

1N-32 CK

51193

21P.



The Ohio State University

ENGINEERING CALCULATIONS FOR COMMUNICATIONS

SATELLITE SYSTEMS PLANNING

C. H. Reilly  
C. A. Levis  
O. M. Buyukdura  
C. A. Mount-Campbell

The Ohio State University

**ElectroScience Laboratory**

Department of Electrical Engineering  
Columbus, Ohio 43212

Technical Report 718688-1  
Grant NAG3-159  
November 1986

(NASA-CR-180106) ENGINEERING CALCULATIONS  
FOR COMMUNICATIONS SATELLITE SYSTEMS  
PLANNING Interim Report, 15 Jan. - 11 Jul.  
1986 (Ohio State Univ.) 21 p CSCL 17B

N87-16198

Unclas  
G3/32 43299

National Aeronautics and Space Administration  
Lewis Research Center  
21000 Brookpark Rd.  
Cleveland, Ohio 44135

## NOTICES

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

<b>REPORT DOCUMENTATION PAGE</b>	<b>1. REPORT NO.</b>	<b>2.</b>	<b>3. Recipient's Accession No.</b>
<b>4. Title and Subtitle</b> Engineering Calculations for Communications Satellite Systems Planning		<b>5. Report Date</b>	
<b>7. Author(s)</b> C.H. Reilly, C.A. Levis, O.M. Buyukdura, C.A. Mount-Campbell		<b>6.</b>	
<b>9. Performing Organization Name and Address</b> The Ohio State University ElectroScience Laboratory 1320 Kinnear Road Columbus, Ohio 43212		<b>8. Performing Organization Rept. No.</b> 718688-1	
<b>12. Sponsoring Organization Name and Address</b> National Aeronautics and Space Administration Lewis Research Center 21000 Brookpark Rd., Cleveland, Ohio 44135		<b>10. Project/Task/Work Unit No.</b>	
<b>15. Supplementary Notes</b>		<b>11. Contract(C) or Grant(G) No.</b> (C) (G) NAG3-159	
<b>16. Abstract (Limit: 200 words)</b>  In this report, we analyze observed solution times for the extended gradient and cyclic coordinate search procedures. The times used in the analysis come from computer runs made during a previously-reported experiment conducted to assess the quality of the solutions to a BSS synthesis problem found by the two search methods. The results of a second experiment with an FSS test problem are also presented. We summarize computational results for mixed integer programming approaches for solving FSS synthesis problems. A promising new heuristic algorithm is described. We discuss a new synthesis model for orbital arc allotment optimization. Research plans for the near future are also presented.		<b>13. Type of Report &amp; Period Covered</b> Interim Report 15 Jan 1986/11 July 1986	
<b>17. Document Analysis a. Descriptors</b>  Satellite Fixed Service Orbie  Frequency Assignment Interference		<b>14.</b>	
<b>b. Identifiers/Open-Ended Terms</b>		<b>19. Security Class (This Report)</b> Unclassified	
<b>c. COSATI Field/Group</b>		<b>21. No. of Pages</b> 20	
<b>18. Availability Statement</b> A. Distribution is unlimited; approved for public release.		<b>20. Security Class (This Page)</b> Unclassified	
		<b>22. Price</b>	

## TABLE OF CONTENTS

LIST OF TABLES	iii
I. PURPOSE	1
II. SOLUTION TIME ANALYSIS OF THE EXTENDED GRADIENT AND CYCLIC COORDINATE SEARCH ALGORITHMS	1
III. ADDITIONAL EXPERIMENTATION WITH THE EXTENDED GRADIENT AND CYCLIC COORDINATE SEARCH ALGORITHMS	5
IV. RESULTS FOR INTEGER PROGRAMMING SYNTHESIS MODELS AND THE BENDERS' DECOMPOSITION APPROACH	9
V. A PERMUTATION ALGORITHM FOR SATELLITE SYNTHESIS	10
VI. SYNTHESIS MODELS FOR ARC ALLOTMENT OPTIMIZATION	12
VII. PLANS FOR THE NEXT INTERIM	14
REFERENCES	15

## LIST OF TABLES

Table 1	Factors and Factor Levels	6
Table 2	Summary of Significant Effects	8

## **I. PURPOSE**

The purpose of this grant is to develop methods and procedures, including computer codes, for performing engineering calculations which will be useful for the United States delegations to international administrative conferences concerning satellite communications. During the interim 15 January 1986 to 11 July 1986, attention has been directed almost exclusively toward Fixed Satellite Service (FSS) issues since this service will be a major topic at the World Administrative Radio Conference in 1988 (WARC-88).

## **II. SOLUTION TIME ANALYSIS OF THE EXTENDED GRADIENT AND CYCLIC COORDINATE SEARCH ALGORITHMS**

In our last interim report [9], we described an experiment conducted to determine which of two search techniques, an extended gradient search (EGS) or a cyclic coordinate search (CCS), performs better when solving satellite synthesis problems formulated as nonlinear programs as suggested in [6]. A complete presentation of the experiment can be found in a recent technical report [11].

So far, our analysis of the search methods has dealt almost entirely with the quality of the solutions found by the methods. However, we are also interested in the time required to obtain solutions with the search algorithms. Here, we analyze the solution times observed for the 64 computer runs made during our experiment. To simplify our analysis, we assume that all service areas have the same number of test points and that all satellites transmit signals on the same number of frequency channels.

We use the following notation for our solution time analysis:

$n$  = number of satellites

$p$  = number of test points in a service area

$f$  = number of frequencies at which each satellite transmits signals

$t_A$  = time to perform an off-axis angle calculation

$t_I$  = time to compute a single-entry carrier-to-interference (C/I) ratio at one test point for one intended channel and one interfering channel

$t_Z(n)$  = time to evaluate the objective function for an  $n$ -satellite problem.

$t_G(n)$  = time to evaluate the gradient of the objective function for an  $n$ -satellite problem

We begin our solution time analysis by considering the worst-case performance of the two algorithms. In the worst case, every iteration of the EGS algorithm, as implemented by us, would involve 13 10-point line searches. The total time for an  $m$ -iteration run of the EGS is therefore bounded above by:

$$(130m+1)n(n-1)t_A + (130m+1)n(n-1)pf^2t_I + (130m+1)t_Z(n) + mt_G(n) \quad (1)$$

With our implementation of the CCS, every cycle (iteration) would involve 5 searches of 10 points for each location variable and each frequency variable in the worst case. Hence, the total solution time for an  $m$ -cycle run of the CCS is bounded above by:

$$(200m+1)n(n-1)t_A + (200m+1)n(n-1)pf^2t_I + (100mn+1)t_Z(n) \quad (2)$$

Upon examining (1) and (2), we see that the coefficients of  $t_I$  dominate these worst-case total time expressions. We can estimate the worst-case solution time ratio for the two methods using the coefficients of  $t_I$  which appear in (1) and (2):

$$\text{Worst-case time ratio} = \frac{\text{CCS worst case}}{\text{EGS worst case}} = \frac{20}{13}$$

This ratio is valid for comparing an m-iteration EGS run and an m-cycle CCS run.

In all of the computer runs made so far, neither of the search methods has exhibited worst-case behavior, at least in terms of solution time. In the CCS runs made in our experiment, a total of 876 line searches were performed in 381 cycles. If the CCS had exhibited consistent worst-case behavior, then 1905 line searches would have been conducted. Hence, the performance of the CCS can be summarized as 46.0 percent of the worst case.

A total of 941 line searches were performed in 438 iterations of the EGS. True worst-case performance would have involved 5694 line searches. Therefore, the observed behavior of the EGS is 16.5 percent of the worst case.

We can estimate the solution time ratio for an m-cycle CCS run and an m-iteration EGS run as follows:

$$\text{Estimated solution time ratio} = \frac{20 * 46.0\%}{13 * 16.5\%} = 4.3$$

The actual average solution time ratio was approximately 5.6.



The difference between the estimated and actual solution time ratios is due, at least in part, to the fact that we have considered only the coefficients of  $t_1$  in our worst-case total time expressions (1) and (2). Also, every service area did not have the same number of test points, as we have assumed.

We can conclude from our analysis that it is not surprising that the CCS consumed more computation time than the EGS in our experiment, given the worst-case total time expressions (1) and (2). It is interesting to note that each cycle of the CCS behaved more like a worst-case cycle than each iteration of the EGS did. This may be due to the fact that the EGS identifies a promising search direction for each line search, while the CCS blindly searches in coordinate directions only. The chance that an improved solution is found in any line search is therefore likely to be greater in the case of the EGS.

The results of our experiment clearly showed that better synthesis solutions were found by the CCS [9,11]. We see that there is a cost, namely additional computing time, which must be paid in order to find these better solutions. This additional cost can be quantified. A cycle of the CCS is expected to take between 4 and 6 times as long as an iteration of the EGS.

The analysis described above addresses the issue of solution time. An analysis of the quality of the solutions found with these two methods is described in [11]. By comparing the solution times for  $m$ -iteration EGS runs and  $m$ -cycle CCS runs, we do not mean to imply that such runs would yield synthesis solutions of comparable quality. In fact, we would expect the solutions found via the CCS to be considerably better than those found with the EGS [11].

### III. ADDITIONAL EXPERIMENTATION WITH THE EXTENDED GRADIENT AND CYCLIC COORDINATE SEARCH ALGORITHMS

In our last interim report [9] and in a recent technical report [11], we described an experiment conducted to determine which of two search methods, an extended gradient search procedure (EGS) or a cyclic coordinate search algorithm (CCS), performs more reliably on satellite synthesis problems formulated as nonlinear programs as suggested in [6]. The objective function in this formulation seeks to maximize the smallest aggregate C/I ratio calculated for any test point in any service area. We also sought to identify the factors that affect the performance of the search methods most when solving synthesis problems. The test problem we used in our experiment consisted of seven BSS satellites, each serving a different South American administration.

Earlier, we had exercised the search methods on a second test problem which consists of eight FSS satellites, all serving the Eastern U.S. [10]. We describe another experiment using this second test problem in this report. This experiment was conducted in order to see what effects, if any, allowing a service area to be served by multiple satellites would have on our conclusions from the first experiment.

We included five factors in our experiment. Each factor had one of two levels, a low level or a high level, in each computer run made. The factors and factor levels are listed below in Table 1.

**Table 1**  
**Factors and Factor Levels**

Factor	Factor Levels	
	Low	High
A-Algorithm	EGS	CCS
B-Location Spacing	0.1°	1.0°
C-Starting Locations	centered at 80°	spaced from 100°
D-Arc Length	74°-100°	60°-100°
E-Run Length	5 CPU minutes or 10 iterations	10 CPU minutes or 20 iterations

Since no cross-polarized antenna discrimination patterns were known to us at the time of our experiment, we used co-polarized FSS discrimination patterns and assumed all of the satellites used the same frequency for signal transmissions. Hence, there are no factors directly related to frequency in this experiment. It was determined that collocating the satellites at any longitude prevented the EGS from finding improved solutions. The components of the gradient would be the same in this case, and the EGS would move the satellites to the same new locations. No separation between satellites could ever be achieved with this initial solution configuration. As a result, the minimum initial separation between satellites was set at 0.1°, instead of the 0° used in our earlier experiment.

The experimental design was a 1/2-fraction of a  $2^5$  factorial design [1]. A total of 16 runs were made. Each factor was set at both of its levels in eight of the runs.

The results of this experiment are almost identical to those of our first experiment. The CCS again outperformed the EGS; however, the magnitude of the difference in the average worst aggregate C/I ratios for the two methods was about 28 percent smaller in the second experiment. The average worst aggregate C/I ratio at any test point in any service area for the CCS runs was 27.755 dB (range: 24.79 dB to 30.39 dB). For the EGS runs, the average worst aggregate C/I ratio was 11.9775 dB (range: -7.44 dB to 25.42 dB).

The same factors and factor combinations that had the most significant effects on the performance of the search methods in the first experiment also tended to be significant in the second experiment. The eight most significant effects in each of the experiments are shown in Table 2.

**Table 2**  
**Summary of Significant Effects**

Factor(s)	Second Experiment		First Experiment	
	Effect	Rank	Effect	Rank
Mean	19.87dB	1	30.68dB	1
Algorithm(A)	15.78dB	2	21.91dB	2
Location Spacing(B)	8.60dB	3	-4.58dB	6
A/B	-8.40dB	4	4.58dB	6
B/Starting Locations(C)	5.54dB	5	-4.60dB	5
A/B/C	-5.49dB	6	5.35dB	4
C	-5.45dB	7	-1.29dB	10
A/C	5.37dB	8	-0.53dB	25
Arc Length(D)	0.25dB	15	7.97dB	3
Frequency Spectrum	N/A	N/A	4.26dB	8

The main conclusion that should be drawn from Table 2 is that the choice of an algorithm is the most critical factor in the successful application of these search methods. From the statistics presented on the average worst C/I ratios and the positive effect of the algorithm factor, it is clear that the better choice is the CCS.

Table 2 also suggests that the choice of the CCS offsets the significant main effects of location spacing and starting locations and their interaction effect. For example, the location spacing main effect

was 8.60 dB in the second experiment and -4.58 dB in the first experiment [11]. The algorithm-location spacing interaction effect was -8.40 dB in experiment 2 and 4.58 dB in experiment 1. The opposite signs on the location spacing main effects indicate that greater initial spacing between satellites ( $1.0^\circ$  versus  $0.1^\circ$ ) led to better final solutions when the satellites were identical, that is, in experiment 2. However, initial solutions in which nonidentical satellites were collocated ( $0^\circ$  versus  $1.0^\circ$ ) led to better final solutions in experiment 1.

In light of our two experiments, we conclude that the CCS is a considerably more reliable solution technique for satellite synthesis problems formulated as suggested in [6]. As pointed out in Section II of this report, the CCS will require more solution time than the EGS, but the payoff in terms of solution quality suggests that the extra solution time may be a worthwhile investment.

#### **IV. RESULTS FOR INTEGER PROGRAMMING SYNTHESIS MODELS AND THE BENDERS' DECOMPOSITION APPROACH**

In a dissertation [4] to be completed soon by a Ph.D. student supported by this grant, attempts to solve several satellite synthesis test problems, ranging in size from 10 to 26 satellites, are described. These synthesis problems are formulated as mixed integer programs. Each problem was solved twice, once with the objective of minimizing the total deviation between desired and prescribed satellite locations and once with the objective of minimizing the maximum deviation from a desired location. In most cases, a proven optimal solution was not

found by a branch-and-bound algorithm [3] or by Benders' decomposition [2] before some criterion for run termination was met. Feasible solutions were identified in all but a few cases, however.

The cost of solving large satellite system synthesis problems for a global optimum by applying either a branch-and-bound method or Benders' decomposition may be prohibitive. Yet, both approaches have been reasonably successful at identifying feasible solutions to our test problems. They may still have considerable merit when viewed as heuristic approaches dedicated to finding feasible solutions. These approaches may also be valuable when solving small synthesis problems, for example, if large synthesis problems are decomposed into smaller, easier-to-solve problems.

The computational results for the mixed integer programming and Benders' decomposition approaches will be summarized in a forthcoming technical report [5].

## **V. A PERMUTATION ALGORITHM FOR SATELLITE SYNTHESIS**

In this section, we briefly describe a new heuristic method for finding solutions to satellite synthesis problems. A more complete description of the method can be found in the dissertation by Gonsalvez [4] and in a forthcoming technical report [5].

The new heuristic method is a switching, or permutation, algorithm. An initial ordering of the satellites is selected, usually based on the satellites' assumed desired locations. For any given ordering, the satellite synthesis model used with the Benders' decomposition approach

is reduced to a linear program [4,7]. The linear program associated with the selected ordering is solved. Next, all possible permutations of  $k$  adjacent satellites are systematically considered. Any permutations that lead to immediately improved solutions, as measured by the objective function, are made. The method continues until no more groups of  $k$  adjacent satellites can switch positions and produce an improved solution. At that time, the method is terminated, or  $k$  is incremented and the process is repeated. The switching method has been implemented for  $k=2$ ,  $k=3$ ,  $k=4$ ,  $k=5$ , and  $k$  increasing from 2 to 5.

Feasible solutions to all of our test problems, including a 59-satellite problem based on the OASTS2G1 scenario, have been found with the switching method. Larger values of  $k$  ( $k=4$  or  $5$ ) and the increasing  $k$  seem to produce the best results. Feasible solutions tend to be found quickly; many improved feasible solutions are typically found. Solutions known to be optimal were found for some of the problems with the switching method.

The switching method has a major advantage over the other solution strategies studied because location-dependent satellite separations, instead of constant, conservative separations, can be enforced. It is therefore less likely that a solution that is truly feasible would be overlooked simply because of the conservative nature of the satellite separations used previously. The use of these location-dependent separations could not be incorporated into any of the other integer programming models we have investigated without considerably complicating the models or the solution procedures.



So far, we have used the switching method with the objective functions of minimizing the total deviation and the maximum deviation between desired and prescribed satellite locations. The switching method can be used with other objective functions because the method itself does not exploit the structure of the synthesis models with the objective functions mentioned above.

## **VI. SYNTHESIS MODELS FOR ARC ALLOTMENT OPTIMIZATION**

The models for satellite system synthesis that we had developed so far on this grant have been point assignment models. We have recently begun to consider a synthesis model which allots a segment of the geostationary orbital arc to each satellite, to each administration, or to a group of administrations. We will use the word "satellite" throughout this presentation with the understanding that "administration" or "group of administrations" could be used in its place. Our arc allotment model also accommodates the deployment of multiple satellites in an allotted arc segment.

Our mixed integer programming model for the point assignment problem can be modified so that arcs can be allotted to satellites. We have chosen the maximization of the shortest arc segment allotted to any satellite as the objective function for this model. The decision variables for satellite locations are no longer needed because specific locations for satellites are not determined. Instead, decision variables for the eastern and western limits of each satellite's allotted arc are included. Parameters for the satellites' desired

locations are not used in our arc allotment model. But, the parameters pertaining to feasible arc restrictions and minimum required orbital spacings are used.

The required satellite separations that are enforced for point assignment problems are also enforced in arc allotment problems. Rather than specifying at least how far satellite locations must be removed from one another, these required separations specify how far removed the nearest points on two allotted arcs must be.

Some of the satellite separations have to be modified in the arc allotment problem. If the required satellite separation for two satellites is  $0^\circ$ , then the arcs allotted to those satellites can overlap. To allow for this possibility, the zero-valued required satellite separations are replaced with large (very) negative numbers in the constraints which enforce the separation of arcs. If this were not done, the arcs allotted to satellites that can be collocated without causing excessive interference could at most have a common endpoint.

The basic arc allotment model can be modified so that the length of the shortest weighted arc allotted to a satellite is maximized. Then the length of the arc allotted to satellites can be based on each satellite administration's population or anticipated communications traffic.

The work completed so far in the area of arc allotments is summarized in a working paper [8]. We believe that the arc allotment model is a good candidate for the application of the switching heuristic [4,5]. A model for the arc allotment problem that is amenable to solution by Benders' decomposition has also been formulated.

## VII. PLANS FOR THE NEXT INTERIM

Our plans for the interim from 12 July 1986 to 11 January 1987 are focused in two areas related to FSS system synthesis. First, we plan to study the minimum required satellite separation concept in greater depth. Our current separation calculation assumes all satellites are identical, all antenna beams are elliptical, and each satellite transmits signals at one frequency. Furthermore, we include only the down-link in our calculations. We plan to investigate how the required satellite separation calculation might change if the up-link is included and if there are nonhomogeneous satellite systems, shaped antenna beams, and transmissions at multiple frequencies.

We also plan to continue our investigation into alternate models and solution techniques for satellite synthesis. We are studying a variety of objective functions for point assignment synthesis models. A model for the allotment of orbital arc segments has been formulated. We will attempt to determine if any of the candidate models have advantages, primarily computational advantages, over the others. The potential applicability of the switching heuristic to other satellite synthesis models will be investigated.

Should the need arise to shift our attention to problems of greater immediate importance, as determined by NASA, we will redirect our efforts accordingly.

## REFERENCES

- [1] Anderson, V. and R. McLean, DESIGN OF EXPERIMENTS -A REALISTIC APPROACH, Marcel Dekker, Inc., New York, 1974.
- [2] Benders, J., "Partitioning procedures for Solving Mixed-Variables Programming Problems", NUMERISCHE MATHEMATIK, vol. 4, pp. 238-252, 1962.
- [3] Garfinkel, R. and G. Nemhauser, INTEGER PROGRAMMING, John Wiley and Sons, 1972.
- [4] Gonsalvez, D., "On Orbital Allotments for Geostationary Satellites", Ph.D. Dissertation, The Ohio State University, December, 1986.
- [5] Gonsalvez, D., C. Reilly, and C. Mount-Campbell, "On Orbital Allotments for Geostationary Satellites", Technical Report for Grant NAG 3-159, forthcoming.
- [6] Levis, C., C. Martin, D. Gonsalvez, and C. Wang, "Engineering Calculations for Communications Satellite Systems Planning", The Ohio State University, ElectroScience Laboratory, Interim Report 713533-3 for Grant NAG 3-159, June 1983.
- [7] Mount-Campbell, C., C. Reilly, and D. Gonsalvez, "A Mixed Integer Linear Programming Formulation for the FSS Synthesis Problem Using Minimum Required Pair-Wise Separations", The Ohio State University, Department of Industrial and Systems Engineering, Working Paper 1986-006.
- [8] Reilly, C., "A Satellite System Synthesis Model for Orbital Arc Allotment Optimization", The Ohio State University, Department of Industrial and Systems Engineering, Working Paper 1986-016.
- [9] Reilly, C., O. Buyukdura, C. Levis, and C. Mount-Campbell, "Engineering Calculations for Communications Satellite Systems Planning", The Ohio State University, ElectroScience Laboratory, Interim Report 716548-5 for Grant NAG 3-159.
- [10] Reilly, C., C. Levis, C. Mount-Campbell, D. Gonsalvez, C. Wang, and Y. Yamamura, "Engineering Calculations for Communications Satellite Systems Planning", The Ohio State University, ElectroScience Laboratory, Interim Report 716548-2 for Grant NAG3-159.

- [11] Reilly, C., C. Mount-Campbell, D. Gonsalvez, C. Martin, C. Levis, and C. Wang, "Broadcasting Satellite Service Synthesis Using Gradient and Cyclic Coordinate Search Procedures", The Ohio State University, ElectroScience Laboratory, Technical Report 716548-4 for Grant NAG 3-159, February 1986.