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EFFECTS OF THE OCEANS ON POLAR MOTION: EXTENDED INVESTIGATIONS

Semi-annual Status Report

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SEMI-ANNUAL REPORT

This progress report for grant NAG 5-145/Supplement 4 covers the period January through August, 1986. The reporting period includes July and August 1986 as the result of a two-month no-cost extension, requested February 19, 1986 and subsequently approved. Most of the research discussed herein concerns the pole tide, the oceanic response to the Chandler wobble. The reader should consult earlier progress reports for relevant background material.

In my Supplement 3 research, I developed a self-consistent dynamic theory of the pole tide in non-global oceans (oceans with continents present). The major goal of Supplement 4 was to extend that theory to the case of turbulent oceans. By the beginning of this reporting period, a method had been found for expressing the tide current velocities in terms of the tide height--with all variables expanded in spherical harmonics; see previous reports for details. I was then able to combine all tide equations into a single, non-differential matrix equation involving only the unknown tide height. This equation could be written

$$\underline{\underline{\mathcal{B}}} \cdot \vec{T} = M_p \vec{b}_p + M_R^* \vec{b}_R$$

where $\vec{T} = \{T_\ell^n\}$ is the collection of spherical harmonic coefficients of the tide height, and M_p and M_R are the prograde and retrograde amplitude components of the wobble that creates the tide; the matrix $\underline{\underline{\mathcal{B}}}$ and vectors $\vec{b}_p$, $\vec{b}_R$ as found in the case of turbulent oceans are given in the appendix.

The pole tide must be constrained so that no tidewater flows across continental boundaries. During the reporting period this constraint, rephrased as in previous Supplements' research, was derived for the case of turbulent oceans; with the tide velocities expressed in terms of the tide height, the result is

$$\tilde{\underline{\underline{\mathcal{B}}}} \cdot \vec{T} = M_p \tilde{\vec{b}}_p + M_R^* \tilde{\vec{b}}_R$$

with $\tilde{\underline{\underline{\mathcal{B}}}}$, $\tilde{\vec{b}}_p$, $\tilde{\vec{b}}_R$ also given in the appendix.

These two matrix equations were combined in the manner devised for the Supplement 3 equations ("method (iii)" discussed below); simple matrix inversion then yielded the constrained solution. Such analytical equations for a tide in turbulent, non-global oceans have never before been derived, and represent an important development in the field. Programs to construct and invert the matrix equations were written (by this time I had become somewhat proficient at vectorized FORTRAN); preliminary results have been obtained and will be discussed near the end of this report.

Prior to this reporting period, my Supplement 3 research appeared to have demonstrated that the pole tide in non-global oceans (without turbulence) modifies the Chandler wobble severely, causing appreciable damping and also lengthening the Chandler period by about 50% more than a static ("equilibrium") pole tide would. One of the two major difficulties with these results, however, was the failure of the tide solutions to globally conserve mass (see previous reports); equivalently, the degree 0, order 0 tide height coefficient (T_o^0) failed to vanish. Although such a problem is commonly encountered in most tide theories, I felt it important that my tide solutions have a much smaller magnitude T_o^0. During the reporting period, three methods were developed to achieve that goal:

(i) Out of the nominal 36 equations (tide + boundary condition equations) whose solution would yield T_ℓ^n through degree and order 5, one of the equations was replaced with an additional constraint that $\int_{\text{oceans}} T ds = 0$ (tide mass is conserved in the oceans), or equivalently

$$\sum_{\ell,n} T_\ell^n (\theta_\ell^n)^* = 0$$

where θ_ℓ^n are the harmonic coefficients of the ocean function;

(ii) Same as (i) but with a second equation replaced by the constraint $\int_{\text{land}} T ds=0$ (total tide mass on land is zero), or equivalently,

$$\sum_{\substack{\ell,n \\ \ell \geq 1}} T_\ell^n (\theta_\ell^n)^* = 0 \;;$$

(iii) The number of boundary condition equations included in the matrix equation was increased.

The result of method (i) was to significantly reduce the magnitude of T_o^0, changing the other T_ℓ^n to a lesser degree. Since the two extra constraints combined are equivalent to requiring $T_o^0 = 0$, it is perhaps not surprising (but also not very satisfying) that method (ii) achieved a zero T_o^0.

The most reliable results were obtained with method (iii). 36 tide equations and 36 boundary condition equations, each involving the 36 unknowns (T_o^0 through T_5^5), were combined to produce an overdetermined matrix equation of the form

$$\hat{\underline{\underline{\mathcal{B}}}} \cdot \vec{T} = M_p \hat{\vec{b}}_p + M_R^* \hat{\vec{b}}_R$$

with $\hat{\underline{\underline{\mathcal{B}}}}$ a 72 x 36 matrix (72 equations in 36 unknowns). A least-squares approach led to the solution

$$\vec{T} = [(\hat{\underline{\underline{\mathcal{B}}}}^T)^* \cdot \hat{\underline{\underline{\mathcal{B}}}}]^{-1} \cdot (\hat{\underline{\underline{\mathcal{B}}}}^T)^* \cdot [M_p \hat{\vec{b}}_p + M_R^* \hat{\vec{b}}_R].$$

In addition to this method being physically sound and mathematically consistent, a much reduced T_o^0 was obtained.

Method (iii) was chosen for the Supplement 4 tide solutions as well.

The other major difficulty with the Supplement 3 results was a variety of characteristics of the results--insensitivity to imposed wobble frequency, "zonality" of tide height coefficients, and so on--that suggested something was "wrong" with the solution. This possibility was reinforced when I began, during this reporting period, to compute net tide heights associated with the coefficients; significantly non-zero tide heights were found on land. I thus devoted a full summer month to re-deriving all of the Supplement 3 theory; another month was spent checking the entire FORTRAN code. No errors were found in either the theory or the programming. By the end of the reporting period, I had begun to realize that most of the characteristics I had judged "undesirable" were in fact consistent with the tide being close to static! The resolution of these difficulties, which turned out to require only a reinterpretation of existing results plus a small, simple additional computation, will be discussed in the next semi-annual status report.

Preliminary results of the Supplement 4 research showed the effects of turbulence to be small. The fact that the solutions shared the same characteristics as the Supplement 3 results--despite the theoretical differences in approach (see discussion above or see previous report)--strengthened my conviction of the need to simply reinterpret the Supplement 3 results.

During this reporting period, my grant-related activities also included supervision of an undergraduate senior, S. Webb. Her project related to my original grant topic, the rotation of the coupled ocean--solid earth system. Her goal, determining the length of day changes associated with the coupling strengths implied by my original work, was not achieved during the one semester of her efforts (plans for her to continue during Summer 1986 were cancelled when she got the opportunity to work at the Lunar and Planetary Sciences Institute in Houston).

In May 1986 I attended the Spring American Geophysical Union meeting and presented a talk entitled "The Self-Consistent Dynamic Pole Tide in Non-global Oceans"; a copy of the abstract is attached. During summer 1986, my article, "New Aspects of the Equilibrium Pole Tide," coauthored with D. J. Steinberg, was published in GJRAS; copies have already been sent to NASA. Final revisions were made during spring 1986 to the manuscript "Another Look at North Sea Pole Tide Dynamics" (coauthored with J. R. Preisig); it was then accepted for publication.

The Self-Consistent Dynamic Pole Tide in Non-global Oceans

S.R. DICKMAN (Dept. of Geological Sciences, State University of New York, Binghamton, NY 13901)

The pole tide is the oceans' response to the Chandler wobble. The possibility of a dynamic response was previously investigated for global oceans, using Laplace tidal equations augmented with linearized bottom friction [Dickman 1985]. A spherical harmonic approach allowed the equations to be expressed as a single non-differential matrix equation, with the coefficients of the tide height as the unknowns. In order to obtain a self-consistent determination of the tide and its effects on wobble, this equation was solved simultaneously with the Liouville equation, which governs the modifications of the Chandler period and damping by the tide.

The previous theory has now been extended to the case of non-global oceans. The matrix equation is complicated significantly by the variable ocean depth, which was also expanded in spherical harmonics. The equation remains linear in the wobble amplitude forcing the tide, however, and this argues against the possibility [Carter, 1981] of frequency modulation of the Chandler period by a dynamic oceanic response.

With continents present, the tide must be constrained so that no water flows across coastlines; such constraints can also be expressed in matrix form. **Preliminary** self-consistent solutions have been obtained for oceans described by very limited harmonic expansions, for a variety of bottom friction strengths. 1) The solutions reduce correctly to those of Dickman 1985 when the oceans are treated as global. 2) For some combinations of ocean basin configuration and friction, a rotational resonance intensifying wobble appears. 3) There are problems in globally conserving mass, though application of the boundary constraints lessens the problem. 4) The oceans appear to dissipate significant wobble energy, and lengthen the Chandler period by ~50% more than a static response would.

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APPENDIX

Finally, defining

$$\vec{I}_{1P} = [\underline{\underline{\mathcal{Z}}} + \underline{\underline{\zeta}} \cdot \underline{\underline{\mathcal{Z}}}^{-1} \cdot \underline{\underline{\zeta}}]^{-1} \cdot \vec{\beta}_P + [\underline{\underline{\mathcal{Z}}} + \underline{\underline{\zeta}} \cdot \underline{\underline{\mathcal{Z}}}^{-1} \cdot \underline{\underline{\zeta}}]^{-1} \cdot \underline{\underline{\zeta}} \cdot \underline{\underline{\mathcal{Z}}}^{-1} \cdot \vec{\alpha}_P$$

$$\vec{I}_{1R} = \qquad '' \qquad \cdot \vec{\beta}_R \qquad\qquad '' \qquad\qquad \cdot \vec{\alpha}_R$$

and

$$\underline{\underline{E}} = \qquad '' \qquad \cdot \underline{\underline{\nu}} \qquad\qquad '' \qquad\qquad \cdot \underline{\underline{\eta}}$$

we can write

$$\vec{V} = \underline{M}_P \vec{I}_{1P} + \underline{M}_R^* \vec{I}_{1R} + \underline{\underline{E}} \cdot \vec{T} \, ;$$

similarly,

$$\vec{U} = \underline{M}_P \vec{I}_{2P} + \underline{M}_R^* \vec{I}_{2R} + \underline{\underline{F}} \cdot \vec{T}$$

if we define

$$\vec{I}_{2P} = \underline{\underline{\mathcal{Z}}}^{-1} \cdot \vec{\alpha}_P - \underline{\underline{\mathcal{Z}}}^{-1} \cdot \underline{\underline{\zeta}} \cdot \vec{I}_{1P}$$

$$\vec{I}_{2R} = \quad '' \quad \cdot \vec{\alpha}_R \qquad '' \quad \cdot \vec{I}_{1R}$$

$$\underline{\underline{F}} = \quad '' \quad \cdot \underline{\underline{\eta}} \qquad '' \quad \cdot \underline{\underline{E}} \, .$$

If we let

$$a_{11} = ({}_-\varepsilon_{p\xi}) D_j^{s} \; ; \quad a_{1(-1)} = -({}_+\varepsilon_{p\xi}) D_j^{-s} \; ; \quad a_{(-1)1} = ({}_-\varepsilon_{p\xi}) \hat{D}_j^{s} \; ;$$

$$a_{(-1)(-1)} = -({}_+\varepsilon_{p\xi}) \hat{D}_j^{-s} \; ; \quad a_{10} = -({}_+\varepsilon_{js}) D_j^{-s-1} + ({}_0\varepsilon_{js} + {}_0\varepsilon_{p\xi}) C_j^{s} + ({}_-\varepsilon_{js}) D_j^{s-1} \; ;$$

and $$a_{(-1)0} = -({}_+\varepsilon_{js}) \hat{D}_j^{-s-1} + ({}_0\varepsilon_{js} + {}_0\varepsilon_{p\xi}) \hat{C}_j^{s} + ({}_-\varepsilon_{js}) \hat{D}_j^{s-1} \; ;$$

note that these $a_{..}$ are functions of $p, \xi, j,$ and s ; also

$$\chi_0 = \Lambda_0(\ell, n) = ia\underline{\sigma}\left[1 - \hat{C}^n_{\ell+1} C^n_\ell - \hat{C}^n_\ell C^n_{\ell-1}\right]$$

$$\chi_{(-2)} = \Lambda_{-2}(\ell'=\ell+2, n'=n) = -ia\underline{\sigma}\hat{C}^n_{\ell+2}\hat{C}^n_{\ell+1}$$

$$\chi_2 = \Lambda_2(\ell'=\ell-2, n'=n) = -ia\underline{\sigma}\, C^n_{\ell-2} C^n_{\ell-1}$$

with $\chi_{(\cdot)}$ functions of ℓ and n as well as $\underline{\sigma}$; then we can finally (....) write continuity as

$$\boxed{\chi_0 \underline{T}^n_\ell + \chi_{(-2)} \underline{T}^n_{\ell+2} + \chi_2 \underline{T}^n_{\ell-2} = \sum_{r,k} \underline{v}^k_r \sum_{j,s} \tilde{\underline{h}}^s_j \left[-i(s+k)\right] A^{nsk}_{\ell j r} + \sum_{p,q} \underline{u}^q_p \sum_{j,s} \tilde{\underline{h}}^s_j \sum_{\substack{r'=-1\\ r'\neq 0}}^{1} \sum_{k'=-1}^{1} a_{r'k'} A^{n(s+k')(q-k')}_{\ell(j+r')p}}$$

At last! we are ready to substitute in the expressions for $\underline{u}^q_p$ and $\underline{v}^k_r$ given on page [2…] (or page [23]). After re-arranging and grouping terms, we find

$$\chi_0 \underline{T}^n_\ell + \chi_{(-2)} \underline{T}^n_{\ell+2} + \chi_2 \underline{T}^n_{\ell-2} + \sum_{\ell',n'} \sum_{p,q} \Big\{ E^{qn'}_{p\ell'} \underline{T}^{n'}_{\ell'} \sum_{j,s} \tilde{\underline{h}}^s_j \left[i(s+q)\right] A^{nsq}_{\ell j p} - F^{qn'}_{p\ell'} \underline{T}^{n'}_{\ell'} \sum_{j,s} \tilde{\underline{h}}^s_j \Big[\sum_{r'=-1,1} \sum_{k'=-1}^{1} a_{r'k'} A^{n(s+k')(q-k')}_{\ell(j+r')p}\Big]\Big\}$$

$$= \underline{M}_P (b_P)^n_\ell + \underline{M}^*_R (b_R)^n_\ell$$

where

$$(b_P)^n_\ell = \sum_{p,q} \Big\{ (\mathcal{I}_{1P})^q_p \sum_{j,s} \tilde{\underline{h}}^s_j \left[-i(s+q)\right] A^{nsq}_{\ell j p} + (\mathcal{I}_{2P})^q_p \sum_{j,s} \tilde{\underline{h}}^s_j \Big[\sum_{r'=-1,1} \sum_{k'=-1}^{1} a_{r'k'} A^{n(s+k')(q-k')}_{\ell(j+r')p}\Big]\Big\}$$

$$(b_R)^n_\ell = \qquad \mathcal{I}_{1R} \qquad\qquad \mathcal{I}_{2R}$$

Note that these latter vectors can be written

$$(b_P)_\ell^n = \sum_{j,s} \tilde{h}_j^s \sum_{p,q} \left\{ (I_{1P})_p^q (-is-iq) A_{\ell jp}^{nsq} + (I_{2P})_p^q \sum_{r'=-1,1} \sum_{k'=-1}^{1} a_{r'k'} A_{\ell(j+r')p}^{n(s+k')(q-k')} \right\}$$

$$(b_R)_\ell^n = \qquad I_{1R} \qquad\qquad I_{2R}$$

Furthermore, if we define

$$_P\tilde{a}_{rk} = (I_{2P})_p^q a_{rk} \quad , \quad _R\tilde{a}_{rk} = (I_{2R})_p^q a_{rk} \qquad \begin{pmatrix} r=-1,1 \\ k=-1,0,1 \end{pmatrix}$$

also

$$_P\tilde{a}_{00} = (I_{1P})_p^q (-is-iq) , \quad _R\tilde{a}_{00} = (I_{1R})_p^q (-is-iq)$$

and

$$_P\tilde{a}_{rk} \equiv 0 \quad , \quad _R\tilde{a}_{rk} \equiv 0 \qquad \text{FOR} \quad r=0,\ k=\pm 1$$

then we can simply write

$$(b_P)_\ell^n = \sum_{j,s} \tilde{h}_j^s \sum_{p,q} \left\{ \sum_{r=-1}^{1} \sum_{k=-1}^{1} {_P\tilde{a}_{rk}} A_{\ell(j+r)p}^{n(s+k)(q-k)} \right\}$$

$$(b_R)_\ell^n = \sum_{j,s} \tilde{h}_j^s \sum_{p,q} \left\{ \sum_{r=-1}^{1} \sum_{k=-1}^{1} {_R\tilde{a}_{rk}} A_{\ell(j+r)p}^{n(s+k)(q-k)} \right\}$$

To compactly express the left-hand side of continuity, define

$$\mathbb{B}_{\ell\ell'}^{nn'} = \delta_{nn'} \left[\chi_0(\ell,n) \delta_{\ell'\ell} + \chi_{(-2)}(\ell,n) \delta_{\ell'(\ell+2)} + \chi_2(\ell,n) \delta_{\ell'(\ell-2)} \right]$$
$$+ \sum_{j,s} \tilde{h}_j^s \left[\sum_{p,q} \left\{ E_{p\ell'}^{qn'} (is+iq) A_{\ell jp}^{nsq} - F_{p\ell'}^{qn'} \sum_{r=-1,1} \sum_{k=-1}^{1} a_{rk} A_{\ell(j+r)p}^{n(s+k)(q-k)} \right\} \right]$$

Then the continuity equation can be written

$$\sum_{\ell',n'} \mathbb{B}_{\ell\ell'}^{nn'} T_{\ell'}^{n'} = \underline{M}_P (b_P)_\ell^n + \underline{M}_R^* (b_R)_\ell^n$$

or

$$\underline{\underline{B}} \cdot \vec{T} = \underline{M}_P \vec{b}_P + \underline{M}_R^* \vec{b}_R$$

so that

$$\vec{T} = \underline{M}_P (\underline{\underline{B}}^{-1} \cdot \vec{b}_P) + \underline{M}_R^* (\underline{\underline{B}}^{-1} \cdot \vec{b}_R)$$

The result, after the usual re-grouping, is

$$\underline{\underline{\tilde{B}}} \cdot \vec{T} = \underline{M}_I \vec{\tilde{b}}_I + \underline{M}^*_R \vec{\tilde{b}}_R$$

or

$$\sum_{\ell', n'} \tilde{B}^{nn'}_{\ell\ell'} \underline{T}^{n'}_{\ell'} = \underline{M}_I (\tilde{b}_I)^n_\ell + \underline{M}^*_R (\tilde{b}_R)^n_\ell$$

where

$$(\tilde{b}_I)^n_\ell = \sum_{j,s} \underline{h}^s_j \sum_{p,q} \Big\{ \Big[(a_{10} - {}_0\varepsilon_{pq} C^s_j) A^{n\,s\,q}_{\ell(j+1)p} + (a_{(-1)0} - {}_0\varepsilon_{pq} \hat{C}^s_j) A^{n\,s\,q}_{\ell(j-1)p} \Big] (\mathcal{I}_{2I})^q_p - \Big[is A^{nsq}_{\ell jp} \Big] (\mathcal{I}_{1I})^q_p \Big\}$$

$$(\tilde{b}_R)^n_\ell = \qquad \mathcal{I}_{2R} \qquad \mathcal{I}_{1R}$$

and

$$\tilde{B}^{nn'}_{\ell\ell'} = \sum_{j,s} \underline{h}^s_j \sum_{p,q} \Big\{ is A^{nsq}_{\ell jp} E^{qn'}_{p\ell'} - \Big[(a_{10} - {}_0\varepsilon_{pq} C^s_j) A^{n\,s\,q}_{\ell(j+1)p} + (a_{(-1)0} - {}_0\varepsilon_{pq} \hat{C}^s_j) A^{n\,s\,q}_{\ell(j-1)p} \Big] F^{qn'}_{p\ell'} \Big\}.$$

--- supercomputer, here we come!!

All quantities not defined in this appendix may be found in earlier progress reports and grant proposals.

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