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Application of Physical Parameter Identification to Finite Element Models

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Abstract

A time domain technique for matching response predictions of a structural dynamic model to test measurements is developed. Significance is attached to prior estimates of physical model parameters and to experimental data. The Bayesian estimation procedure allows confidence levels in predicted physical and modal parameters to be obtained. Structural optimization procedures are employed to minimize an error functional with physical model parameters describing the finite element model as design variables.The number of complete FEM analyses are reduced using approximation concepts, including the recently developed convoluted Taylor series approach. The error function is represented in closed form by converting free decay test data to a time series model using Prony's method. The technique is demonstrated on simulated response of a simple truss structure.

1 Introduction

This work is motivated by the need to have at hand accurate structural dynamic models for the design of robust vibration suppression control systems. Knowledge of modal properties is of course necessary for control system design. Possession of an accurate physical model of the structure can also be beneficial. Such an instance is when one anticipates making modifications to the structure to incorporate control actuators in primary load paths for the purpose of reducing dynamic response. The physical model will allow one to predict the new modal properties of the perturbed structure. A modal model alone or even a mass and stiffness representation would not allow this.

A second motivation is the possibility of performing "on–orbit" structural system identification where the only test equipment available is the control system actuators and sensors. The number of sensors will necessarily be limited, nowhere near the scores or hundreds of accelerometers which are commonly used to saturate modal survey test articles. A prior structural model can provide valuable information which may then be supplemented by data acquired from the control system to provide response predictions more accurate than available from either analysis or test alone.

A beneficial side effect of the Bayesian parameter estimation approach is the availability of uncertainty estimates on both physical parameters and modal quantities such as frequencies and mode shapes. Variances on derived response quantities such as gain factors may also be obtained. These statistical estimates provide mean values and ranges which may be used to design the control system and test it for stability and robustness across the range of possible physical and modal parameter values.

1.1 Background

Previous work at TRW on system identification for the design of vibration suppression control systems has focused on the Maximum Likelihood approach. In the Maximum Likelihood approach no weight is given to prior parameter estimates nor is uncertainty assigned to the measurement data. Model parameters are adjusted so as to best fit the experimental data. This estimation procedure has been applied in the frequency domain to a flat plate experiment [1] and in the time domain to the Large Space Structure Truss Experiment (LSSTE) [2]. As applied in the time domain, the test article was excited by the control actuators using a fast sine sweep or "chirp" across the frequency range of interest. Motion was then allowed to decay. Physical parameter estimation was implemented by minimizing the error

residual between observed and predicted response at the control sensors. This error residual was defined as the integral over the duration of the test, both forced and unforced, of the square of the difference between measured and predicted response. As reported, the perturbation of physical parameters allowed the accurate matching of response in amplitude, frequency and phase over the duration of the experiment. The structural model was a set of discrete equations of motion and was not represented directly in the estimation procedure as a finite element model. The parameters which could be identified were as a result limited to inertia properties of the top plate, the modulus of elasticity and modal damping ratios.

Recent advances in the state of the practice of structural optimization techniques make it possible to consider the extension of physical parameter estimation to finite element models of at least moderate size. In particular, linking MSC/NASTRAN Design Sensitivity Analysis [3] with the Automated Design Synthesis (ADS) code [4] has enabled structural optimization to proceed on a production basis. Approximation concepts [5, 6] have allowed the reduction of the number of full structural analyses required for a model optimization to single digit levels. At TRW, structural models having up to 10,000 static and 400 dynamic degrees of freedom have been weight-optimized with minimum frequency constraints using up to 200 design variables. The availability of eigenvector derivatives [7] has enabled the prediction and minimization of vibratory response using the approximation concepts approach [8]. Physical parameter identification can be implemented using these proven methods by employing uncertain parameters as design variables and minimizing an appropriate error functional.

It was found in Reference 2 that a significant portion of the computation time was spent integrating the equations of motion for each trial design and for each perturbed design required for finite difference gradient calculations. A closed form representation for the error functional would increase the computational efficiency of the estimation process while enhancing numerical stability. Recognizing that linear models predict free decay response as a sum of exponentially damped sinusoids and that one can extract these same components from free decay test data employing Prony's method, such a closed form error functional has now been derived. The prior filtering of the test data using Prony's method has the advantage that frequency, mode shape and damping data are available to some extent and may be compared to prior model predictions even before the estimation procedure formally begins.

1.2 System Identification Methods

The system identification field is vast and the options one encounters in choosing an approach are numerous. Perhaps the greatest advantage of the approach adopted

herein is its ability to go directly from test data to an improved analytical model. This approach is by no means advocated as superior in all respects to others. It is one more tentative step in the direction of physical parameter identification and, in fact, some valuable lessons have been learned in its implementation. Some discussion of where this approach falls in the larger framework of system identification is in order.

The common starting point for system identification and model verification for most aerospace structures is the modal survey. This procedure is used to determine the system's normal modes, frequencies and modal damping ratios. This effort is undertaken independent of any prior knowledge of the model other than in the selection of instrumentation type and location and choice of modal identification procedure. Numerous modal identification techniques are available in either the frequency or the time domain [9]. Having a set of test modes, these are frequently corrected to enforce orthogonality to an analytic mass matrix using a technique such as Gram-Schmidt orthogonalization, or the Targoff method [10]. In this approach the analytic mass matrix is assumed perfect and the corrected modes are taken as linear combinations of test modes.

A procedure which might be labelled the Baruch–Berman method [11, 12] may also be employed to generate an improved analytic stiffness and mass matrix which exactly reproduces the measured mode shapes and frequencies while producing minimal changes to the analytic model in a weighted least squares sense. This procedure has been applied numerous times with considerable success. The predicted changes to the analytic mass and stiffness matrices have been used to guide the improvement of the physical finite element model in a heuristic manner. One criticism of the Baruch–Berman method is the generation of stiffness matrices with coupling between degrees of freedom which appear to be physically unconnected. Unrealistic mass changes have also been observed. Although the method generates a mass and stiffness model which generates the test modes, additional modes within the test bandwidth can also arise from the model.

A promising answer to the infeasible coupling problem has been provided by Kabe [13]. Assuming the test modes have been orthogonalized to the analytic mass matrix, an improved stiffness matrix is formed which preserves the topology of and minimizes the perturbation to the original analytic matrix, while exactly matching the test frequencies and modes supplied to the algorithm. The technique has not seen application in practice due to very large computational requirements.

Having in one's possession an improved stiffness and mass representation of a structure is not in all cases sufficient to allow the analysis process to proceed. In many cases the test configuration is not the field configuration. The structure in

question may be altered or coupled to other components, which would make a test verified physical finite element model valuable. The Bayesian technique [14, 15] is a general framework which allows one to match the analytic predictions of a model to test data while minimizing the changes to the original model. Uncertainties in the form of standard deviations may be assigned to one's original parameter estimates and to the measurement data. The resulting set of revised parameters will be provided with a statistical estimate of confidence given the additional knowledge provided by the test. The changes in estimated parameters vis-a-vis the original estimates can be used to infer the adequacy of the test and or the functional form of the analytic representation [16]. This method has been applied successfully to estimate a small bi-linear model using transient test data [17]. It is the intent of this work to extend the Bayesian technique in an efficient manner to finite element model estimation.

2 Estimation Procedure

An overview of the estimation procedure is given in the flow chart of Figure 1. The process consists of: 1) Test; 2) Prony analysis; 3) Finite Element Analysis; 4) Construction of an approximate problem; 5) Optimization; and 6) Bayesian statistical analysis. Steps 3–5 are repeated iteratively until the approximate problem converges to closely resemble the actual problem.

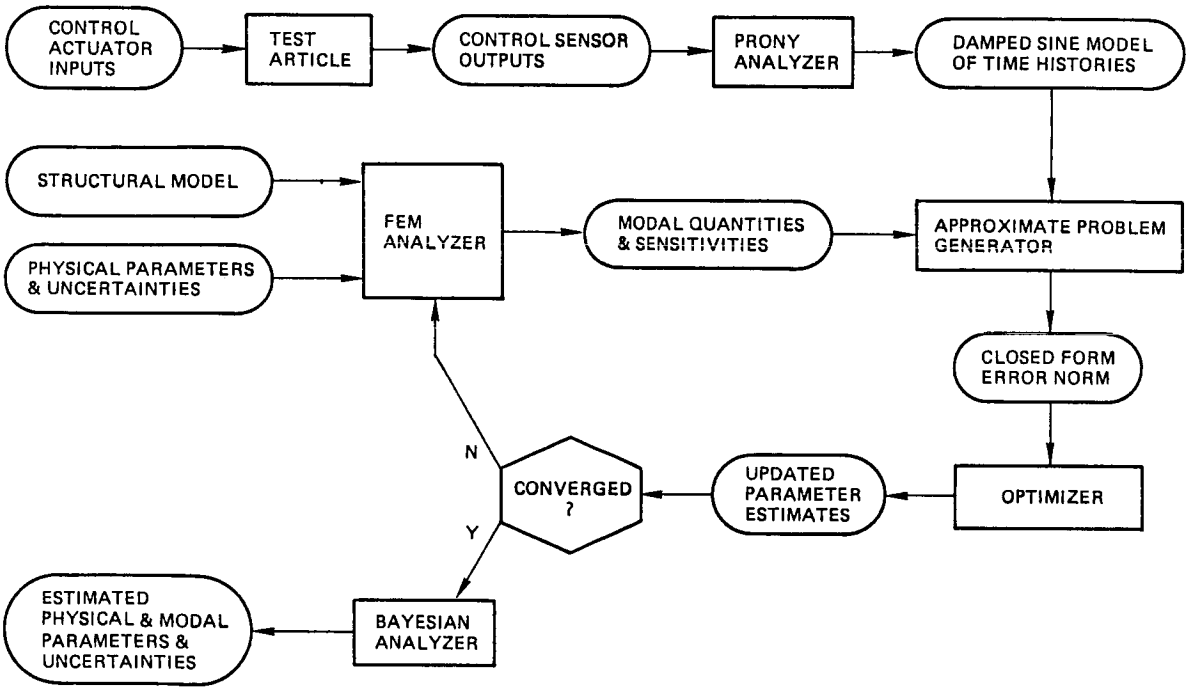

Figure 1: Parameter Estimation Using Structural Optimization Techniques

The test portion of the structural parameter estimation procedure is implemented by subjecting the test article to a series of force load events $\{F_k(t)\}$, $k = 1, \ldots, N_{lc}$ using combinations of control actuators. Free decay time histories $u^e_{jk}(t)$, $j = 1, \ldots, N_s$; $k = 1, \ldots, N_{lc}$ are measured at each of the N_s control sensors in each of the N_{lc} load conditions. This experimental data serves as the basis for the estimation procedure. Assuming that the structure behaves in a linear fashion, each of the recorded time histories can be represented as the sum of a finite number of

significant damped sinusoidal components as follows:

$$u^e_{jk}(t) \simeq \bar{u}_{jk}(t) \equiv \sum_{m=1}^{M_{jk}} \bar{A}_{mjk} \cos(\bar{\omega}_{mjk_d} t + \bar{\theta}_{mjk}) e^{-\bar{\zeta}_{mjk}\bar{\omega}_{mjk} t} \tag{1}$$

Zero time is assumed to occur at the end of the excitation process and at the beginning of the free decay. The subscript d denotes damped frequency as defined in Equation 2. The barred quantities are meant to represent the result of filtering experimental data. In this work Prony's method of extracting damped sinusoidal components was employed [18]. The particular implementation of this algorithm [19] has been found to operate quite well in the presence of considerable noise. The choice of the number of components in the time series model is done interactively until the user feels a good fit has been obtained. Pre-test analysis using the prior analytic model can serve as a guide to what frequency components may be expected at each sensor due to each load condition. Non-linearities and noise will in some sense be removed from the data in this process. One could of course employ the actual time history data in the optimization procedure. This would remove the necessity of making an assumption on the form of the time series model. It would also allow the data recorded during the excitation process to be fit to the analytic structural model. However, the computational burden is greatly increased by having to filter the data literally thousands of times during the optimization process. The insight gained through the estimated Prony model components will also be lost.

The linear analytic model is formed as a function of the variable parameter set $\mathbf{D}$. This model will also predict response at the sensors in the form of damped sinusoids. The response will be at the N damped natural frequencies of the system which are given in terms of the undamped natural frequencies ω_n and the modal damping ratios ζ_n as follows:

$$\omega_{n_d} \equiv \omega_n \sqrt{1 - \zeta_n^2} \tag{2}$$

The predicted response at each of the sensors j in each of the load conditions k as a function of parameters $\mathbf{D}$ is thus

$$u_{jk}(t; \mathbf{D}) = \sum_{n=1}^{N} A_{njk} \cos(\omega_{njk_d} t + \theta_{njk}) e^{-\zeta_{njk}\omega_{njk} t} \tag{3}$$

We note that the amplitude, frequency, phase and damping ratio are all functions of the design variables and that the amplitude and phase are also functions of the excitations applied prior to time zero.

The Bayesian estimation procedure requires an error residual set $\mathbf{E}$ which is defined here as the root mean square difference between predicted and observed response over the duration of the experiment T, i.e.

$$E_{jk} \equiv \sqrt{\frac{1}{T}\int_0^T [\bar{u}_{jk}(t) \; - \; u_{jk}(t;\mathbf{D})]^2 \; dt} \tag{4}$$

Given the representation of both experimental and analytic response as sums of damped sinusoids, this error functional can be written in closed form. The algebra and the test process will be considerably simplified if the excitations applied to the structure are simple static loads applied at the actuators and released at time zero. Assuming the structural model is described adequately by a set of N real normal modes, $\{\phi_n\}$, the amplitude components in the damped sinusoidal representation will be

$$A_{njk} = \phi_{jn}\frac{1}{\omega_n^2}\{\phi_n\}^T\{F_k\} \tag{5}$$

and the phase angles θ_{njk} will be identically zero. Using pre-test predictions the input into expected modes of interest could be maximized by choosing the force vectors in proportion to modal response at the actuators. The number of independent combinations of force vectors is limited to the number of actuators, and additional modes may also be excited in each twang test. More load cases than actuators can nevertheless be employed as this will merely over-determine the test. One might hope that the phase components identified from the test, $\bar{\theta}_{mjk}$, will also be identically zero. This will invariably not be the case. These phase components may be discarded if their magnitude is small. Large measured phase components will indicate a problem in the test procedure, errors in the Prony analysis, or perhaps non-linearities in the test article.

2.1 Bayesian Estimation

The Bayesian estimation procedure may most easily be understood as the minimization of a performance index represented as the weighted sum square of error residuals between observed and predicted response plus a weighted sum square of error residuals between prior and adjusted parameter values, as defined in Equation 6. One is thus seeking to minimize a combination of measurement error and parameter error. If one weights measurements more heavily by assigning greater confidence (i.e. a smaller variance) to them , the design will then tend to match the measurements more strongly. If one weights one's initial estimates of the parameters more heavily by assigning them a smaller uncertainty then the adjusted parameters will tend to move less from the prior estimates.

The observation weighting function may be a full matrix if separate measurements are statistically correlated. The usual assumption is that the measurements are uncorrelated, resulting in a diagonal observation weight matrix. The parameter weighting matrix may also be full if the prior parameter estimates are correlated. This situation will present itself when data sets are considered sequentially, with the adjusted parameter set arising from one batch of data used as the prior model for the next batch. The prior model for the subsequent estimations will then usually exhibit a full covariance matrix. Allowing for the full statistical correlation of both measurements and parameters the problem is defined: Find that $\mathbf{D}^\star$ which minimizes the performance index

$$\Upsilon(\mathbf{D}) \equiv \mathbf{E}(\mathbf{D})^T \mathbf{W}_e \mathbf{E}(\mathbf{D}) \; + \; (\mathbf{D} - \mathbf{D}^o)^T \mathbf{W}_{D^o} (\mathbf{D} - \mathbf{D}^o) \tag{6}$$

The subscript e denotes experimental data and the superscript o denotes original or prior parameter estimate. The weighting matrices $\mathbf{W}$ are the inverse of the covariance matrices $\mathbf{S}$. Thus the larger the uncertainty in a given measurement or a given parameter estimate, the smaller will be its assigned weight in the estimation procedure. The test covariance $\mathbf{S}_e$ and the prior parameter covariance $\mathbf{S}_{D^o}$ are usually assumed to be diagonal matrices with diagonal elements equal to the square of the standard deviation of the measurement or parameter in question. Thus the weight assigned will be $W_{ii} = 1/\sigma_i^2$ where σ is the standard deviation. The covariance of the final best fit parameter estimate will be

$$\mathbf{S}_{D^\star} \; = \; \left[\mathbf{W}_{D^o} + \mathbf{T}^T \mathbf{W}_e \mathbf{T}\right]^{-1} \; = \; \left[\mathbf{S}_{D^o}^{-1} + \mathbf{T}^T \mathbf{S}_e^{-1} \mathbf{T}\right]^{-1} \tag{7}$$

In this case, $\mathbf{T}$ is a sensitivity matrix representing the rate of change of the error residuals with respect to the variable parameter set, i.e.

$$\mathbf{T} \equiv \left.\frac{\partial \mathbf{E}}{\partial \mathbf{D}}\right|_{\mathbf{D}^\star}$$

The sensitivities must be evaluated for the optimum parameter set. The standard deviation of an improved estimated parameter can be derived from the diagonal elements of the covariance matrix as follows: $\sigma_{D_i^\star} = \sqrt{S_{D_{ii}^\star}}$.

A useful feature of the Bayesian approach is its ability to provide variance, or uncertainty, estimates of modal parameters such as frequency or mode shape [20]. The uncertainty bounds placed on the initial parameter estimates and the measurement data were propagated through the estimation procedure to provide uncertainties on the estimated parameter set as given in Equation 7. The optimum estimated parameter set $\mathbf{D}^\star$ can in turn be used to provide estimates of expected

response quantities and their uncertainties. These expected response quantities will be obtained by evaluating the system's describing equations using the estimated parameter set. For example, expected natural frequencies, $\omega_n^\star$, and mode shapes, $\{\phi_n\}^\star$, may be derived by performing an eigensolution on the matrix equations of motion using the optimum parameter set. Variance estimates may be obtained using the sensitivities of the response quantities in question with respect to parameters evaluated at the optimum. Using natural frequency as an example, its variance would be:

$$S_{\omega_n} = \left\{ \frac{\partial \omega_n}{\partial \mathbf{D}} \right\}^T [S_{D^\star}] \left\{ \frac{\partial \omega_n}{\partial \mathbf{D}} \right\} \tag{8}$$

The ability to predict a range of natural frequencies due to uncertainties in model parameters could prove beneficial, especially in the design of control systems for on-orbit structures which cannot be fully tested on the ground.

2.2 Approximation Concepts

The ability to match the response of the structural model to the measured test data hinges on one's ability to efficiently compute the response for a large number of varied parameter combinations. Simply performing a complete eigensolution at each design iteration becomes excessively costly for all but the smallest systems. The approximation concepts approach [5] which has evolved in the structural optimization field solves the problem by defining an approximate problem based on solution of one structural eigenproblem, which is then submitted to the optimization process (see Figure 1). Having an optimum design (parameter set) for the approximate problem, the structure is re-analyzed, and the process continues until the approximate problem, and hence the design, converges. The number of complete structural analyses is thus minimized. The key to implementation is the construction of a highly accurate approximate problem.

In this work, the natural frequencies and mode shapes were modeled as approximate functions of the design variables using analytic gradient infromation. Eigenvector derivative calculations are the most costly portion of the design process. Nelson's method [21] was implemented here using over 200 lines of Direct Matrix Abstraction Programming (DMAP) [7]. This is the most efficient of the exact methods, but it still requires a full matrix decomposition for each eigenvector and a back-substitution for each design parameter. Given approximate eigenvalue and eigenvector functional relationships, $\tilde{\omega}_n(\mathbf{D})$ and $\{\tilde{\phi}_n(\mathbf{D})\}$, respectively, the objective function Υ was computed in closed form for all design perturbations. The objective function has a complex algebraic form, which makes computation of its

sensitivities by finite difference attractive from both computational and programming standpoints. The objective is also a highly non-linear function of frequency and mode shapes, so use of a quasi-linear approximation would degrade the approximate problem. The convoluted first order Taylor series expansion, derived by Woo [6], is used to approximate the eigen-parameters. This approximation, for any given function f, is

$$f(\mathbf{D}) \approx \tilde{f}(\mathbf{D}) \equiv f(\mathbf{D}_o) + \nabla f(\mathbf{D}_o)(\mathbf{D} - \mathbf{D}_o)\left(\frac{\mathbf{D}}{\mathbf{D}_o}\right)^p \tag{9}$$

The choice of the parameter p is obviously important in determining the character of the approximation. A choice of $p = 0$ is simply a linear Taylor series approximation. A choice of $p = -1$ turns out to be the same as linear Taylor series with respect to reciprocal design variables $1/D_i$. Schmit first proposed reciprocal design variables as providing a high-quality linearization of structural response quantities. Choosing the sign of p as shown below results in an approximation which consistently either under-predicts or over-predicts a linear approximation.

$$\text{sign}(p) = \begin{cases} -\text{sign}(\nabla f) & ; \quad \text{underpredictor} \\ +\text{sign}(\nabla f) & ; \quad \text{overpredictor} \end{cases}$$

Choosing the sign of p in this fashion one can guarantee conservativism in the design process, always over-predicting response and under-predicting stiffness. Woo has shown that an approximate problem may be constructed which is always convex, possessing no local minima, and with the attendant increases in efficiency to be gained using optimizers tailored to convex problems.

This is not our intent here. The parameter identification process is not one of conservatism, but of making a best estimate. The value of p chosen here in the extrapolation of eigenvalues was thus chosen to reflect the general law of diminishing returns (under–prediction vis-a-vis a linear extrapolation) and to fall somewhere between the linear and reciprocal variable assumptions. The value $p = -\frac{1}{2}\text{sign}(\nabla\omega_n^2)$ was found to accelerate the convergence of the approximate problems quite well. A value of $p = 0$ was used for eigenvector extrapolation as it is not at all clear whether one should over or under predict mode shape quantities.

3 Simulation Results

A simple ten-bar truss structure as shown in Figure 2 was chosen to test the performance of the estimation procedure. This structure was chosen to represent an optical pointer in some sense. The motions of the top element, axial, lateral and rotational, are taken to represent optical sensors. An attempt to perform parameter identification using just these sensors was made. An initial design was chosen to represent an analysts best estimate of the structure's characteristics. This design was perturbed to obtain a baseline design which represents the actual structure in the field. The baseline structure was excited and it's Prony characteristics were identified. Using the initial design as a starting point, the estimation procedure was used to match the response of the model to the "measured" response of the baseline model. The estimated parameter set was found to be significantly closer to the baseline parameter set for parameters which had response sensitivity. Parameters which did not produce significant response sensitivity were found to move in the right direction slightly. It may be noted that the difference from the baseline of all parameters was well within the final estimate of their standard deviations. Thus some parameters' estimates were significantly improved, while none were degraded.

The initial design was chosen such that all members had the same area of 0.1 in^2. Damping ratios in the lowest six modes were assumed to be 1%. The baseline model was then constructed by perturbing the initial design as follows: upper legs were decreased 10% in area, lower legs increased 20%, diagonals increased 10% and horizontals decreased 40%; damping in the first lateral mode remained at 1%, damping in the first axial and second lateral modes was increased to 1.5% and damping in higher modes was increased to 2%. Parameter values for the initial, the perturbed baseline and the final estimated designs are presented in Table 1. Initial assumed standard deviations and final estimated standard deviations are also given. The initial standard deviations were arrived at by assuming stiffness estimates were accurate to ±40% and damping estimates were accurate to only ±400%.

Two separate "twang tests" were run on the perturbed model by applying and suddenly releasing 1,000 pound axial and lateral forces on the top element. Time histories of axial and lateral displacement and rotational response were recorded at the top of the truss. Axial motions were found to be de-coupled from lateral and rotational motions. The Prony components identified from the simulated tests are summarized in Table 2. The first axial and lateral modes were the only strong signals in the time histories, causing the parameter estimation process to be dominated by matching these components. The second and third lateral modes were clearly identified but had such small amplitudes that the estimation process all but ignored

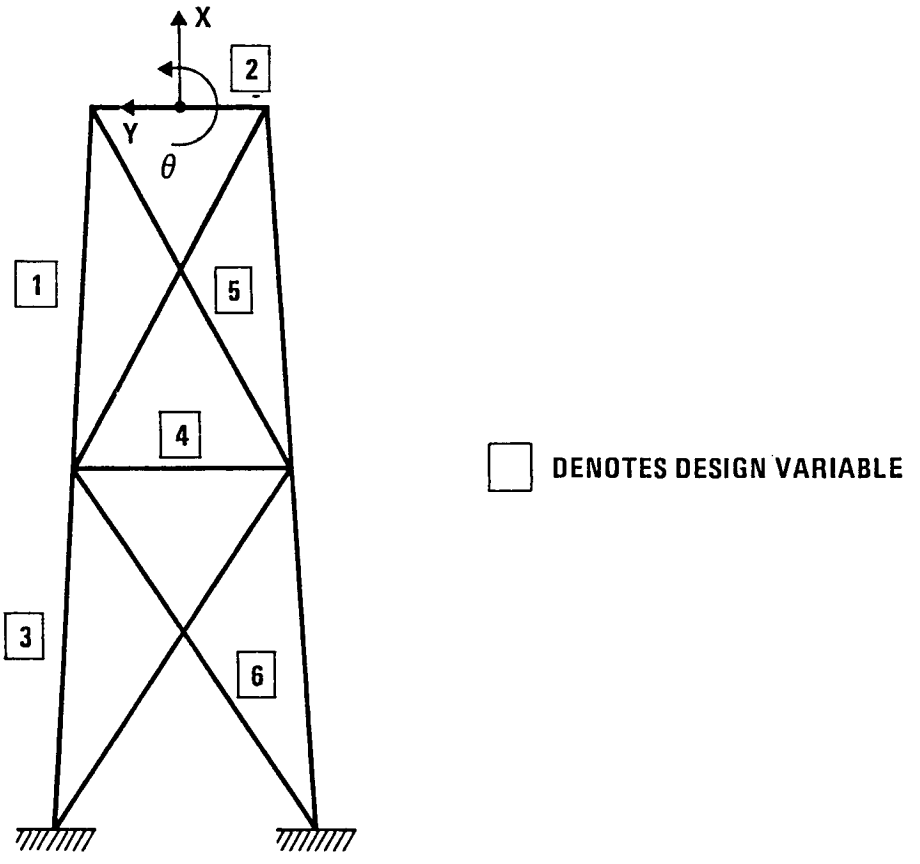

Figure 2: Ten Bar Truss Model

their presence. The Prony analyzer did not always identify the exact damping ratio as may be seen in the second component of measurement 2 which should have had $\varsigma = 1.5\%$. The higher than actual damping estimate was compensated for by the estimation of an amplitude greater than actual. The resultant integral of amplitude over the 2 second duration of the experiment was thus similar. The phase components of the identified signals were close enough to zero or π that they were ignored.

The three measurements were assigned initial standard deviations of approximately 5% of their peak values. The initial RMS error residuals and the final RMS error residuals for the estimated model are given in Table 3. The parameter estimation process came very close to eliminating the errors between observed and predicted response. A comparison of baseline, initial and estimated modal frequencies is given in Table 4. The first axial and lateral modes were matched quite well in frequency to the baseline. Their standard deviations were also reduced significantly. The second and third lateral modes, which were weakly present in the signals due to the lateral load case, were not estimated as strongly. As was seen in Table 1, the damping ratios for these weakly present modes were not estimated strongly either.

Table 1: Initial, Baseline and Estimated Parameter Sets for Ten Bar Truss

Parameter	Description	Initial Design	Baseline Design	Estimated Design	Initial σ_{D^0}	Estimated σ_{D^*}
A_1 (in^2)	Upper Legs	.1	.08	.0756	.04	.0332
A_2	Upper Horiz.	.1	.06	.0975	.04	.0400
A_3	Lower Legs	.1	.12	.1245	.04	.0244
A_4	Lower Horiz.	.1	.06	.0652	.04	.0287
A_5	Upper Diags.	.1	.11	.1023	.04	.0342
A_6	Lower Diags.	.1	.11	.1001	.04	.0328
ς_1 (%)	1st Lateral	1.0	1.0	1.0005	4.0	2.58
ς_2	2nd Lateral	1.0	1.5	1.175	4.0	3.88
ς_3	1st Axial	1.0	1.5	1.537	4.0	3.96
ς_{4+}	Higher Modes	1.0	2.0	1.024	4.0	3.99

The optimization process converged in four NASTRAN finite element analyses. An iteration history of frequencies in the first axial and lateral modes is given in Table 5. The frequencies actually computed in the FEM analyzer are contrasted with those predicted from the previous iteration by the convoluted Taylor series. Also shown are the portions of the objective function Υ due to the measurement error residuals and that due to the deviation of the parameter estimates from the initial parameter values. These are the square root of the first term in Equation 6 (measurement error) and the square root of the second term (parameter error). Note that the parameter error does not involve prediction error and hence is not extrapolated using convoluted Taylor series. Measurement error is an implicit function of modal quantities and hence it is also extrapolated indirectly. Note that the measurement errors were not predicted nearly as well as the frequencies. This is due to the non-linear nature of the objective. A small frequency or phase shift can produce large differences in error residuals. In any case, convergence to the optimum in only four finite element analyses must be considered extraordinary.

Table 2: Identified Prony Components

Time History	Sensor	Load Case	Amplitude	Frequency (Hz)	Damping %	Phase (Radians)
1	Axial (in)	Axial	.03149	15.292	1.521	-3.03
2	Lateral (in)	Lateral	.063791	3.664	1.002	.0142
			.0041	12.67	3.475	.1255
			.000886	26.267	2.54	.169
3	Rotation (rad)	Lateral	.008759	3.664	1.002	.0148
			.000360	12.655	1.224	.0961
			.000260	26.245	1.978	-2.975

Table 3: RMS Error Residuals

Time History	Initial Error	Final Error	Improvement %
1	.00685	.000192	97.2
2	.4547	.00232	99.5
3	.00595	.000189	96.8

Table 4: Model Frequencies in Hz

Mode	Description	Initial Design	Baseline Design	Estimated Design	Estimated $\sigma^\star$
1	1st Lateral	3.448	3.664	3.663	0.142
2	2nd Lateral	12.49	12.67	12.21	1.242
3	1st Axial	15.09	15.31	15.30	0.102
4	3rd Lateral	26.60	26.25	25.39	2.778

Table 5: Iteration History

FEM Analysis	Computation Type	Lateral Frequency	Axial Frequency	Measurement Error	Parameter Error
1	Actual	3.4479	12.492	19.78	0.0
2	Actual	3.6440	11.489	2.524	1.062
	Convoluted	3.6615	11.432	.6676	
3	Actual	3.6563	12.125	.9369	1.238
	Convoluted	3.6614	12.096	.5015	
4	Actual	3.6631	12.206	.4067	1.240
	Convoluted	3.6631	12.206	.4068	

4 Conclusions

The parameter identification procedure outlined here has been shown to work quite well on a small test case. A limited amount of displacement data was used to back out physical parameters which one might not expect to be identifiable. Use of acceleration data would be expected to be more beneficial as it would allow higher modes than the first to be identified strongly, thus providing more data to the estimation procedure. Application to larger models and actual test data is in progress. It is expected that the largest difficulties will arise in three areas. First is the Prony process itself. Problems in separating closely spaced modes have been encountered. Close modes sometimes appear as one mode with a higher or lower damping ratio than is actually present. Sorting actual modes from noise modes is also a difficulty. The second problem is computation time for the eigenvector derivatives. This may be overcome using more approximate methods and by selective computation for only those modes contributing heavily to response. The third difficulty is encountered by the analyst in choosing an appropriate model of the system and in choosing the appropriate parameters in that model to vary. The appropriate parameters in this simulated test case were self-evident. Experience will certainly be the major determinant in solving this last problem.

The time domain estimation procedure is applicable to forcing functions other than step functions. However computation of amplitude and phase would be considerably more difficult for more complex force time histories. Impulse loading would however be simple to analyze and would excite higher modes more than twang tests. The use of impulses should be explored as an alternative to twang tests.

The use of a frequency domain procedure analogous to the current time domain procedure should also be explored. One could identify the poles and zeroes of a structure excited by white noise using an Auto Regressive–Moving Average (ARMA) time series model. Efficient lattice filter algorithms have been developed to do this. The power spectral density of this measured model could be matched to the PSD of the predicted model by integrating the square of the difference over the frequency domain. The integration can be performed in closed form using residue theory. This frequency domain scheme may be able to more strongly estimate the higher frequency modes and thus extract more information about the physical model.

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