COMPUTER-AIDED DESIGN AND DISTRIBUTED SYSTEM
TECHNOLOGY DEVELOPMENT FOR LARGE SPACE STRUCTURES

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CHARACTERISTICS OF LARGE SPACE SYSTEMS

Proposed large space structures have many characteristics that make them diffi-
cult to analyze and control. They are highly flexible – with components mathemati-
cally modeled by partial differential equations or very large systems of ordinary
differential equations. They have many resonant frequencies, typically low and
closely spaced. Natural damping may be low and/or improperly modeled. Coupled with
stringent operational requirements of orientation, shape control, and vibration
suppression, and the inability to perform adequate ground testing, these character-
istics present an unconventional identification and control design problem to the
systems theorist.

This presentation describes some of the research underway within Langley's
Spacecraft Control Branch, Guidance and Control Division aimed at developing theory
and algorithms to treat large space structures systems identification and control
problems. The research areas to be considered are Computer-Aided Design Algorithms,
and Systems Identification and Control of Distributed Systems.

- ● Highly flexible

- ● Many resonant frequencies

- ● Low natural damping

- ● Stringent operational requirements
    - ● Orientation
    - ● Shape
    - ● Vibration suppression

- ● Limited ground testing

The established, tested computer-aided design system entitled "Optimal Regulator Algorithms for the Control of Linear Systems" (ORACLS) is being updated and modified so as to more easily accommodate the numerically difficult characteristics of large space systems lumped models. Modifications include greater use of LINPACK software (ref. 1) and inclusion of more robust Riccati equation algorithms (ref. 2). The ORACLS package (ref. 3) is also being expanded to allow multivariable frequency domain analysis and modern approaches to order reduction.

OPTIMAL REGULATOR ALGORITHMS FOR THE CONTROL OF LINEAR SYSTEMS

A modern control theory design package
for time-invariant linear systems

- Modular construction

- Efficient numerical methods

- Unified   Continuous and discrete systems
  Constant and time - varying gains
  Deterministic and stochastic

- Quadratic synthesis methods

- Cosmic, NASA TP 1106, Marcel Dekker

FREQUENCY RESPONSE ANALYSIS

A general purpose multivariable frequency domain analysis package (FREQ) has been constructed to be compatible with ORACLS. Given a multivariable unity gain feedback loop around a cascaded design system and dynamic compensator, the FREQ package has options to compute (1) a variety of transfer matrices, with loop broken at input or output, (2) singular values and vectors for the matrices, and (3) multivariable Bode-like plots with maximum/minimum singular values plotted against frequency. Typical use of this package is in the analysis/design of compensators for spillover control.
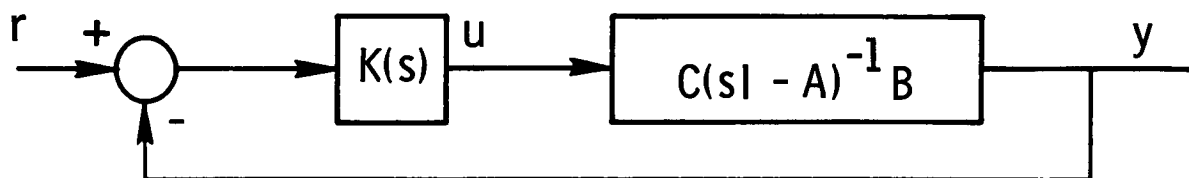
- **Transfer matrices at** $s = j\omega$

Loop gain	$G_0 = KC(sI - A)^{-1}B$
Closed-loop	$G_0(I + G_0)^{-1}$
Return difference	$I + G_0$
Inverse ret. diff.	$I + G_0^{-1}$
Sensitivity	$(I + G_0)^{-1}$

- **Calculates singular values/vectors**

- **Combines ORACLS, EISPACK, LINPACK software**

ADVANCED MULTIVARIABLE ALGORITHMS

Further needs in large space structures control design have motivated other algorithm development. Algorithms for computing multivariable invariant zeros (ref. 4) and for treating controllability/observability are included since finite element models can have transmission zeros near closely spaced open-loop poles if sensors and actuators are not properly selected. High-order design models and/or high-order compensators typically occur and require order reduction techniques (ref. 5). It is anticipated that these and other algorithms will be collected into a new computer-aided design package (ORACLS II) motivated by the needs of large space structures controller design.

- ● Multivariable zeros and relative controllability/observability

    - ● Applied to sensor/actuator selection

- ● Multivariable frequency response package (FREQ)

    - ● Applied to spillover control

- ● Algorithms based on Hankel-Norm Theory

    - ● Reduced order model/compensator

- ● Combine algorithms into new CAD package

    - ● ORACLS II

SYSTEMS IDENTIFICATION AND CONTROL OF DISTRIBUTED SYSTEMS

Although there are currently many theoretical and numerical difficulties associated with control laws designed with partial differential equation models, it is felt that sensor/actuator technology and on-board computer capability will eventually be improved to the point where distributed parameter methodology can be applied to large space structures. Anticipating these developments, the Spacecraft Control Branch is supporting research into systems identification and control theory and algorithms based on partial differential equation models. The approach currently being considered is to apply spline-based Galerkin projection approximation methods (ref. 6) and multivariable identification/control theory to models generic to large space structures.

HOOP—COLUMN APPLICATION

A particular example of this research is given by the derivation of a parameter and state estimation procedure for distributed systems which was demonstrated with a model generic to the hoop-column antenna. The reflector surface of the antenna is assumed to be approximated by the static two-dimensional stretched membrane equation with appropriate boundary conditions and variable stiffness. The problem considered was to estimate the system state (u) and stiffness (E) from given applied force and displacement measurements.
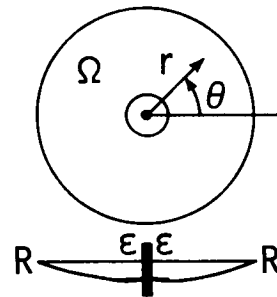
● Static two-dimensional stretched membrane

$$-\frac{1}{r}\frac{\partial}{\partial r}\left[rE(r,\theta)\frac{\partial u}{\partial r}(r,\theta)\right] - \frac{\partial}{\partial \theta}\left[\frac{E(r,\theta)}{r^2}\frac{\partial u}{\partial \theta}(r,\theta)\right] = f(r,\theta)$$

Over $\Omega = \left[\varepsilon, R\right] \times \left[0, 2\pi\right]$
with boundary conditions

$$u(\varepsilon,\theta) = u_0, \qquad u(r,o) = u(r,2\pi)$$

$$u(R,\theta) = 0$$

● Given f(r,θ) and measured displacements

$u_m\left(r_i, \theta_j\right)$, $(i = 1, \ldots, L_r$; $j = 1, \ldots, L_\theta)$, estimate

E (r, θ) and u (r, θ) within Ω

SYSTEM IDENTIFICATION APPROACH

The general approach for identification can be described as follows. After formulating a distributed model for the dynamic system, some mathematical approximation technique, such as finite elements or splines, is used to project the identification problem onto a finite-dimensional subspace. The finite-dimensional identification problem is then solved within the subspace. The approximation is successfully refined and the identification problem repeatedly solved to produce a sequence of estimates to be analysed for convergence. This approach, when specialized to the hoop-column application, is outlined below. Details of the study may be found in reference 7.

General

- Distributed parameter formulation

- Project onto finite dimensional subspace

- Solve identification problem within subspace

- Successively increase subspace dimension and solve identification problem to generate sequence of estimates

Hoop-column application

- Stretched membrane equation

- Galerkin projection with linear spline basis functions Parameterize $E(r,\theta)$ via cubic splines

- Output error identification technique

- Numerical algorithm

The same general approach is being investigated for controller design. When applied to dynamic systems, the spline-based Galerkin approximation procedure produces high-order state equations. If linear quadratic regulator theory is applied for control law design, the related difficulty of solving high-order Riccati equations arises. In control applications, wherein only the regulator gain is required, direct application of the Chandrasekhar technique (ref. 8) is extremely difficult due to numerical stiffness problems. Recent results from a NASA-sponsored research grant (NAG-1-517) with Brown University have produced a new hybrid Chandrasekhar-type algorithm for approximating the steady-state gain matrix independently of the Riccati equations.

● **Linear quadratic regulator problem**

$$\dot{x} = Ax + Bu$$

$$J = \int_0^\infty (x^T C^T C x + u^T u)\, dt$$

$$A^T P + PA - PBB^T P + C^T C = 0$$

$$u = -Kx = -B^T Px$$

● **Chandrasekhar algorithm**

$$-\frac{d}{dt} P(t) = A^T P(t) + P(t)A$$
$$- P(t)BB^T P(t) + C^T C, \quad P(t_f) = 0$$

$$\frac{d\,K(t)}{dt} = -B^T L^T(t) L(t), \quad K(t_f) = 0$$

$$\frac{d\,L(t)}{dt} = -L(t)\left[A - BK(t)\right], \quad L(t_f) = C$$

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In order to begin the hybrid algorithm, the standard Chandrasekhar algorithm is employed to obtain an initial stabilizing gain for application of the Newton-Kleinman algorithm for solving the algebraic Riccati equation. After several steps through the Newton-Kleinman sequence, enough data are obtained to rewrite the recursion equation in an alternate form giving an update formula for the gain matrix. Banks and Ito at Brown University have discovered a way to compute the gain update (K_{i+1}) without computing Z_i.

- Chandrasekhar equations give starting value for Newton-Kleinman

- Newton-Kleinman

$$(A-BK_i)^T P_i + P_i (A-BK_i) + K_i^T K_i + C^T C = 0$$

$$K_{i+1} = B^T P_i$$

- Alternate form -

$$(A-BK_i)^T Z_i + Z_i (A-BK_i) + (K_i - K_{i-1})^T (K_i - K_{i-1}) = 0$$

$$K_{i+1} = K_i + B^T Z_i$$

Can compute update without computing Z_i

Apply the (Smith) bilinear transformation to the Z_i equation. A sequential solution can then be written for the tranformed equation from which the update algorithm can be derived.

- **Bilinear transformation (Smith) –**

$$X = Z_i \qquad\qquad U = (I - r\widetilde{A})^{-1}(I + r\widetilde{A})$$

$$\widetilde{A} = A - BK_i \qquad\qquad Y = 2r(I - r\widetilde{A})^T D^T D(I - r\widetilde{A})^{-1}$$

$$D = K_i - K_{i-1} \qquad X = U^T X U + Y$$

- **Sequential solution –**

$$X_{k+1} = U^T X_k U + Y$$

$$X_{k+1} - X_k = U^T(X_k - X_{k-1})U$$

- **Algorithm –**

$$X_{k+1} = X_k + 2r M_{k+1}^T M_{k+1}$$

$$M_{k+1} = M_k U$$

$$B^T X_{k-1} = B^T X_k + 2r\, B^T M_{k+1}^T M_{k+1}$$

NUMERICAL RESULTS

For a distributed parameter example, consider the parabolic system below with boundary control. The control input u(t) is to be chosen to minimize J(u) subject to the dynamic equation and boundary condition constraints. After discretizing with linear splines and Galerkin projection, an Nth order linear quadratic regulator problem is obtained. The index N increases with refinement of the linear spline approximation. For N=10, CPU times are comparable between the Potter (ref. 8) and hybrid methods. However, as N increases, the hybrid algorithm excels.

$$\bullet \quad \frac{\partial v(x,t)}{\partial t} = \frac{\partial^2 v(x,t)}{\partial x^2} \; , \; x \varepsilon \; (0,1)$$

$$v(0,x) = \phi(x), \frac{\partial v}{\partial x}(t,0) = u(t), \; \frac{\partial v}{\partial x}(t,1) = 0$$

$$J(u) = \int_0^\infty \left(\left| cv(t) \right|^2 + \left| u(t) \right|^2 \right) dt$$

$$cv(t) = \int_0^\infty (1+x)v(t,x) \, dx$$

	Potter	Hybrid
N = 10	0.162	0.197
N = 40	6.3	1.29

STABILITY AUGMENTATION BY BOUNDARY-FEEDBACK CONTROL

Another approach to distributed-parameter formulation of the control problem is being considered by Balakrishnan (ref. 9) for application to the Spacecraft Control Laboratory Experiment (SCOLE) configuration (ref. 10). An abstract wave formulation is developed as a nonlinear wave equation in Hilbert space. The system is shown to be controllable, and a feedback control law is developed assuming point actuators and sensors at the boundaries. The control law is shown to be strongly stable and robust to parameter uncertainties.

- $M \ddot{x}(t) + A x(t) + K(\dot{x}(t)) + B u(t) = 0$

    M: mass matrix

    A: $D \to L_2 [0,L]^3 \times R^{14}$, differential operator

    K: nonlinear function

    u(t): applied moments, proof mass forces

    $$x = \left[u_\phi(.),\ u_\theta(.),\ u_\psi(.),\ u_\phi(0^+),\ u_\theta(0^+),\ u_\phi(L^-),\ u_\theta(L^-), \right.$$
    $$u'_\phi(0^+),\ u'_\theta(0^+),\ u_\psi(0^+),\ u'_\phi(L^-),\ u'_\theta(L^-),\ u_\psi(L^-),$$
    $$\left. u_\phi(s_2),\ u_\theta(s_2),\ u_\phi(s_3),\ u_\theta(s_3) \right]^T$$

    $u_\phi(s),\ u_\theta(s)$: displacements; $u_\psi(s)$: rotation

    $s_2,\ s_3$: proof mass actuator locations

- State space form: $\dot{y}(t) = \bar{A} y(t) + \bar{K}(y(t)) + \bar{B} u(t)$

- Control law: $u(t) = -P \bar{B}^* y(t)$; $P > 0$

DAMPING MODELS

The mathematical modeling of damping mechanisms is a little understood, most important aspect of systems identification of large space structures. Traditionally, damping is superimposed in linear viscous form on modal dynamic equations obtained from finite element software. Unfortunately, none of the expected forms of energy dissipation is linear viscous (ref. 11). Research employing partial differential equation dynamic models indicates that damping operators should have nonlocal structure.

● **Little understood**

● **Recognized as most important aspect**

● **Linear-viscous**

$$M\ddot{\hat{u}} + D\dot{\hat{u}} + K\hat{u} = \hat{f}$$

● **None of expected damping is linear-viscous**

● **PDE damping operators can require nonlocal structure**

Using the torsional beam equation to model the Spacecraft Control Laboratory Experiment (SCOLE) (ref. 10), it has been found that, for proportional damping to occur, consideration of the total beam length in the damping operator formulation is required. Future research under the Brown University grant (NAG-1-517) will consider other forms of damping operators for an Euler-Bernoulli beam model. The investigation will include damping with nonlocal interaction terms and fading memory formulations where, for example, stress is a functional of the history of deformation and prior times.

- ## Scale model (Balakrishnan, UCLA) for infinite beam with torsion equation

$$\frac{\partial^2 u}{\partial t^2} + 2\zeta D \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0$$

$$D\frac{\partial u}{\partial t}(t,x) = \frac{1}{\pi}\frac{\partial}{\partial t}\int_{-\infty}^{\infty} \frac{\frac{\partial}{\partial \zeta}u(t,\zeta)}{(x-\zeta)}\,d\zeta$$

gives proportional damping

- ## Investigating (Banks, Brown Univ.)

$$u_{xxt} - \int_{x-\delta}^{x+\delta} h(z-x)\,u_{xxt}(z,t)\,dz$$

and

$$\frac{\partial^2}{\partial x^2}\left\{ \int_0^t k(t,s)\,\phi\left(u_{xx}(s,x)\right)\,ds \right\}$$

In Euler-Bernoulli beam model

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