N87-16751, D9-46
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1986

NASA/ASEE SUMMER FACULTY FELLOWSHIP PROGRAM

MARSHALL SPACE FLIGHT CENTER
THE UNIVERSITY OF ALABAMA

A SIMPLE MODEL FOR THE INTERACTION OF PLANETARY AND SYNOPTIC-SCALE WAVES IN THE TROPOSPHERE

Prepared by: Lee E. Branscome, Ph.D.

Academic Rank: Associate Professor

University and Department: University of Miami
Rosenstiel School of Marine and Atmospheric Science
Division of Meteorology and Physical Oceanography

NASA/MSFC:
Laboratory: Systems Dynamics
Division: Atmospheric Sciences
Branch: Fluid Dynamics

NASA Colleague: Timothy L. Miller, Ph.D.

Date: August 18, 1986

Contract No.: NGT 01-002-099
The University of Alabama

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Lee E. Branscome
Associate Professor of Meteorology
and Physical Oceanography
University of Miami
Miami, Florida

ABSTRACT

The time-mean circulation of the troposphere during the Northern Hemisphere winter is dominated by planetary-scale stationary waves superposed on longitudinal-mean westerly flow. The movement and generation of weather-making, synoptic-scale transient waves are controlled by this large-scale circulation. The interannual variability of planetary-scale waves determines the severity of the winter season over broad geographic regions through such factors as frequency of storms, precipitation and temperature.

The transient waves or eddies play an important role in maintaining the heat and angular momentum balance of the atmosphere. By maintaining the westerly flow in mid-latitudes, these waves are responsible for the mean zonal flow over large-scale topography that forces, in part, the stationary planetary waves. In addition, the precipitation and subsequent latent heat release in the organized "storm tracks",i.e., preferred regions of transient wave activity, form a longitudinally-varying pattern of diabatic heating that also forces stationary waves. On the other hand, the transient, baroclinic waves damp the planetary-scale waves by the down-gradient flux of sensible heat.

A conceptual model that incorporates these feedbacks between the two scales is developed and guides the formulation of a quantitative low-order model which retains a few basic physical processes and wave components. In the low-order model the fluxes of sensible heat and momentum and latent heat release by transient eddies are parameterized in terms of the evolving large-scale circulation. Radiative forcing and damping, planetary-scale variations in moisture and topography and Ekman dissipation are also included. Testing of this model is in progress. The results will help interpret experiments from sophisticated general-circulation models and ascertain the need for satellite observations of precipitation.

ACKNOWLEDGMENTS

Thanks to my NASA Colleague, Tim Miller, my summer visit to Marshall has been pleasant and productive. I appreciate his hospitality and patient helpfulness. I would also like to thank George Fichtl for allowing me to use his nice office, for which I feel fortunate. Thanks also to Charlie Shafer for the use of his IBM PC and to other members of the Atmospheric Sciences Division who have contributed their thoughts about my project and lent a helping hand.

I thank Mike Freeman and other co-ordinators of the Summer Fellowship for selecting me for this summer's program. I am glad that I accepted this kind offer of summer support and opportunity to participate in Marshall's research activities.

INTRODUCTION

The time-mean circulation of the mid-latitude troposphere during the Northern Hemisphere winter is dominated by westerly (eastward) flow and stationary waves of planetary scale (see Fig. 1a). The movement and generation of synoptic-scale (1000-4000 km), high-frequency (2-6 day period) transient waves are controlled by the large-scale circulation (see Fig. 1b and Background section below). These synoptic-scale waves, which have significant circulation throughout the troposphere, are manifested at the surface by the mid-latitude storms or cyclones that determine the daily weather.

The planetary-scale waves have substantial interannual variability and their positions and amplitudes averaged over a given winter can deviate significantly from the climatological norm (see Fig. 2). Since these waves direct weather-making cyclone activity, the wintertime precipitation and temperature distributions can also deviate from normal patterns, thus determining the severity of the winter season over broad geographic regions.

Therefore, it is of considerable importance to understand the mechanisms responsible for forcing the planetary waves to improve weather prediction, seasonal forecasts and climate models. Because the transient waves have transport properties that modify the global-scale circulation, we suspect that they play a crucial role in the dynamics of the planetary waves. We anticipate that feedbacks or interactions between the two scales of motion continuously operate.

One method for evaluating the controlling feedbacks relies on simple "low-order" models which retain only a few physical processes and wave components. These models allow us to isolate the dominant mechanisms that control planetary wave behavior. This approach to the problem will be developed in this study.

OBJECTIVES

The objectives of this study were to:

1) develop a conceptual model for the generation of planetary-scale waves and their interaction with synoptic-scale waves or eddies

2) develop an idealized, quantitative low-order model to simulate the basic feedbacks of this system

3) develop a computer code to solve the equations of the quantitative model

4) compare the modeling with observed properties of the atmosphere and ascertain the need for space-based observations of certain atmospheric parameters related to these feedbacks or interactions.

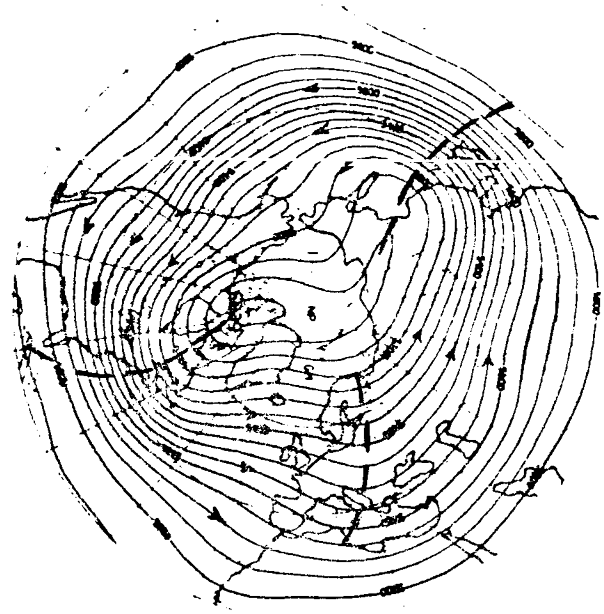


Fig. 1a

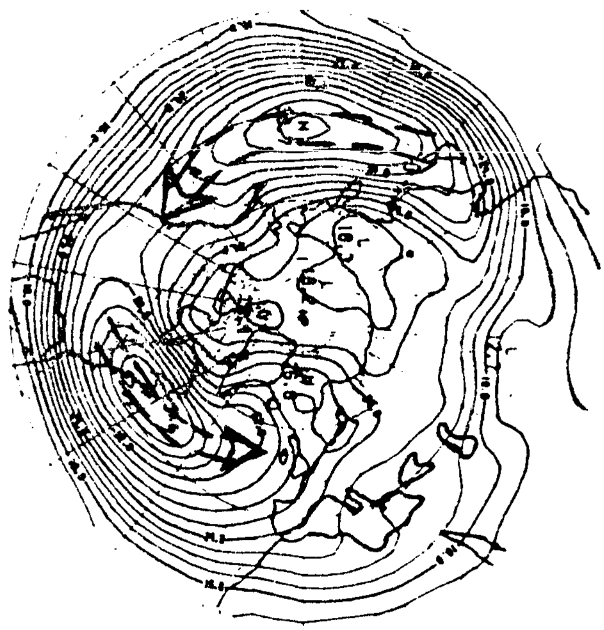


Fig. 1b

Fig. 1. a) Time-mean geopotential height of 500 mb (averaged over 9 winters). Proportional to sin(latitude)·streamfunction. Dashed lines show major stationary wave troughs.
b) Geopotential variance for synoptic-scale waves at 500 mb. Dashed arrows show major storm/cyclone tracks. (from Blackmon, 1976).

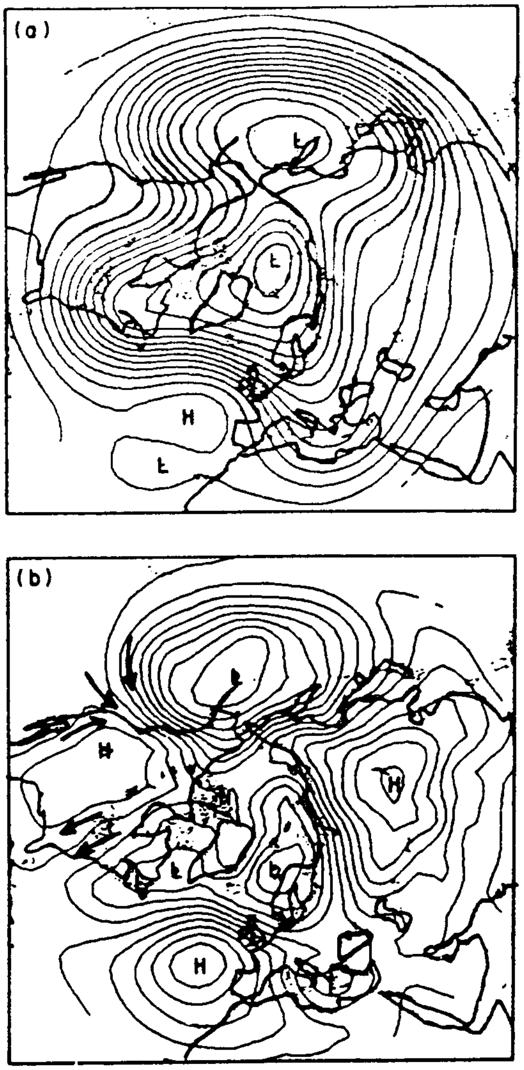


Fig. 2. a) Mean geopotential height for January 1981 at 500 mb. Note amplified ridge-trough-ridge pattern over N. America-Atlantic. b) As in a. except for 1000 mb (similar to surface pressure). Arrows show direction of near-surface flow over N. America. This pattern led to unusually cold January in the Eastern U.S. with warm, dry weather along the Pacific coast of N. America. (from Wallace and Blackmon 1983).

BACKGROUND

1. The role of transient eddies in the global circulation

The latitudinal variation of incoming solar radiation, primarily in the visible wavelengths, produces a latitudinal temperature gradient with the tropics being warmer than high latitudes. This contrast in heating introduces "available" potential energy into the atmospheric system and provides the energy for atmospheric and, consequently also, oceanic motions. As with many fluid systems with spatial contrasts of some parameter related to energy (here, the relationship is between temperature contrast and the potential energy in a hydrostatic fluid), eddy motions appear, esp. through an instability process, and attempt to eliminate the gradient of the energetic parameter.

In the case of the atmosphere, this instability process is called baroclinic instability since it relies on the baroclinic nature of the fluid. The differential solar heating enhances the baroclinicity (essentially, the temperature contrast along a constant pressure surface), while the baroclinic eddies or waves attempt to eliminate the thermal contrast by transporting sensible heat down the temperature gradient. The waves derive their energy from the energy which is introduced into the system by the insolation.

The overall heat balance of the global atmosphere is depicted in Fig. 3. Relatively large amounts of solar radiation are absorbed by the earth in low latitudes. The baroclinic waves, which are the same transient waves discussed in the Introduction, transport heat poleward. At high latitudes the energy is emitted to space in the form of infrared radiation. These high latitudes are actually warmer than they would be if no heat was transported poleward and the temperature was determined only as a result of a purely radiative balance at those latitudes. In fact, these latitudes emit more radiation (in the form of infrared radiation) than they absorb (in the form of solar radiation).

An additional characteristic of the baroclinic instability process is that these waves also transport angular momentum poleward. This transport plays an essential role in the maintenance of the observed latitudinal distribution of surface winds. Fig. 4 shows the basic angular momentum balance of the earth-atmosphere system. Middle (low) latitudes have westerly (easterly) surface winds.

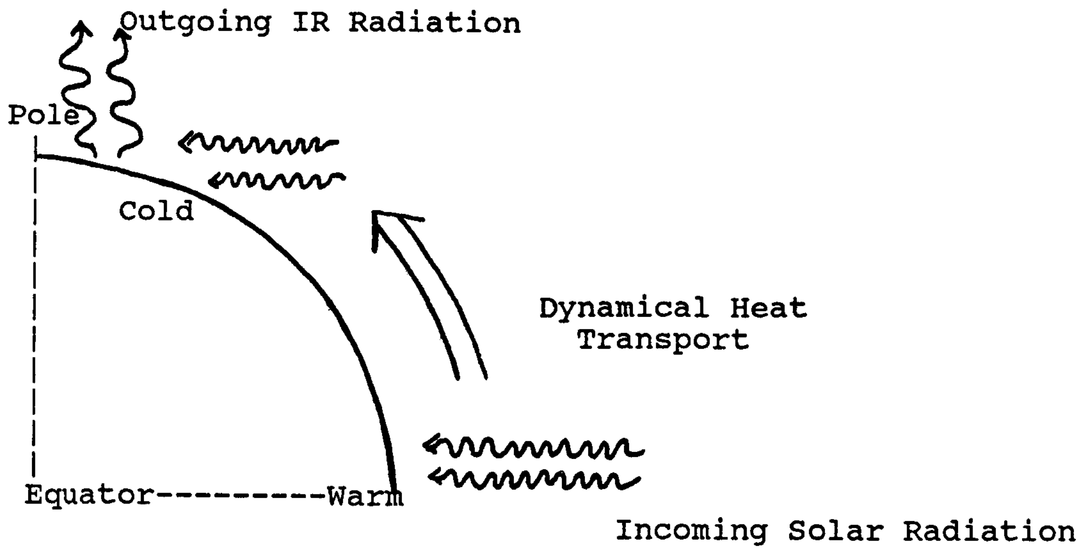


Fig. 3. Schematic diagram of global heat balance

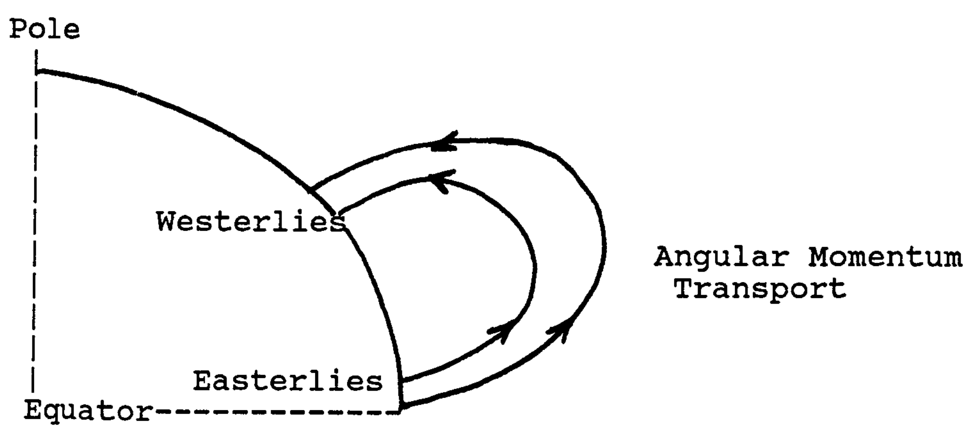


Fig. 4 Schematic diagram of atmosphere-Earth angular momentum balance

Because the atmosphere is rotating faster (slower) than the earth in middle (low) latitudes, it loses (gains) angular momentum to (from) the earth through frictional drag at the surface. The baroclinic waves are partly responsible for this surface wind distribution by providing the primary mechanism for the transport of angular momentum into mid-latitudes.

In addition, baroclinic eddies also transport water vapor poleward. From experience we know that southerly winds in advance of a cold front and associated cyclone are warm and humid, whereas the northerly winds behind the front are cold and dry. Averaged over the entire weather system, this correlation implies the northward transport of water vapor. The source of the water vapor is primarily evaporation from the subtropical ocean surface. By the Clausius-Clapeyron relation, the saturation vapor pressure increases rapidly with air temperature, allowing the warm southerly flow which obtains moisture from the subtropics to carry large amounts of water vapor poleward.

Part of the baroclinic instability process includes a correlation between upward motion and relatively warm air. This correlation allows potential energy existing in the thermal contrasts of the wave field to be converted into eddy or wave kinetic energy. A consequence of this uplifting of warm air in a moist atmosphere is adiabatic expansion and cooling of air parcels and, inevitably, condensation and precipitation. This precipitation is most evident in the cyclonic (counterclockwise rotation in the Northern Hemisphere) portion of the synoptic-scale wave. This condensation involves the release of latent heat and, therefore, introduces an additional heating mechanism, one that is dependent on the activity of the transient baroclinic waves.

We saw in Fig. 1b that the synoptic-scale, transient wave activity is largely confined to "storm tracks" over the northern oceans. Very reliable estimates of precipitation over the oceans are not available. We expect, however, that precipitation and latent heat release are maximized in these oceanic storm tracks, esp. since a ready source of moisture from the subtropical ocean is present. Thus, the maxima in latent heat release in these storm tracks introduces <u>longitudinal asymmetries</u> in diabatic heating.

In summary, baroclinic wave activity controls the strength and sign of the surface wind in a long-term mean. In addition, baroclinic waves extract potential energy from the larger-scale circulation and, in doing so, act to eliminate horizontal temperature contrasts of the larger scales. The storm systems of these waves produce precipitation and planetary-scale variations in diabatic heating averaged over several storm occurrences.

2. "Storm tracks"

As seen briefly in the Introduction and Fig. 1b, the planetary-scale circulation controls the movement and generation of the synoptic-scale waves. The nature of this control is not well-understood theoretically, but some general comments from observations and basic theory are possible.

First, the behavior of mid-latitude atmospheric waves with horizontal wavelengths of about 1000 km or longer is governed by "quasi-qeostrophic" dynamics (see Appendix for brief description of the quasi-geostrophic system). The phase speed of a linear wave in the presence of a background mean zonal (east-west) flow can be derived from the conservative forms of (A.1-3):

$$c, \text{ phase speed} = \bar{u} - \frac{\beta}{k^2 + l^2 + m^2 f^2/N^2} \qquad (1)$$

where $\bar{u}$ is the mean zonal velocity, k,l,m the zonal, latitudinal and vertical wavenumbers, respectively, and β the latitudinal derivative of the Coriolis parameter which, along with the remaining variable, N, is defined in the Appendix.

From (1) we see that as the wavenumbers increase, the wave phase speed is closer to the mean zonal velocity since the second term in (1) , which is always positive, becomes smaller. The shorter waves are more likely to be "steered" by the large-scale velocity field. Therefore, as evidenced by Fig. 1, the synoptic-scale waves are guided in their movement by the planetary-scale circulation. The planetary-scale waves are more likely to be slowly propagating or stationary. The second term which involves the wave's advection of planetary vorticity is larger for these waves and tends to cancel the mean flow advection of the wave vorticity as represented by the first term in (1).

An additional consideration is the generation of synoptic-scale disturbances. As mentioned above, they obtain their energy from the large-scale temperature contrasts. These contrasts are maximized in long-term time means at the base of the planetary-scale troughs. Thus, transient synoptic-scale disturbances appear to be preferentially generated in these regions and are then carried downstream by the large-scale velocity field.

3. Mid-latitude variability

Some observational and modelling analyses (e.g., Horel and Wallace, 1981; Pitcher et al., 1983) suggest that large-scale, mid-latitude circulation anomalies are strongly influenced by changes in atmospheric forcing from the earth's surface, esp. through fluctuations in tropical sea surface temperatures (SST's). SST anomalies from the climatological norm, esp., the elevated temperatures during the El Niño event, alter the pattern of tropical atmospheric heating and may induce considerable changes in the mid-latitude large-scale circulation.

This hypothesis omits the significant interaction of synoptic and planetary-scale waves in mid-latitudes. Recent modelling and observational studies indicate that persistent circulation anomalies, which last for two weeks or longer and are fixed in longitude (so-called "blocks"), are initiated, maintained and, perhaps, destroyed by transient, synoptic-scale disturbances (Dole, 1983; Mullen, 1985; Colucci, 1985). Indeed, the large-scale circulation often shifts dramatically from one persistent pattern to another within a few days. This rapid transition cannot be easily explained by the slow evolution of tropical SST's which transpires over a few months or more.

4. Directions for modelling interaction of synoptic and planetary waves

It will be our premise that synoptic-scale waves continuously interact with the planetary-scale circulation. To successfully model the planetary-scale waves over a few days or on a climatic basis, the effects of the synoptic-scale eddies must be included. In the following section we will consider mechanisms which are responsible for the generation of planetary-scale waves. Accounting for the nature of synoptic eddy transports as discussed above, we will develop a conceptual model for the interaction of these two scales of motion.

Our direction in this project will be to understand the interactions of the two scales in a climatological sense. That is, we are interested not in a short-range prediction of the circulation, but, instead, in what mechanisms operate in the time mean to maintain the observed pattern of stationary planetary waves seen in Fig. 1a and the time-mean transports by the synoptic-scale eddies.

CONCEPTUAL MODEL

1. Mechanisms for generating planetary waves

a. Topography

Two mechanisms for generating planetary-scale waves are considered here in the context of the preceding discussion. First, steady surface flow over large-scale topography (e.g., Rockies, Himalayas) will generate stationary planetary-scale waves. This concept has been treated extensively (one of the earliest papers was by Charney and Eliassen, 1949; a recent excellent review is given by Held, 1983) though not in the context of interaction with synoptic-scale eddies.

Fig. 5 shows the basic idea in terms of the quasi-geostrophic vorticity equation (A.1) which accounts for changes in the absolute vorticity of a fluid parcel in the presence of vertical motion. Westerly flow over a mountain barrier decreases (increases) the vorticity on the upwind (lee) side by the vertical contraction (stretching) of vertically-oriented vortex tubes. The reduced (increased) vorticity on the upwind (lee) side creates an anticyclonic ridge (cyclonic trough), forming a standing wave pattern.

The forcing for this wave depends on the strength of the vertical velocity created by the surface flow over the topography. Specifically, the vertical velocity (i.e., the velocity component parallel to the gravitational force) at the earth's surface is given by the kinematic boundary condition:

$$W(z=h) = \underset{\sim}{V_h} * \nabla h \qquad (2)$$

where $h=h(x,y)$ is the topographic height and $\underset{\sim}{V_h}$ is the surface horizontal velocity.

As noted in the Background section, the time-mean surface wind distribution is largely controlled by the angular momentum transport by baroclinic transient (i.e., synoptic-scale) waves. Therefore, these waves have an indirect influence on the strength of the topographically-forced stationary waves. Thus, a key element of any interactive model will be this angular momentum transport.

b. Diabatic heating

Longitudinal asymmetries in diabatic heating, as represented by the non-conservative term, Q, in (A.2), can also induce stationary planetary waves. In the presence of a

MECHANISMS FOR GENERATING PLANETARY WAVES

Flow over Topography

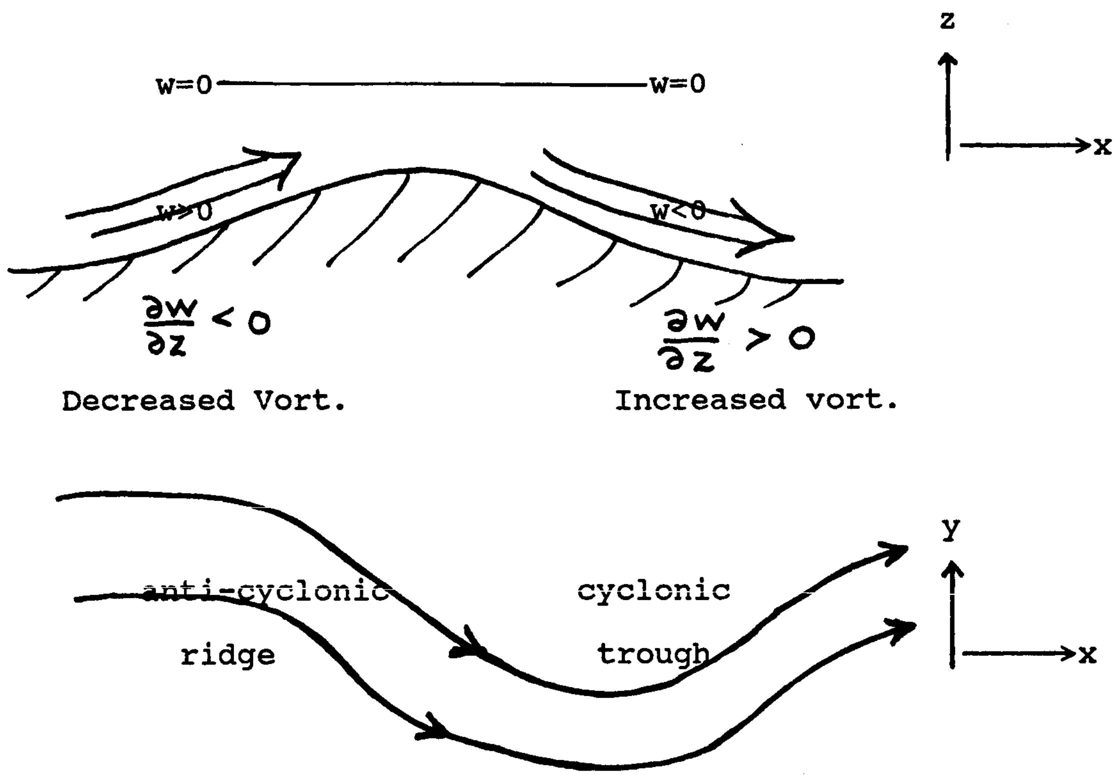


Fig.5. Forcing of standing planetary wave by steady westerly flow over large-scale topography

wave-like perturbation of Q, the dominant balance in the thermodynamic equation (A.2) for time-mean planetary waves is between mean zonal advection of the wave's temperature pertubation and the diabatic heating (Held, 1983). This balance is depicted in Fig. 6 which shows that the temperature field leads the diabatic heating field by roughly 90°.

The thermal-wind equation (A.3) for this hydrostatic, geostrophic motion shows that a relatively cold region is associated with a decrease of the streamfunction with height. For a wave-like perturbation this implies an increase of the Laplacian of the streamfunction (i.e., the relative vorticity) with height. Thus, a trough or cyclonic circulation intensifies with altitude in a cold region.

The appearance of longitudinal asymmetries in precipitation in mid-latitudes encourages the formation of stationary planetary waves through this mechanism. The strength of this forcing is dependent on the longitudinal distribution of moisture and cyclone (synoptic-scale wave) activity. Again, interaction between the two types of waves is possible through a indirect means.

c. Damping by synoptic-scale eddies

As suggested above, synoptic-scale eddies play a crucial role in the generation of planetary waves. On the other hand, these waves act to destroy temperature contrasts and will damp the planetary waves through the heat flux occurring in the baroclinic instability process. This effect must also be incorporated in an interactive model. The observational analysis of Holopainen (1983) indicates that this damping mechanism is an important means of limiting the planetary-wave amplitude.

2. Feedback diagram

Fig. 7 shows a schematic diagram which describes the means by which planetary waves are forced and interact with the transient baroclinic waves. It is not intended to be a complete diagram of the general circulation processes of the atmosphere, but many important elements are included. To a limited extent, one can picture the feedbacks as exchanges of energy from one form or one horizontal scale to another. The arrow directly connecting the stationary planetary waves with synoptic transient waves represents the loss of energy to these smaller scales through the thermal damping of the baroclinic instability process.

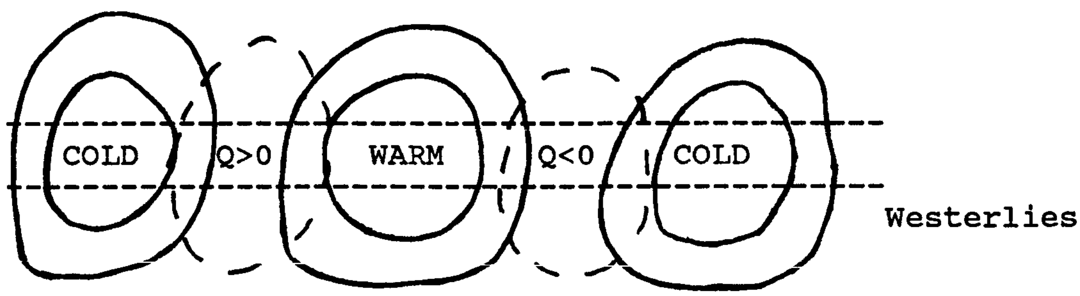
MECHANISMS FOR GENERATING PLANETARY WAVES

Diabatic Heating

Dominant balance in thermodynamic equation for these waves:

$$[u] \frac{\partial}{\partial x}\left(\frac{g\theta'}{\theta_o}\right) = Q$$

advection of temp. wave by westerly flow — heating/cooling



Cold regions (i.e., $\frac{\theta'}{\theta_o} < 0$) ===> $-\frac{\partial \psi}{\partial z}$ ===> $\frac{\partial \nabla^2 \psi}{\partial z} > 0$

Relative vorticity increases with altitude (i.e., cyclonic circulation intensifies with height).

Fig. 6. Forcing of planetary waves by longitudinal asymmetries in diabatic heating

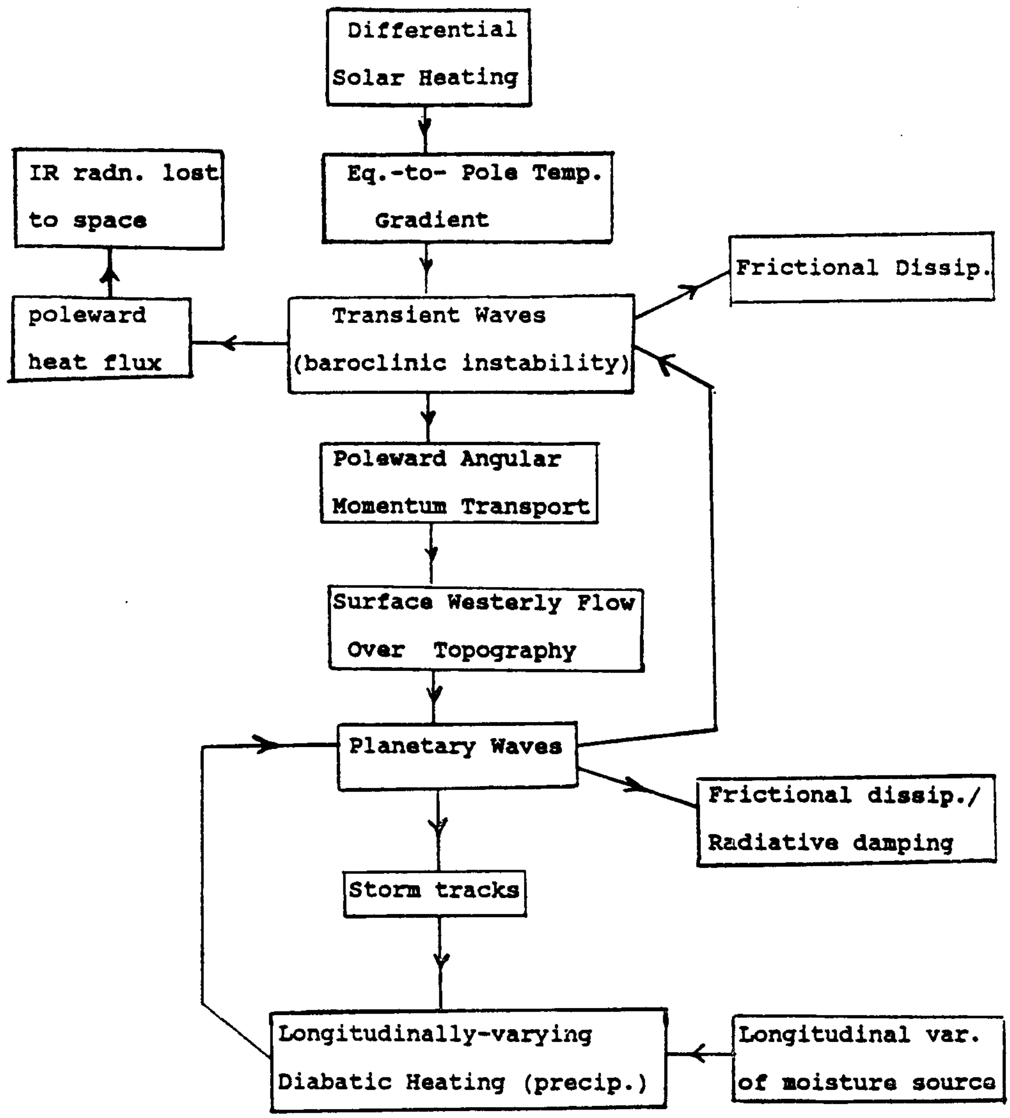


Fig. 7. Feedback diagram summarizing conceptual model.

QUANTITATIVE "LOW-ORDER" MODEL

1. Related studies

The approach will follow similar low-order models which retain only a limited number of horizontal and vertical modes in a spectral representation of the evolving fields. For example, Charney and Straus (1980) used a low-order model to examine the nonlinear equilibrium states of the zonally-averaged zonal flow and a single zonal wave that exist in presence of radiative forcing, Ekman dissipation and planetary-scale topography. An extension of their study (Reinhold and Pierrehumbert, 1982) included an additional, shorter wave and showed that synoptic-scale waves play an essential role in the types of planetary-scale wave configurations that can occur.

The formulation here is similar to a study by White and Green (1982) who also examined the forcing of planetary-scale waves in the presence of transient eddy fluxes. Like this project they also parameterized the effects of the transient eddies rather than expliciting calculating individual synoptic-wave events. However, serious problems arise as a result of their physical modelling.

They exclude diabatic heating caused by latent heat release in transient eddies and specify a heating wave which is independent of evolving flow parameters (i.e., non-interactive forcing). We anticipate considerable interaction between the planetary-scale circulation and the transient eddies responsible for the latent heating.

Their parameterization of the eddy sensible heat flux is quasi-linear and diffusive and damps the shortest modes most strongly. This mechanism reduces the amplitudes of waves which are primarily forced by topography and allows a dominance of the longest modes (planetary wavenumber one) which are primarily forced by diabatic heating. An improved parameterization is used here which depends on the instability characteristics of the large-scale circulation and does not preferentially damp shorter scales.

Lastly, radiative heating is specified in their model and is independent of the temperature field. Therefore, no radiative damping of the planetary-scale thermal waves is included. In addition, poleward dynamical heat flux is effectively predetermined by the specification of radiative heating, prohibiting interaction between transient eddy heat flux and the radiative heating.

2. Basic equation and spectral representation

We will combine many aspects of the conceptual model in a quantitative "low-order" model of the general circulation. First, we use (A.1-3) to find a single equation for ψ, i.e., the quasi-geostrophic potential vorticity equation,

$$\frac{d}{dt}\left\{ \nabla^2\psi + f + f^2 \frac{\partial}{\partial z}\left(N^{-2}\frac{\partial\psi}{\partial z}\right)\right\} = \mathcal{F} + f\frac{\partial}{\partial z}\left(QN^{-2}\right) \qquad (3)$$

The terms on the right-hand side of (3) represents sources/sinks. Note that, in the absence of these terms, the quantity in the brackets (quasi-geostrophic potential vorticity) is conserved with the motion.

Eq. (3) is a nonlinear PDE and has no general closed-form, analytic solutions. First, we model the vertical variation of ψ in (3) by assuming

$$\psi(x,y,z,t) = \psi_B(x,y,t)\,F_0(z) + \psi_T(x,y,t)\,F_1(z) \qquad (4)$$

where the functions F_0 and F_1 are shown in Fig. 8.

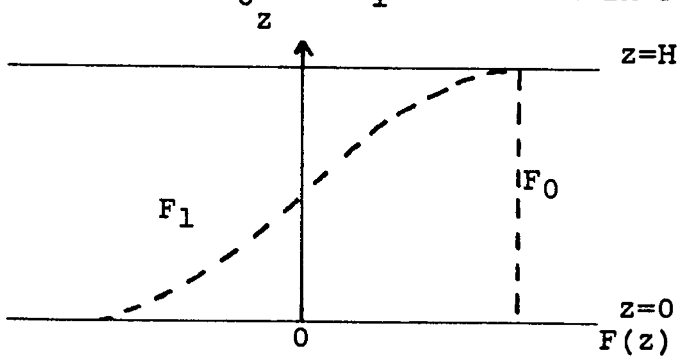


Fig. 8 Vertical modes, $F_i(z)$

The barotropic component of the streamfunction, ψ_B, has no temperature gradients associated with it, whereas the baroclinic component, ψ_T, has horizontal temperature gradients, even at the upper and lower boundaries of the model. This latter feature makes this choice of vertical modes more realistic for atmospheric modelling than the standard two-mode approach (used primarily in ocean models) which has no boundary temperature gradients. This modal representation produces two equations for ψ_B and ψ_T that are mathematically analogous to the standard two-level model of Phillips (1954) that is widely used in atmospheric modelling. The modal formulation has the advantage of easily projecting vertically continuous parameterizations of transient eddy heat and momentum fluxes and latent heating onto each vertical mode. It also ensures that energy is conserved in the presence of topographic forcing and is lost through Ekman damping (properties not necessarily guaranteed in a two-level model).

By substituting (4) into (3) and projecting the equation onto the two vertical modes, we find evolution equations for ψ_B and ψ_T. Upper and lower boundary terms, which appear upon taking the projections, can be expressed in terms of the vertical velocity at the boundaries. We assume that W(z=H)=0 and W(z=0) will be determined by flow over topography and Ekman pumping. These equations are

$$\frac{\partial}{\partial t}\nabla^2\psi_B + J(\psi_B,\nabla^2\psi_B) + J(\psi_T,\nabla^2\psi_T) + \beta\frac{\partial\psi_B}{\partial x} = \langle \mathcal{F} \rangle - \frac{fW(0)}{H} \tag{5}$$

$$\frac{\partial}{\partial t}(\nabla^2\psi_T - \lambda^2\psi_T) + J(\psi_T,\nabla^2\psi_B) + J(\psi_B,\nabla^2\psi_T - \lambda^2\psi_T) + \beta\frac{\partial\psi_T}{\partial x} = -\frac{fW(0)F_1(0)}{H} + \langle \mathcal{F} F_1\rangle - (f/N)^2\langle Q\,dF_1/dz\rangle \tag{6}$$

where J is the x-y Jacobian, < > indicates vertical average and $\lambda^2 = 12.4\ (f/NH)^2$.

The horizontal structures of ψ_B and ψ_T are represented by an eigenmode expansion in a periodic channel centered in mid-latitudes:

$$\psi_B = \sum_i \Psi_i(t)\ G_i(x,y) \qquad \psi_T = \sum_i \Theta_i(t) G_i(x,y)$$

where the eigenmodes are found from the equation $\nabla^2 G_i = -a_i^2\ G_i$ with G_i periodic in x over the length L_x, $dG_i/dx=0$ and $d[G_i]/dy=0$ at $y=0,L_y$. The brackets indicate a zonal (x) average. The latter two conditions ensure no meridional velocity or stress at the northern and southern sidewalls of the channel.

We truncate the series in G to resolve only the longitudinal mean flow and planetary scales. The effects of synoptic-scale waves will be parameterized. For initial calculations we retain only 6 of the gravest orthonormal modes:

$G_1 = 2\cos y'$ $\qquad$ $G_2 = 2\cos 2y'$
$G_3 = 2\sin nx \sin y'$ $\qquad$ $G_4 = 2\cos nx \sin y'$

$G_5 = 2 \sin nx \cos 2y'$ $\quad G_6 = 2 \cos nx \sin 2y'$

where $y' = \pi y/L_y$ and $n = 2\pi/L_x$. In effect, this truncation leads to a 12-parameter model for the six components of ψ_B and ψ_T.

3. Vertical velocity at the lower boundary

The vertical velocity, W(z=0), is given by (2), representing the topographic forcing, plus an additional "Ekman pumping" term. The latter term is due to vertical motions induced by convergence in the frictional boundary layer near the surface. Convergence (divergence) in the boundary layer occurs when the interior geostrophic flow has cyclonic (anticyclonic) vorticity. The convergence (divergence) induces upward (downward) motion at the top of the boundary layer which decreases (increases) the interior geostrophic vorticity by the vertical contraction (stretching) of vortex tubes in the interior (see (A.1)). The total vertical velocity is

$$W(z=0) = J(\psi(z=0), h) + k\nabla^2\psi(z=0)$$

where $k = (A_v/2f)^{1/2}$ and A_v is the eddy viscosity.

4. Radiative heating

We parameterize radiative processes by linear relaxation to the temperature of radiative equilibrium, T_e, i.e., the temperature field in the absence of dynamical heat transports. T_e is assumed to be independent of longitude (x) with its largest value at the southern boundary, decreasing montonically to the north. The difference in T_e across the channel is specified and determines the strength of the differential solar heating. Larger values will force stronger dynamical heat transports. However, as the transports increase, the temperature contrast across the channel will decrease and reduce the energy available to the baroclinic transient eddies, hindering further poleward heat flux. The increased temperature in high latitudes leads to stronger radiative cooling and a restoration of the meridional temperature gradient which drives more eddy heat transport. Thus, the heat flux and radiative processes form an interactive system.

The zonal-mean temperature field (equivalently, the ψ_T field; see (A.3)) relaxes to T_e in the absence of dynamical heat fluxes on a time scale of τ, usu. taken as 20 days. The thermal field of the planetary wave is radiatively damped on the same time scale. These radiative terms appear in the source term, Q.

5. Topography

As a initial test, we limit the topography to the G_4 mode. That is, we take $h(x,y)=h_0G_4(x,y)$ where $4h_0$ is the height difference between the lowest and highest points, typically, 3-4 km. Later versions of the model will examine topographic forcing on various scales.

6. Parameterization of transient eddy fluxes

a. Eddy source terms

The remaining contributions to the source/sink terms in (5) and (6) originate from unresolved transient-eddy activity. Those terms are

$$\mathcal{F} = -\nabla * (\overline{\tilde{v}'\zeta'})$$

$$Q = -\nabla * (\overline{\tilde{v}'\partial\psi'/\partial z}) + Q_c$$

where ζ is relative vorticity and $\overline{(\ )}$ is a time average over a period long compared to the synoptic time scale (2-6 days) and the prime is the deviation from that average. Thus, the correlations represent synoptic-eddy fluxes of vorticity and heat. The quantity, Q_c, is the latent heating due to eddy activity.

b. Heat flux

The approach here will follow Branscome's (1983) parameterization of meridional eddy heat flux which is based on baroclinic instability theory on a mid-latitude beta-plane. This parameterization is generalized to a two-dimensional heat-flux vector. Observing the down-gradient nature of the heat flux, we write this formula as

$$\overline{\tilde{v}'\theta'} = -\{AgNH^2/(\theta f^2)\}\ |\nabla\theta|\ \nabla\theta\ (1+\Upsilon)^{-2}\exp(-z/d)$$

where $\Upsilon = \beta H\theta N^2/(fg|\nabla\theta|)$, a dimensionless related to the instability of the baroclinic flow; $d= H/(1+\Upsilon)$, the depth of heat flux; and A=0.6, a correlation constant determined by instability theory and/or observations (see Branscome, 1983). The parameters inside the brackets are assumed constant in the model, while the remaining factors vary in x, y, and t as the thermal gradient (i.e., ψ_T) in the planetary-scale circulation evolves. After relating θ to ψ_T this parameterization enters (6) through the Q term. Though the heat flux is down-gradient, this formulation is highly nonlinear (not linear diffusion) and projections of the full flux field onto each G_i mode of ψ_T must be made.

c. Vorticity flux

The primary influence of the vorticity flux in the dynamics of the planetary-scale circulation comes through the meridional transport. This component is responsible for the poleward angular momentum transport which drives the surface zonal wind. The zonal flux acts to damp the planetary-scale stationary waves, but it is less effective than the transient eddy heat flux (apparent in analysis of Holopainen, 1983). Therefore, we model only the zonal-mean meridional transport, i.e., we take

$$\mathcal{F} = - \partial[\overline{v'\zeta'}]/\partial y.$$

It can be shown from the definition of the potential vorticity that

$$[v^*q^*] = [v^*\zeta^*] + (f/N)^2\, \partial[v^*\partial\psi^*/\partial z]/\partial z \qquad (7)$$

where * indicates deviation from the zonal mean and q is the potential vorticity as defined in (3). Furthermore, we find $[v^*\zeta^*] = - \partial[u^*v^*]/\partial y$. Integrating (7) in y and z and imposing the condition that $[u^*v^*]=0$ at $y=0,L_y$, we find

$$\int_0^H \int_0^{L_y} [v^*q^*]\, dy\, dz = \int_0^{L_y} (f/N)^2 [v^*\partial\psi^*/\partial z]\Big|_0^H dy. \qquad (8)$$

That is, the volume integral of potential vorticity flux is proportional to the difference of the upper and lower boundary integrals of the heat flux.

Next, we assume that the potential vorticity flux is down-gradient. This assumption is a good one for baroclinically unstable waves as seen from observations and theory (Edmon et al., 1980). For quasi-geostrophic motion this flux can then be written as $[v^*q^*] = -K_q\, \partial[q]/\partial y$ where K_q is a mixing coefficient of potential vorticity. Here we assume that $K_q=K_0 \sin y'$ so that no potential vorticity flux occurs at the sidewalls. This choice maximizes the mixing in the center of the channel where the meridional temperature gradient is strongest, i.e., where we expect maximum eddy activity. The amplitude, K_0, of the mixing coefficient is determined by substituting the down-gradient q-flux into the constraint (8). The mixing coefficient depends on the strength of the heat flux and a volume-average potential vorticity gradient:

$$K_0 = \frac{- \int_0^{L_y} (f/N)^2\, [v^*\partial\psi^*/\partial z]\Big|_0^H dy}{\int_0^H \int_0^{L_y} \partial[q]/\partial y\, \sin y'\, dy\, dz}$$

After finding K_0, which is dependent on the evolving fields, we can find $[v^*\zeta^*]$ from (7). Treating these quantities as time, as well as zonal, means, we can find an expression for $\mathcal{F}$. The flux $[v^*\zeta^*]$ includes both the transient eddy flux, $[v'\zeta']$, and stationary eddy flux, $[v^*\zeta^*]$. Of course, we are only parameterizing the former quantity since the latter is explicitly computed in the model as part of the predicted field of ψ.

d. Condensational heating

The flux of water vapor by transient eddies is $\overline{\underset{\sim}{v}'m'}$ where m is the specific humidity. The quantity, m, is a strong function of temperature and here we assume $m=m(T)$. That is, we assume that variations of specific humidity in a transient eddy are entirely dependent on temperature so that $m' = m(T)-m(\overline{T})=R_h(m_s(T)-m_s(T))$ where R_h is the relative humidity and m_s is the saturated value of m. Then, m_s can be found from the Clausius-Clapeyron equation. The relative humidity is assumed to depend on geographic location only so that we can introduce a global-scale variation representing the continent-ocean moisture contrasts.

The water vapor flux can be related to the eddy heat flux by expanding m_s' in terms of temperature deviation so that

$$\overline{\underset{\sim}{v}'m'} = R_h \frac{\partial m_s}{\partial T}(\overline{T})\ \overline{T'\underset{\sim}{v}'}.$$

A similar expression was derived by Leovy (1973) for use in planetary atmospheres modelling.

We are interested in modelling latent heat release in regions of precipitation. Thus, we only consider regions where convergence of $\underset{\sim}{v}'m'$ occurs, implying a loss of water vapor through condensation. In these regions, latent heat is released and we obtain only positive contributions to diabatic heating. We implicitly assume that evaporation and associated cooling has previously occurred at the ocean surface and does not affect the mid-latitude planetary-scale circulation. The contribution to diabatic heating is

$$Q_c = \frac{gL_v}{2fC_pT}\left(|\nabla * \overline{\underset{\sim}{v}'m'}| - \nabla * \overline{\underset{\sim}{v}'m'}\right)$$

where C_p is the specific heat of moist air and L_v latent heat of condensation. Note that Q_c is zero or positive.

7. Model equations

Without further approximation, we would obtain a set of 12 coupled nonlinear, first-order differential equations for the evolution of the components of ψ_B and ψ_T. However, these equations can be simplified by scaling (5)-(6) and neglecting small terms. First, scale ψ by UL and x and y by L where $U \sim 20$ m sec^{-1} and $L \sim 6000$ km for these scales of motion. The nonlinear and local derivative terms in (5) are small ($O(U/\beta L^2)=0.03$) compared to the advection of planetary vorticity by these waves (the $\beta \partial \psi_B/\partial x$ term). In (6) the terms involving the relative vorticity are also small by the same reasoning. However, the terms involving λ^2 must be retained since $U \lambda^2/\beta$ is O(1).

Thus, we arrive at six predictive equations for the spectral components of ψ_T and six diagnostic equations relating ψ_B and ψ_T. It is not instructive to present the equations for the individual spectral components. However, it is relevant to consider the basic forms in terms of the zonal mean and wave components.

The zonal-mean form of (5) is

$$< \mathcal{F} > - \frac{kf}{H} \frac{d^2}{dy^2}[\psi_o] - f\,[J(\psi_o^*, h)] = 0$$

where $\psi_o = \psi(z=0)$. This equation expresses a balance amongst transient eddy vorticity flux, Ekman dissipation of the zonal mean flow and mountain torque induced by the planetary wave. The wave form of (5) is

$$\beta \frac{\partial \psi_B^*}{\partial x} = \frac{-kf}{H} \nabla^2 \psi_o^* - \frac{f}{H} J([\psi_o], h)$$

which represents a balance amongst planetary vorticity advection by the waves, Ekman dissipation and forcing by zonal flow over topography.

The zonal-mean form of (6) is

$$\frac{\partial}{\partial t}[\psi_T] + [J(\psi_B^*, \psi_T^*)] = \frac{kf}{\lambda^2 H} \frac{d^2}{dy^2}[\psi_o]\, F_1(0) + \frac{f}{\lambda^2 H}[J(\psi_o^*, h)] + (f/N\lambda)^2 <[Q] dF_1/dz>$$

expressing changes in the zonal-mean temperature by planetary wave heat fluxes, adiabatic heating/cooling caused by zonal-mean vertical velocity that is induced by Ekman pumping and mountain torque, and zonal-mean diabatic heating. The contribution from the vertical variation of the eddy vorticity flux in (6) is small compared to the other terms and has been neglected.

The wave form of (6) is

$$\frac{\partial \psi_T^*}{\partial t} + J(\psi_B^*,[\psi_T]) + J([\psi_B],\psi_T^*) - \lambda^{-2}\beta \frac{\partial \psi_T^*}{\partial x} =$$

$$\frac{kf}{\lambda^2 H}\nabla^2\psi_o^* + \frac{f}{\lambda^2 H} J([\psi_o],h) + (f/N\lambda)^2 \langle Q^* dF_1/dz \rangle$$

in which the change in waves' thermal field is determined by wave advection of the mean temperature field, mean flow advection of the wave temperature field, adiabatic heating/cooling induced by vertical variation in planetary vorticity advection, Ekman pumping and zonal-mean flow over topography, and, lastly, longitudinally-varying diabatic heating. The latter term includes damping by thermal radiation and transient eddy sensible heat flux and forcing by longitudinal variation in latent heating.

These equations retain many fundamental feedbacks described in the conceptual model. The complete set of equations without the scaling approximations is also of interest and the simplified model should be compared with the complete formulation. For example, the simplified set does not allow for baroclinic instability of the planetary-scale waves. We have implicitly assumed that this process is not vital in generating the stationary planetary waves.

8. Current status and future plans

The code for numerically integrating the equations has been developed and is currently in the testing stage. Each process is being included in the model on an individual basis to test the code and examine the impact of the various processes on the circulation.

In the future, the model will be tested with modifications to the eddy flux parameterizations. It will also be expanded to allow a less severe spectral truncation. A seasonal variation of solar heating could also be investigated.

CONCLUSIONS AND RECOMMENDATIONS

This report describes a model for the interaction of planetary and synoptic-scale waves based on some basic observations and theory. The intention is to investigate the effects of transient eddy (synoptic-scale) transports on the planetary-scale waves. Much of the synoptic-scale activity occurs over the northern oceans which are areas of sparse ground-based observations. Our knowledge of cumulative precipitation over these regions is unreliable, although its effect on the planetary-scale circulation may be significant on a climatological and seasonal basis.

Models like the one described in this report will be useful in determining the impact of synoptic-scale waves on the larger-scale circulation. This model will complement other models with better spatial resolution and more elaborate physical parameterizations. The strengths of the low-order model are in its ease of interpretation and numerical efficiency in long-term or extensive sets of experiments.

The eventual goal is to ascertain and understand the dominant physical processes which operate in the extratropical, large-scale circulation. In doing so, we can identify what atmospheric parameters (e.g., precipitation over the oceans) must be observed by remote sensing techniques and what accuracy and frequency in the measurements must be achieved to allow successful modelling of the circulation.

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APPENDIX

Quasi-geostrophic Dynamics

The quasi-geostrophic approximation to the equations of motion is suitable for motions whose vertical component of relative vorticity, as determined by horizontal velocity in a reference frame rotating with the Earth, is small compared to the Earth's vorticity (twice the angular velocity, Ω). We scale the relative velocity by U and the horizontal gradients by L. Geostrophic balance between the Coriolis torque and pressure gradient force is the lowest-order diagnostic balance in the horizontal momentum equations if

$$\text{Rossby Number} = \frac{U}{2\Omega L} \ll 1.$$

At the next order we obtain a predictive equation for the vorticity of the geostrophic (lowest-order) horizontal velocity:

$$\frac{d}{dt}(\nabla^2\psi + f) = f\frac{\partial W}{\partial z} + \mathcal{F} \qquad \text{(A.1)}$$

where ψ is the geostrophic streamfunction, f planetary vorticity=$2\Omega \sin\phi$, ϕ latitude, W vertical velocity, z vertical co-ordinate, $\nabla^2\psi$ relative vorticity, t time and $\mathcal{F}$ a source/sink of vorticity. The total derivative involves advection by the horizontal geostrophic motion only. Eq. (A.1) states that changes in absolute or total vorticity are caused by vertical stretching of vortex tubes (a result of angular momentum conservation in a rotating system) and sources/sinks.

The potential temperature, θ, (the temperature an air parcel would have if brought to the surface adiabatically) is governed by the thermodynamic equation:

$$\frac{d}{dt}\left(\frac{g\theta'}{\theta_o}\right) + N^2W = Q \qquad \text{(A.2)}$$

where g is gravitational acceleration, $N=(g\, d\ln\theta_o/dz)^{1/2}$ the Brunt-Väisälä frequency, $\theta'$the deviation from the global mean θ_o, Q diabatic sources/sinks. For hydrostatic motion (an excellent approximation for these motions) the streamfunction and θ' are related by the thermal-wind equ.:

$$\frac{g\theta'}{\theta_o} = f\frac{\partial\psi}{\partial z} \qquad \text{(A.3)}$$