by Carton and Wahr (1986). This further indicates that additional nontidal forcing mechanisms should be considered, and there are limited meteorological possibilities.

2. METEOROLOGICAL FORCING

We wish to review the basis for considering meteorological forcings at this low frequency. It has been known for some time that the sea level atmsopheric pressure in the North Atlantic has a quasi-periodic variation near 14 months, with an amplitude of several tenths of a millibar (see Lamb, 1972, p. 222-224). This is associated with an oscillation in the position of the Icelandic low (Maksimov et al., 1967), and studies by Holland and Murty (1970) and Bryson and Starr (1977) show that at some time during the cycle, a pressure gradient of several tenths of a millibar directed northward exists over both the North and Baltic Seas. It was shown by Starr (1983) that this phenomenon could not be the result of direct polar wobble forcing on the atmosphere. However, there is a quasi-biennial oscillation in atmospheric pressure with a period of 26-28 months, which is known to influence the position of the Icelandic low (Angell and Korshover, 1974). Since the atmosphere is a nonlinear system with a strong annual forcing, it may be expected that some response will appear at the difference of the annual and quasi-biennial frequencies, giving rise to the 14 month oscillation. Although a complete explanation of this phenomenon has not been given, the pressure oscillation in the North Atlantic is well established. The work of Lamb (1973) and Hohn (1973) shows that the climatology of the North Sea is greatly influenced by changes in the Icelandic low and resulting westerly winds.

A time series of monthly mean sea level pressures for the period 1899-1970 was examined by Bryson and Starr (1977) for each of 180 grid points in the Northern Hemisphere. For an assumed period of 437 days, sea level pressure patterns were shown at various times in the cycle. Although the spatial resolution of the grid points is too coarse to determine a detailed structure of this pressure distribution over the North Sea, it is seen that at some time during the cycle (Bryson and Starr, 1977, Fig. 10) there is a pressure gradient directed northward, of magnitude

$$\frac{\Delta P}{\Delta Y} = \frac{0.4 \text{ mb}}{600 \text{ Km}} \qquad (1)$$

As will be shown later, it is this pressure gradient that will have the greatest influence on sea levels along the continental coast.

Now we shall consider how this meteorological forcing affects sea level. The North Sea has been the object of extensive modeling over the past twenty years, because of a need to predict storm surges and pollution dispersal on longer time scales. There are three effects that must be considered and an excellent description is given by Timmerman (1977). First, the wind stress acts tangentially on the sea surface, and so the force per unit mass on a water column is inversely proportional to the depth. Secondly, the horizontal gradient of atmospheric pressure acts directly on the sea surface, and this forcing is independent of depth. Thirdly, the effect of winds and atmospheric pressure on the deep ocean and continental shelf sets up a surge at the opening to the North Sea, which affects the interior flow regime as a boundary condition, as is the case with the tides. The work of Timmerman (1977, p. 47-49) shows that for the conditions of mean monthly winds over the shallow southern North Sea, the wind stress is the dominant effect, and subsequently will be the only forcing considered in the equations of motion.

We can assume that the mean monthly surface winds are in geostrophic equilibrium with the mean monthly atmospheric sea level pressure gradient. Since this relationship is linear we can compute the change in wind directly from the change in pressure gradient during the course of the 14 month oscillation. It is often assumed that boundary layer friction results in the surface wind being turned by up to 30° toward low pressure, and decreased in speed by perhaps 20%. However, since the surface isobar patterns for this oscillation are only approximately known, we shall simply use the geostrophic relationship to compute the surface wind. Then for the oscillation in the north-south pressure gradient about the long term average (eq. 1) the magnitude of the eastward wind deviation is computed to be

$$u_a = \frac{1}{\rho_a f} \frac{\Delta P}{\Delta Y} = 0.47 \ \mathrm{ms}^{-1} \tag{2}$$

where $\rho_a = 1.2$ Kg m^{-3} is the air density and $f = 1.19 \times 10^{-4}$ s^{-1} is the coriolis parameter.

The wind stress on the sea surface is parameterized by a bulk aerodynamic formula (Hellerman, 1967, Hellerman and Rosenstein, 1983) with a quadratic resistance law,

$$\begin{aligned} \tau_x &= \rho_a C_D (u_a^2 + v_a^2) u_a \\ \tau_y &= \rho_a C_D (u_a^2 + v_a^2) v_a \end{aligned} \tag{3}$$

where the drag coefficient $C_D = 2.6 \times 10^{-3}$ is for winds greater than 6.7 m s^{-1}, which applies to annual average winds over the North Sea. The annual average wind stress over the North Sea is shown by Hellerman (1967). It has little spatial variability and a representative value is

$$\begin{aligned} \tau_x &= 0.62 \ \text{dynes cm}^{-2} \\ \tau_y &= 0.21 \ \text{dynes cm}^{-2} \end{aligned} \tag{4}$$

so that from equations (3) we can calculate that the annual average winds must be

$$\begin{aligned} u_a &= 13.71 \ \mathrm{m\ s}^{-1} \\ v_a &= 4.65 \ \mathrm{m\ s}^{-1} \end{aligned} \tag{5}$$

Since the wind stress on the sea is proportional to the square of the wind speed, it is important to note that the oscillation in the momentum transfer during the 14 month cycle will be proportional to the difference of the squares (as opposed to the square of the difference) of the maximum and average winds. We now add the amplitudes of the wind oscillation (eq. 2) to the long term average wind (eq. 5), and then use eq. (3) to calculate the maximum wind stress during the cycle

$$\begin{aligned} \tau_x &= 0.66 \ \text{dynes cm}^{-2} \\ \tau_y &= 0.22 \ \text{dynes cm}^{-2} \end{aligned} \tag{6}$$

The amplitude of the oscillation in momentum transfer is the difference of these maximum (eq. 6) and average (eq. 4) values, and is

$$\begin{aligned} \Delta\tau_x &= 0.04 \text{ dynes cm}^{-2} \\ \Delta\tau_y &= 0.01 \text{ dynes cm}^{-2} \end{aligned} \tag{7}$$

It is seen that both the annual average and 14 month oscillations in wind stress have large westerly components. This oscillation in wind stress must drive a 14 month oscillation in sea level, and so these values (eq. 7) must be used as the forcing for the ocean model.

3. DYNAMICS

Numerical models have been developed that consider the North Sea and Northwest European continental shelf as a system, and determine the dependent sea response to changing atmospheric presssure, winds, and external tides and storm surges from the deep ocean. The more comprehensive models are three dimensional, and include realistic coastal geometry and bottom topography, expressed in spherical coordinates. The dynamical equations may include the nonlinear advective terms, nonlinear bottom friction, and lateral boundary friction (see Nihoul and Ronday, 1976; Sundermann and Lenz, 1983, for a review).

We shall follow a greatly simplified approach, since we are presently considering a 14 month cycle described quantitatively by monthly mean values of the wind stress. It is desired to show that at the time of maximum wind stress, the dynamical interaction of the forced flow with the geometry and bottom topography produces the observed gradient in sea level along the Dutch coast. The wind stress forcing may be taken to be spatially uniform and stationary in time. Certainly any wind stress curl is not the dominant forcing mechanism. Since the response of the shallow North Sea to changing winds is on the order of several days, we may consider the mean monthly flow to be in a steady state.

A number of investigations have considered steady, uniform wind forcing on the North Sea. The work of Lauwerier (1960, a, b), and Furnes (1980) shows that the basic features of a wind forced circulation can be modeled by steady state linear dynamics for a rectangular bay deepening to the north. Accordingly, we shall represent the geometry with a rectangle of dimensions 500 Km by 600 Km opening to the deep ocean at the north. The depth H(y) increased from 20 m to 100 m with an exponential depth profile (Fig. 1). Since the sea is of limited longitudinal extent the coriolis parameter is taken as a constant.

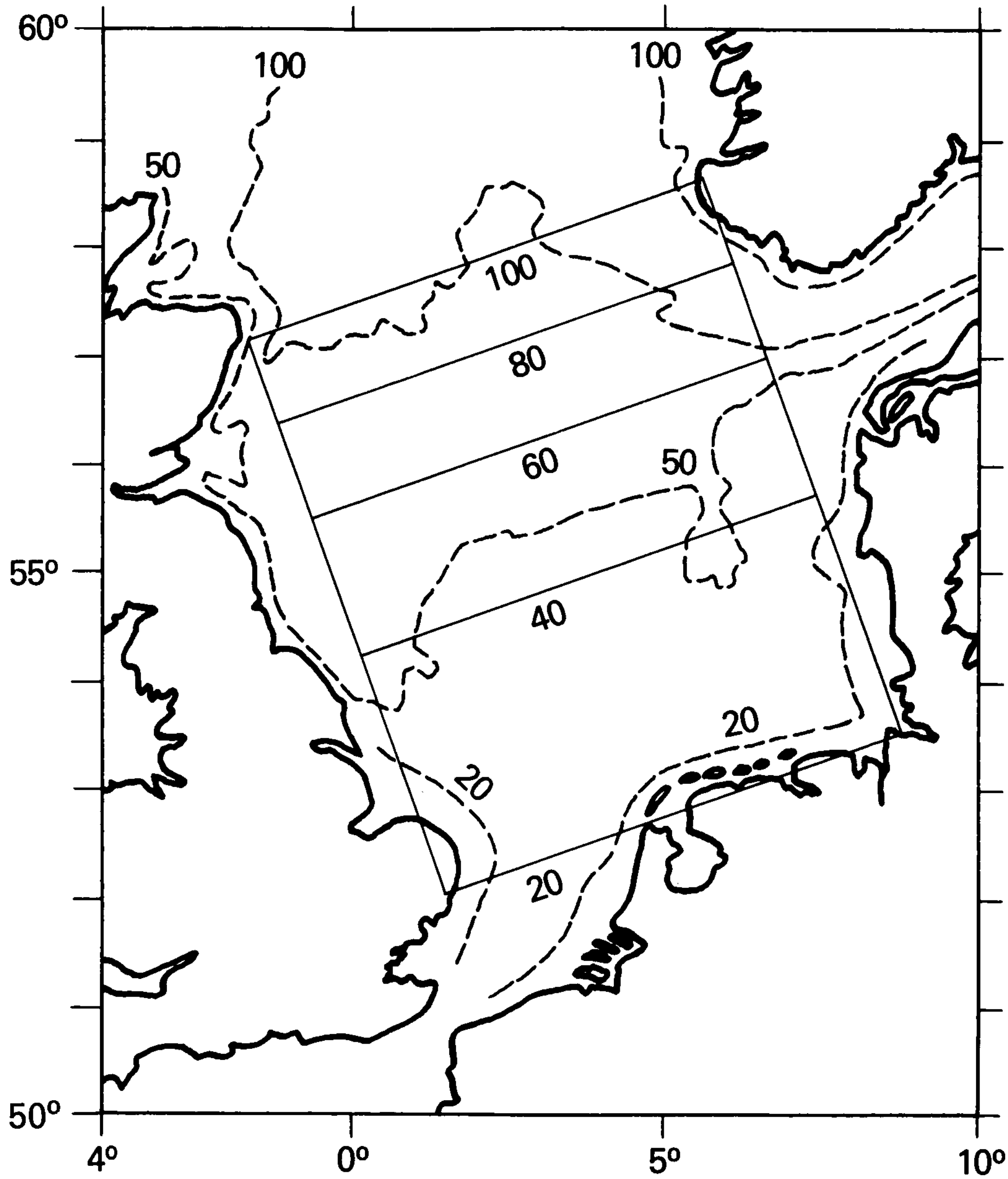

Fig. 1 North Sea model with exponential depth profile. Depths in meters.

Our problem is to describe the dynamics of a residual circulation, which is typically defined as the mean velocity field averaged over a long enough period (at least several days) to cancel transitory wind currents and tidal oscillations. This feature has been studied because of a desire to model pollution dispersion in the North Sea, and may be considered for winds averaged over several days, or seasonal and annual periods. Diurnal tidal velocities are typically two orders of magnitude greater than velocities due to long term average winds, and because of nonlinearity, the effect of this diurnal variation does not average out over longer time scales, but creates what is called a tidal stress. However, the work of Ronday (1975) and Nihoul and Ronday (1975) shows that the broad features of the wind forced residual circulation can be modeled without the nonlinear advective terms. More recently, residual circulations were calculated for seasonal and annual average winds in the North Sea by Davies (1982, 1983) who neglected these advective terms in the equations of motion.

We can use the linearized, vertically integrated equations of motion, with the assumption that the sea level variations are small compared to the depth. The horizontal momentum equations show a balance between the coriolis, pressure gradient, wind stress, and bottom friction stress

$$-fHV = -gH \frac{\partial Z}{\partial X} + \frac{1}{\rho}(\Delta\tau_X - F_X) \tag{8}$$

$$fHU = -gH \frac{\partial Z}{\partial Y} + \frac{1}{\rho}(\Delta\tau_y - F_y) \tag{9}$$

and the continuity equation is that for horizontally nondivergent flow

$$\frac{\partial}{\partial X}(HU) + \frac{\partial}{\partial Y}(HV) = 0 \tag{10}$$

(All symbols defined in an appendix.)

The wind stress values are given by equation (7) and represent the difference between the maximum momentum input during the cycle, and the long term average. We note that the orientation of the model North Sea (Fig. 1) is not aligned directly north–south. Furthermore, we recall that the atmospheric pressure gradient was oriented only approximately northward. Since we will only be able to develop a model for the wind stressed sea level that is qualitative correct in configuration and of the correct order of magnitude, we shall consider only the westerly component of the wind stress. This is the case of the wind blowing parallel to the depth contours. We may write

$$\frac{\Delta\tau_X}{\rho} = T\epsilon, \quad \Delta\tau_y = 0 \tag{11}$$

where $T = 0.04\ \mathrm{cm}^2\ \mathrm{s}^{-2}$ and the direction cosine $\epsilon = 1$. This form allows us to follow the influence of the wind stress forcing throughout the derivations.

Generally, bottom friction is best described by a quadratic resistance law. However, we are using a vertically integrated velocity which will be greater than the velocity just above the sea bed. The work of Groen and Groves (1962), Heaps (1978) and Nihoul (1980) shows that for long term wind driven residual circulations in shallow seas where tidal currents are large, the residual bottom stress is approximately linear in the residual velocity. Then the friction terms may be exressed by

$$\frac{F_x}{\rho} = KU \qquad \frac{F_y}{\rho} = KV \tag{12}$$

and the value of the coefficient K will subsequently be specified.

We nondimensionalize the coordinates by dividing by the width L = 500 Km, so that

$$\begin{aligned} x &= X/L \qquad 0 \leqslant x \leqslant 1 \\ y &= Y/L \qquad 0 \leqslant y \leqslant 1.2 \end{aligned} \tag{13}$$

The exponential law of depth can be written

$$H(y) = h_o \exp(sy) \tag{14}$$

where $h_o = 20$ m, and

$$s = \frac{1}{H}\frac{dH}{dy} = \frac{1}{1.2} \log\left(\frac{100}{20}\right) = 1.34 \tag{15}$$

The velocities and tide height can be nondimensionalized by the change of variables

$$u = \frac{fh_o}{T} U, \qquad v = \frac{fh_o}{T} V \tag{16}$$

$$\zeta = \frac{gh_o}{TL} Z \tag{17}$$

The two dimensional horizontally nondivergent form of the continuity equation allows us to define the nondimensional stream function

$$u \exp(sy) = \frac{\partial \psi}{\partial y} \tag{18}$$

$$v \exp(sy) = -\frac{\partial \psi}{\partial x} \tag{19}$$

and then write the horizontal momentum equations (8, 9) in the form:

$$\frac{\partial \psi}{\partial x} = -\frac{\partial \zeta}{\partial x} \exp(sy) + \epsilon - R\frac{\partial \psi}{\partial y} \exp(-sy) \tag{20}$$

$$\frac{\partial \psi}{\partial y} = -\frac{\partial \zeta}{\partial y} \exp(sy) + R \frac{\partial \psi}{\partial x} \exp(-sy) \tag{21}$$

where the nondimensional friction coefficient is given by

$$R = \frac{K}{f h_o} = 0.2 \tag{22}$$

corresponding to an assumed value of $K = 0.05$ cm s^{-1}. Most linear friction coefficients are of magnitude 10^{-1} cm s^{-1} for depths of 50 meters. However, since the depth $h_o = 20$ m used here for nondimensionalization is smaller than the average depth of the North Sea, the corresponding value of K is also reduced. Although linear friction coefficients are typically adjusted to give the desired model results, the value used here is close to those commonly assumed in similar modelling studies.

A vorticity equation is formed by cross differentiating the momentum equations and eliminating the sea level height to obtain

$$R\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right) - 2 Rs \frac{\partial \psi}{\partial y} - s \exp(sy) \frac{\partial \psi}{\partial x} = -\epsilon s \exp(sy) \tag{23}$$

Although the wind stress forcing is spatially constant, an inhomogeneous term remains in the vorticity equation as a consequence of the structure of the vertically integrated equations of motion with variable bottom topography. Because the function exp (sy) multiplies the first derivative term in x, the method of separation of variables cannot be applied to this equation, and so finite differencing methods will be employed.

Since we have an inhomogeneous second order elliptic equation on a rectangular domain, a unique solution can be found once the appropriate boundary conditions are specified. The physical requirement that the normal component of velocity vanishes at a coastal boundary yields the conditions on three sides

$$\psi = 0 \text{ at } x = 0, 1 \text{ and } y = 0. \tag{24}$$

The open boundary condition for a semi-enclosed sea is a study in itself, because the condition specified along the open boundary will determine the interior solution. Either the tide height, velocity, or some relation between them must be specified. If the velocity or height fields are known sufficiently well from observations this data can be used. However, small changes in the slope of the sea level can have a great effect on the current system. For seas with idealized geometry, the sea level is often set equal to zero at the open boundry (Lauwerier, 1960a, b; Timmerman, 1977). More recent numerical models have tended to remove the open boundary from the opening of the sea to the deep ocean side of the continental shelf, so that the semi-enclosed sea and continental shelf are considered together. A number of time dependent numerical models specify a radiation condition (Davies 1982, 1983) between the velocity and tide height at the open boundary,

$$V = \frac{\sqrt{gH}}{H}\,(\bar{Z} - Z) \tag{25}$$

where $\bar{Z}$ is that tide height which would exist at the edge of the deep ocean in the absence of the adjoining sea. This is based on the requirement that waves generated at the interior boundaries will propagate outward.

We are modeling a steady state residual current system for a semi-enclosed sea, for which we need the inflow and outflow. According to Nihoul (1973, 1980), this model will depict the broad trends of the residual circulation, but the data at the open boundary is insufficient to apply directly to a model. Then we shall construct a consistent open boundary condition based on the dynamics of the forcing and conservation of mass. Since we are modeling a steady state, there must be zero mass flux across the open boundary, or

$$\int_{x=0}^{x=1} v\,dx = \int_{x=0}^{x=1} \frac{\partial\psi}{\partial x}\,dx = 0 \text{ at } y = 1.2 \tag{26}$$

and this requirement is met by the present boundary conditions (24). Then it remains to determine the values of ψ along the open boundary. In order to decide this, we shall first consider the structure of the vorticity equation with a small parameter multiplying the highest order derivatives. This immediately leads us to speculate that intense boundary currents will be part of the solution somewhere along the boundary. From the sign of the first derivative term in x, we know that the intensification of the flow solution will be at the eastern boundary (Nayfeh, 1973, p. 122-124; Pedlosky, 1979, p. 253-260).

It is instructive to consider this type of problem with asymptotic methods. The form of equation (23) shows a balance between the first derivative term in x, and the inhomogensous forcing term. An approximate particular solution based on these terms alone is

$$\psi = \epsilon x \text{ at } y = 1.2. \tag{27}$$

Physically, this represents a balance between the wind stress forcing and coriolis terms in the horizontal momentum equations, and is the Ekman flow at right angles to the wind. It represents a wind forced inflow across the open boundary that would exist if no physical boundaries existed. To complete the specification of our open boundary condition, we must find that outflow velocity across the open boundary that results from the interaction of the forced flow with the interior boundaries of the North Sea, which must be a homogeneous solution to equation (23). The assumption is made that variations in x are larger than variations in y (reasonable, since the opening is far removed from the southern boundary). This is equivalent to finding a solution to equation (23) with the asymptotic stretching transformation (Nayfeh 1973, p. 122-124; Pedlosky, 1979, p. 253-260)

$$\xi = \frac{1-x}{R} \tag{28}$$

Physically, this depicts an eastern boundary current where friction must become important in the balance of forces in the horizontal momentum equation (21), while a geostrophic balance exists between the coriolis force and pressure gradient force in the horizontal momentum equation (20).

The resulting homogenous solution is

$$\psi = c_1 + c_2 \exp\left[-\frac{s(1-x)}{R}\exp(1.2s)\right], \quad y = 1.2 \tag{29}$$

where c_1 and c_2 are arbitrary constants. The two solutions (27) and (29) may be added, and the boundary conditions (24) applied to determine c_1 and c_2. The resulting solution for the flow across the open boundary, neglecting terms that remain exponentially small everywhere, is

$$\psi = \epsilon \left\{ x - \exp\left[-\frac{s(1-x)}{R}\exp(1.2s)\right] \right\}, \quad y = 1.2 \tag{30}$$

Over most of the boundary there is the inflow forced by the wind stress, but there is outflow in an intense eastern boundary current. A similar exponential boundary current could have been obtained by assuming a radiation condition with the outflow velocity proportional to the interior sea level height.

Now that we have specified the value of ψ continuously along the boundaries, we have the Dirichlet boundary conditions for an elliptic equation on a rectangular domain. It is a straightforward matter to obtain a solution by the method of relaxation (Haltiner, 1971, p. 113-115) and the resulting stream function is shown in Fig. 2 with its eastward intensification. The height field can be found by numerical integration of the horizontal momentum equations, and the results are shown in Fig. 3. The wind forced sea levels are negligible except along the southern and eastern boundaries where they support boundry currents. In particular, the model shows the pronounced sea level gradient along the southern boundary.

4. DISCUSSION

We now discuss the dynamics of the solution in the North Sea. In the deeper northern part,

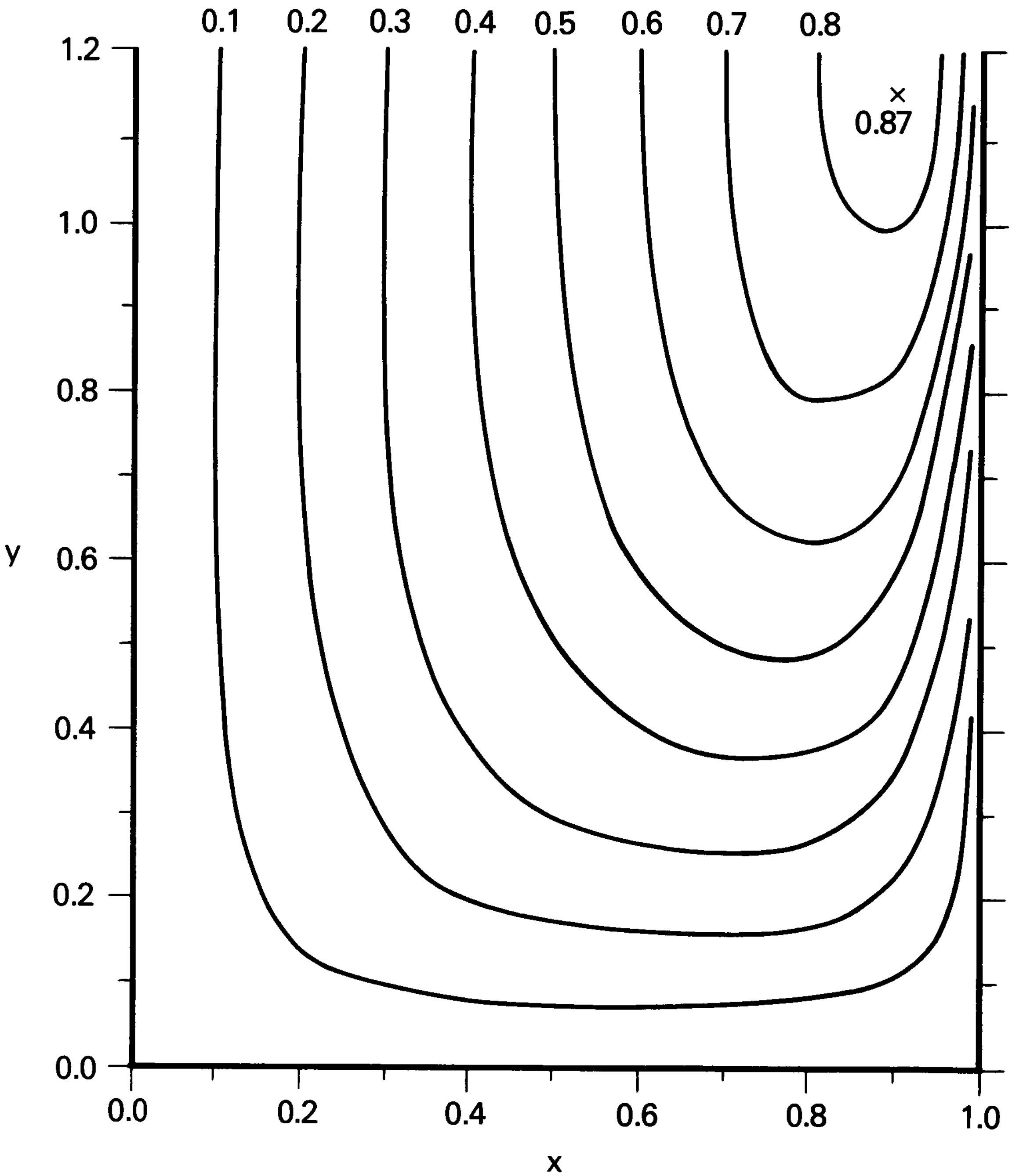

Fig. 2 The stream function solution.

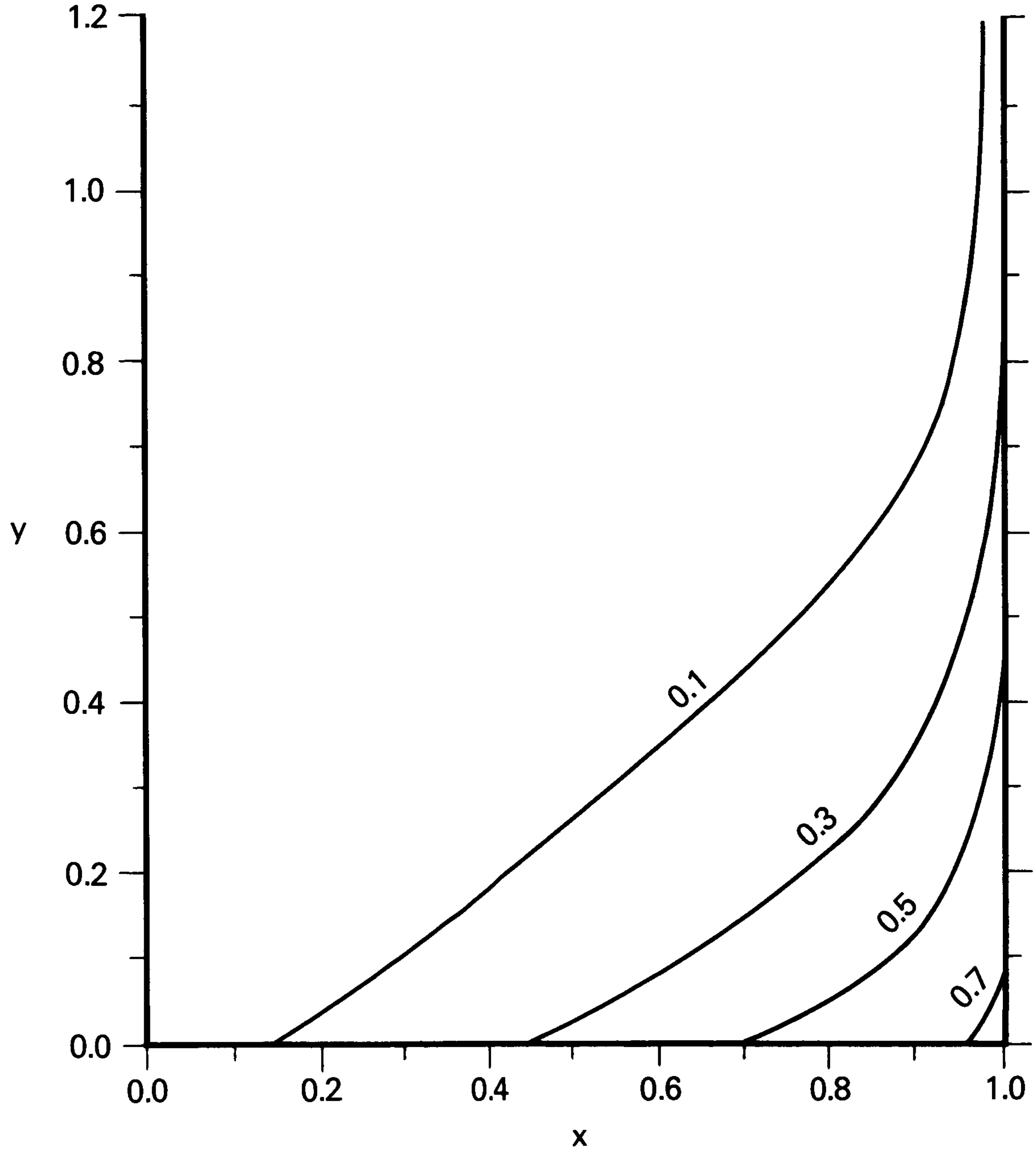

Fig. 3 The sea level heights in cm. The sea level at (0,1.2) was set equal to zero.

there is the inflow from Ekman transport due to the westerly wind. The dominant counterclockwise circulation pattern over the majority of the domain is determined by the northward slope of the bottom topography, at right angles to the wind. Larger pressure gradients can be set up in shallow water, and so the effect of wind and bottom topography is to pile up water in the German Bight. This excess water is removed by a narrow boundary current flowing northward along the Danish coast.

It is desirable to compare the results obtained here with those of other studies, particularly as they apply to the sea level gradient along the Dutch and Danish coasts. One of the earlier studies of the effect of wind direction and bottom topography on North Sea dynamics was done by Lauwerier (1960a, b). An exponential law of depth was used and the tide height was set equal to zero at the opening to the sea. He concluded that the contribution of westerly winds and northward deepening bottom topography greatly influences the sea level along the Dutch coast, with the greatest effect resulting from northwest winds. Storm surges resulting from strong westerly winds have been investigated with more advanced numerical models, and show a counterclockwise circulation over the North Sea with large sea level gradients along the Dutch coast (Fischer 1959, Fig. 3a; Mathisen and Johansen 1983, Fig. 13, 14).

The residual circulation studies of the North Sea are especially applicable to our problem, since they deal with flows on longer time scales. Residual circulations that result from westerly winds have a counterclockwise circulation and increasing sea level gradient along the Dutch coast, according to the studies of Maier-Reimer (1977, Fig. 8), Prandle (1978, Fig. 9), and Backhaus and Maier-Reimer (1983, Fig. 5-7). The residual circulation studies of Pingree and Griffiths (1980) and Davies (1982, 1983) are of particular significance because of the completeness of their models. Davies used the annual and seasonal wind stress values of Hellerman (1967) which have a strong westerly component. The horizontal advection terms in the equations of motion were neglected and the residual currents were calculated for the surface, bottom, and depth mean currents, as well as the resulting sea level contours. Pingree and Griffiths determined that the linearized equations could adequately describe the residual circulation and computed the resulting currents and sea levels for a southwest wind. The comparison of these meteorologically forced sea levels (Davies 1983, Fig. 3-6; Pingree and Griffiths, 1980, Fig. 2, 3) with the observed pole tide height along the Dutch coasts (Wunsch 1974, Fig. 2,3) reveals a remarkable similarity. There is the sharp sea level gradient along the Dutch coast, with the maximum sea level height nearly constant along the Danish coast.

We argue the case of dynamic similarity for the results of the pole tide and the annual and seasonal wind forced circulations. It has been shown that the horizontal advective terms can be neglected and bottom friction can be linearized, and still realistically represent the residual circulation in the North Sea. Since the dynamical system is then linear, the tide height amplitude is proportional to the wind stress forcing. Pingree and Griffiths used southwesterly wind stress of magnitude 1.6 dynes cm^{-2} and obtained a maximum set up along the Danish coast of 28 cm. Davies used Hellerman's westerly annual (0.53 dynes cm^{-2}) and winter (0.86 dynes cm^{-2}) wind stresses and obtained sea level set ups of 12 cm and 20 cm, respectively, at this same location. In our problem we assumed a difference in wind stress of 0.04 dynes cm^{-2} between the maximum in the 14 month cycle and the long term average, and obtained a maximum set up of 0.74 cm in the southeast corner of the model domain. The velocities in the eastern boundary current of our model are of the order 0.5 cm s^{-1}, while those in the models of Pingree and Griffiths and Davies are of the order of 5 - 10 cm s^{-1}. These results are seen to be consistent with the principle of dynamic similarity, considering the different geometries used for the North Sea models. It should be noted that observational studies (Pattullo et al., 1955) show that both the mean annual sea level and the amplitude of the seasonal oscillation show a steep gradient along the Dutch coast.

Although the model presented here shows the large sea level gradient increasing eastward along the Dutch coast, it has the sea level decreasing northward along the Danish coast. This is in contrast to other more advanced models and actual observations, where the sea level increases slightly along the Danish coast. The most probable explanation is the effect of realistic bottom topography. The streamlines in our model are lines of equal mass transport. Since the model depth increases northward the mass transport can increase without an increase in velocity in the boundary current. In the real case the North Sea depth decreases eastward (Fig. 1). Then the current that flows northward along the Danish coast is in a region of nearly constant depth (20 - 30 m), and so must increase velocity northward in order to remove water from the German Bight. The coastal sea level must then increase northward to keep the current in geostrophic balance, and would be closer to the observed height of 2.5 cm. It is this sea level set up forced by the 14 month cycle in wind stress that has been referred to as the anomolously large or enhanced pole tide in the North Sea.

Since the model used here depends on the inflow and outflow specified along the open boundary, we wish to consider how well it represents the real situation. An observational study of the residual current inflow and outflow for the North Sea was done by Riepma (1980). He concluded that for persistent westerly winds there is inflow along the western side of the opening of the North Sea, a pile up of water in the German Bight, and outflow in the deep Norwegian Trench. This was explained by the models of Furnes (1980) and Davies and Heaps (1980). Both the observational evidence and the numerous residual circulation studies indicate that at the North Sea opening there is inflow in the western part and outflow in an eastern boundary current.

We now discuss the pole tide in the Baltic Sea where the amplitude of the pole tide increases to five times the equilibrium value, almost three centimeters. The Baltic resembles more an enclosed sea rather than a bay opening to the ocean. Furthermore, it does not have a pronounced bottom slope as does the North Sea, but 60 m may be taken as an average uniform depth. We consider what is known about the meteorological effects on sea level in the Baltic where the sea level rises from the southern Baltic proper to the northern Gulf of Bothnia. This phenomenon has been studied in detail by Lisitzin (1974, p. 128-137) who determined that the most significant cause is the mean wind stress set up (15 cm) and its seasonal oscillation (10 cm), with lesser constributions from the static air pressure effect and density differences. Similar results were found by Rossiter (1967, p. 295-296) who concluded that the sea level gradients in the North and Baltic seas can be attributed to the meteorological effects.

The Baltic region also has a pronounced quasi-periodic 14 month atmospheric pressure oscillation, of the same general magnitude and direction as that of the North Sea (Bryson and Starr, 1977; Holland and Murty, 1970). Both regions are affected by a quasi-periodic 14 month oscillation of the Icelandic low. Again we invoke the principle of dynamic similarity to explain the recorded sea levels of the 14 month pole tide in the Baltic as a consequence of wind stressed set up. The westerly winds are responsible for the annual, seasonal, and 14 month oscillations in the sea level increasing northward from the Baltic to the Gulf of Bothnia. The annual average pressure gradient of 4 mb over this region results in an annual average set up of about 15 cm (Rossiter, 1967, Fig. 9). The 14 month pressure oscillation has a gradient of 0.4 mb and results in a set up of about 3 cm.

In our model we have neglected any interchange of water between the North and Baltic Seas. According to Stigebrandt (1984) there is a 14 month sea level oscillation with amplitude 1.7 cm in the Kattegat. However, numerical modeling studies of Pingree and Griffiths (1980) and Davies (1982, 1983) show that while part of the boundary current flowing northward along the Danish coast flows eastward around northern Denmark, this flow then reverses direction and flows outward in the Norwegian Trench. While some water exchange must take place between the North and Baltic Seas, it is the wind stress that is the dominant forcing mechanism in each, and to a good first approximation the sea level set up in each may be computed independently of the other.

Two conclusions can be drawn from the foregoing research. The nonequilibrium pole tide observed at many coastal stations is influenced by the 14 month oscillation in wind stressed sea level.

This accounts for the large reported values of the pole tide admittance. Secondly, the energy dissipated by these coastal currents supporting the sea level heights is predominantly that energy transferred to the ocean by the wind stress, and not the energy of the pole tide forced by the Chandler wobble.

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APPENDIX: LIST OF SYMBOLS

C_D = 2.6×10^{-3} wind drag coefficient

F_x, F_y bottom stress positive earthward, northward

f = 1.19×10^{-4} s^{-1} coriolis parameter

g = 980 cm s^{-2} acceleration of gravity

H = ocean depth

h_o = 20 m

K = 0.05 cm s^{-1} bottom friction coefficient

L = 500 Km

P = sea level atmospheric pressure

R = 0.2 nondimensional friction coefficient

s = 1.34

T = 0.04 cm^2 s^{-2}

U, u = current velocity positive eastward

V, v = current velocity positive northward

u_a, v_a = wind velocity positive eastward, northward

X, x = eastward coordinates

Y, y = northward coordinates

Z = sea level

ϵ = 1

ζ = nondimensional sea level

ρ = 1.0 g cm^{-3} water density

ρ_a = 1.2 Kg m^{-3} air density

τ_x, τ_y = wind stress positive eastward, northward

ψ = nondimensional stream function

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16. Abstract

From published research it is known that a quasi-periodic 14 month atmospheric pressure oscillation of a few tenths of a millibar exists in the region of the North and Baltic Seas. At some time in the cycle the associated wind stress has a westerly component that drives a circulation in the North Sea. The results of a dynamical model and comparisons with several North Sea residual circulation studies show that a large sea level gradient results along the Dutch coast. It is this feature that has been referred to as the enhanced pole tide. The dynamic similarity of this pole tide in the North and Baltic Seas to the annual and seasonal wind forced circulations is considered. It is inferred that the large deviations of the pole tide from equilibrium at coastal stations are the result of this sea level set up forced by the 14 month wind stress cycle.

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