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ESTIMATION OF BIAS ERRORS IN MEASURED  
AIRPLANE RESPONSES USING MAXIMUM  
LIKELIHOOD METHOD

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VLADISLAV KLEIN  
DAN R. MORGAN

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National Aeronautics and  
Space Administration

Langley Research Center  
Hampton, Virginia 23665

## SUMMARY

A procedure for compatibility check of measured airplane responses is presented. This procedure includes estimation of bias errors in the measured data in terms of constant measurement biases and scale factors, and a comparison of reconstructed responses with those measured. The model relating airplane states and outputs is based on six-degree-of-freedom kinematic equations. In these equations the input variables are replaced by their measured values which are assumed to be without random errors.

A maximum likelihood method is used as the estimation technique. The resulting algorithm is verified with simulated data and data from flight testing. The results from simulated data show that the increased number of unknown parameters and the correlation among them can degrade the accuracy of the estimates; however, moderate measurement noise level in the input variables has only a small effect on the estimates. The maximum likelihood estimates from flight data were compared with those obtained by using an extended Kalman filter and a nonlinear fixed-interval smoother. This comparison showed no major differences in results of all three techniques.

## SYMBOLS AND ABBREVIATIONS

A	sensitivity matrix
$a_x, a_y, a_z$	longitudinal, lateral, and vertical accelerations, m/sec <sup>2</sup>
$b_y$	constant bias error in variable y
E	expected value
g	acceleration due to gravity, m/sec <sup>2</sup>
h	altitude, m
h( )	nonlinear output vector used to represent measurement system
J	cost function
M	information matrix
$M_{MOD}$	modified information matrix, see Eq. (17)
$m_{ij}$	element of inverse information matrix
N	number of data points
n	measurement - noise vector
$n_m$	number of measured output variables
$n_p$	number of unknown parameters
p, q, r	roll, pitch, and yaw velocities, rad/sec or deg/sec
R	measurement - noise covariance matrix
s( )	standard error estimate
t	time
u, v, w	longitudinal, lateral, and vertical airspeed components, m/sec
V	true airspeed, m/sec
x	state vector
$x_b, y_b, z_b$	linear position coordinates of aircraft, m

$x_\alpha, y_\alpha, z_\alpha$	position coordinates of $\alpha_v$ wind vane with respect to aircraft center of gravity, m
$x_\beta, y_\beta, z_\beta$	position coordinates of $\beta_v$ wind vane with respect to aircraft center of gravity, m
y	output vector
W	transformation matrix
z	measurement vector
$\alpha$	angle of attack, rad or deg
$\beta$	sideslip angle, rad or deg
$\delta_{ij}$	Kronecker delta
$\epsilon_1, \epsilon_2$	convergence criteria, see Eq. (16)
$\eta$	input vector
$\theta$	vector of unknown parameters
$\theta$	pitch angle, rad or deg
$\lambda_y$	scale factor error of variable y
v	residual vector
$\xi$	process - noise vector
$\sigma^2$	variance
$\phi$	roll angle, rad or deg

Subscripts:

b	body axes
E	measured quantity
N	nominal value
O	initial value
R	uncorrected for bias error
v	wind vane

**Matrix Exponents:**

T transpose matrix

-1 inverse matrix

**Mathematical Notation:**

• over symbols denotes derivative with respect to time

^ over symbol denotes estimated value

$\Delta$  incremental value

**Abbreviations:**

EKF extended Kalman filter

ML maximum likelihood

NFIS nonlinear fixed-interval smoother

## INTRODUCTION

For more than fifteen years there have been numerous attempts to estimate airplane states from measured flight data. This estimation is possible because of well known kinematic equations relating the airplane states and output variables. In many cases the estimated states have been used for data compatibility checks, that is, for comparison of measured and predicted response variables of an airplane. Because the measured data are corrupted by random and systematic errors, it was recognized that the state estimates should be combined with the estimation of unknown biases (parameters) to obtain satisfactory results. Various methods for state and parameter estimation were applied. They can be divided into two groups:

1. Methods for estimating separately unknown parameters and states (refs. 1 to 6). For parameter estimation the maximum likelihood or nonlinear least squares techniques are used. For state estimation the system is assumed to be deterministic. That is, the state variables are simply obtained by the integration of model equations.

2. Methods for estimating states and parameters simultaneously using an extended Kalman filter (refs. 7 and 10), or nonlinear smoother (ref. 11). A review of various approaches to the problem of airplane state estimation is presented in reference 12.

The purpose of this report is (1) to develop a maximum likelihood algorithm applicable to general motion of an airplane; (2) to compile an efficient computer program based on this algorithm; and (3) to verify both the algorithm and program on simulated and real flight data. The report starts with the formulation of model equations and estimation techniques. Then several examples are presented. When the real flight data are analyzed the maximum likelihood results are compared with those obtained by using an extended Kalman filter and nonlinear smoother.

## MODEL EQUATIONS

The mathematical model used for the data compatibility check is described by three sets of kinematic equations with the state variables consisting of three linear velocities  $u$ ,  $v$ , and  $w$ ; three Euler angles  $\phi$ ,  $\theta$ , and  $\psi$ ; and three linear positions  $x_b$ ,  $y_b$ , and  $z_b$ . The input variables in these equations are the linear accelerations  $a_x$ ,  $a_y$ , and  $a_z$  and angular velocities  $p$ ,  $q$ , and  $r$ . The form of the kinematic (state) equations considered can be found in various references (see, e.g. reference 9).

The following variables are measured:

1. The inputs to the system  $a_x$ ,  $a_y$ ,  $a_z$ ,  $p$ ,  $q$ , and  $r$ .
2. The airspeed  $V$ , two incidence angles  $\beta_v$  and  $\alpha_v$ , three Euler angles  $\phi, \theta$ , and  $\psi$  and altitude  $h = -z_b$ .

These variables represent the output of the system. The measured variables  $z$  are corrupted by systematic and random errors. It is assumed that each of them can be expressed as

$$z = (1 + \lambda_y) y + b_y + n_y \quad (1)$$

where  $y$  is the true value of the output,  $\lambda_y$  is the unknown scale factor error,  $b_y$  is the constant bias error, and  $n_y$  is the measurement noise. It is further assumed that the scale factor error is equal to zero for all the input variables. This assumption will simplify an estimation procedure for remaining scale factor and bias errors.

The system of state equations is simplified by deleting the equations for  $x_b$  and  $y_b$ . Then, replacing the input variables in the remaining state equations by their measured values results in the following set of state equations

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} 0 & (r_{R,E} - b_r) & -(q_{R,E} - b_q) & 0 \\ -(r_{R,E} - b_r) & 0 & p_{R,E} - b_p & 0 \\ q_{R,E} - b_q & -(p_{R,E} - b_p) & 0 & 0 \\ \sin \theta & -\cos \theta \sin \phi & -\cos \theta \cos \phi & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ h \end{bmatrix} + \begin{bmatrix} -g \sin \theta \\ g \cos \theta \sin \phi \\ g \cos \theta \cos \phi \\ 0 \end{bmatrix} + \begin{bmatrix} a_{xR,E} - b_{ax} + n_x \\ a_{yR,E} - b_{ay} + n_y \\ a_{zR,E} - b_{az} + n_z \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & w & -v & 0 \\ -w & 0 & u & 0 \\ v & -u & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} n_p \\ n_q \\ n_r \\ 0 \end{bmatrix} \quad (2a)$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \frac{\sin \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta} \end{bmatrix} \begin{bmatrix} p_{R,E} - b_p + n_p \\ q_{R,E} - b_q + n_q \\ r_{R,E} - b_r + n_r \end{bmatrix} \quad (2b)$$

The output equations take the form

$$\begin{aligned}
 V_R &= (1 + \lambda_v) \sqrt{u^2 + v^2 + w^2} + b_v \\
 \beta_{vR}^* &= (1 + \lambda_\beta) \tan^{-1} \left[ \frac{v + r_{R,E^x\beta} - p_{R,E^z\beta}}{u - r_{R,E^y\beta} + q_{R,E^z\beta}} \right] + b_\beta \\
 \beta_{vR} &= (1 + \lambda_\beta) \tan^{-1} \left[ \frac{v + r_{R,E^x\beta} + p_{R,E^z\beta}}{u - r_{R,E^y\beta} + q_{R,E^z\beta}} \right] \cos \alpha_{vR,E} + b_\beta \\
 \beta_R &= (1 + \lambda_\beta) \sin^{-1} \frac{v}{V} + b_\beta \\
 \alpha_{vR} &= (1 + \lambda_\alpha) \tan^{-1} \left[ \frac{w - q_{R,E^x\alpha} + p_{R,E^y\alpha}}{u - r_{R,E^y\alpha} + q_{R,E^z\alpha}} \right] + b_\alpha \\
 h_R &= (1 + \lambda_h) h \\
 \phi_R &= (1 + \lambda_\phi) \phi + b_\phi \\
 \theta_R &= (1 + \lambda_\theta) \theta + b_\theta \\
 \psi_R &= (1 + \lambda_\psi) \psi
 \end{aligned} \tag{3}$$

In Eq. (3)  $\beta_v$  and  $\alpha_v$  are the incidence angles measured by the vane. The sideslip angle  $\beta^*$  is measured in the  $Ox_b y_b$  plane, whereas  $\beta$  is measured between the wind vector and its projection on the  $Ox_b z_b$  plane. These two angles are therefore defined as

$$\begin{aligned}
 \beta^* &= \tan^{-1} \frac{v}{u} \\
 \beta &= \sin^{-1} \frac{v}{V}
 \end{aligned} \tag{4}$$

Sometimes the measured sideslip angle has been corrected for the c.g. offset prior to the compatibility check. In such case  $\beta_R$  can be computed from the simplified equation as indicated above. In the output equations for  $h_R$  and  $\psi_R$  the constant bias terms are omitted because the reference values for  $\psi$  and  $h$  can be selected arbitrarily. Further, in Eqs. (2) and (3),  $R$  indicates the variable uncorrected for bias errors and index  $E$  measured variable. Finally,  $x_\alpha, y_\alpha, z_\alpha$  and  $x_\beta, y_\beta, z_\beta$  are the position coordinates of  $\alpha$  and  $\beta$  vane with respect to airplane center of gravity.



## PROBLEM STATEMENT

The general form of state equations for a given system can be written as

$$\dot{x}(t) = \zeta [x(t), \eta(t), \theta_1] + g [x(t)] \xi(t), \quad x(0) = x_0 \quad (5)$$

and the discrete form of the measured equations as

$$z(i) = h [x(i), \eta(i), \theta_1] + n(i), \quad i = 1, 2, \dots, N \quad (6)$$

where  $x$ ,  $\eta$ , and  $z$  are the state, input and measurement vector respectively,  $\theta_1$  is the vector of unknown biases and scale factor errors,  $x_0$  is the vector of unknown initial conditions,  $\xi$  and  $n$  are the process and measurement noise vectors respectively, and  $N$  is the number of data points. The compatibility check can be now formulated as an identification problem which involves the estimation of state and output variables, unknown parameters  $\theta_1$  and  $x_0$ , and covariance matrices of  $\xi$  and  $n$ , from measured data.

The postulated model equations represent a nonlinear stochastic system with state-dependent process noise and with nonlinear output equations with an additive measurement noise. The state estimation in this case would be an extremely difficult problem. The separate estimation of unknown parameters would be equally complicated because of the resulting form of the sensitivity equations. For these reasons, possible simplification of the problem will be considered.

## ESTIMATION METHOD

The state and parameter estimation problem outlined above can be reduced to parameter estimation only by neglecting the process noise altogether. The state estimation is thus replaced by the integration of the state equations with the estimated values for initial conditions and bias errors. For the parameter estimation the maximum likelihood (ML) method is applied. The measurement noise in equation (6) is assumed to be zero-mean, uncorrelated, and gaussian, i.e.,

$$E\{n(i)\} = 0, \text{ and } E\{n(i)n^T(j)\} = R\delta_{i,j}$$

where the symbol  $\delta_{i,j}$  is the Kronecker delta.

The ML method finds a set of parameters by minimizing the log - likelihood function (see, e.g. reference 14).

$$J(\theta_1, \sigma_y^2) = -\frac{1}{2} \sum_{i=1}^N v^T(i) R^{-1} v(i) - \frac{N}{2} \ln |R| \quad (7)$$

where

$$\begin{aligned}\theta &= [\theta_1 \ x_0]^T \\ v(i) &= z(i) - h [x(i), \eta(i), \hat{\theta}_1]\end{aligned}\quad (8)$$

$\alpha_y^2$  is the variance of the measurement noise in output variables and  $N$  is number of data points. Minimizing (7) for parameters in  $R$  gives the estimate of measurement-noise covariance matrix as

$$\hat{R} = \frac{1}{N} \sum_{i=1}^N v(i) v^T(i) \quad (9)$$

The estimates of the remaining unknown parameters are given by the root of the equation

$$\left. \frac{\partial J(\theta)}{\partial \theta} \right|_{\theta = \hat{\theta}} = 0 \quad (10)$$

for  $R$  replaced by  $\hat{R}$ . This root can be found by modified Newton-Raphson iteration technique (ref. 14).

It is well known that under the above mentioned assumptions the final estimates of unknown parameters are consistent, asymptotically unbiased, and asymptotically efficient with

$$E\{(\hat{\theta} - \theta) (\hat{\theta} - \theta)^T\} > \sum_{i=1}^N [A_i^T \hat{R}^{-1} A_i]^{-1} \quad (11)$$

In this expression  $A^T$  is the transpose of the sensitivity matrix with the elements

$$A_{k\ell}^T = \frac{\partial v_k}{\partial \theta_\ell}, \quad \begin{cases} k = 1, 2, \dots, n_m \\ \ell = 1, 2, \dots, n_p \end{cases}$$

where  $n_m$  and  $n_p$  are the numbers of measured output variables and unknown parameters in  $\theta$  respectively.

#### COMPUTING ALGORITHM

The block diagram of computing procedure for the ML estimation of unknown parameters is presented in Figure 1. The measured data are considered in the form of digitized time histories of input and output variables with sampling interval  $\Delta t$ . The state and output equations are given by Eqs. (2) and (3) respectively with the unknown biases and scale factors

$$\theta_1^T = [b_{ax}, b_{ay}, b_{az}, b_p, b_q, b_r, b_v, b_\beta, b_\alpha, b_\phi, b_\theta, \lambda_v, \\ \lambda_\beta, \lambda_\alpha, \lambda_h, \lambda_\phi, \lambda_\theta, \lambda_\psi]$$

and unknown initial conditions

$$x_0^T = [u_0, v_0, w_0, h_0, \phi_0, \theta_0, \psi_0]$$

For the integration of state equations and computing output time histories the starting values of unknown parameters must be specified. The biases and scale factors are usually set equal to zero. The starting value of initial conditions are obtained from measured variables at  $t=0$  as

$$\begin{aligned} u_0 &= V_{R,E}(0) \cos \alpha_{R,E}(0) \cos \beta_{R,E}(0) \\ v_0 &= V_{R,E}(0) \sin \beta_{R,E}(0) \\ w_0 &= V_{R,E}(0) \sin \alpha_{R,E}(0) \cos \beta_{R,E}(0) \\ h_0 &= h_{R,E}(0) \\ \phi_0 &= \phi_{R,E}(0) \\ \theta_0 &= \theta_{R,E}(0) \\ \psi_0 &= \psi_{R,E}(0) \end{aligned} \tag{12}$$

Using the modified Newton-Raphson iterative technique the unknown parameters obtained from

$$\hat{\theta} = \theta_N + \Delta \hat{\theta} \tag{13}$$

where  $\theta_N$  are the starting values and

$$\Delta \hat{\theta} = \left[ \sum_{i=1}^N A_i^T R^{-1} A_i \right]^{-1} \left[ \sum_{i=1}^N A_i^T R^{-1} v_i \right]$$

The transpose of the sensitivity matrix  $A$  is given in Table I, where the crosses indicate sensitivities  $\partial y_k / \partial \theta_\ell = -\partial v_k / \partial \theta_\ell$  computed by a numerical method of reference 14. The remaining sensitivities in Table I are equal to a

known constant or zero, or are formed by the computed output variables. The estimates of elements in the measurement noise covariance matrix  $R$  are obtained from Eq. (9) and the residuals from Eq. (8).

For  $t=0$  the output equations for  $\phi_R$  and  $\theta_R$  give

$$\begin{aligned} b_\phi &= \phi_R(0) - (1+\lambda_\phi) \phi_0 \\ b_\theta &= \theta_R(0) - (1+\lambda_\theta) \theta_0 \end{aligned} \quad (15)$$

It means that for  $\lambda_\phi$  and  $\lambda_\theta$  a linear relationship exists between  $\phi_0$  and  $b_\phi$ , and  $\theta_0$  and  $b_\theta$ . The algorithm therefore provides two options for  $b_\phi$  and  $b_\theta$  being either estimated or computed from Eq. (15).

The iteration process is completed when certain stopping criterion is met. In this report two criteria proposed in reference 13 are adopted, i.e.,

$$\frac{\Delta\theta}{\theta} \leq \epsilon_1$$

$$\frac{\Delta J}{J} \leq \epsilon_2$$

where  $\epsilon_1$  and  $\epsilon_2$  are specified, for example  $\epsilon_1 = \epsilon_2 = .001$ . These two criteria should be met simultaneously. When the iteration is completed the correlation matrix of unknown parameters is computed from

$$\bar{M}_{MOD}^{-1} = \bar{W}^{-1} W \quad (16)$$

where

$$\begin{aligned} \bar{M}^{-1} &= \{m_{ij}\} \\ W &= \{1/\sqrt{m_{ii}}\} \end{aligned}$$

The residuals can be examined using their time histories, autocorrelation functions and power spectral densities (ref. 9).

#### EXAMPLES USING SIMULATED DATA

The method developed in this study was applied to simulated data to check the accuracy of the computing algorithm and estimated parameters. Two sets of simulated data, one representing the longitudinal and the other representing the lateral motion of an airplane, were used. The time histories of the input and output variables are shown in Figures 2 and 3. The sampling interval for these time histories was  $\Delta t = 0.05$  sec. The measurement noise with standard errors given in Table II was added to the simulated data.

The results of three cases using the longitudinal data with measurement noise only in the output variables are presented in Table III. In Case 1 the initial conditions were fixed on true values, in Case 2 on values corrupted by measurement noise. In Case 3 the initial conditions were estimated. The degradation in the accuracy of the estimated parameters from Case 1 to Cases 2 and 3 is apparent. It is probably caused by incorrect values of initial conditions (Case 2), by an increased number of unknown parameters (Case 3) and high correlation (greater than 0.9) between some parameters. The expected high correlation between parameters  $b_{\theta}$  and  $\theta_0$  (see Eq. 15) did not materialize.

A similar approach was adopted in the next three cases, where both the input and output variables were corrupted by measurement noise and the number of unknown parameters was increased. The results in Table IV do not show any substantial deterioration in the accuracy of the estimated parameters when compared with the previous results in Case 2 and 3.

In Table V two sets of results from the analysis of lateral data are given. These sets differ in the two values of simulated bias errors in the variables  $\beta$ ,  $p$ ,  $r$ , and  $\phi$ . The different values of bias errors did not change the differences between the true and mean values of estimated parameters but changed the standard errors of the parameters and the correlation between them. The accuracy was worse than that obtained from the longitudinal data. The estimation was then repeated with the initial conditions as the unknown parameters. This attempt, however, failed because of the estimation procedure divergence. The analysis of lateral data indicated a possibility of identifiability problems with this type of data. To avoid that, the design of an optimal maneuver for more accurate estimates and with less sensitivity to the number of unknown parameters should be investigated.

#### ANALYSIS OF FLIGHT DATA

Three sets of measured flight data were analyzed. Two of them represented a longitudinal motion of an airplane, the last set was obtained from a combined maneuver with predominantly lateral motion. The sampling interval for all data was  $\Delta t = 0.05$  sec. The measured output variables  $V_R$ ,  $\alpha_R$ , and  $\theta_R$  in the first run are presented in Figure 4. The resulting estimates which include the parameter mean values, their standard errors (Cramer-Rao lower bound) and standard errors of the measurement noise in the output variables are summarized in Table VI. In Case 1 the vector of unknown parameters was postulated as

$$\theta^T = [b_{ax}, b_{az}, b_{\alpha}, b_v, b_{\alpha'}, \lambda_v, \lambda_{\alpha'}, u_0, w_0, \theta_0]$$

As can be seen from the results, three pairs of estimated parameters are highly correlated, and the parameters  $b_{ax}$  and  $\theta_0$ , have large standard errors. As the next step, therefore, the parameters  $b_{ax}$ ,  $w_0$  and  $\lambda_v$  were fixed on their estimated values. The estimation of the remaining parameters was then repeated

in Case 2. The new results indicate no change in the mean values but lower standard errors of the estimates. The predicted time histories of the output variables are compared with those measured in Figure 4. The agreement is very good in all variables plotted.

The time histories in the second run had similar form as those in the first one. The estimated parameters are contained in the last two columns of Table VI. All parameters from both runs agree well with the exception of the parameter  $b_v$ . This disagreement could be caused by the elimination of  $\lambda_v$  in the vector of unknown parameters in the second run. As in Run 1 the results also exhibit high correlation between some parameters.

In order to further validate the results, the ML estimates from Run 1 were compared in Table VII with those obtained by a nonlinear-fixed-internal smoothing (NFIS) technique (ref. 12) and by an extended Kalman filter (EKF) (ref. 9). For the ML and NFIS estimation the initial values of unknown bias and scale factor errors were set equal to zero whereas for the EKF these values were made equal to the ML estimates because of the slow convergence of the filter. The estimates from all the three techniques agree well. The main differences are seen only in parameters  $b_v$  and  $\theta_0$ . The parameter  $\theta_0$  was estimated with poor accuracy in all cases. The reason for the disagreement in  $b_v$  could be due to insufficient excitation of the airspeed during the airplane motion.

Data from the third maneuver were analyzed assuming only the bias errors in variables  $a_y, a_z, p, q, r, \beta, \phi$  and  $\theta$ . After reviewing the estimates, some of the less important terms were eliminated. As a result of that, the vector of unknown parameters was postulated as

$$\theta^T = [b_{ay}, b_{az}, b_q, b_r, b_\beta, u_0, v_0, w_0, \phi_0, \theta_0]$$

with  $b_\phi$  and  $b_\theta$  computed from Eq. (15). The measured and predicted time histories of output variables are plotted in Figure 5. Then, as in the previous case, the ML estimates were compared with those using NFIS and EKF techniques. The results are given in Table VIII. The agreement between the ML and EKF estimates is good. This could be, however, due to the use of ML estimates as starting values for the EKF technique. Therefore, more thorough checks should be made for better assessment of results from both methods. The main differences between ML and NFIS techniques are in the parameters  $b_g$  and initial conditions  $v_0, \phi_0$  and  $\theta_0$ . These differences can be caused by poor accuracy of these estimates and different values of the remaining parameters.

## CONCLUDING REMARKS

A maximum likelihood method was developed for the estimation of initial conditions and bias errors in measured airplane responses. In the development it was assumed that the input variables to the system represented by linear accelerations and angular velocities are measured without random errors. The model relating airplane state, input and output variables is based on six-degree-of-freedom kinematic equations and on output equations specifying the measured variables.

The resulting technique was first applied to a limited number of simulated data runs to check the accuracy of the computing algorithm and estimated parameters. It was demonstrated that the increased number of unknown parameters and the correlation among them can degrade the accuracy of the estimates. At the same time it was observed that a moderate noise in measured inputs has only a small effect on the accuracy of the results. The lateral maneuver analyzed provided less accurate parameter estimates than the longitudinal one. The increased number of unknowns in the lateral case resulted in a divergence of the estimation procedure. The results from simulated data indicate a need for a design of an optimal maneuver for more accurate estimates and with less sensitivity to the number of unknown parameters.

The maximum likelihood method developed for this report was also applied to the analysis of real flight data. Two longitudinal maneuvers similar in form were analyzed first. The resulting parameter estimates from both maneuvers were in good agreement. Then the maximum likelihood estimates were compared with those obtained by a nonlinear-fixed-interval smoother and an extended Kalman filter. The comparison of the three techniques showed no main differences in the results. Similar conclusions were obtained from the analysis of a lateral maneuver with a strong longitudinal coupling. All the comparisons served as a verification of the maximum likelihood technique presented in the report.

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TABLE I. - TRANSPOSE OF SENSITIVITY MATRIX

	$V_R$	$\beta_{VR}$	$\alpha_{VR}$	$h_R$	$\phi_R$	$\theta_R$	$\psi_R$
$b_{ax}$	x	x	x	x	0	0	0
$b_{ay}$	x	x	x	x	0	0	0
$b_{az}$	x	x	x	x	0	0	0
$b_p$	x	x	x	x	x	x	x
$b_q$	x	x	x	x	x	x	x
$b_r$	x	x	x	x	x	x	x
$b_v$	1	0	0	0	0	0	0
$b_\beta$	0	1	0	0	0	0	0
$b_\alpha$	0	0	1	0	0	0	0
$b_\phi$	0	0	0	0	1	0	0
$b_\theta$	0	0	0	0	0	1	0
$\lambda_v$	v	0	0	0	0	0	0
$\lambda_\beta$	0	$\beta_v$	0	0	0	0	0
$\lambda_\alpha$	0	0	$\alpha_v$	0	0	0	0
$\lambda_h$	0	0	0	h	0	0	0
$\lambda_\phi$	0	0	0	0	$\phi$	0	0
$\lambda_\theta$	0	0	0	0	0	$\theta$	0
$\lambda_\psi$	0	0	0	0	0	0	$\psi$
$u_0$	x	x	x	x	0	0	0
$v_0$	x	x	x	x	0	0	0
$w_0$	x	x	x	x	0	0	0
$h_0$	0	0	0	$1+\lambda_h$	0	0	0
$\phi_0$	x	x	x	x	$1+\lambda_\phi$	x	x
$\theta_0$	x	x	x	x	x	$1+\lambda_\theta$	x
$\psi_0$	0	0	0	0	0	0	$1+\lambda_\psi$

x indicate sensitivities computed by a numerical method

TABLE II. - STANDARD ERRORS OF  
MEASUREMENT NOISE

Variable	Standard Error of Measurement Noise of Variable
$a_x, \text{ m/sec}^2$	0.1
$a_y, \text{ m/sec}^2$	0.1
$a_z, \text{ m/sec}^2$	0.1
$p, \text{ rad/sec}$	0.002
$q, \text{ rad/sec}$	0.002
$r, \text{ rad/sec}$	0.002
$V, \text{ m/sec}$	1.0
$\beta, \text{ rad}$	0.002
$\alpha, \text{ rad}$	0.002
$\phi, \text{ rad}$	0.002
$\theta, \text{ rad}$	0.002
$\psi, \text{ rad}$	0.002

TABLE III. - EFFECT OF MEASUREMENT NOISE IN OUTPUT VARIABLES ON  
PARAMETER ESTIMATES FOR SIMULATED DATA OF LONGITUDINAL MOTION

Parameter	True Value	Case 1		Case 2		Case 3	
		$\hat{\theta}$	$s(\hat{\theta})$	$\hat{\theta}$	$s(\hat{\theta})$	$\hat{\theta}$	$s(\hat{\theta})$
$b_y$ , m/sec	1.0	1.0 a	.11	2.5 a	.14	.6 a	.26
$b_\alpha$ , rad	0.002	.0020	.00022	.0000	.00025	.0023 a	.00031
$b_\theta$ , rad	0.002	.0020	.00018	.0042	.00020	.0020	.00093
$\lambda_v$	0.1	.101 a	.0011	.085 a	.0018	.104 a	.0029
$\lambda_\alpha$	0.1	.1003	.00079	.1016	.00085	.1001	.00080
$u_0$ , m/sec	58.94	58.94 b		59.698 b		58.94	.086
$w_0$ , m/sec	6.16	6.16 b		6.298 b		6.13 a	.022
$\theta_0$ , rad	0.000086	0.000086 b		.00203 b		.0001	.00093

a Parameter with high correlation  
b Fixed parameter

TABLE IV. - EFFECT OF MEASUREMENT NOISE IN INPUT AND OUTPUT VARIABLES  
ON PARAMETER ESTIMATES FOR SIMULATED DATA OF LONGITUDINAL MOTION

Parameter	True Value	Case 1		Case 2		Case 3	
		$\hat{\theta}$	$s(\hat{\theta})$	$\hat{\theta}$	$s(\hat{\theta})$	$\hat{\theta}$	$s(\hat{\theta})$
$b_{ax}, m/sec^2$	0.1	.161 a	.0078	.112 a	.0062	.147 a	.0089
$b_{az}, m/sec^2$	0.1	.092	.0013	.096	.0013	.093	.0017
$b_q, rad/sec$	0.002	.002035 <sup>a</sup>	.0000036	.002100	.0000035	.002034	.0000036
$b_v, m/sec$	1.0	.12 a	.13	2.3 a	.15	.7 a	.21
$b_\alpha, rad$	0.002	.0016	.00078	.0010	.00087	.0006	.00080
$b_\theta, rad$	0.002	.0073 a	.00078	.0033 a	.00066	.0059	.00085
$\lambda_v$	0.1	.099 a	.0017	.087 a	.0019	.104 a	.0023
$\lambda_\alpha$	0.1	.102 a	.0013	.1000	.00088	.1014	.00090
$u_0, m/sec$	58.94	58.94 b		59.698 b		58.82	.040
$w_0, m/sec$	6.16	6.16 b		6.298 b		6.24	.047
$\theta_0, rad$	.000086	.000086 b		.00203 b		.0038 a	.00085

a Parameter with high correlation  
b Fixed parameter

TABLE V. - EFFECT OF MEASUREMENT NOISE ON PARAMETER ESTIMATES FOR SIMULATED DATA OF LATERAL MOTION

Parameter	Case 1			Case 2		
	True Value	$\hat{\theta}$	$s(\hat{\theta})$	True Value	$\hat{\theta}$	$s(\hat{\theta})$
$b_{ay}$ , m/sec <sup>2</sup>	0.1	.120	.0048	0.1	.12	.023
$b_p$ , rad/sec	0.002	.002007	.0000033	0.02	.020007	.0000035
$b_r$ , rad/sec	0.002	.00183	.000019	0.02	.019830 <sup>a</sup>	.0000019
$b_\beta$ , rad	0.002	-.008	.0071	0.02	.010	.0067
$b_\phi$ , rad	0.002	-.0016	.00063	0.02	.0164	.00027
$\lambda_\beta$	0.1	.102	.0013	0.1	.102	.0013

<sup>a</sup> Parameter with high correlation

TABLE VI. - ESTIMATES OF PARAMETERS FROM LONGITUDINAL FLIGHT DATA OF TWO DIFFERENT RUNS

Parameter	RUN 1				RUN 2	
	Case 1		Case 2		$\hat{\theta}$	$s(\hat{\theta})$
	$\hat{\theta}$	$s(\hat{\theta})$	$\hat{\theta}$	$s(\hat{\theta})$		
$b_{ax}$ , g units	.004 a	.0065	.004 c			
$b_{az}$ , g units	-.1240	.00040	-.1240	.00021	-.1257	.00022
$b_q$ , deg/sec	1.792	.0019	1.792	.0018	1.827 a	.00060
$b_v$ , m/sec	-1.2 a	.30	-1.24 a	.098	-.5 a	.16
$b_{\alpha}$ , deg	2.6 a	.19	2.631	.0036	1.5 a	.21
$b_{\theta}$ , deg	-12.7 b		-12.72 b		-12.87 b	
$\lambda_v$	.018 a	.0077	.018 c			
$\lambda_{\alpha}$	-.073	.0027	-.073	.0026	-.081	.0034
$u_0$ , m/sec	35.5	.10	35.5 a	.10	37.2 a	.16
$w_0$ , m/sec	6.0 a	.13	6.0 c		6.5 a	.12
$\theta_0$ , deg	-2. a	.37	-.19	.025	7.60 a	.025
$s(v)$ , m/sec	.260		.260		.338	
$s(\alpha)$ , deg	.280		.280		.331	
$s(\theta)$ , deg	.748		.748		1.19	

- a Parameters with high correlation
- b Computed from  $\theta_0$  and  $\theta_{R,E}^{(0)}$
- c Fixed parameter

TABLE VII. - ESTIMATES OF PARAMETERS FROM LONGITUDINAL FLIGHT DATA USING THREE DIFFERENT METHODS

Parameter	ML		NFIS		EKF	
	$\hat{\theta}$	$s(\hat{\theta})$	$\hat{\theta}$	$s(\hat{\theta})$	$\hat{\theta}$	$s(\hat{\theta})$
$b_{az}$ , g units	-.1240	.00021	-.122	.0035	-.123	.0035
$b_q$ , deg/sec	1.792	<sup>a</sup> .0018	1.8	.14	1.74	.042
$b_v$ , m/sec	1.24	<sup>a</sup> .098	.02	.40	-.28	.075
$b_\alpha$ , deg	2.631	.0036	2.34	.34	2.6	.10
$b_\theta$ , deg	-12.72	<sup>b</sup>	-13.03	.10	-12.64	.033
$\lambda_\alpha$	-.073	.0026	-.050	.015	-.09	.012
$u_0$ , m/sec	35.5	<sup>a</sup> .10	37.8	.40	35.5	.82
$w_0$ , m/sec	6.0	<sup>c</sup>	6.1	.27	6.1	.39
$\theta_0$ , deg	-.19	<sup>a</sup> .025	.7	.30	.4	.72
$s(v)$ , m/sec	.260		.177			
$s(\alpha)$ , deg	.280		.158			
$s(\theta)$ , deg	.748		1.08			

- <sup>a</sup> Parameters with high correlation
- <sup>b</sup> Computed from  $\theta_0$  and  $\theta_{R,E}(0)$
- <sup>c</sup> Fixed parameter



TABLE VIII. - ESTIMATES OF PARAMETERS FROM FLIGHT DATA USING THREE DIFFERENT METHODS

Parameter	ML		NFIS		EKF	
	$\hat{\theta}$	$s(\hat{\theta})$	$\hat{\theta}$	$s(\hat{\theta})$	$\hat{\theta}$	$s(\hat{\theta})$
$b_{ay}$ , g units	-.003 <sup>a</sup>	.0045	-.018	.0063	-.003	.0035
$b_{az}$ , g units	-.1801	.00041	-.172	.0024	-.181	.0035
$b_q$ , deg/sec	3.152	.0065	3.07	.050	3.14	.042
$b_r$ , deg/sec	1.414	.0038	-1.9	.13	-1.43	.056
$b_\beta$ , deg	.7	.10	1.2	.18	.83	.094
$b_\phi$ , deg	-3.5 <sup>b</sup>		-3.9	.43	-3.94	.031
$u_0$ , m/sec	35.20	.037	36.91	.038	35.3	.16
$v_0$ , m/sec	-.47	.084	.1	.17	-.4	.25
$w_0$ , m/sec	7.63	.029	7.14	.092	7.4	.39
$\phi_0$ , deg	.2 <sup>a</sup>	.24	-1.8	.58	.2	1.6
$\theta_0$ , deg	.70	.044	.2	.17	.7	.60
$s(V)$ , m/sec	.300		.511			
$s(\beta)$ , deg	.435		.206			
$s(\alpha)$ , deg	.854		.182			
$s(\phi)$ , deg	2.078		1.053			
$s(\theta)$ , deg	1.105		.961			
$s(\psi)$ , deg	.728					

<sup>a</sup> Parameter with high correlation

<sup>b</sup> Computed from  $\phi_0$ ,  $\theta_0$  and  $\phi_{R,E}(0)$ ,  $\theta_{R,E}(0)$

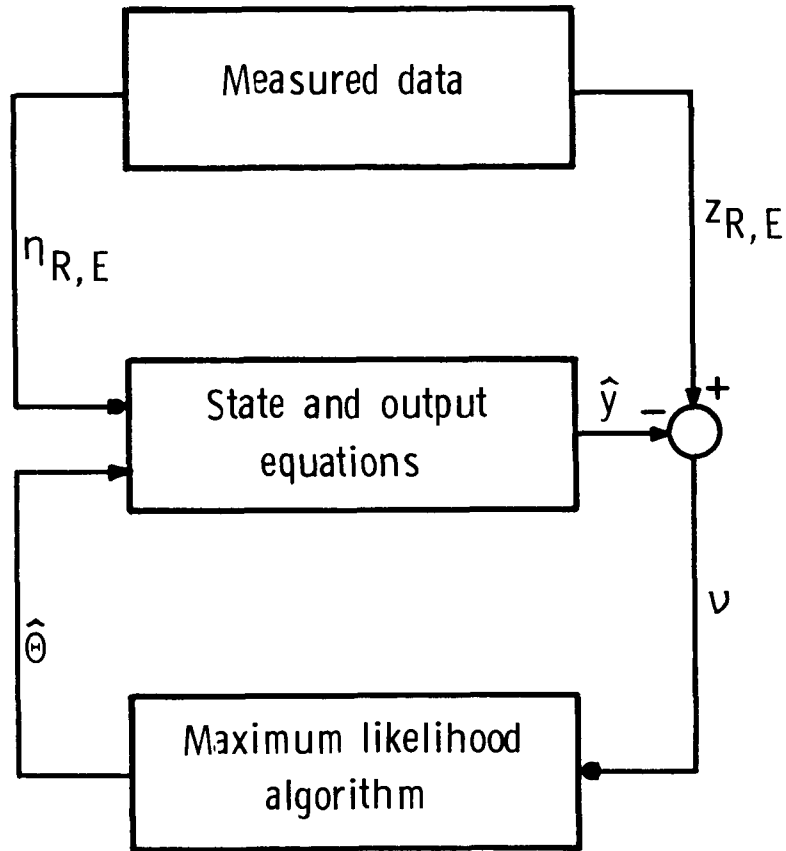


Figure 1. - Block diagram of computing procedure for ML estimation of unknown parameters.

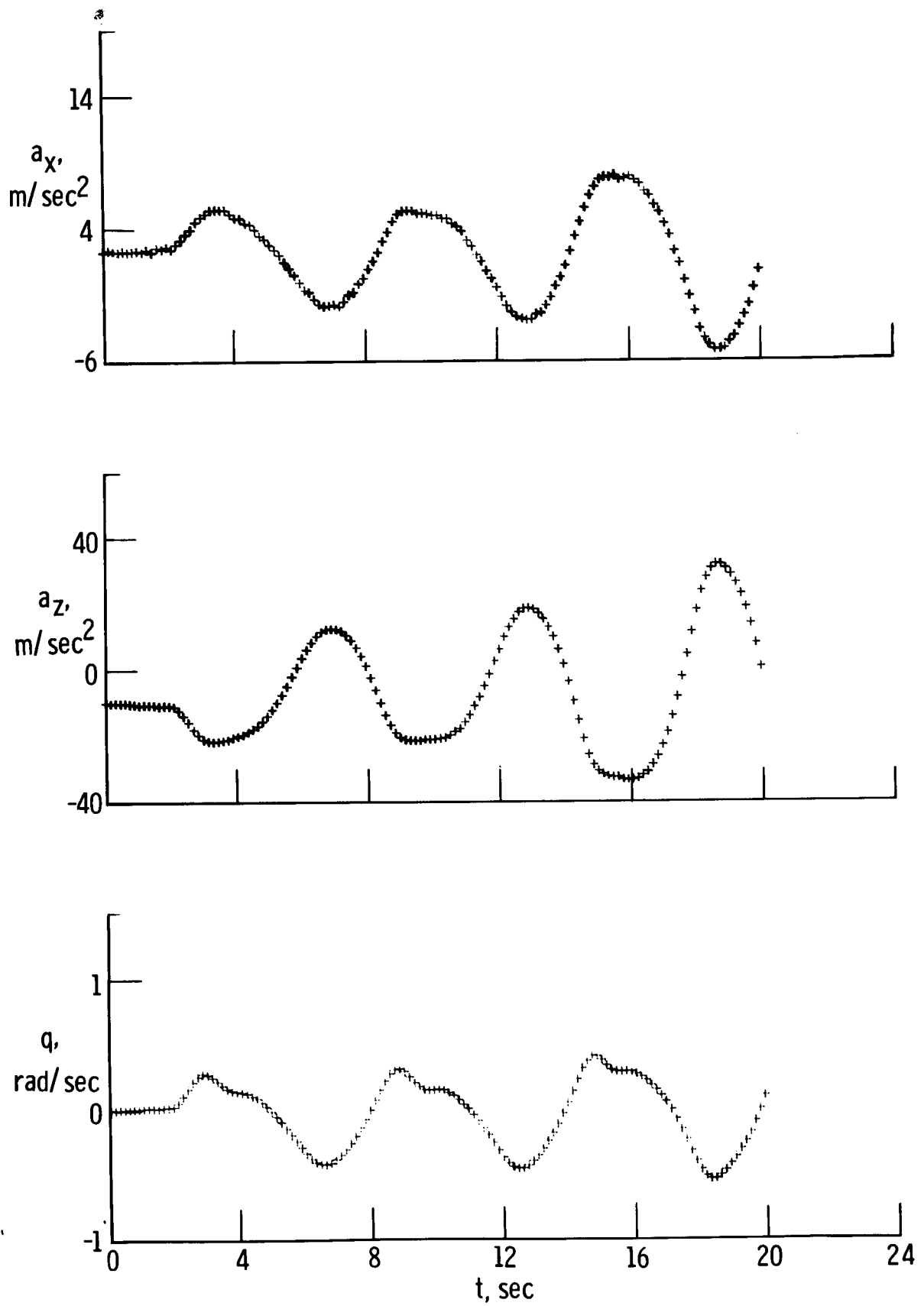


Figure 2. - Simulated longitudinal motion of an airplane.

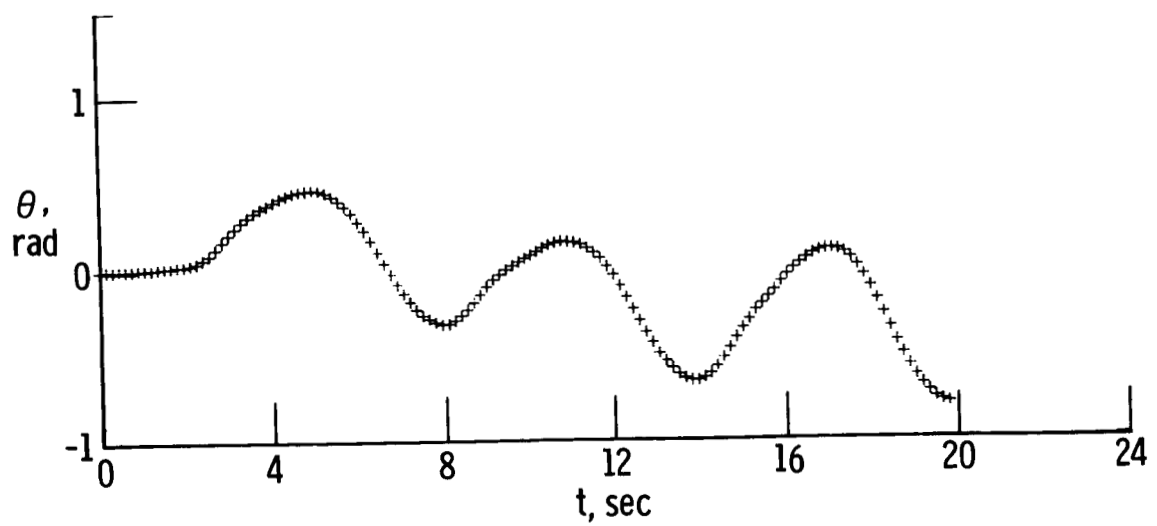
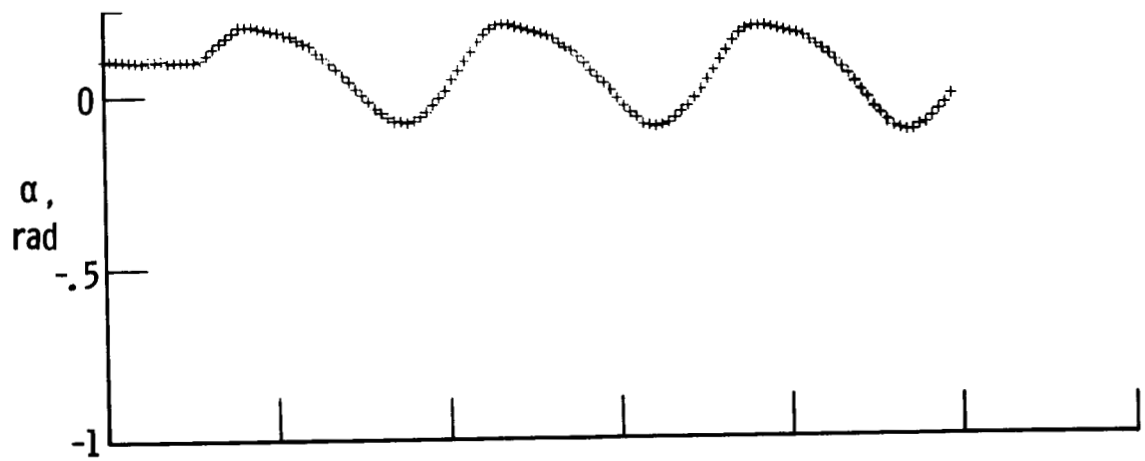
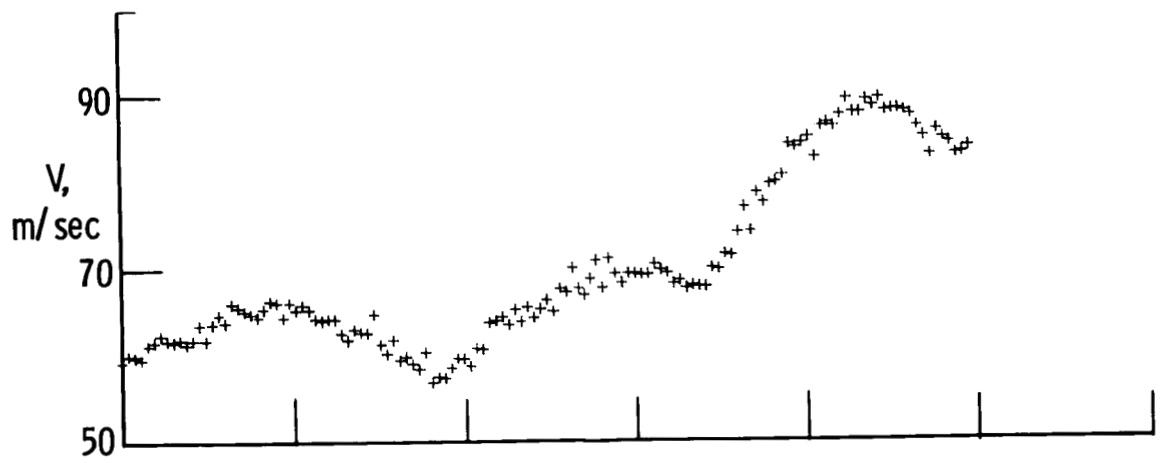


Figure 2. - Concluded

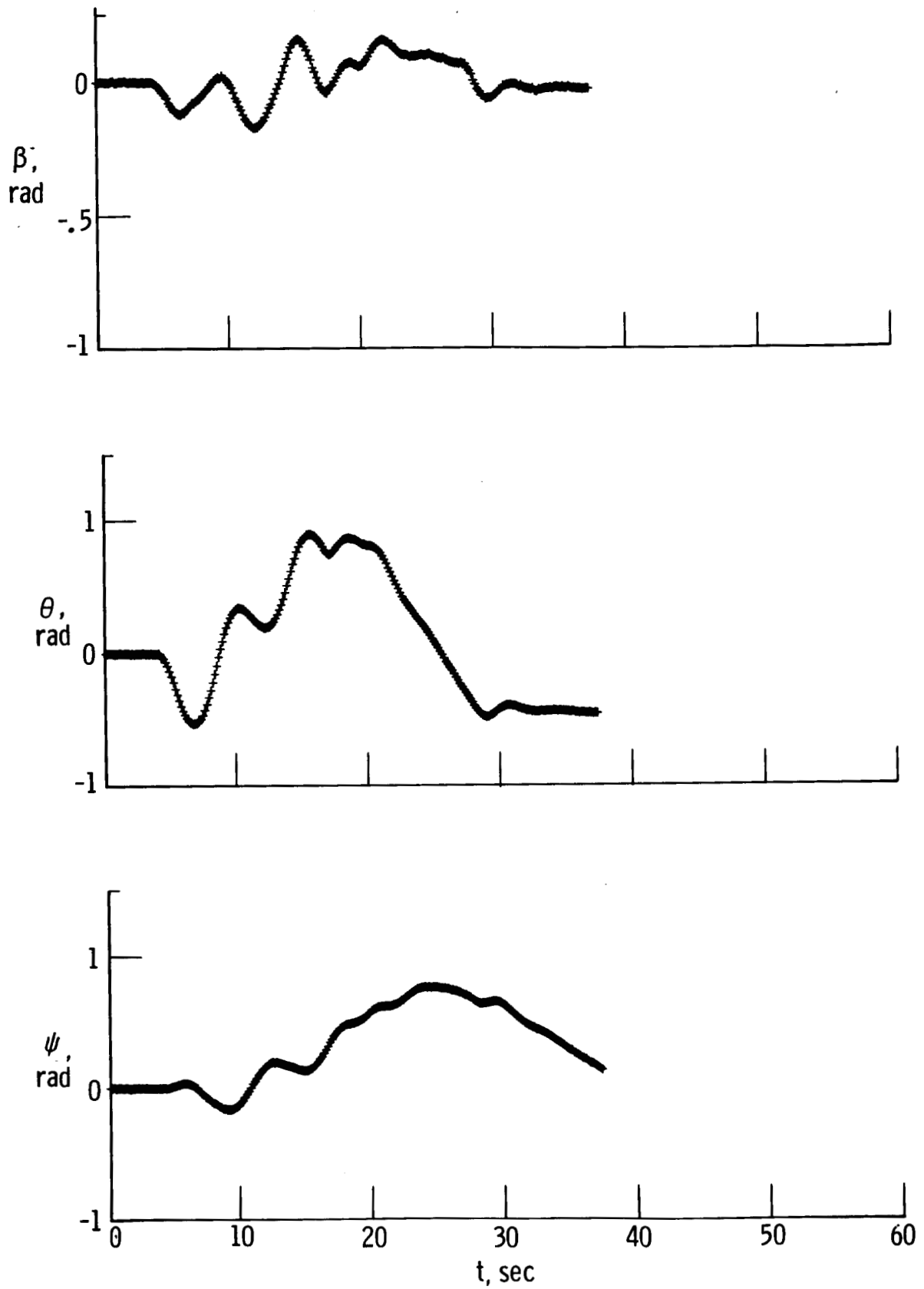


Figure 3. - Simulated lateral motion of an airplane

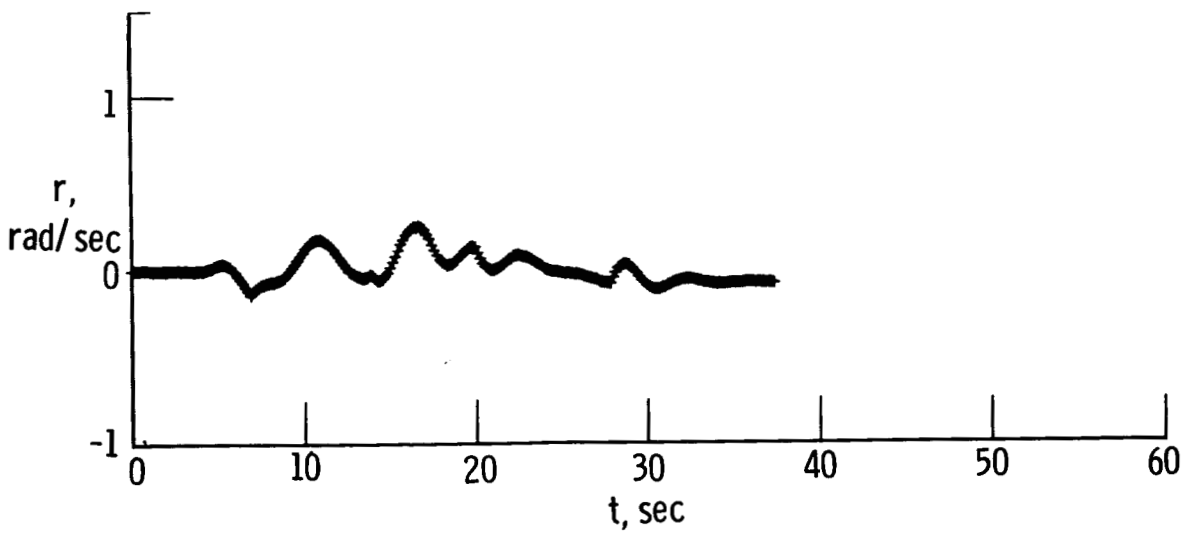
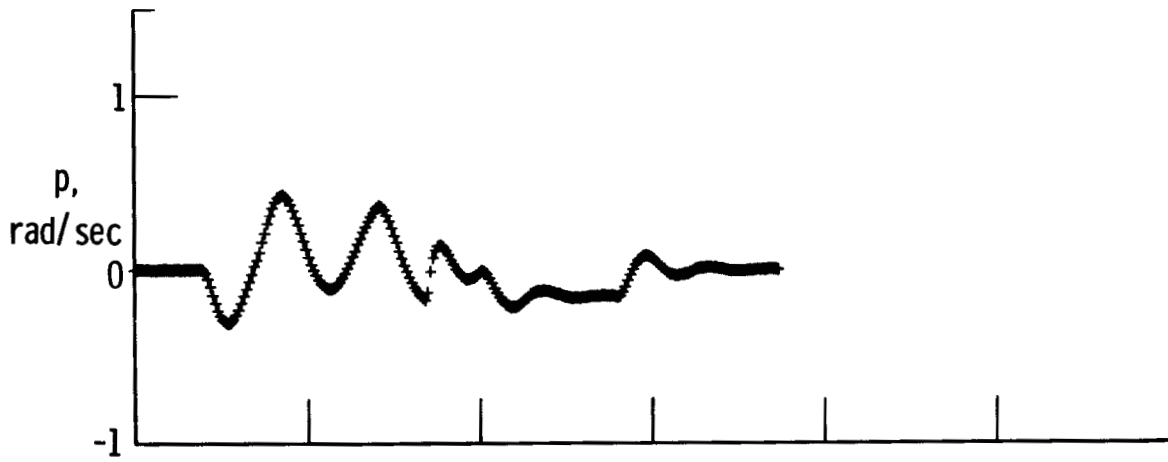
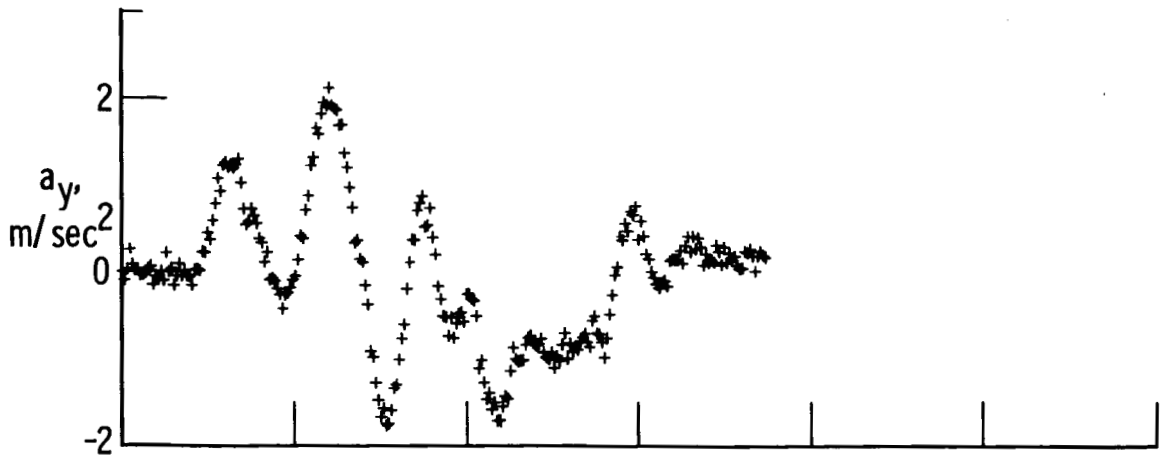


Figure 3. - Concluded

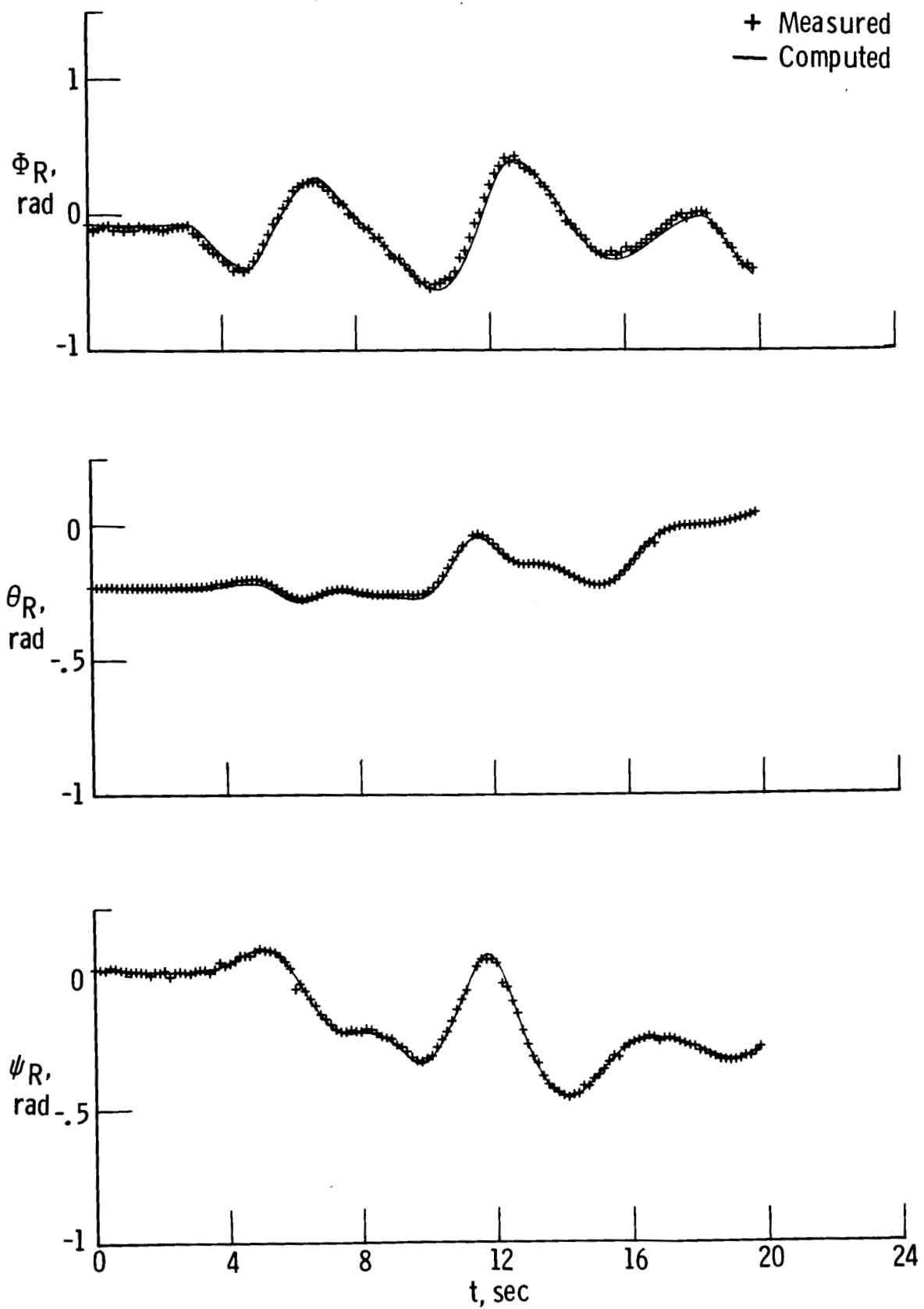


Figure 4. - Comparison of measured time histories in longitudinal maneuver with those computed.

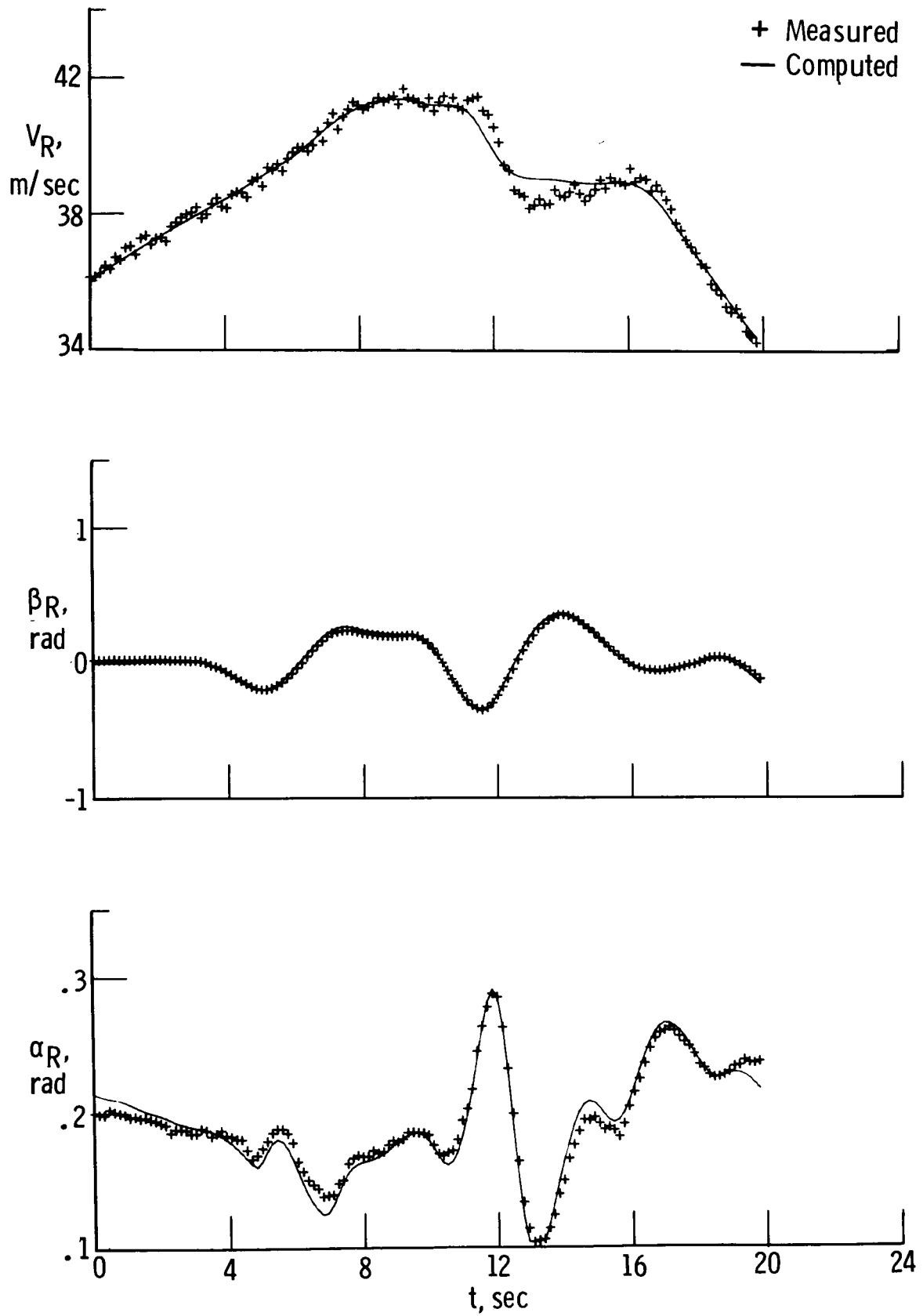


Figure 5. - Comparison of measured time histories in combined maneuver with those computed.



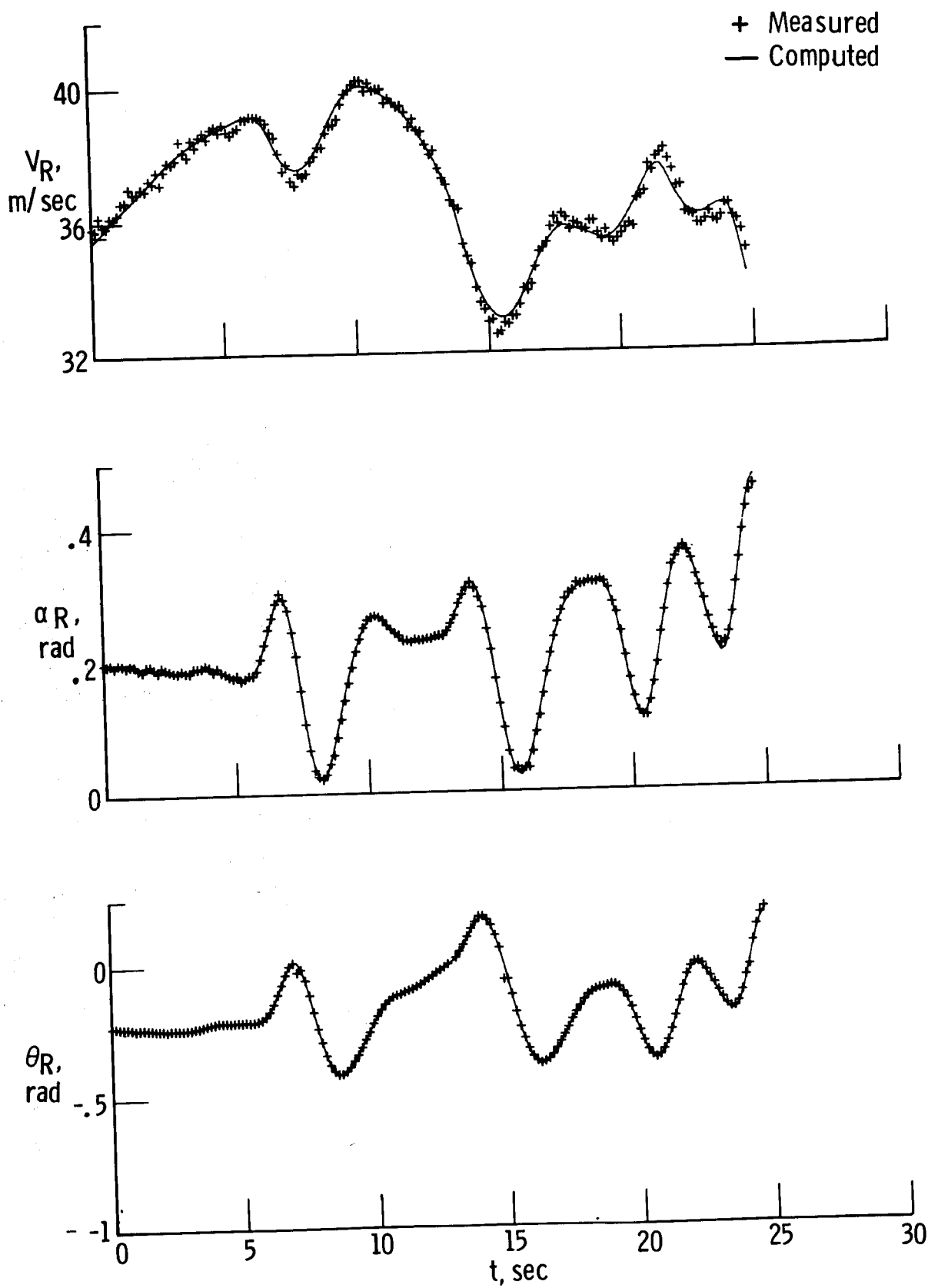


Figure 5. - Concluded

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