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REMOTE BOUNDARY CONDITIONS FOR UNSTEADY MULTIDIMENSIONAL AERODYNAMIC COMPUTATIONS

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ABSTRACT

We discuss the behavior of gas dynamic flows which are perturbations of a uniform stream in terms of information transfer across artificial (computational) boundaries remote from the source of disturbance. A set of boundary conditions are derived involving vorticity, entropy, and pressure-velocity relationships derived from bicharacteristic equations.

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1. INTRODUCTION

A recurring frustration in Computational Fluid Dynamics is the apparent difficulty of giving numerical expression to very simple statements. Typical of this situation is the specification, in aerodynamic problems, that the flow is uniform at large distances. The problem is caused by the fact that the outer limit of the computational domain never is truly at infinity, because it can always be reached by a numerical signal. Therefore, merely specifying uniform conditions on an outer boundary results in an overconstrained problem; signals which do reach the outer boundary are liable to be reflected from it and may completely corrupt the interior solution.

There has been a long search for effective absorbing boundary conditions, but none so far has found universal acceptance, and many practical codes make use of empirical procedures. In this note, we indicate the fallacy in three current practices and advocate a new procedure which may be less objectionable and can be applied to unsteady flow in any number of dimensions.

We remark that there is no problem when the flow is supersonic at infinity. It is then both simple and correct to prescribe everything at inflow and nothing at outflow. Upper and lower boundaries can be treated as rigid walls remote enough that reflected waves do not impinge on the region of interest. Our treatment, therefore, concentrates on the subsonic case where the difficulties are twofold. The boundary conditions must lead to a well-posed problem, so as to avoid the instabilities associated with overconstraint, and they should also be an accurate statement of the physics so that they can be applied at fairly small distances. In the absence of rigorous analysis (which is difficult, see [1] for a recent review), we hope that the second property will imply the first. We derive physically correct equations which express the passage of different kinds of information across the boundary. If these are differenced so that the numerical information used is taken from the proper domain of dependence, we assume that stability will be assured.

2. SOME UNSOUND PRACTICES

2.1 SPECIFY FLOW DIRECTION AT INFLOW

Consider Figure 1. The proposal is to set $v = 0$ on AB. This means that AB cannot contribute to the circulation $\oint \underset{\sim}{u} \cdot d\underset{\sim}{s}$ around ABCD. The circulation should be independent of the shape or distance of the integration contour (provided the flow is inviscid and all shocks are inside the contour). It is understandable, therefore, that with this method lifting forces are usually underestimated [2]. A cure, applicable to steady two-dimensional aerofoil flows, is to match the direction to that found in a far-field analytic solution whose circulation would produce the lift measured on the aerofoil at each iteration. No such procedure is available for three-dimensional or unsteady flows.

2.2 SPECIFY PRESSURE AT OUTFLOW

This is an essentially empirical procedure which often works quite well. However, it would not be applicable in a three-dimensional flow with shed vorticity. In steady incompressible flow, for example, we should satisfy Bernoulli's equation

$$p + \frac{1}{2}\rho_0(u^2 + v^2 + w^2) = \text{const.} \tag{2.1}$$

so that substantial values of v, w imply that p must vary.

2.3 APPLY ONE-DIMENSIONAL WAVE ANALYSIS

Assume, reasonably, that the flow near the outer boundary is a small perturbation of the free stream. Also assume, less reasonably, that the flow in a radial direction resembles a one-dimensional flow. Introduce a radial coordinate r and a velocity component u_r in that direction. These assumptions together imply that

$$(\frac{\partial}{\partial t} + a_0 \frac{\partial}{\partial r})(p + \rho_0 a_0 u_r) = 0 \tag{2.2a}$$

$$(\frac{\partial}{\partial t} - a_0 \frac{\partial}{\partial r})(p - \rho_0 a_0 u_r) = 0. \tag{2.2b}$$

Now consider steady flow, and consider two grid points as shown in Figure 1. Connecting b to i through equation (2.2a) leads to

$$(p_b - p_i) = -\rho_0 a_0 (u_{rb} - u_{ri}). \tag{2.3a}$$

Connecting b to the free stream by (2.2b) leads to

$$(p_b - p_0) = \rho_0 a_0 (u_{rb} - u_{r0}). \tag{2.3b}$$

These two equations may be solved for p_b, u_{rb}. However, they imply that

$$\frac{p_b - p_i}{p_0 - p_b} = - \frac{u_{rb} - u_{ri}}{u_{r0} - u_{rb}} . \tag{2.4}$$

We expect that in the far field both p and u_r decay monotonically to their free stream values. In that case both fractions in (2.4) should be positive, but here we force them to have opposite signs. Careful examination of the output from codes using this method reveals this non-monotone behavior at the boundary.

The observation made in this section originates with Dr. Cedric Lytton, of the Royal Aircraft Establishment, Farnborough, United Kingdom.

3. THE NEW PROPOSALS

3.1. SPECIFY ENTROPY AT INFLOW AND OUTFLOW

This is not of course a new proposal. It is perhaps the only widespread current practice that is truly unobjectionable. For it to be valid, we merely have to draw the outer boundary far enough away that no shockwaves intersect it. Since entropy is constant along particle paths in smooth flows, we should specify $s = s_0$ at points of inflow, and extrapolate s from the interior at points of outflow.

3.2 SPECIFY VORTICITY AT INFLOW AND OUTFLOW

This is commonly done where vorticity is used to formulate the problem (e.g., vorticity-streamfunction treatment of Navier-Stokes equations). The author does not know of its use for solving the Euler equations in conservation form. It allows us to specify the tangential velocity components on the boundary, equal to their free stream values at inflow or extrapolated from the interior at outflow. Probably it is sufficient to make a zeroth order extrapolation, but the convection laws of vorticity

$$\frac{\partial \underset{\sim}{\omega}}{\partial t} + (\underset{\sim}{u}\cdot\nabla)\underset{\sim}{\omega} + (\underset{\sim}{\omega}\cdot\nabla)\underset{\sim}{u} = \frac{\nabla p \times \nabla \rho}{\rho^2}$$

could be used instead.

In either case, the numerical implementation of this condition should probably be through the expression for the integrated vorticity in a cell

$$\frac{1}{V}\int \underset{\sim}{\omega}\, dv = \frac{1}{V}\int \underset{\sim}{u} \times \underset{\sim}{n}\, ds. \tag{3.1}$$

Then in a finite-volume analysis, say, there will be boundary cells with one exterior face. All other faces carry fluxes determined by the interior scheme, so that enforcing (3.1) will determine the tangential component(s) of velocity on the exterior face. Thus we have (n - 1) boundary conditions in n dimensions.

3.3 USE BICHARACTERISTIC ANALYSIS ON THE ACOUSTIC WAVES

(a) The Two-Dimensional Case

We seek solutions of the Euler equations at large distances, assuming an expansion of the form

$$\rho = \rho_0 + \varepsilon\rho_1 + \cdots$$

$$p = p_0 + \varepsilon p_1 + \cdots$$

$$u = u_0 + \varepsilon u_1 + \cdots$$

$$v = \varepsilon v_1 + \cdots$$

Insertion of these expansions into the complete equations gives

$$\frac{\partial\rho_1}{\partial t} + u_0\frac{\partial\rho_1}{\partial x} + \rho_0\left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y}\right) = 0 \qquad (3.2a)$$

$$\frac{\partial u_1}{\partial t} + u_0\frac{\partial u_1}{\partial x} + \frac{1}{\rho_0}\frac{\partial p_1}{\partial x} = 0 \qquad (3.2b)$$

$$\frac{\partial v_1}{\partial t} + u_0\frac{\partial v_1}{\partial x} + \frac{1}{\rho_0}\frac{\partial p_1}{\partial y} = 0 \qquad (3.2c)$$

$$\frac{\partial p_1}{\partial t} + u_0\frac{\partial p_1}{\partial x} + \rho_0 a_0^2\left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y}\right) = 0. \qquad (3.2d)$$

These equations are partly uncoupled, in that ρ_1 does not appear in the last three but could be found after these have been solved. From now on we exclude the first equation. An equation holding in a characteristic plane can be obtained by multiplying (3.2b) and (3.2c) by $\rho_0 a_0 \cos\theta$, $\rho_0 a_0 \sin\theta$ respectively and adding them to (3.2d). The result is (we have dropped the first-order suffixes)

$$\left[\frac{\partial p}{\partial t} + (u_0 + a_0\cos\theta)\,\frac{\partial p}{\partial x} + a_0\sin\theta\,\frac{\partial p}{\partial y}\right]$$

$$+ \rho_0 a_0 \cos\theta\left[\frac{\partial u}{\partial t} + (u_0 + a_0\sec\theta)\,\frac{\partial u}{\partial x}\right] \tag{3.3}$$

$$+ \rho_0 a_0 \sin\theta\left[\frac{\partial v}{\partial t} + u_0\,\frac{\partial v}{\partial x} + a_0\operatorname{cosec}\theta\,\frac{\partial v}{\partial y}\right] = 0.$$

It is easy to check that the differential operators inside each bracket all act in one plane, which is the property that qualifies (3.3) as a characteristic equation. A revealing rearrangement, however, is the following

$$\left[\frac{\partial}{\partial t} + (u_0 + a_0\cos\theta)\,\frac{\partial}{\partial x} + a_0\sin\theta\,\frac{\partial}{\partial y}\right]\{p + \rho_0 a_0(u\cos\theta + v\sin\theta)\}$$

$$+ \rho_0 a_0^2\left[\sin\theta\,\frac{\partial}{\partial x} - \cos\theta\,\frac{\partial}{\partial y}\right](u\sin\theta - v\cos\theta). \tag{3.4}$$

Here the first operator acts along a particular bicharacteristic (TP in Figure 2) on the sum of pressure plus $\rho_0 a_0$ times the component of velocity in the direction θ. The second operator acts only in space, perpendicularly to the direction θ, on the velocity component in its own direction (PQ). Writing u_r for the velocity in direction SP, and u_θ for the velocity in direction PQ, we have

$$(p + \rho_0 a_0 u_r)_P - (p + \rho_0 a_0 u_r)_T + \rho_0 a_0\,\frac{[(u_\theta)_Q - (u_\theta)_P]}{\tan\theta} = 0.$$

Even in a steady flow, this does not reduce to equation (2.2a). To employ such a relationship as a boundary condition, we need to choose a specific value of θ. There are various tempting choices, but we may take a lead from the work of Bayliss and Turkel [3]. They showed that under the transformation

$$\eta = y, \quad \xi = \frac{x}{\beta}, \quad \tau = \beta a_0 t + M_0 \frac{x}{\beta} \tag{3.5}$$

where $M_0 = u_0/a_0$ and $\beta = (1 - M_0^2)^{1/2}$, the system (3.2) implies that p obeys a regular wave equation

$$p_{\tau\tau} - p_{\xi\xi} - p_{\eta\eta} = 0 \tag{3.6}$$

or, equivalently,

$$p_{\tau\tau} - p_{RR} - \frac{1}{R} p_R - \frac{1}{R^2} p_{\phi\phi} = 0 \tag{3.7}$$

where

$$R^2 = \xi^2 + \eta^2 = \frac{x^2}{\beta^2} + y^2$$

and

$$\tan\phi = \frac{\eta}{\xi} = \frac{\beta y}{x} .$$

Now at large distances, the last term in (3.7) tends to be small. For example, if p is given by the well-known separable solution

$$p(\tau, R, \phi) = e^{ik\tau}\cos(n\phi)J_n(kR) \tag{3.8}$$

the orders of successive terms in (3.7) are $R^{-1/2}$, $R^{-1/2}$, $R^{-3/2}$, $R^{-5/2}$. Therefore, we truncate (3.7) to

$$p_{\tau\tau} - p_{RR} - \frac{1}{R} p_R = 0 \tag{3.9}$$

with relative error R^{-1}. We observe that the trial solution

$$p = f(\tau - R)/R^{1/2} \tag{3.10}$$

which represents a decaying outgoing solution, satisfies (3.9) up to the order of the neglected term. Also, (3.10) satisfies exactly

$$p_\tau + p_R + \frac{p}{2R} = 0. \tag{3.11}$$

Note that this equation holds along an <u>outgoing</u> bicharacteristic of (3.9). (This seems a little strange at first, but considering the analogous analysis of the one-dimensional case in Appendix A makes it seem natural). In fact, (3.11) is selecting, out of all the local bicharacteristics at a point, that one which coincides with a global bicharacteristic (see Figure 3). For our purposes, that is the most useful bicharacteristic because along it we can write an equation derived from global considerations.

Under the transformation inverse to (3.5), the bicharacteristics of (3.9) must become the bicharacteristics of the system (3.2). The distinguished bicharacteristic equation (3.11) becomes

$$\frac{\partial p}{\partial t} + \frac{\beta^2 a_0}{\beta R - M_0 x} \left[x \frac{\partial p}{\partial x} + y \frac{\partial p}{\partial y} + \frac{1}{2} (p - p_0) \right] = 0 \tag{3.12}$$

which is one form of the boundary condition recommended by Bayliss and Turkel [3] to suppress incoming radiation. Here also, we take (3.12) to be the equation which will determine pressure on the boundary. The differential operator in (3.12) coincides with the bicharacteristic operator in (3.3) if we choose

$$\sin\theta = \frac{\beta^2 y}{\beta R - M_0 x} . \tag{3.13}$$

Other useful forms of this result are

$$\sin\theta = \frac{(M_0 x + \beta R)y}{x^2 + y^2}$$

$$\cos\theta = \frac{x - \beta M_0 R}{\beta R - M_0 x} = \frac{\beta x R - M_0 y^2}{x^2 + y^2} .$$

With this choice of θ, equation (3.2) can be written

$$\begin{aligned}
&\frac{\partial p}{\partial t} + \frac{\beta^2 a_0}{\beta R - M_0 x}\left[x \frac{\partial p}{\partial x} + y \frac{\partial p}{\partial y}\right] \\
&+ \rho_0 a_0 \frac{x - \beta M_0 R}{\beta R - M_0 x}\left[\frac{\partial u}{\partial t} + \frac{\beta^3 R\, a_0}{x - \beta M_0 R}\frac{\partial u}{\partial x}\right] \\
&+ \rho_0 a_0 \frac{\beta^2 y}{\beta R - M_0 x}\left[\frac{\partial v}{\partial t} + u_0 \frac{\partial v}{\partial x} + \frac{(\beta R - M_0 x)a_0}{\beta^2 y}\frac{\partial v}{\partial y}\right] = 0.
\end{aligned} \tag{3.14}$$

Combining (3.3) and (3.14) leads to

$$\begin{aligned}
&(x - \beta M_0 R)\frac{\partial u}{\partial t} + \beta^3 R\, a_0 \frac{\partial u}{\partial x} + \beta^2 y\left[\frac{\partial v}{\partial t} + u_0 \frac{\partial v}{\partial x}\right] \\
&+ a_0(\beta R - M_0 x)\frac{\partial v}{\partial y} = \frac{p - p_0}{2\rho_0} .
\end{aligned} \tag{3.15}$$

Since (3.12) and (3.14) both hold along an outgoing bicharacteristic, it should be a stable numerical procedure to evaluate the spatial derivatives from inside the boundary in both cases; hence in (3.15) also.

Equation (3.15) can be used to update the boundary value of the linear combination

$$(x - \beta M_0 R)u + \beta^2 yv. \tag{3.16}$$

Since our proposal is to use vorticity to update the component of velocity tangential to the boundary, we need to ensure that these two conditions are independent. In other words, the boundary contour must never lie in the direction

$$\frac{dy}{dx} = \frac{\beta^2 y}{x - \beta M_0 R} = f(M_0,\ y/x). \tag{3.17}$$

These prohibited directions are sketched in Figure 5. In the limit $M_0 \to 0$ these directions are radial; as $M_0 \to 1$ they are horizontal in the left half-plane, in the right half-plane they are tangential to circles that are centered on the y-axis and pass through the origin. Clearly, there is little temptation to construct any boundary curve that follows these directions.

(b) The Three-dimensional Case

No new ideas are involved here, but the procedure is harder to visualize. However, we can simply repeat the formalism of the two-dimensional case, adjoining to (3.2) the additional equation

$$\frac{\partial w_1}{\partial t} + u_0 \frac{\partial w_1}{\partial x} + \frac{1}{\rho_0} \frac{\partial p_1}{\partial z} = 0$$

and taking the three-dimensional divergence in (3.2d).

Then a characteristic combination is

$$\begin{aligned}
&\left[\frac{\partial p}{\partial t} + (u_0 + a_0\cos\theta)\frac{\partial p}{\partial x} + a_0\sin\theta\cos\phi\,\frac{\partial p}{\partial y} + a_0\sin\theta\sin\phi\,\frac{\partial p}{\partial z}\right] \\
&+ \rho_0 a_0\cos\theta\,\left[\frac{\partial u}{\partial t} + (u_0 + a_0\sec\theta)\frac{\partial u}{\partial x}\right] \\
&+ \rho_0 a_0\sin\theta\cos\phi\,\left[\frac{\partial v}{\partial t} + u_0\frac{\partial v}{\partial x} + a_0\operatorname{cosec}\theta\sec\phi\,\frac{\partial v}{\partial y}\right] \\
&+ \rho_0 a_0\sin\theta\sin\phi\,\left[\frac{\partial w}{\partial t} + u_0\frac{\partial w}{\partial x} + a_0\operatorname{cosec}\theta\operatorname{cosec}\phi\,\frac{\partial w}{\partial z}\right].
\end{aligned} \tag{3.18}$$

It may be checked that all four operators act within one three-dimensional space, which is defined by the equation

$$\frac{x}{t}\cos\theta + \frac{y}{t}\sin\theta\cos\phi + \frac{z}{t}\sin\theta\sin\phi = u_0 + a_0\cos\theta. \tag{3.19}$$

Visualized in the space $(x/t,\ y/t,\ z/t)$ this is a plane surface (see Figure 6) which touches a sphere whose radius is a_0 and whose center is at $(u_0,\ 0,\ 0)$. The line OP is the direction along which pressure is differentiated in (3.18). It is bicharacteristic in the sense that it is the intersection of planes having parameters $(\theta \pm d\theta)$, $(\phi \pm d\phi)$.

Again, we will select specific values of θ, ϕ by appeal to the far field analysis. The transformation (3.5) (with $\zeta = z$) produces

$$p_{\tau\tau} - p_{\xi\xi} - p_{\eta\eta} - p_{\zeta\zeta} = 0. \qquad (3.20)$$

At large distances, solutions of this equation have the form

$$p = f(\tau - R)/R \qquad (3.21)$$

where

$$R^2 = \xi^2 + \eta^2 + \zeta^2 = \frac{x^2}{\beta^2} + y^2 + z^2. \qquad (3.22)$$

These solutions obey the differential equation

$$p_\tau + p_R + \frac{p}{R} = 0. \qquad (3.23)$$

As before, this can be transformed back into an equation along the bicharacteristic, allowing pressure to be updated thus

$$\frac{\partial p}{\partial t} + \frac{\beta^2 a_0}{\beta R - M_0 x} [x \frac{\partial p}{\partial x} + y \frac{\partial p}{\partial y} + z \frac{\partial p}{\partial z} + p - p_0] = 0. \qquad (3.24)$$

The differential operator here acts along the bicharacteristic in (3.18) if we choose as before

$$\cos\theta = \frac{x - \beta M_0 R}{\beta R - M_0 x} \qquad (3.25)$$

and also

$$\tan\phi = \frac{z}{y} . \qquad (3.26)$$

Using these expressions, we convert (3.18) into

$$
\begin{aligned}
&\frac{\partial p}{\partial t} + \frac{\beta^2 a_0}{\beta R - M_0 x}\left[x \frac{\partial p}{\partial x} + y \frac{\partial p}{\partial y} + z \frac{\partial p}{\partial z}\right] \\
&+ \rho_0 a_0 \frac{x - \beta M_0 R}{\beta R - M_0 x}\left[\frac{\partial u}{\partial t} + \frac{\beta^3 R\, a_0}{x - \beta M_0 R}\frac{\partial u}{\partial x}\right] \\
&+ \rho_0 a_0 \frac{\beta^2 y}{\beta R - M_0 x}\left[\frac{\partial v}{\partial t} + u_0 \frac{\partial v}{\partial x} + \frac{(\beta R - M_0 x) a_0}{\beta^2 y}\frac{\partial v}{\partial y}\right] \\
&+ \rho_0 a_0 \frac{\beta^2 z}{\beta R - M_0 x}\left[\frac{\partial w}{\partial t} + u_0 \frac{\partial w}{\partial x} + \frac{(\beta R - M_0 x) a_0}{\beta^2 z}\frac{\partial w}{\partial z}\right] = 0.
\end{aligned}
\tag{3.27}
$$

Now we combine the two bicharacteristic equations (3.24) and (3.27) to obtain

$$
\begin{aligned}
&(x - \beta M_0 R)\left[\frac{\partial u}{\partial t} + \frac{\beta^3 R\, a_0}{x - \beta M_0 R}\frac{\partial u}{\partial x}\right] \\
&+ \beta^2 y\left[\frac{\partial v}{\partial t} + u_0 \frac{\partial v}{\partial x} + \frac{(\beta R - M_0 x)}{\beta^2 y} a_0 \frac{\partial v}{\partial y}\right] \\
&+ \beta^2 z\left[\frac{\partial w}{\partial t} + u_0 \frac{\partial w}{\partial x} + \frac{(\beta R - M_0 x)}{\beta^2 z} a_0 \frac{\partial w}{\partial z}\right] \\
&\qquad\qquad = \frac{p - p_0}{\rho_0}\,.
\end{aligned}
\tag{3.28}
$$

This equation allows us to update the velocity component

$$(x - \beta M_0 R)u + \beta^2(yv + zw).$$

The condition that avoids having this component parallel to the boundary is the same as in two dimensions.

4. DISCUSSION

The intention announced in the introduction has been carried out. A set of boundary conditions, sufficient in number to determine the flow, have been obtained from physical considerations. In all cases, it is possible to represent the conditions as finite-difference formulae involving the proper domains of dependence. Nevertheless, there is a possible hazard associated with expressing the inflow boundary condition in terms of vanishing vorticity. Effectively this is a derivative condition which fails to communicate what the velocity vector at infinity actually is. Although the statement is true, it is incomplete. In fact, if the initial data for the problem is close to uniform flow, our boundary conditions take the form of specifying that no outside influence creates any changes. With a conservative scheme, total momentum within the computational domain will change only through boundary effects which have been allowed for. What might happen is a slow drift away from the desired velocity, which could probably be stabilized by prescribing a constant velocity magnitude, say, at one point.

5. ACKNOWLEDGEMENTS

I am happy to have had useful discussions on this problem with Clive Albone at the Royal Aircraft Establishment, Farnborough, and with Eli Turkel at ICASE.

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APPENDIX A

The One-dimensional Case

One-dimensional acoustic flow is governed by the pair of equations

$$p_t + \rho_0 a_0^2 u_x = 0 \tag{A.1}$$

$$u_t + \frac{1}{\rho_0} p_x = 0 \tag{A.2}$$

from which we may deduce the wave equation

$$p_{tt} - a_0^2 p_{xx} = 0 \tag{A.3}$$

with its general solution

$$p = f(x - a_0 t) + g(x + a_0 t). \tag{A.4}$$

If there are no incoming waves at large x, then $g = 0$, and p satisfies

$$p_t + a_0 p_x = 0. \tag{A.5}$$

This corresponds to equations (3.11) or (3.23) in the text. We also have the characteristic equations

$$\left(\frac{\partial}{\partial t} + a_0 \frac{\partial}{\partial x}\right)(p + \rho_0 a_0 u) \tag{A.6}$$

$$\left(\frac{\partial}{\partial t} - a_0 \frac{\partial}{\partial x}\right)(p - \rho_0 a_0 u). \tag{A.7}$$

At the outer boundary, we discard (A.7), which should carry no information inward, and retain (A.6) which propagates the inner solution outward. Combining (A.6) with (A.5) yields

$$u_t + a_0 u_x = 0 \tag{A.8}$$

so that (A.5) and (A.6) are two outgoing characteristic equations, stating that p and u remain constant on lines $dx = a_0 dt$. Thus we see that the assumption of no incoming waves enables us to write two outgoing characteristic equations, one for p and one for u. This is precisely what happens in the main body of the text.

To test these ideas in one dimension, a simple code was written to solve (A.1), (A.2) on a grid $(0,1,\ldots,N + 1)$. At points 1 through N, the solution was updated by a first-order upwind scheme based on the characteristic variables $(p \pm \rho_0 a_0 u)$. At $0, N + 1$ both p and u were updated by first order upwind schemes using data from $(0,1)$ or $(N,N + 1)$. The initial data consisted of an internal disturbance superposed on a uniform state. The disturbance passed cleanly and stably through the boundary with no reflections whatsoever.

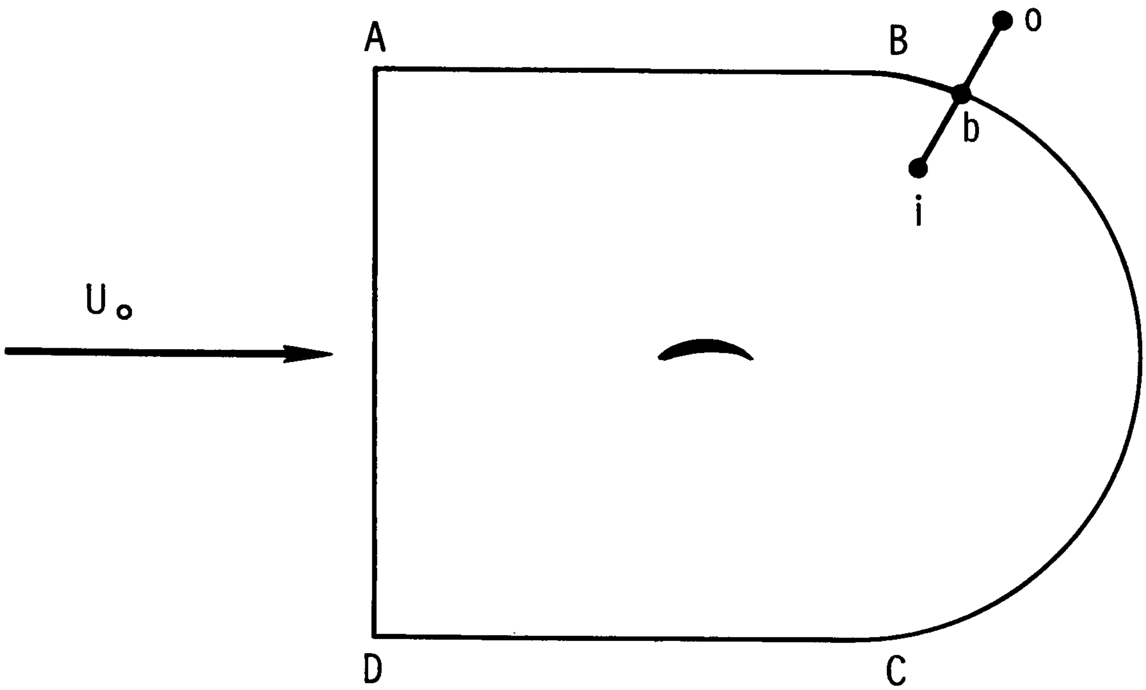

Figure 1. Aerofoil and computational boundary

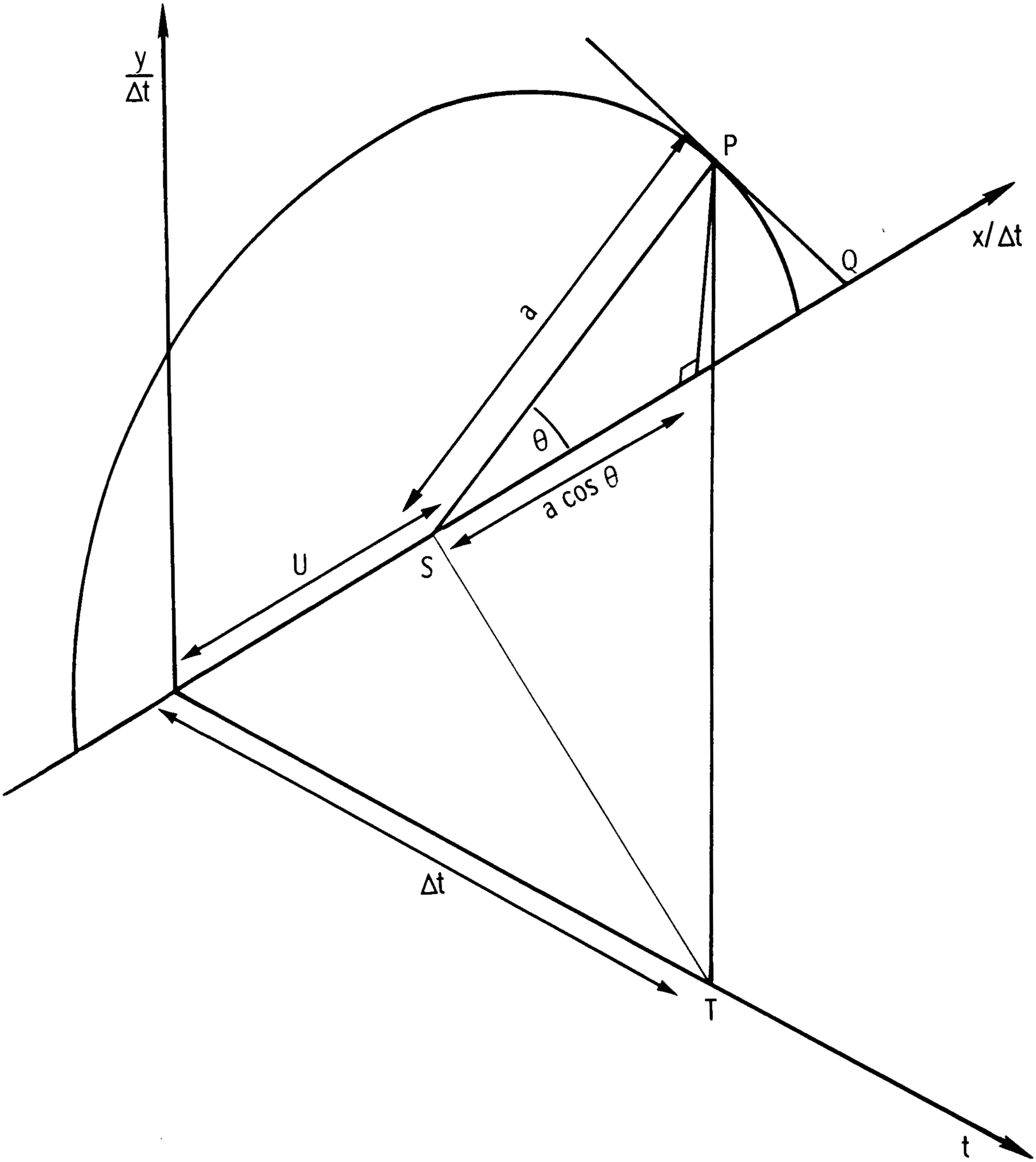

Figure 2. Geometry of two-dimensional bicharacteristics

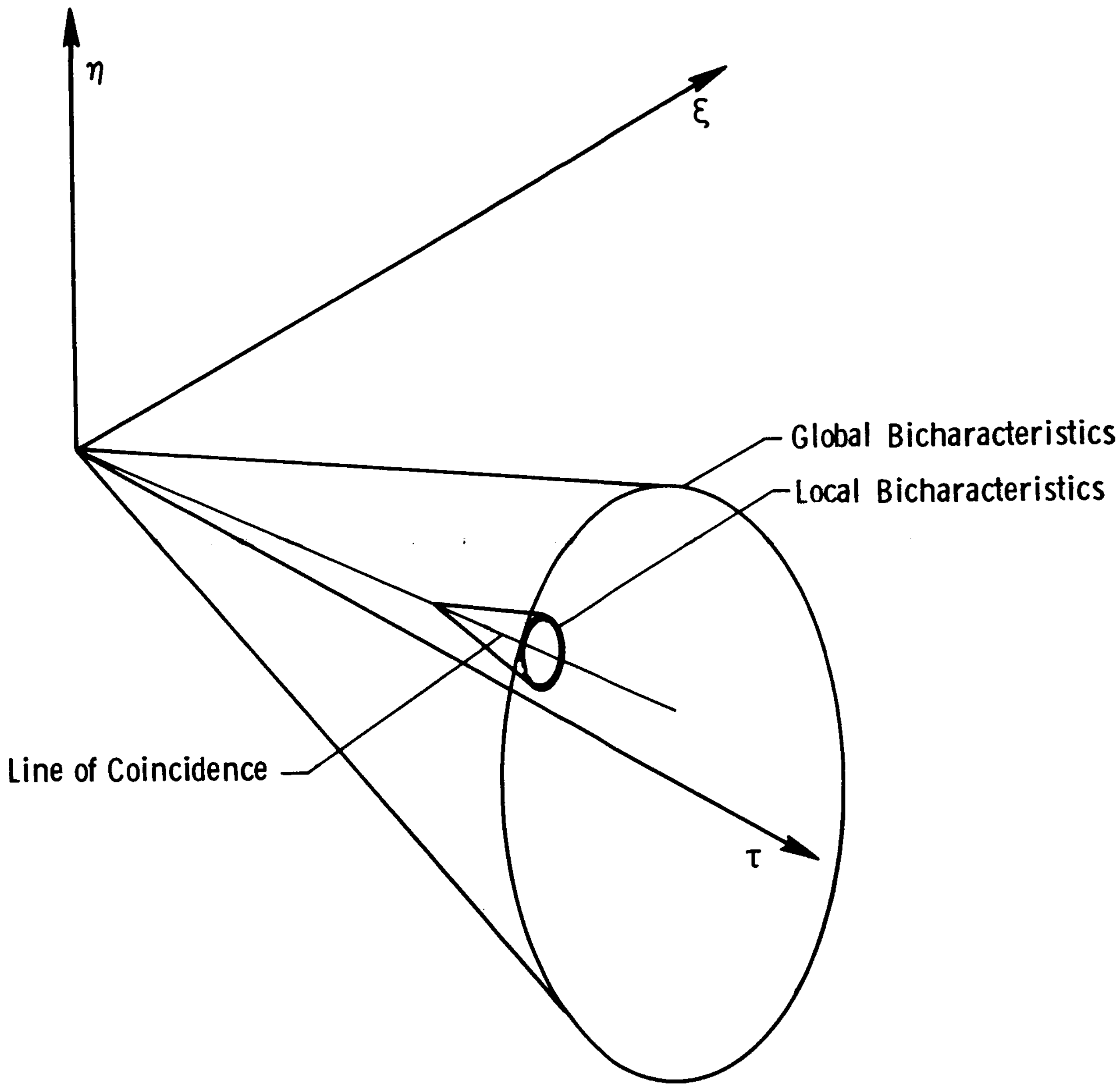

Figure 3. Global and local bicharacteristics

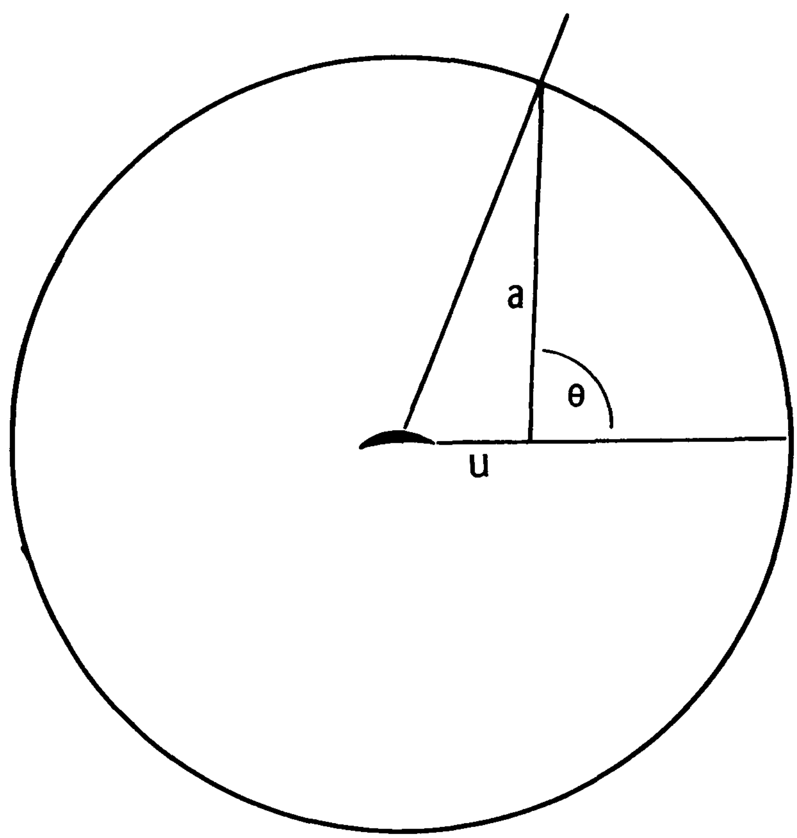

Figure 4. The choice of θ which selects the radial bicharacteristic

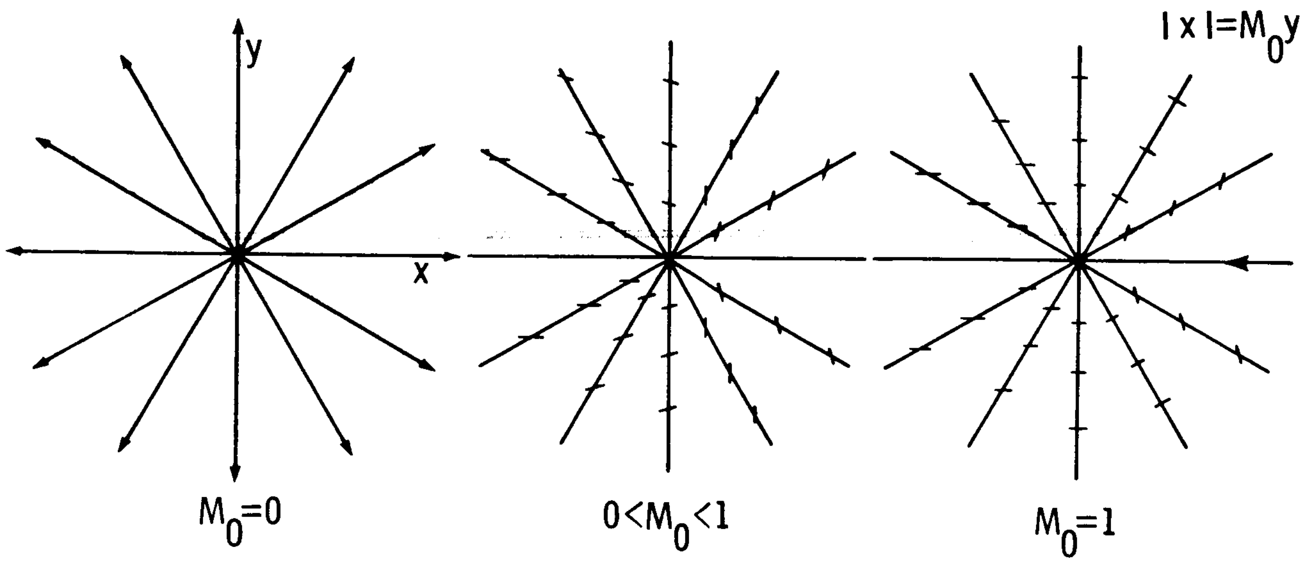

Figure 5. The forbidden boundary directions

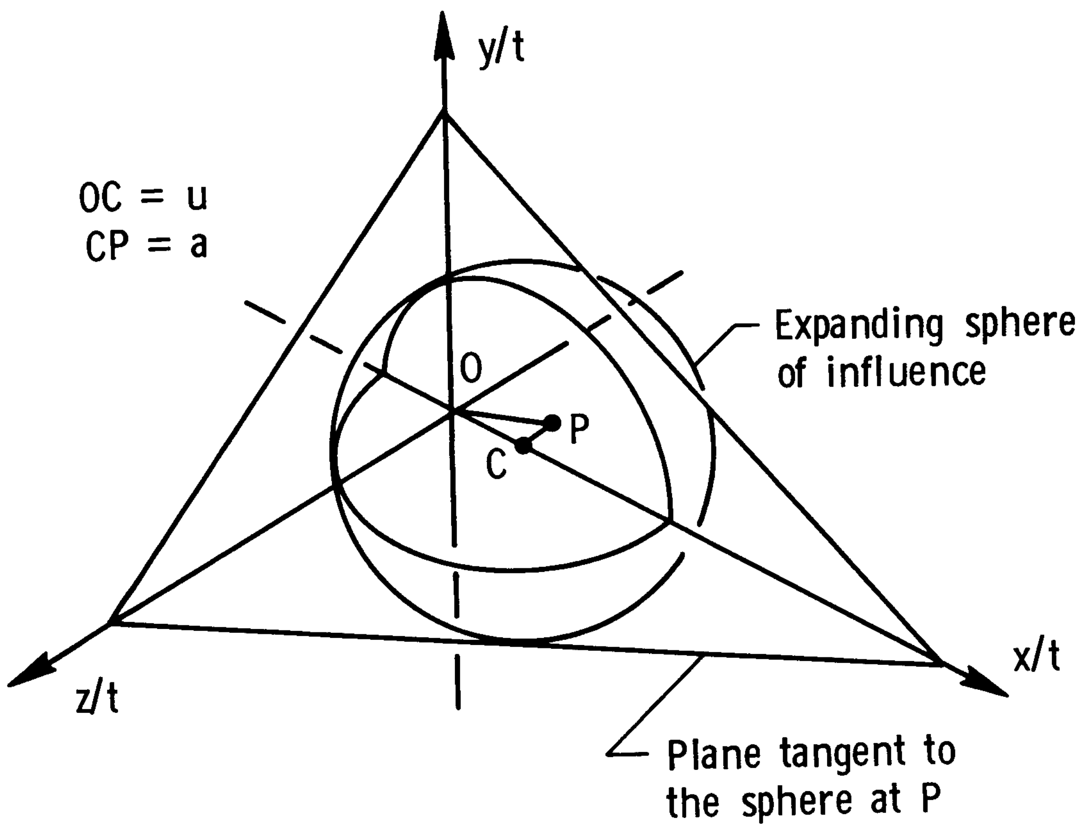

Figure 6. Geometry of three-dimensional bicharacteristics

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16. Abstract

We discuss the behavior of gas dynamic flows which are perturbations of a uniform stream in terms of information transfer across artificial (computational) boundaries remote from the source of disturbance. A set of boundary conditions are derived involving vorticity, entropy, and pressure-velocity relationships derived from bicharacteristic equations.

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