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SPACECRAFT ATTITUDE CONTROL MOMENTUM REQUIREMENTS ANALYSIS

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ABSTRACT

The relationship between attitude and angular momentum control requirements is derived for a fixed attitude, Earth orbiting spacecraft with large area articulating appendages. Environmental effects such as gravity gradient, solar radiation pressure, and aerodynamic forces arising from a dynamic, rotating atmosphere are examined. It is shown that, in general, each environmental effect contributes to both cyclic and secular momentum requirements both within and perpendicular to the orbit plane. The gyroscopic contribution to the angular momentum control requirements resulting from the rotating, Earth oriented spacecraft is also discussed. Special conditions are described where one or more components of the angular momentum can be made to vanish, or become purely cyclic. Computer generated plots for a candidate Space Station configuration are presented to supplement the analytically derived results.

GLOSSARY OF SYMBOLS

$\bar{a}$	(3×1) aerodynamic torque acting on spacecraft (LVLH coordinates)
A_i	body X, Y, and Z spacecraft projected areas
AMCD	Angular Momentum Control Device
C_D	spacecraft coefficient of drag
$\bar{F}_i^A$	(3×1) aerodynamic forces acting on the body YZ(i=1), XZ(i=2), and XY(i=3) planes
$\bar{F}_i^S$	(3×1) solar forces acting on the body YZ(i=1), XZ(i=2), and XY(i=3) planes
$\bar{g}$	(3×1) gravity gradient torque acting on spacecraft (LVLH coordinates)
$\bar{g}^A$	(3×1) gravity gradient torque due to articular part acting on spacecraft (LVLH coordinates)
$\bar{H}$	(3×1) spacecraft angular momentum (inertial coordinates)
$\bar{H}_A$	(3×1) aerodynamic angular momentum contribution (inertial coordinates)
$\bar{H}_C$	(3×1) control device angular momentum (inertial coordinates)
I	(3×3) identity matrix
IOP	in the orbit plane
$\mathbb{I}$	(3×3) spacecraft inertia tensor (inertial coordinates)
$\mathbb{I}_A$	(3×3) articular part inertia tensor (articular coordinates)
$\mathbb{I}_b$	(3×3) spacecraft inertia tensor (body coordinates)
$\mathbb{I}_{ij}$	spacecraft inertia tensor element (body coordinates)
LVLH	local vertical local horizontal

(GLOSSARY OF SYMBOLS cont'd.)

m_A	articular part mass
$\bar{p}$	(3×1) position vector from spacecraft cg to articular part cg (LVLH coordinates)
POP	perpendicular to the orbit plane
$\hat{r}$	(3×1) unit vector down (LVLH coordinates)
$\bar{r}_i$	(3×1) cp-cg offsets on the body YZ(i=1), XZ(i=2), and XY(i=3) planes
$\bar{s}$	(3×1) solar torque acting on spacecraft (LVLH coordinates)
t	time
$\bar{T}_A$	(3×1) aerodynamic torque acting on spacecraft (inertial coordinates)
$\bar{T}_C$	(3×1) control torque applied to spacecraft (inertial coordinates)
$\bar{T}_E$	(3×1) environmental torque acting on spacecraft (inertial coordinates)
$\bar{T}_G$	(3×1) gravity gradient torque acting on spacecraft (inertial coordinates)
$\bar{T}_S$	(3×1) solar torque acting on spacecraft (inertial coordinates)
T_a^b	(3×3) transformation matrix from articular part to body coordinates
T_b^L	(3×3) transformation matrix from body to LVLH coordinates
T_L^i	(3×3) transformation matrix from LVLH to inertial coordinates
$\hat{v}_i$	X, Y, and Z components of spacecraft unit velocity vector (body coordinates)

V	magnitude of spacecraft velocity vector
V_i	X, Y, and Z components of spacecraft velocity (LVLH coordinates)
ρ	atmospheric density
ψ	ordered Euler angle about body Z (yaw)
Θ	ordered Euler angle about body Y (pitch)
ϕ	ordered Euler angle about body X (roll)
$\bar{\omega}$	(3×1) spacecraft angular velocity (inertial coordinates)
ω_0	orbit rate

CHAPTER 1

INTRODUCTION

In response to a variety of proposed nominal attitude profiles to be flown by the Space Station, the NASA Langley Research Center (LaRC) Space Station Office (SSO) has undertaken an analysis of Space Station attitude equations of motion, environmental effects, and control requirements and implications.

When using angular momentum control devices (AMCD's) such as Control Moment Gyros (CMG's) to control attitude, it is necessary to examine the time history of the angular momentum requirements. Secular trends indicate a need for periodic desaturation or momentum "dumps." Peak values of cyclic components impact the CMG sizing requirements. Therefore, both terms are critical when specifying "optimal" attitudes from a controls point of view.

A variety of attitude profiles, most of which are labeled torque equilibrium attitude (TEA), have been presented to the Level B Systems Integration Board (SIB). These have included principal to body axis rotations, 2-axis (pitch, roll) angular attitudes, and 3-axis (pitch, roll, yaw) attitudes, all with associated momentum dumping strategies (continuous, discrete, periodic, gravity gradient assisted, etc.) One purpose of this study was to ascertain if constant TEA solutions could be utilized to eliminate or minimize frequent angular momentum dumping requirements, and what associated penalties, if any, are incurred. In order to accomplish this, an understanding of the contributions of gyroscopic, gravity gradient, aerodynamic, and solar effects on cyclic and secular angular momentum requirements was necessary. These effects were derived analytically, and demonstrated using the Articulated Rigid Body Control Dynamics (ref.1) (ARCD) module of the IDEAS^2 Software (ref. 2) to simuluate Space Station control requirements for different TEA solutions.

CHAPTER 2

DERIVATION OF THE CONTROL MOMENTUM REQUIREMENT

2.0 The General Equation (Inertial Coordinates)

The rotational equation of motion for a spacecraft in Earth orbit is given by

$$\dot{\bar{H}} = \sum \bar{T}_E + \bar{T}_C \tag{1}$$

where $\bar{H}$ = (3×1) spacecraft angular momentum,

$= \mathbb{I} \, \bar{\omega}$

$\mathbb{I}$ = (3×3) spacecraft inertia tensor,

$\bar{\omega}$ = (3×1) spacecraft angular velocity,

$\sum \bar{T}_E$ = (3×1) sum of environmental torques acting on the spacecraft,

$\bar{T}_C$ = (3×1) control torque applied to the spacecraft (e.g., to maintain a specified attitude).

For Earth oriented, fixed attitude spacecraft, the angular velocity $\bar{\omega}$ is constant in both magnitude (equal to the orbit rate ω_o) and direction (perpendicular to the orbit plane), so the time rate of change of the angular momentum with respect to inertial space is given by

$$\dot{\bar{H}} = \dot{\mathbb{I}} \, \bar{\omega} \tag{2}$$

With $\mathbb{I}(t)$ and $\bar{\omega}$ specified, once the environmental torques are computed over the orbit, Eq. (1) can be solved for the control torque $\bar{T}_C$ which must be provided by the spacecraft attitude control system.

The momentum equation governing an AMCD is given by

$$\dot{\bar{H}}_C = -\bar{T}_C \tag{3}$$

where $\bar{H}_C$ = (3×1) AMCD angular momentum.

Substituting Eqs. (1) and (2) into (3), defining the gyroscopic torque to be $\dot{\mathbf{I}}\,\bar{\omega}$, and assuming that the environmental torques consist of gravity gradient torque $\bar{T}_G$, aerodynamic torque $\bar{T}_A$, and solar torque $\bar{T}_S$, the angular momentum requirement at time t is given by

$$\bar{H}_C = \int_o^t \Big[\underbrace{-\dot{\mathbf{I}}\,\bar{\omega}}_{\text{gyroscopic}} \quad \underbrace{+ \bar{T}_G + \bar{T}_A + \bar{T}_S}_{\text{environmental}} \Big]\, d\tau \tag{4}$$

where Eq. (4) is expressed in inertial coordinates thus avoiding integration in a rotating coordinate frame.

2.1 Rotation From Local Vertical Coordinates

The terms on the right-hand side of Eq. (4) may be expressed in more meaningful coordinates by using transformations. If local vertical local horizontal (LVLH) coordinates are defined to have a Z-axis positive down toward the center of the Earth, Y-axis perpendicular to the orbit plane (positive in the minus orbit angular momentum direction), and X-axis completing the right-handed coordinate system (positive in the velocity vector direction), then a transformation T_L^i from LVLH to inertial coordinates is given by

$$T_L^i = \begin{bmatrix} \cos \omega_o t & 0 & -\sin \omega_o t \\ 0 & 1 & 0 \\ \sin \omega_o t & 0 & \cos \omega_o t \end{bmatrix} \tag{5}$$

Notice that at time zero, the LVLH coordinate frame coincides with the inertial coordinate frame. A transformation T_b^L from body to LVLH coordinates using a Z(yaw)-Y(pitch)-X(roll) ordered Euler transformation (ψ, θ, ϕ) is given by

$$T_b^L = \begin{bmatrix} c\theta c\psi & -c\phi s\psi + s\phi s\theta c\psi & s\phi s\psi + c\phi s\theta c\psi \\ c\theta s\psi & c\phi c\psi + s\phi s\theta s\psi & -s\phi c\psi + c\phi s\theta s\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \quad (6)$$

where $c\theta = \cos\theta$

$s\theta = \sin\theta$, etc.

At $(\psi,\theta,\phi) = (0,0,0)$ the body axis coincides with LVLH axis. The transformation T_L^i is a sinusoidal function of time. The transformation T_b^L is constant for a fixed attitude spacecraft.

CHAPTER 3

DERIVATION OF THE CONTROL MOMENTUM COMPONENTS

3.0 Gyroscopic Effect

Using the transformations developed in Section 2.1, the gyroscopic torque in inertial coordinates may be expressed as

$$\dot{\mathbb{I}}\,\bar{\omega} = \frac{d}{dt}\left[T_L^i\, T_b^L\, \mathbb{I}_b\, T_b^{L^T}\, T_L^{i^T} \right] \begin{bmatrix} 0 \\ -\omega_o \\ 0 \end{bmatrix} \tag{7}$$

where $\mathbb{I}_b$ = (3×3) spacecraft inertia tensor in body coordinates (which varies with time for an articulating spacecraft),

$$= \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$$

3.1 Environmental Effects

As was the case with the gyroscopic effects, the environmental effects are more easily derived using non-inertial coordinates. The environmental torques in inertial coordinates may be expressed as

$$\bar{T}_G = T_L^i\, \bar{g} = T_L^i \begin{bmatrix} g_x\,(\psi,\theta,\phi,t) \\ g_y\,(\psi,\theta,\phi,t) \\ 0 \end{bmatrix} \tag{8}$$

$$\bar{T}_A = T_L^i\, \bar{a} = T_L^i \begin{bmatrix} a_x\,(\psi,\theta,\phi,t) \\ a_y\,(\psi,\theta,\phi,t) \\ a_z\,(\psi,\theta,\phi,t) \end{bmatrix} \tag{9}$$

$$\bar{T}_S = T_L^i \, \bar{s} = T_L^i \begin{bmatrix} s_x \, (\psi,\theta,\phi,t) \\ s_y \, (\psi,\theta,\phi,t) \\ s_z \, (\psi,\theta,\phi,t) \end{bmatrix} \tag{10}$$

where $\bar{g}$, $\bar{a}$, and $\bar{s}$, are gravity gradient, aerodynamic, and solar torques in LVLH coordinates acting on the spacecraft, respectively. (Notice that, no gravity gradient torque can be generated about the LVLH Z-axis. Also, the X and Y components, g_x and g_y, are time independent for non-articulating spacecraft since $\mathbf{I}_b$ is constant). In LVLH coordinates,

$$\begin{bmatrix} g_x \\ g_y \\ 0 \end{bmatrix} = 3\omega_o^2 \; (\hat{r} \times T_b^L \, \mathbf{I}_b {T_b^L}^T \; \hat{r}) \tag{11}$$

where $\hat{r}$ = unit vector down in LVLH coordinates,

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The aerodynamic torque exerted on the vehicle is dependent upon the aerodynamic forces, coefficient of pressure - center of gravity (cp-cg) offsets, and the attitude. In LVLH coordinates,

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = T_b^L \;\; (\bar{r}_1 \times \bar{F}_1^A + \bar{r}_2 \times \bar{F}_2^A + \bar{r}_3 \times \bar{F}_3^A) \tag{12}$$

where $\bar{r}_1$, $\bar{r}_2$, $\bar{r}_3$ = (3×1) cp-cg offsets looking at the body YZ, XZ, and XY planes, respectively,

$\bar{F}_1^A$, $\bar{F}_2^A$, $\bar{F}_3^A$ = (3×1) time-dependent aerodynamic forces acting on the body YZ, XZ, and XY planes of spacecraft, respectively.

The aerodynamic forces acting on the spacecraft are given by

$$\bar{F}_i^A = -\tfrac{1}{2}\rho V^2 C_D A_i \left| \hat{v}_i \right| \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \hat{v}_3 \end{bmatrix} \tag{13}$$

where
ρ = atmospheric density,
V = magnitude of velocity vector,
C_D = spacecraft coefficient of drag,
A_1, A_2, A_3, = body X, Y, and Z spacecraft projected areas, respectively, and
$\hat{v}_1, \hat{v}_2, \hat{v}_3$ = X, Y, and Z components of unit velocity vector in body coordinates, respectively.

The LVLH aerodynamic torque's dependence on time is due to three effects: the change in atmospheric density due to the solar induced diurnal "bulge", time-dependent projected areas caused by spacecraft articulation, and the orbital inclination with respect to the rotating atmosphere.

For a non-rotating atmosphere, the time dependent aerodynamic force acting on a spacecraft is always along the LVLH X axis for circular orbits and hence time-dependent aerodynamic torques can be generated about the LVLH Y and Z axes only. In real life, a rotating atmosphere causes the aerodynamic force to oscillate slightly in the LVLH XY plane if the spacecraft orbit has a non-zero inclination. This gives rise to a small time-dependent a_x.

A similar expression for the solar torque may be derived as follows

$$\begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix} = T_b^L \left(\bar{r}_1 \times \bar{F}_1^S + \bar{r}_2 \times \bar{F}_2^S + \bar{r}_3 \times \bar{F}_3^S \right) \tag{14}$$

where $\bar{F}_1^S$, $\bar{F}_2^S$, $\bar{F}_3^S$ = (3×1) time-dependent solar forces acting on YZ, XZ, and XY planes of spacecraft, respectively.

The solar torques acting on the Space Station are relatively small compared to gyroscopic, gravity gradient, and aerodynamic torques at near-Earth orbiting altitudes.

3.2 The General Equation (Local Vertical Components)

Substituting Eqs. (5) through (14) into (4) and performing matrix multiplication gives the AMCD angular momentum control requirements for an articulating spacecraft flying at a fixed attitude in inertial coordinates as a function of time t.

$$\bar{H}_C(t) = \bar{H}_C(0) - \mathbb{I}(\tau)\Big|_0^t \bar{\omega} + \int_0^t (\bar{T}_G(\tau) + \bar{T}_A(\tau) + \bar{T}_S(\tau))\,d\tau$$

$$= \bar{H}_C(0) - \left[T_L^i(\tau)\, T_b^L\, \mathbb{I}_b(\tau)\, T_b^{L^T} T_L^{i^T}(\tau) \right]\Big|_0^t \bar{\omega}$$

$$+ \int_0^t T_L^i(\tau)\,(\bar{g}(\tau) + \bar{a}(\tau) + \bar{s}(\tau))\,d\tau$$

Thus,

$$\bar{H}_C(t) = \bar{H}_C(0) - \omega_0 \begin{bmatrix} f_1 \cos \omega_0\tau - f_2 \sin \omega_0\tau \\ f_3 \\ f_1 \sin \omega_0\tau + f_2 \cos_0\omega\, \tau \end{bmatrix}\Bigg|_0^t$$

$$+ \int_0^t \begin{bmatrix} (g_x + a_x + s_x)\cos \omega_0\tau - (a_z + s_z)\sin \omega_0\tau \\ g_y + a_y + s_y \\ (g_x + a_x + s_x)\sin \omega_0\tau + (a_z + s_z)\cos \omega_0\tau \end{bmatrix} d\tau \qquad (15a)$$

where

$$f_1 = c\theta s\psi \left[I_{xx}(c\theta c\psi) + I_{xy}(-c\phi s\psi + s\phi s\theta c\psi) + I_{xz}(s\phi s\psi + c\phi s\theta c\psi) \right]$$

$$+(c\phi c\psi + s\phi s\theta s\psi) \left[I_{xy}(c\theta c\psi) + I_{yy}(-c\phi s\psi + s\phi s\theta c\psi) + I_{yz}(s\phi s\psi + c\phi s\theta c\psi) \right]$$

$$+(-s\phi c\psi + c\phi s\theta s\psi) \left[I_{xz}(c\theta c\psi) + I_{yz}(-c\phi s\psi + s\phi s\theta c\psi) + I_{zz}(s\phi s\psi + c\phi s\theta c\psi) \right] \quad (15b)$$

$$f_2 = c\theta s\psi \left[I_{xx}(-s\theta) + I_{xy}(s\phi c\theta) + I_{xz}(c\phi c\theta) \right]$$

$$+(c\phi c\psi + s\phi s\theta s\psi) \left[I_{xy}(-s\theta) + I_{yy}(s\phi c\theta) + I_{yz}(c\phi c\theta) \right]$$

$$+(-s\phi c\psi + c\phi s\theta s\psi) \left[I_{xz}(-s\theta) + I_{yz}(s\phi c\theta) + I_{zz}(c\phi c\theta) \right] \quad (15c)$$

$$f_3 = c\theta s\psi \left[I_{xx}I_{xy} + I_{xy}I_{yy} + I_{xz}I_{yz} \right] + (c\phi c\psi + s\phi s\theta s\psi) \left[I_{xy}^2 + I_{yy}^2 + I_{yz}^2 \right]$$

$$+ (-s\phi c\psi + c\phi s\theta s\psi) \left[I_{xz}I_{xy} + I_{yz}I_{yy} + I_{yz}I_{zz} \right] \quad (15d)$$

$$g_x = 3\omega_0^2 \left\{ s\theta \left[I_{xx}(c\theta s\psi) + I_{xy}(c\phi c\psi + s\phi s\theta c\psi) + I_{xz}(-s\phi c\psi + c\phi s\theta s\psi) \right] \right.$$

$$-s\phi c\theta \left[I_{xy}(c\theta s\psi) + I_{yy}(c\phi c\psi + s\phi s\theta c\psi) + I_{yz}(-s\phi c\psi + c\phi s\theta s\psi) \right]$$

$$\left. -c\phi c\theta \left[I_{xz}(c\theta s\psi) + I_{yz}(c\phi c\psi + s\phi s\theta c\psi) + I_{zz}(-s\phi c\psi + c\phi s\theta s\psi) \right] \right\} \quad (15e)$$

$$g_y = 3\omega_0^2 \left\{ s\theta \left[I_{xx}(c\theta c\psi) + I_{xy}(-c\phi s\psi+s\phi s\theta c\psi) + I_{xz}(s\phi s\psi+c\phi s\theta c\psi) \right] \right.$$

$$-s\phi c\theta \left[I_{xy}(c\theta c\psi) + I_{yy}(-c\phi s\psi+s\phi s\theta c\psi) + I_{yz}(s\phi s\psi+c\phi s\theta c\psi)\right]$$

$$\left. -c\phi c\theta\left[I_{xz}(c\theta c\psi)+I_{yz}(-c\phi s\psi+s\phi s\theta c\psi)+I_{zz}(s\phi s\psi+c\phi s\theta c\psi)\right] \right\} \quad (15f)$$

$$a_x = c\theta c\psi \left[r_{1_y}F^A_{1_z} - r_{1_z}F^A_{1_y} - r_{2_z}F^A_{2_y} + r_{3_y}F^A_{3_z} \right]$$

$$+(-c\phi s\psi+s\phi s\theta c\psi) \left[r_{1_z} F^A_{1_x} - r_{2_x}F^A_{2_z} + r_{2_z}F^A_{2_x} - r_{3_x}F^A_{3_z} \right]$$

$$+(s\phi s\psi+c\phi s\theta c\psi) \left[-r_{1_y} F^A_{1_x} + r_{2_x}F^A_{2_y} + r_{3_x}F^A_{3_y} - r_{3_y}F^A_{3_x} \right] \quad (15g)$$

$$a_y = c\theta s\psi \left[r_{1_y}F^A_{1_z} - r_{1_z}F^A_{1_y} - r_{2_z}F^A_{2_y} + r_{3_y}F^A_{3_z} \right]$$

$$+(c\phi c\psi+s\phi s\theta s\psi) \left[r_{1_z} F^A_{1_x} - r_{2_x}F^A_{2_z} + r_{2_z}F^A_{2_x} - r_{3_x}F^A_{3_z} \right]$$

$$+(-s\phi c\psi+c\phi s\theta s\psi) \left[-r_{1_y} F^A_{1_x} + r_{2_x}F^A_{2_y} + r_{3_x}F^A_{3_y} - r_{3_y}F^A_{3_x} \right] \quad (15h)$$

$$a_z = -s\theta \left[r_{1_y}F^A_{1_z} - r_{1_z}F^A_{1_y} - r_{2_z}F^A_{2_y} + r_{3_y}F^A_{3_z} \right]$$

$$+s\phi c\theta \left[r_{1_z} F^A_{1_x} - r_{2_x}F^A_{2_z} + r_{2_z}F^A_{2_x} - r_{3_x}F^A_{3_z} \right]$$

$$+c\phi c\theta \left[-r_{1_y} F^A_{1_x} + r_{2_x}F^A_{2_y} + r_{3_x}F^A_{3_y} - r_{3_y}F^A_{3_x} \right] \tag{15i}$$

$$s_x = c\theta c\psi \left[r_{1_y}F^S_{1_z} - r_{1_z}F^S_{1_y} - r_{2_z}F^S_{2_y} + r_{3_y}F^S_{3_z} \right]$$

$$+(-c\phi s\psi + s\phi s\theta c\psi) \left[r_{1_z} F^S_{1_x} - r_{2_x}F^S_{2_z} + r_{2_z}F^S_{2_x} - r_{3_x}F^S_{3_z} \right]$$

$$+(s\phi s\psi + c\phi s\theta c\psi) \left[-r_{1_y} F^S_{1_x} + r_{2_x}F^S_{2_y} + r_{3_x}F^S_{3_y} - r_{3_y}F^S_{3_x} \right] \tag{15j}$$

$$s_y = c\theta s\psi \left[r_{1_y}F^S_{1_z} - r^S_{1_z}F_{1_y} - r_{2_z}F^S_{2_y} + r_{3_y}F^S_{3_z} \right]$$

$$+(c\phi c\psi + s\phi s\theta s\psi) \left[r_{1_z} F^S_{1_x} - r_{2_x}F^S_{2_z} + r_{2_z}F^S_{2_x} - r_{3_x}F^S_{3_z} \right]$$

$$+(-s\phi c\psi + c\phi s\theta s\psi) \left[-r_{1_y} F^S_{1_x} + r_{2_x}F^S_{2_y} + r_{3_x}F^S_{3_y} - r_{3_y}F^S_{3_x} \right] \tag{15k}$$

$$s_z = -s\theta \left[r_{1_y}F^S_{1_z} - r_{1_z}F^S_{1_y} - r_{2_z}F^S_{2_y} + r_{3_y}F^S_{3_z} \right]$$

$$+s\phi c\theta \left[r_{1_z} F^S_{1_x} - r_{2_x}F^S_{2_z} + r_{2_z}F^S_{2_x} - r_{3_x}F^S_{3_z} \right]$$

$$+c\phi c\theta \left[-r_{1_y} F^S_{1_x} + r_{2_x}F^S_{2_y} + r_{3_x}F^S_{3_y} - r_{3_y}F^S_{3_x} \right] \tag{15l}$$

and the f_i represent gyroscopic contributions to the angular momentum requirements and where the I_{ij} are time-dependent for articulating spacecraft.

CHAPTER 4

DISCUSSION OF THE CONTROL MOMENTUM REQUIREMENT COMPONENTS

4.0 General Comments Regarding Secular Momentum Build-Up

The secular angular momentum build-up over one orbit is simply $\bar{H}_C(2\pi/\omega_0) - \bar{H}_C(0)$ where $\bar{H}_C(0)$ represents the initial angular momentum of the AMCD. Secular momentum build-up (and hence momentum dumping) can be avoided if a (ψ,θ,ϕ) attitude can be found such that

$$\bar{H}_C(2\pi/\omega_0) - \bar{H}_C(0) = 0 \tag{16}$$

One interesting phenomenon which can be observed from Eq. (15a) is that a torque which is constant when expressed in body or LVLH coordinates yields only cyclic angular momentum requirements in the inertial X and Z coordinates, i.e., in the orbit plane (IOP) (since the integral from t = 0 to $2\pi/\omega_0$ of K cos $\omega_0 t$ and K sin $\omega_0 t$ equals zero).

4.1 Gyroscopic Component

The second term on the right hand side of Eq. (15a) is the gyroscopic momentum which must be absorbed by the AMCD. For articulating appendages whose motion equals orbit rate (a valid assumption for solar arrays and radiators), $\mathbf{I}(2\pi/\omega_0) = \mathbf{I}(0)$, $f_i(2\pi/\omega_0) = f_i(0)$, and thus it can be seen that gyroscopic effects can never yield secular momentum build-up. An articulating spacecraft in general has non-zero gyroscopic cyclic momentum both IOP and perpendicular to orbit plane (POP); however, for fixed spacecraft (no articulation) the f_i are constant, and hence the gyroscopic momentum will be cyclic IOP and identically zero POP.

Cyclic IOP momentum for non-articulating spacecraft may be eliminated completely by choosing an appropriate (ψ,θ,ϕ) attitude. If the body axes are chosen to be aligned with the principal axes so that all off diagonal inertia tensor terms are zero, then it can be seen that f_1 and f_2 will be identically zero for any $(\psi,\theta,\phi) = (0,\theta,0)$. This indicates that a non-articulating spacecraft has no gyroscopic momentum if flying at an attitude with any principal axis POP. Furthermore, a single degree of freedom (DOF) in pitch exists such that the gyroscopic momentum remains zero.

4.2 Gravity Gradient Component

The gravity gradient momentum contributions in Eq. (15a) are in general secular both IOP and POP due to the articulating parts which cause $\mathbf{I}_b$ to change with time. If no articular parts are present then g_x is constant and the gravity gradient momentum will be cyclic IOP (about x local horizontal only) and secular POP.

If an articular part rotates about the articular part cg at orbital rate, the secular gravity gradient momentum contributed by the articular part may be derived as follows. The gravity gradient torque about the spacecraft cg due to the articular part in LVLH coordinates is given by

$$\begin{bmatrix} g_x^A \\ g_y^A \\ 0 \end{bmatrix} = 3\omega_0^2 \left\{ \hat{r} \times \left[T_b^L T_a^b \mathbb{I}_A T_a^{b^T} T_b^{L^T} + m_A(\bar{p}^T \bar{p}\, I - \bar{p}\, \bar{p}^T) \right] \hat{r} \right\} \tag{17}$$

where T_a^b = (3×3) transformation from articular part to body coordinates,

$\mathbb{I}_A$ = (3×3) articular part inertia tensor in articular coordinates,

m_A = articular part mass,

$\bar{p}$ = (3×1) position vector from spacecraft cg to articular part cg (LVLH coordinates),

I = (3×3) identity matrix.

The transformation T_a^b is dependent on the motion of the articular part. For example, if the articular part rotates about the body Y axis at orbital rate, as might be the case for a solar array, then the transformation from articular part to body coordinates is given by

$$T_a^b = \begin{bmatrix} \cos \omega_0 t & 0 & \sin \omega_0 t \\ 0 & 1 & 0 \\ -\sin \omega_0 t & 0 & \cos \omega_0 t \end{bmatrix} \tag{18}$$

The secular gravity gradient momentum due to the articulating part in inertial coordinates may be evaluated by substituting Eq. (17) into Eq. (15a) and letting $t = 2\pi/\omega_0$. In general, gravity gradient induced secular angular momentum due to part articulation will accumulate both IOP and POP. Pure cyclic momentum requirements IOP can occur only for the special case where all articular parts rotate at orbit rate about an axis coincident with an articular part principal axis and passing through the articular part cg.

For non-articulating spacecraft, the gravity gradient momentum contribution may be eliminated with an appropriate (ψ,θ,ϕ) solution. If body axes are aligned with principal axes then the following properties may be observed:

1) $g_x = 0$ for $(\psi,\theta,\phi) = (0,\theta,0)$ (see Eq. 15e)

2) $g_y = 0$ for $(\psi,\theta,\phi) = (0,0,\phi)$ (see Eq. 15f)

3) $g_x = 0$ and $g_y = 0$ for $(\psi,\theta,\phi) = (\psi,0,0)$

Property 1) indicates that IOP gravity gradient induced momentum requirement may be completely eliminated if any principal axis is aligned POP, with a DOF in pitch existing such that this momentum remains zero. Property 2) indicates that POP gravity gradient momentum may be completely eliminated if any principal axis is aligned with the $\hat{X}$ direction in LVLH coordinates, and a DOF in roll exists such that this momentum remains zero. Property 3) indicates that both IOP and POP gravity gradient momentum may be completely eliminated if any principal axis is aligned with the local vertical and a DOF in yaw exists such that the momentum is zero.

4.3 Aerodynamic Component

The aerodynamic momentum contribution is in general secular both IOP and POP since the LVLH aerodynamic torques change with time. The IOP momentum contribution can be made cyclic if a (ψ,θ,ϕ) solution can be found such that

$$\int_0^{2\pi/\omega_0} (a_x \cos \omega_0\tau - a_z \sin \omega_0\tau)\, d\tau = 0 \tag{19a}$$

$$\int_0^{2\pi/\omega_0} (a_x \sin \omega_0\tau + a_z \cos \omega_0\tau)\, d\tau = 0 \tag{19b}$$

(from Eq. 15a).

The aerodynamic momentum contribution for a spacecraft in a circular orbit may be derived as follows. The velocity vector $\bar{v}$ in body coordinates for a spacecraft in a circular orbit is given by

$$\bar{v} = T_b^{L^T} \begin{bmatrix} V_x \\ V_y \\ 0 \end{bmatrix} = \begin{bmatrix} V_x \, c\theta c\psi + V_y c\theta s\psi \\ V_x(-c\phi s\psi + s\phi s\theta c\psi) + V_y(c\phi c\psi + s\phi s\theta s\psi) \\ V_x(s\phi s\psi + c\phi s\theta c\psi) + V_y(-s\phi c\psi + c\phi s\theta s\psi) \end{bmatrix} \tag{20}$$

where V_x and V_y are the X and Y components of velocity in LVLH coordinates. Substituting Eq. (20) into Eq. (13) gives the aerodynamic forces acting on the spacecraft.

$$\bar{F}_1^A = -\tfrac{1}{2}\rho C_D A_1 \left| V_x c\theta c\psi + V_y c\theta s\psi \right| \begin{bmatrix} V_x c\theta c\psi + V_y c\theta s\psi \\ V_x(-c\phi s\psi + s\phi s\theta c\psi) + V_y(c\phi c\psi + s\phi s\theta s\psi) \\ V_x(s\phi s\psi + c\phi s\theta c\psi) + V_y(-s\phi c\psi + c\phi s\theta s\psi) \end{bmatrix} \tag{21a}$$

$$\bar{F}_2^A = -\tfrac{1}{2}\rho C_D A_2 \left| V_x(-c\phi s\psi + s\phi s\theta c\psi) + V_y(c\theta c\psi + s\phi s\theta s\psi) \right|$$

$$\times \begin{bmatrix} V_x c\theta c\psi + V_y c\theta s\psi \\ V_x(-c\phi s\psi + s\phi s\theta c\psi) + V_y(c\phi c\psi + s\phi s\theta s\psi) \\ V_x(s\phi s\psi + c\phi s\theta c\psi) + V_y(-s\phi c\psi + c\phi s\theta s\psi) \end{bmatrix} \tag{21b}$$

$$\bar{F}_3^A = -\tfrac{1}{2}\rho C_D A_3 \left| V_x(s\phi s\psi + c\phi s\theta c\psi) + V_y(-s\phi c\psi + c\phi s\theta s\psi) \right|$$

$$x \begin{bmatrix} V_x c\theta c\psi + V_y c\theta s\psi \\ V_x(-c\phi s\psi + s\phi s\theta c\psi) + V_y(c\phi c\psi + s\phi s\theta c\psi) \\ V_x(s\phi s\psi + c\phi s\theta c\psi) + V_y(-s\phi c\psi + c\phi s\theta s\psi) \end{bmatrix} \tag{21c}$$

Substituting Eqs. (21a-c) into Eqs. (15f) through (15h) gives the aerodynamic torque in LVLH coordinates acting on a spacecraft in a circular orbit.

$$a_x = \tfrac{1}{2}\rho C_D V_y \Big\{ A_1 \left| V_x c\theta c\psi + V_y c\theta s\psi \right| \Big[r_{1_y}\big[c\theta c\psi(-s\phi c\psi + c\phi s\theta s\psi) - (s\phi s\psi + c\phi s\theta c\psi)c\theta s\psi\big]$$

$$+ r_{1_z}\big[-c\theta c\psi(c\phi c\psi + s\phi s\theta s\psi) + (-c\phi s\psi + s\phi s\theta c\psi)c\theta s\psi\big]\Big]$$

$$+A_2 \left| V_x(-c\phi s\psi + s\phi s\theta c\psi) + V_y(c\phi c\psi + s\phi s\theta s\psi) \right| \Big[r_{2_x}\big[-(c\phi s\psi + s\phi s\theta c\psi)(-s\phi c\psi + c\phi s\theta s\psi)$$

$$+(s\phi s\psi + c\phi s\theta c\psi)(c\phi c\psi + s\phi s\theta s\psi)\big] + r_{2_z}\big[-c\theta c\psi(c\phi c\psi + s\phi s\theta s\psi)$$

$$+(-c\phi s\psi + s\phi s\theta c\psi)c\theta s\psi\big]\Big]$$

$$+A_3 \left| V_x(s\phi s\psi + c\phi s\theta c\psi) + V_y(-s\phi c\psi + c\phi s\theta s\psi) \right| \Big[r_{3_x}\big[-(-c\phi s\psi + s\phi s\theta c\psi)(-s\phi c\psi + c\phi s\theta s\psi)$$

$$+(s\phi s\psi + c\phi s\theta c\psi)(c\phi c\psi + s\phi s\theta s\psi)\big] + r_{3_y}\big[c\theta c\psi(-s\phi c\psi + c\phi s\theta s\psi)$$

$$-(s\phi s\psi + c\phi s\theta c\psi)c\theta s\psi\big]\Big]\Big\} \tag{22a}$$

$$a_y = \tfrac{1}{2}\rho C_D V_x \Big\{ A_1 \left| V_x c\theta c\psi + V_y c\theta s\psi \right| \Big[r_{1_y} \big[c\theta s\psi(s\phi s\psi + c\phi s\theta c\psi) - (-s\phi c\psi + c\phi s\theta s\psi)c\theta c\psi \big]$$

$$+ r_{1_z} \big[-c\theta s\psi(-c\phi s\psi + s\phi s\theta c\psi) + (c\phi c\psi + s\phi s\theta s\psi)c\theta c\psi \big] \Big]$$

$$+ A_2 \left| V_x(-c\phi s\psi + s\phi s\theta c\psi) + V_y(c\phi c\psi + s\phi s\theta s\psi) \right| \Big[r_{2_x} \big[-(c\phi c\psi + s\phi s\theta s\psi)(s\phi s\psi + c\phi s\theta c\psi)$$

$$+ (-s\phi c\psi + c\phi s\theta s\psi)(-c\phi s\psi + s\phi s\theta s\psi) \big] + r_{2_z} \big[-c\theta s\psi(-c\phi s\psi + s\phi s\theta c\psi)$$

$$+ (c\phi c\psi + s\phi s\theta s\psi)c\theta c\psi \big] \Big]$$

$$+ A_3 \left| V_x(s\phi s\psi + c\phi s\theta c\psi) + V_y(-s\phi c\psi + c\phi s\theta s\psi) \right| \Big[r_{3_x} \big[-(c\phi c\psi + s\phi s\theta s\psi)(s\phi s\psi + c\phi s\theta c\psi)$$

$$+ (-s\phi c\psi + c\phi s\theta s\psi)(-c\phi s\psi + s\phi s\theta c\psi) \big] + r_{3_y} \big[c\theta s\psi(s\phi s\psi + c\phi s\theta c\psi)$$

$$- (s\phi s\psi + c\phi s\theta c\psi)c\theta c\psi \big] \Big] \Big\} \tag{22b}$$

$$a_z = \tfrac{1}{2}\rho C_D V_x \Big\{ A_1 \left| V_x c\theta c\psi + V_y c\theta s\psi \right| \Big[r_{1_y} \big[-s\theta(V_x(s\phi s\psi + c\phi s\theta c\psi) + V_y(-s\phi c\psi + c\phi s\theta s\psi))$$

$$- c\phi c\theta(V_x c\theta c\psi + V_y c\theta s\psi) \big] + r_{1_z} \big[s\theta(V_x(-c\phi s\psi + s\phi s\theta s\psi) + V_y(c\phi c\psi + s\phi s\theta s\psi))$$

$$+ s\phi c\theta(V_x c\theta c\psi + V_y c\theta s\psi) \big] \Big]$$

$$+ A_2 \left| V_x(-c\phi s\psi + s\phi s\theta c\psi) + V_y(c\phi c\psi + s\phi s\theta s\psi) \right| \Big[r_{2_x} \big[-s\phi s\theta(V_x(s\phi s\psi + c\phi s\theta c\psi)$$

$$+V_y(-s\phi c\psi+c\phi s\theta s\psi))+c\phi c\theta(V_x(-c\phi s\psi+s\phi s\theta c\psi)+V_y(c\phi c\psi+s\phi s\theta s\psi))\Big]$$

$$+r_{2_z}\Big[s\theta(V_x(-c\phi s\psi+s\phi s\theta c\psi)+V_y(c\phi c\psi+s\phi s\theta s\psi))+s\phi c\theta(V_x c\theta c\psi+V_y c\theta s\psi)\Big]\Big]$$

$$+A_3\Big|V_x(s\phi s\psi+c\phi s\theta c\psi) + V_y(-s\phi c\psi+c\phi s\theta s\psi)\Big|\Big[r_{3_x}\Big[-s\phi c\theta(V_x(s\phi s\psi+c\phi s\theta c\psi)$$

$$+V_y(-s\phi c\psi+c\phi s\theta s\psi))+c\phi c\theta(V_x(-c\phi s\psi+s\phi s\theta c\psi)+V_y(c\phi c\psi+s\phi s\theta s\psi))\Big]$$

$$+r_{3_y}\Big[-s\theta(V_x(s\phi s\psi+c\phi s\theta c\psi)+V_y(-s\phi c\psi+c\phi s\theta s\psi))-c\phi c\theta(V_x c\theta c\psi+V_y c\theta s\psi)\Big]\Big]\Big\} \tag{22c}$$

For an articulating spacecraft, ρ, V_y, A_i, r_{ix}, r_{iy}, and r_{iz} are in general time-dependent (in the case of a non-articulating rigid spacecraft, only ρ and V_y change with time). The aerodynamic momentum contribution in inertial coordinates, $\bar{H}_A(t)$, is given by

$$\bar{H}_A(t) = \int_0^t \begin{bmatrix} a_x(\tau)\cos\omega_0\tau - a_z(\tau)\sin\omega_0\tau \\ a_y(\tau) \\ a_x(\tau)\sin\omega_0\tau + a_z(\tau)\cos\omega_0\tau \end{bmatrix} d\tau \tag{23}$$

where a_x, a_y, and a_z, are defined in Eq. (22). Techniques and special conditions resulting in zero secular momentum aerodynamic contributions are discussed in Section 5.

4.4 Solar Component

The solar momentum contribution is relatively small compared to the gyroscopic, gravity gradient, and aerodynamic momentum contributions for Space Station altitudes. Since LVLH solar torques change with time, the solar momentum contribution is secular both IOP and POP.

CHAPTER 5

RESULTS OF TEST CASES

In order to better understand the concept of momentum management, various attitude solutions for hypothetical test cases were considered using the ARCD program. The reference Initial Operational Capability (IOC) Space Station Configuration (ref. 3), as shown in Fig. 1, was chosen as the subject spacecraft. The station is designed to operate in an Earth-oriented attitude, with the dual keel aligned close to local vertical. Five comparatively massive modules (a U.S. habitation module, a U.S. laboratory module, a logistic module, a Japanese Experiment module, and a European Space Agency module) are located on a porch at the center of the station. A hybrid power supply of solar arrays and solar dynamic dishes articulate to track the sun, and radiators articulate to anti-track the sun. Appendix A contains mass properties for the Space Station Configuration studied. All test cases were flown at an altitude of 250 Nm.

5.1 - Case 1 - Non-Articulating; Constant Density, Non-Rotating Atmosphere

The first case simulated was that of a rigid Space Station flying through a non-rotating atmosphere with constant atmospheric density (4.84E-12 KG/M**3) and no solar pressure. Figure 2 shows a time history of the control momentum which must be supplied by the AMCD in order to maintain an arbitrary attitude of $(\psi,\theta,\phi) = (-7,-1,3)$ in degrees (Case 1A). Since for this case, gyroscopic, gravity gradient, and aerodynamic torques in LVLH coordinates are constant, no IOP secular momentum requirements are generated. For an arbitrary attitude the POP control momentum will in general be secular; however, a single DOF may be used to eliminate this secular momentum. Figure 3 shows a plot of the control momentum required to maintain an attitude of $(\psi,\theta,\phi) = (0,-.36,0)$ (Case 1B). This solution balances the POP gravity gradient momentum shown in Fig. 4 with the POP aerodynamic momentum shown in Fig. 5 to give zero POP control momentum requirements. Furthermore, the Y principal axis could be aligned POP to eliminate the cyclic momentum requirements arising from the gravity gradient and gyroscopic effects.

5.2 Case 2 - Non-Articulating; Variable Density, Non-Rotating Atmosphere

For a second case, a rigid Space Station was simulated flying through a non-rotating atmosphere with a Jachia 1970 (J70) atmospheric density model and no solar pressure. Figure 6 shows the density profile over the orbit ($\bar{f}=230$, $A_p=140$). Figure 7 shows a time history of the control momentum required to maintain the attitude $(\psi,\theta,\phi) = (0,-0.36,0)$ from Case 1B. This case is denoted as Case 2A. Both the POP and IOP control momentum requirements are now secular. This attitude no longer balances the POP gravity gradient with the aerodynamic momentum. The aerodynamic momentum contribution, shown in Fig. 8, is secular IOP due to the presence of a time-varying LVLH aerodynamic torque a_z, shown in Fig. 9. It follows that the

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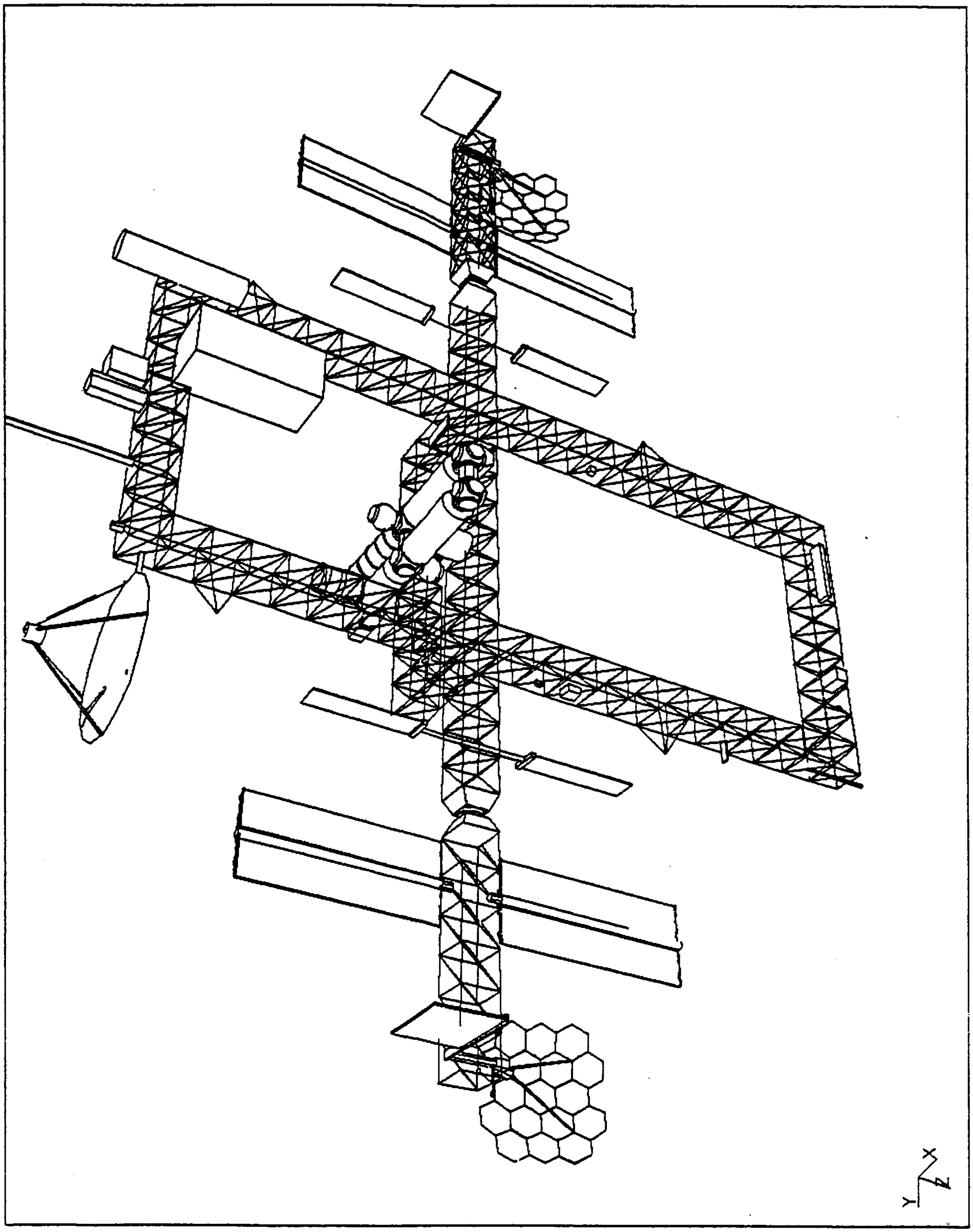


Fig. 1 - Initial Operational Capability Space Station Used in Study

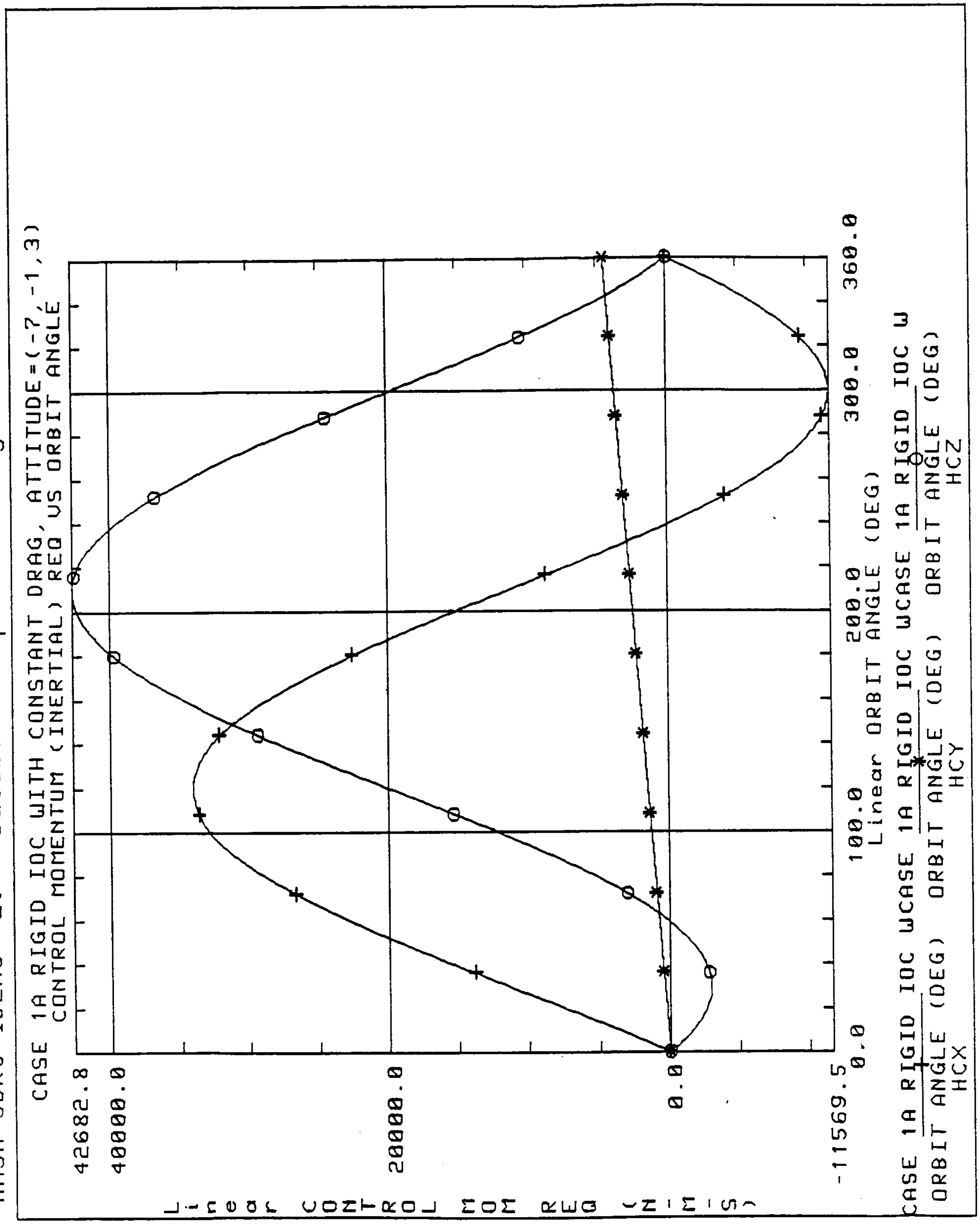
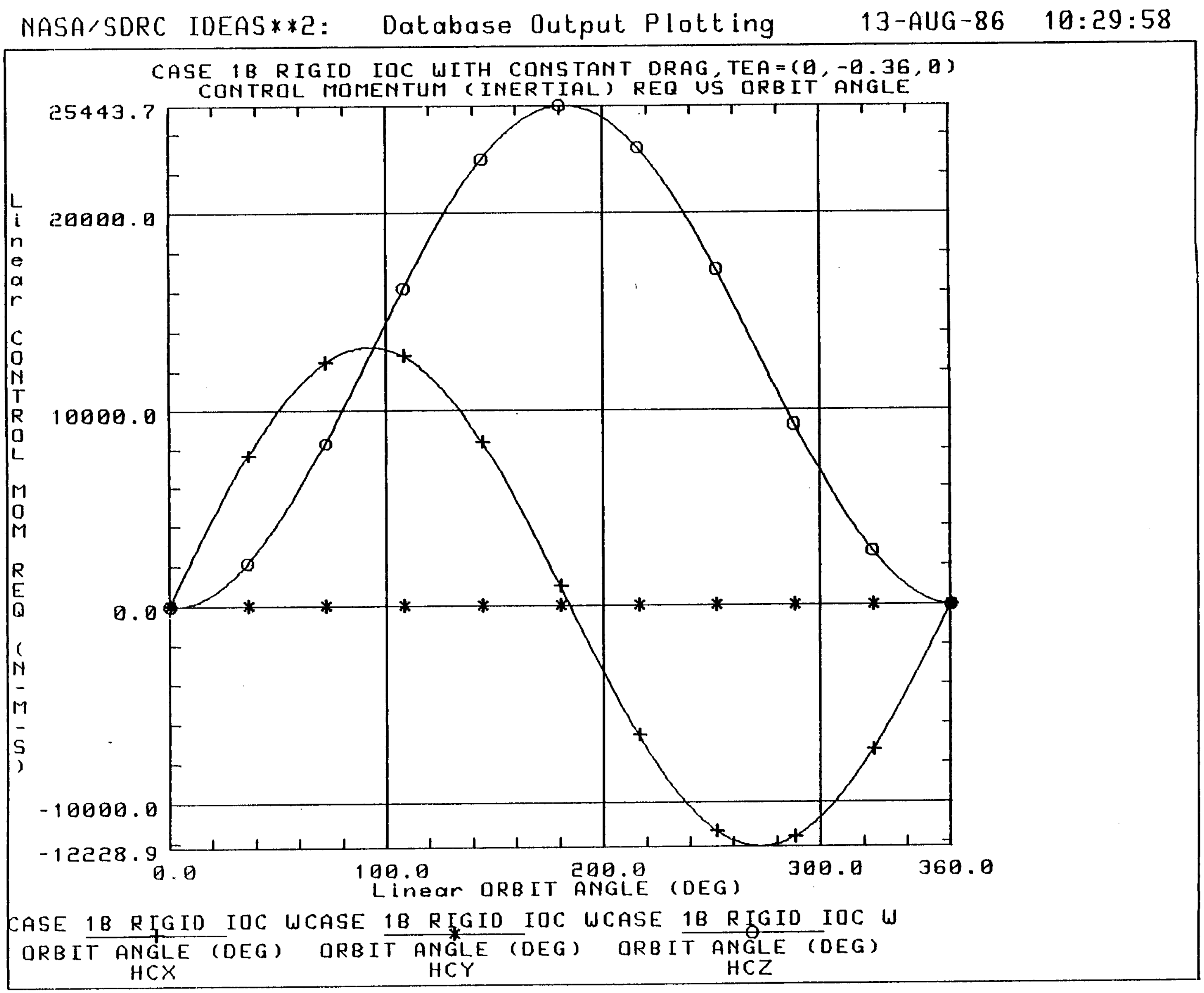

NASA/SDRC IDEAS**2: Database Output Plotting 13-AUG-86 09:27:29
CASE 1A RIGID IOC WITH CONSTANT DRAG, ATTITUDE=(-7,-1,3)
CONTROL MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
42682.8
40000.0
20000.0
0.0
-11569.5
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
Linear CONTROL MOM REQ (N-M-S)
CASE 1A RIGID IOC WCASE 1A RIGID IOC WCASE 1A RIGID IOC W
ORBIT ANGLE (DEG) ORBIT ANGLE (DEG) ORBIT ANGLE (DEG)
HCX
HCY
HCZ

Fig. 2



NASA/SDRC IDEAS**2: Database Output Plotting 13-AUG-86 10:29:58
CASE 1B RIGID IOC WITH CONSTANT DRAG,TEA=(0,-0.36,0)
CONTROL MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
25443.7
20000.0
10000.0
0.0
-10000.0
-12228.9
Linear CONTROL MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 1B RIGID IOC W
ORBIT ANGLE (DEG)
HCX
CASE 1B RIGID IOC W
ORBIT ANGLE (DEG)
HCY
CASE 1B RIGID IOC W
ORBIT ANGLE (DEG)
HCZ

Fig. 3

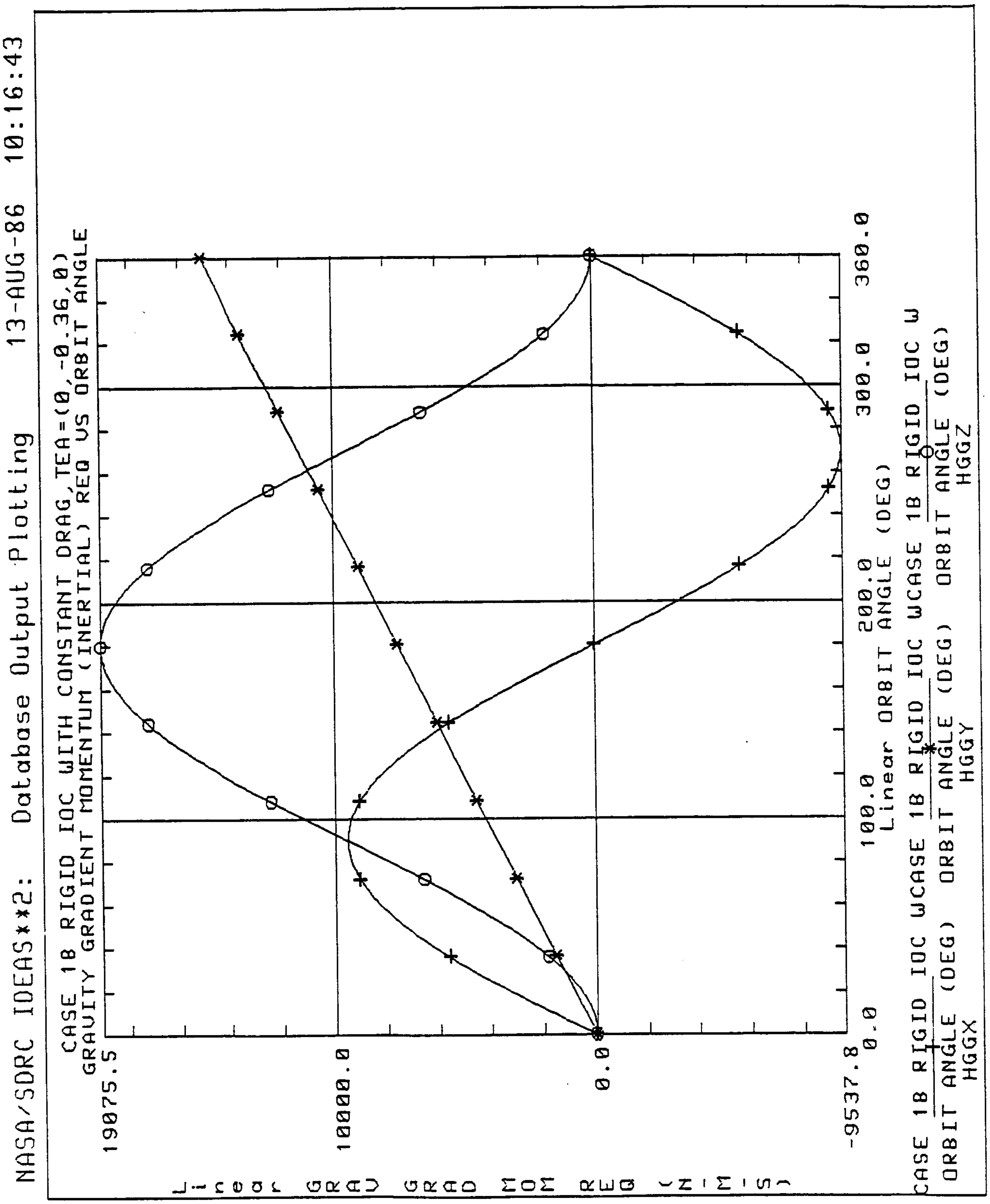

NASA/SDRC IDEAS**2: Database Output Plotting 13-AUG-86 10:16:43
CASE 1B RIGID IOC WITH CONSTANT DRAG, TEA=(0,-0.36,0)
GRAVITY GRADIENT MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
19075.5
10000.0
0.0
-9537.8
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
Linear GRAV GRAD MOM REQ (N-M-S)
CASE 1B RIGID IOC W
ORBIT ANGLE (DEG)
HGGX
CASE 1B RIGID IOC W
ORBIT ANGLE (DEG)
HGGY
CASE 1B RIGID IOC W
ORBIT ANGLE (DEG)
HGGZ

Fig. 4

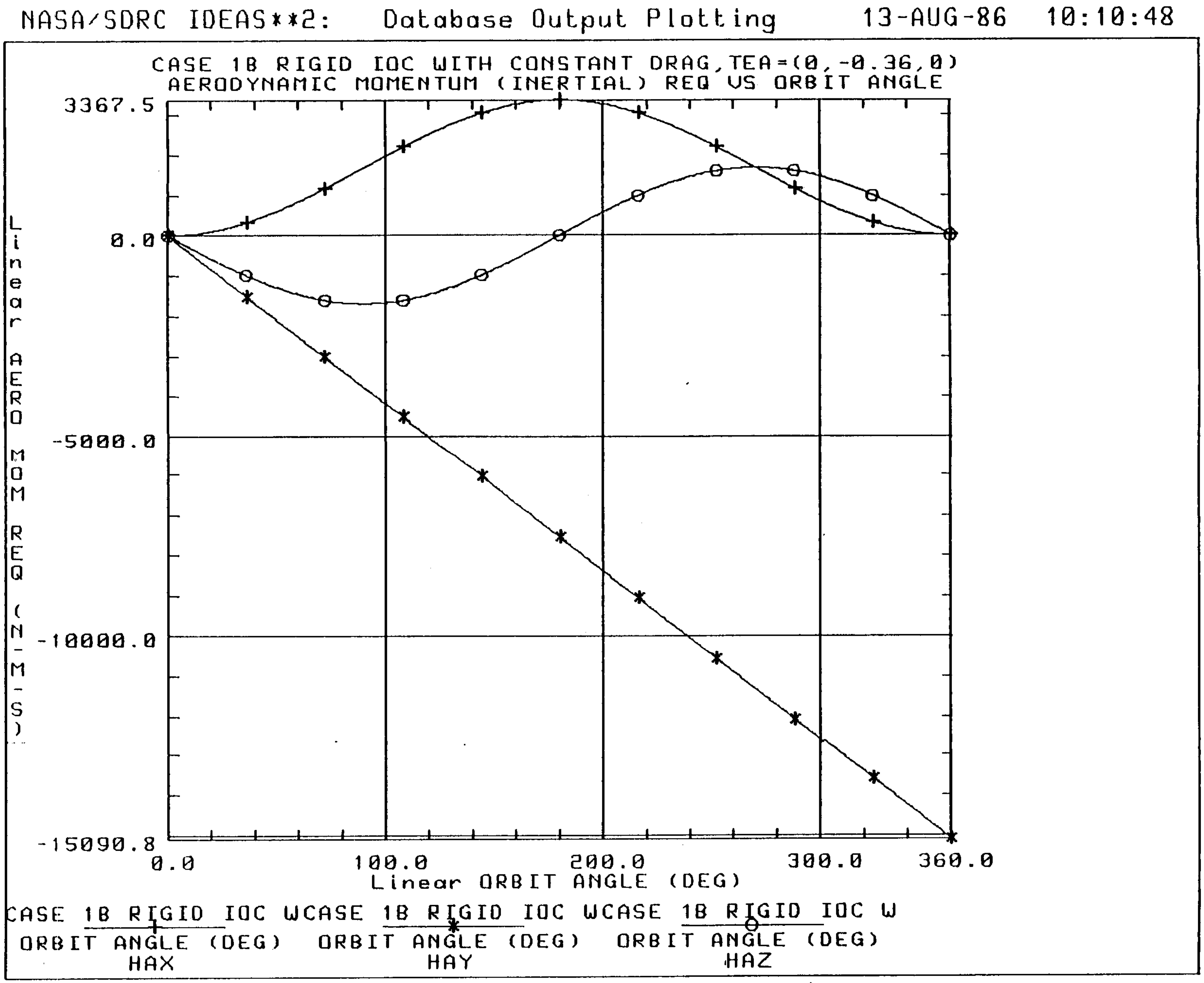
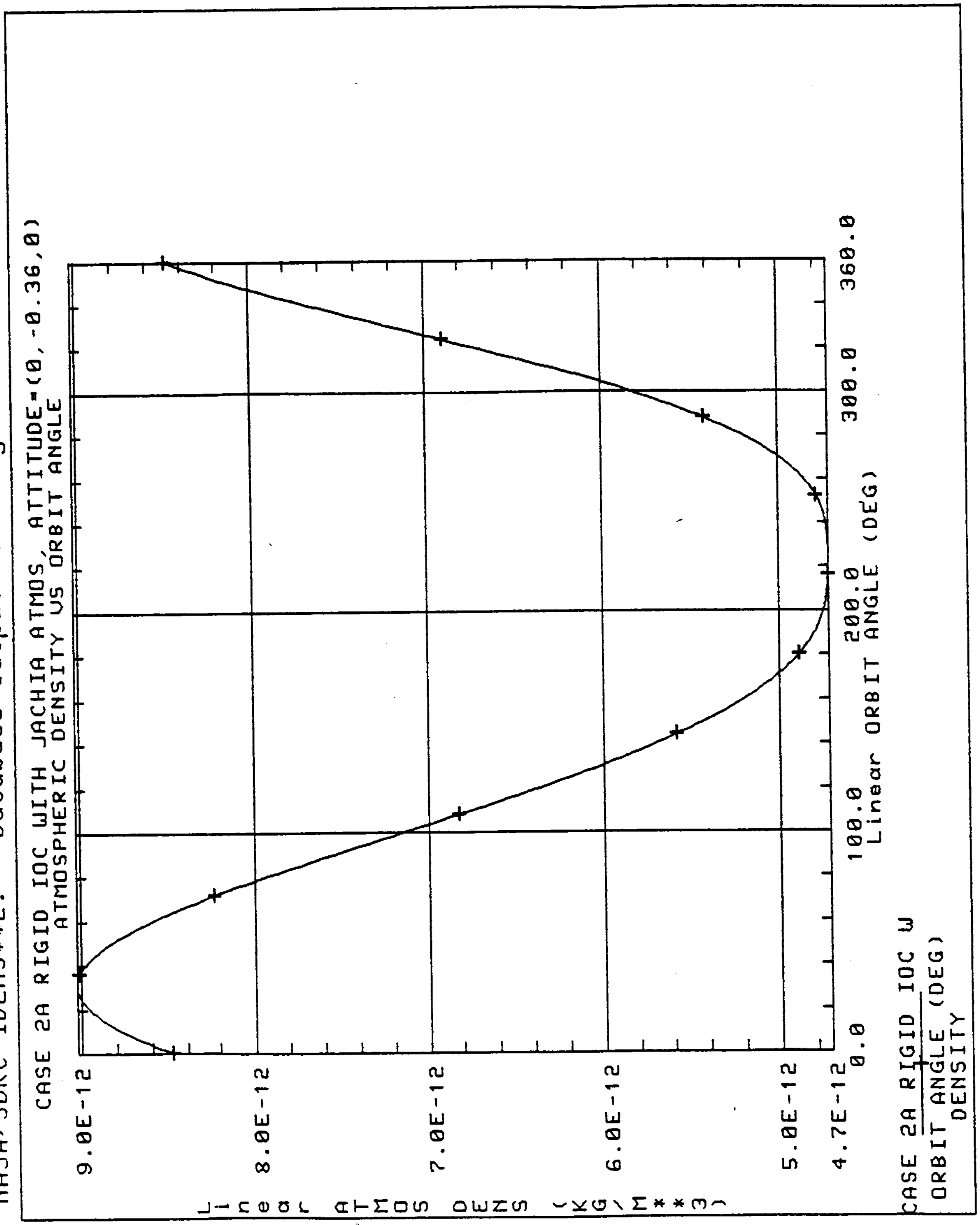

NASA/SDRC IDEAS**2: Database Output Plotting 13-AUG-86 10:10:48
CASE 1B RIGID IOC WITH CONSTANT DRAG,TEA=(0,-0.36,0)
AERODYNAMIC MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
3367.5
0.0
-5000.0
-10000.0
-15090.8
Linear AERO MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 1B RIGID IOC W
ORBIT ANGLE (DEG)
HAX
CASE 1B RIGID IOC W
ORBIT ANGLE (DEG)
HAY
CASE 1B RIGID IOC W
ORBIT ANGLE (DEG)
HAZ

Fig. 5



NASA/SDRC IDEAS**2: Database Output Plotting 23-SEP-86 09:13:01
CASE 2A RIGID IOC WITH JACHIA ATMOS, ATTITUDE=(0,-0.36,0)
ATMOSPHERIC DENSITY VS ORBIT ANGLE
9.0E-12
8.0E-12
7.0E-12
6.0E-12
5.0E-12
4.7E-12
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
Linear ATMOS DENS (KG/M**3)
CASE 2A RIGID IOC W
ORBIT ANGLE (DEG)
DENSITY

Fig. 6

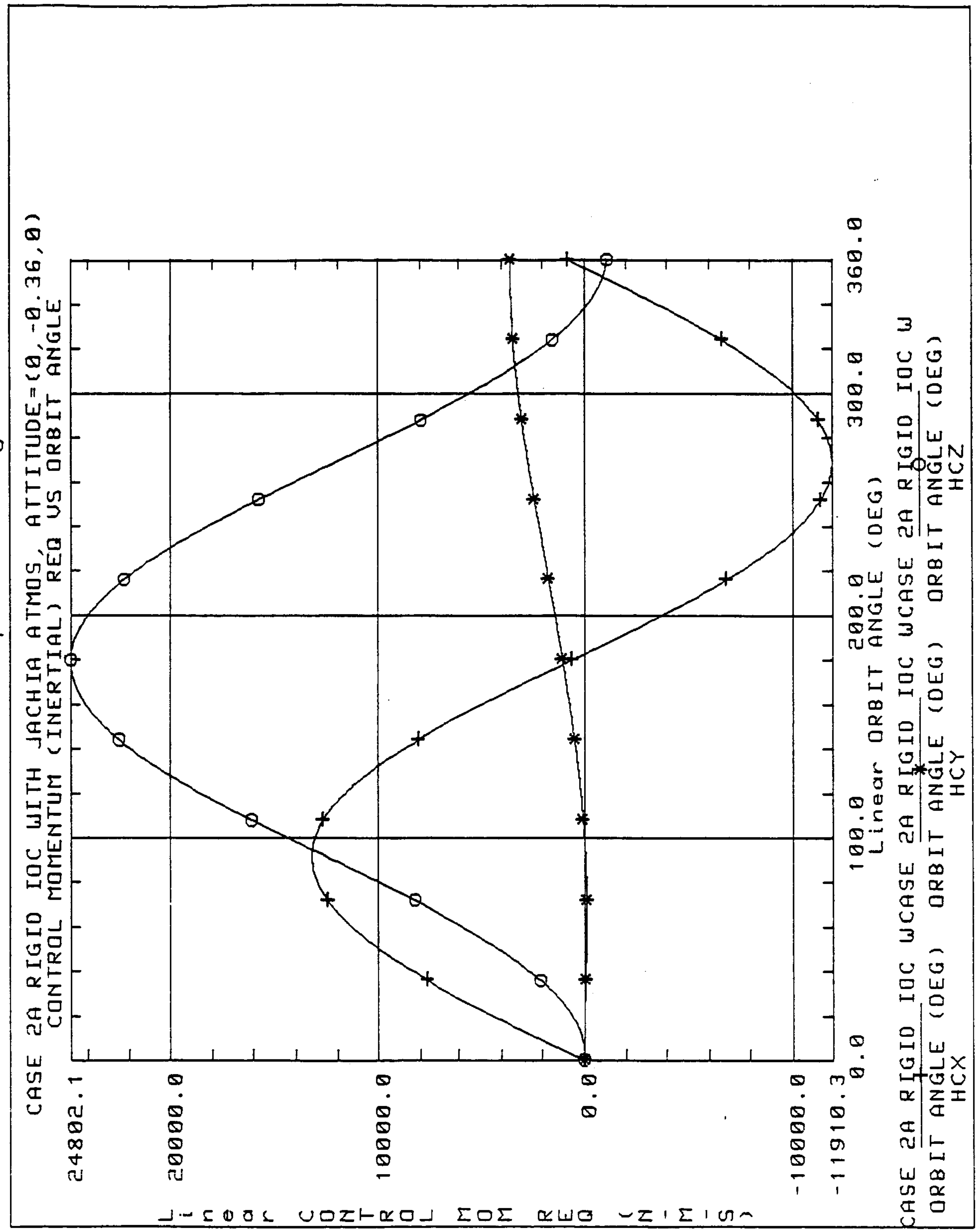

NASA/SDRC IDEAS**2: Database Output Plotting 13-AUG-86 13:20:51
CASE 2A RIGID IOC WITH JACHIA ATMOS, ATTITUDE=(0,-0.36,0)
CONTROL MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
24802.1
20000.0
10000.0
0.0
-10000.0
-11910.3
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
Linear CONTROL MOM REQ (N-M-S)
CASE 2A RIGID IOC WCASE 2A RIGID IOC WCASE 2A RIGID IOC W
ORBIT ANGLE (DEG) ORBIT ANGLE (DEG) ORBIT ANGLE (DEG)
HCX
HCY
HCZ

Fig. 7

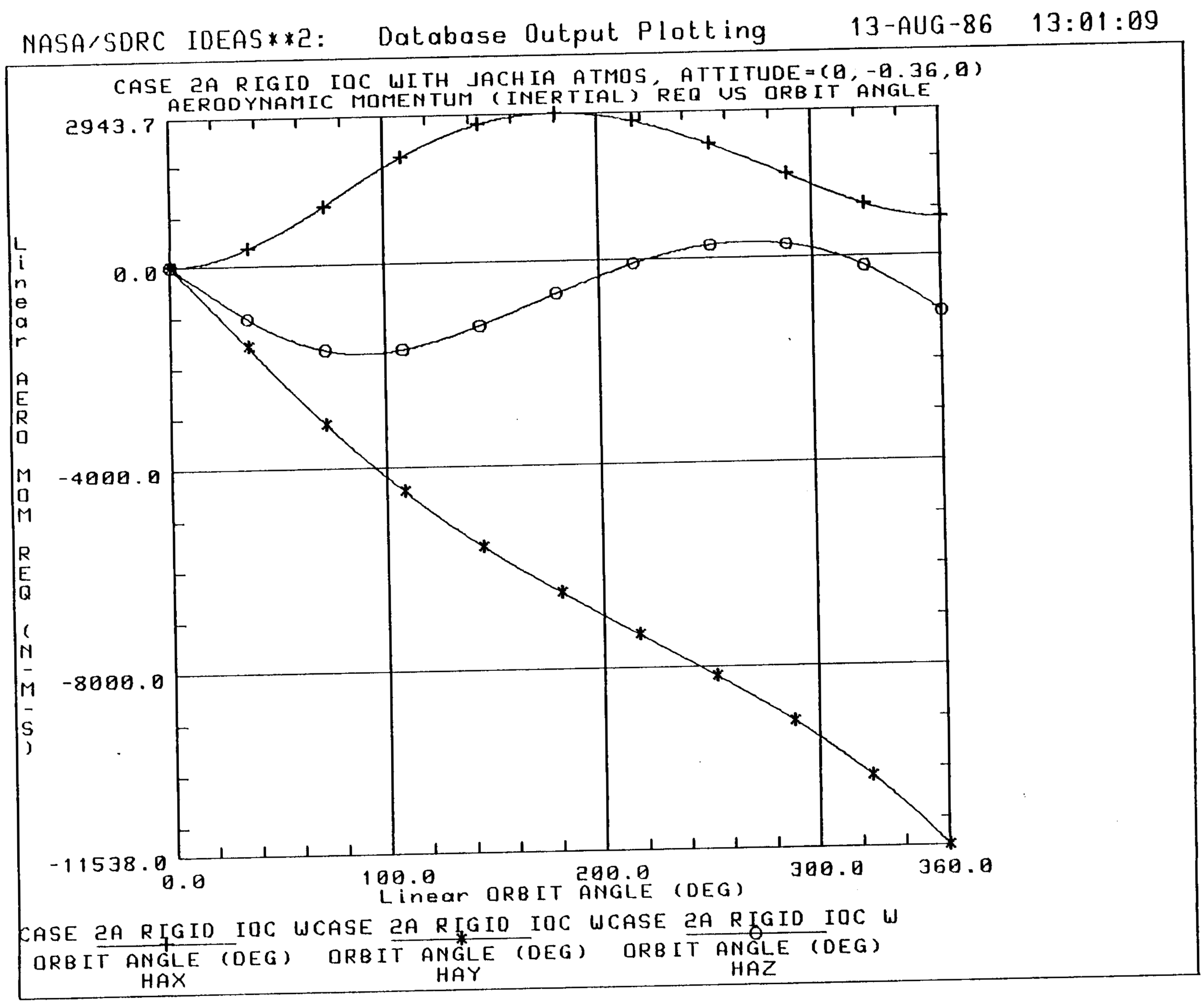

NASA/SDRC IDEAS**2: Database Output Plotting 13-AUG-86 13:01:09
CASE 2A RIGID IOC WITH JACHIA ATMOS, ATTITUDE=(0,-0.36,0)
AERODYNAMIC MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
2943.7
0.0
-4000.0
-8000.0
-11538.0
Linear AERO MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 2A RIGID IOC W
ORBIT ANGLE (DEG)
HAX
CASE 2A RIGID IOC W
ORBIT ANGLE (DEG)
HAY
CASE 2A RIGID IOC W
ORBIT ANGLE (DEG)
HAZ

Fig. 8

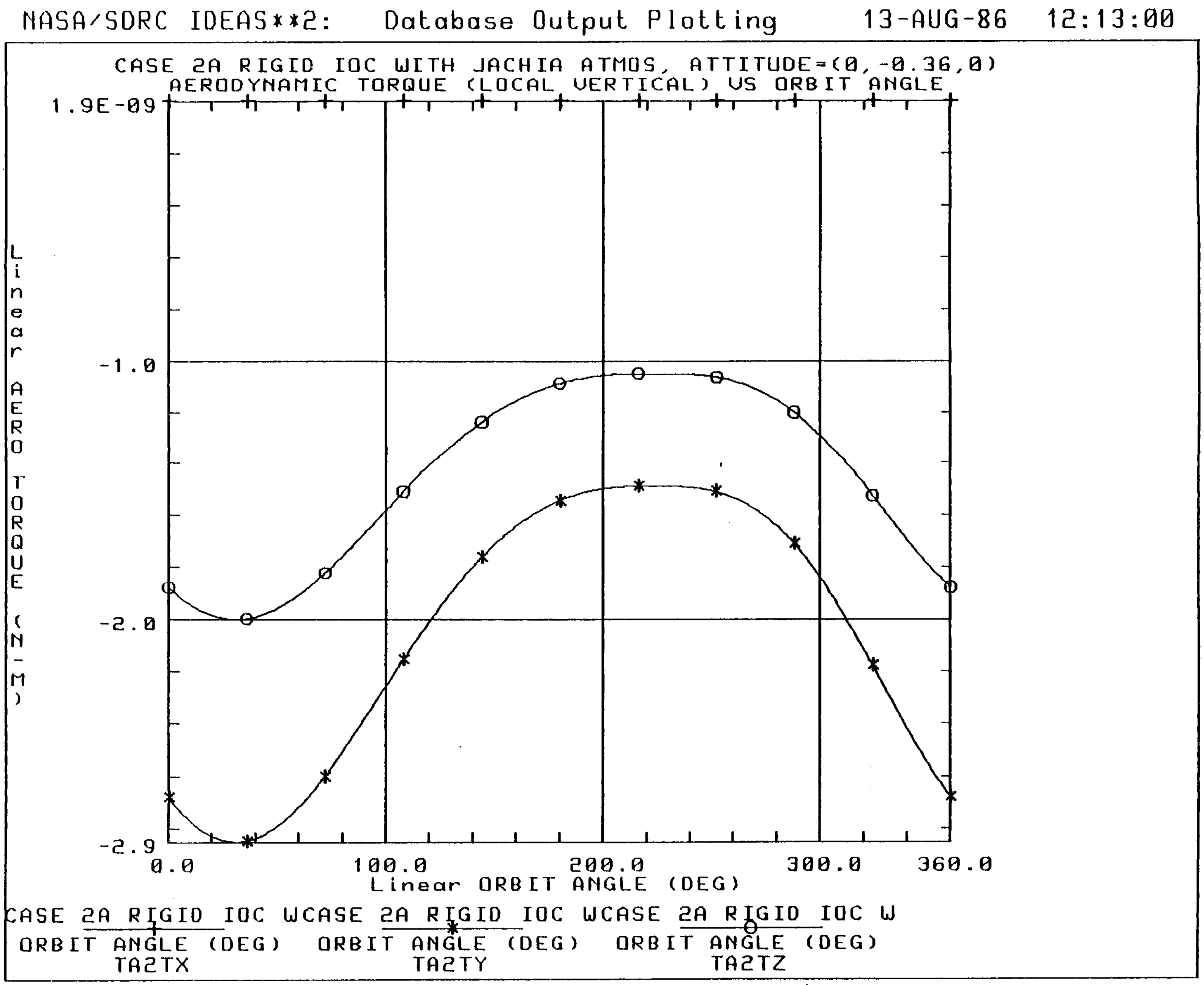

NASA/SDRC IDEAS**2: Database Output Plotting 13-AUG-86 12:13:00
CASE 2A RIGID IOC WITH JACHIA ATMOS, ATTITUDE=(0,-0.36,0)
AERODYNAMIC TORQUE (LOCAL VERTICAL) VS ORBIT ANGLE
1.9E-09
-1.0
-2.0
-2.9
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
Linear AERO TORQUE (N-M)
CASE 2A RIGID IOC W
ORBIT ANGLE (DEG)
TA2TX
CASE 2A RIGID IOC W
ORBIT ANGLE (DEG)
TA2TY
CASE 2A RIGID IOC W
ORBIT ANGLE (DEG)
TA2TZ

Fig. 9

determination of an attitude such that $a_z = 0$ can be used to eliminate secular momentum build-up IOP.

For a non-rotating atmosphere, $V_x = V$ and $V_y = 0$, and from Eq. (22c) the aerodynamic torque a_z is given by

$$a_z = \tfrac{1}{2}\rho V^2 C_D \Big\{ A_1 \left| c\theta c\psi \right| \Big[r_{1_y} \big[-s\theta(s\phi s\psi + c\phi s\theta c\psi) - c\phi c\theta(c\theta c\psi) \big]$$

$$+ r_{1_z} \big[s\theta(-c\phi s\psi + s\phi s\theta c\psi) + s\phi c\theta(c\theta c\psi) \big]$$

$$+ A_2 \left| -c\phi s\psi + s\phi s\theta c\psi \right| \Big[r_{2_x} \big[-s\phi c\theta(s\phi s\psi + c\phi s\theta c\psi) + c\phi c\theta(-c\phi s\psi + s\phi s\theta c\psi) \big]$$

$$+ r_{2_z} \big[s\theta(-c\phi s\psi + s\phi s\theta c\psi) + s\phi c\theta(c\theta c\psi) \big]$$

$$+ A_3 \left| s\phi s\psi + c\phi s\theta c\psi \right| \Big[r_{3_x} \big[s\phi c\theta(s\phi s\psi + c\phi s\theta c\psi) + c\phi c\theta(-c\phi s\psi + s\phi s\theta c\psi) \big]$$

$$+ r_{3_y} \big[-s\theta(s\phi s\psi + c\phi s\theta c\psi) - c\phi c\theta(c\theta c\psi) \big] \Big] \Big\} \tag{24}$$

One possible means of eliminating a_z is to roll the spacecraft so that the cp-cg of pressure offset looking down the LVLH x axis is in the orbit plane (which results in a zero moment arm). By setting $\psi = \theta = 0$ in Eq. (24) it can be seen that

$$\tan \phi = \frac{r_{1_y}}{r_{1_z}} \tag{25}$$

results in $a_z = 0$, and thus, zero secular momentum IOP. For the Space Station under study a $\phi = -35°$ was calculated using Eq. (25). A non-zero pitch angle θ can be utilized to balance the POP gravity gradient momentum with the POP aerodynamic momentum. Then, Eq. (24) can be solved for different values of (ϕ,ψ) such that the $a_z = 0$. Figures 10, 11, and 12 show plots of the control momentum required, aerodynamic momentum, and the aerodynamic torque, respectively, for an attitude of $(\psi,\theta,\phi) = (0,-.92,-34.79)$ [Case 2B]. Observe that the secular momentum requirements have been driven to zero.

Note that a_z can also be eliminated with solutions involving $\psi \neq 0$ by balancing the $\bar{r}_i \times \bar{F}_i^A$ contributions to a_z. For the case of a pure yaw, by setting $\theta = \phi = 0$ in Eq. (24), it can be seen that

$$\tan^2\psi = \frac{-r_{1_y} A_1}{r_{2_x} A_2} \tag{26}$$

results in $a_z = 0$, and hence, the IOP momentum is zero. For the mass properties in Appendix A, a $\psi = 58^0$ was calculated using Eq. (26), and an attitude of $(\psi,\theta,\phi) = (54.0,5.15,0)$ was determined using the program ARCD. Figures 13, 14, and 15 show plots of the control momentum required, aerodynamic momentum, and the aerodynamic torque, respectively, for this attitude (Case 2C). Again note that secular momentum build-up is zero since

$a_z \simeq 0$

5.3 Case 3 - Non-Articulating; Constant Density, Rotating Atmosphere

For a third case, a rigid Space Station was simulated flying through a rotating atmosphere with constant density and no solar pressure. Figure 16 shows a time history of the control momentum required to maintain the attitude $(\psi,\theta,\phi) = (0,-0.36,0)$ [Case 3A] from Case 1B. Although this plot seems to show only cyclic momentum requirements (due to the large scale), the IOP momentum requirements are in fact secular and result from a secular IOP aerodynamic momentum contribution of approximately 250 Nms for this

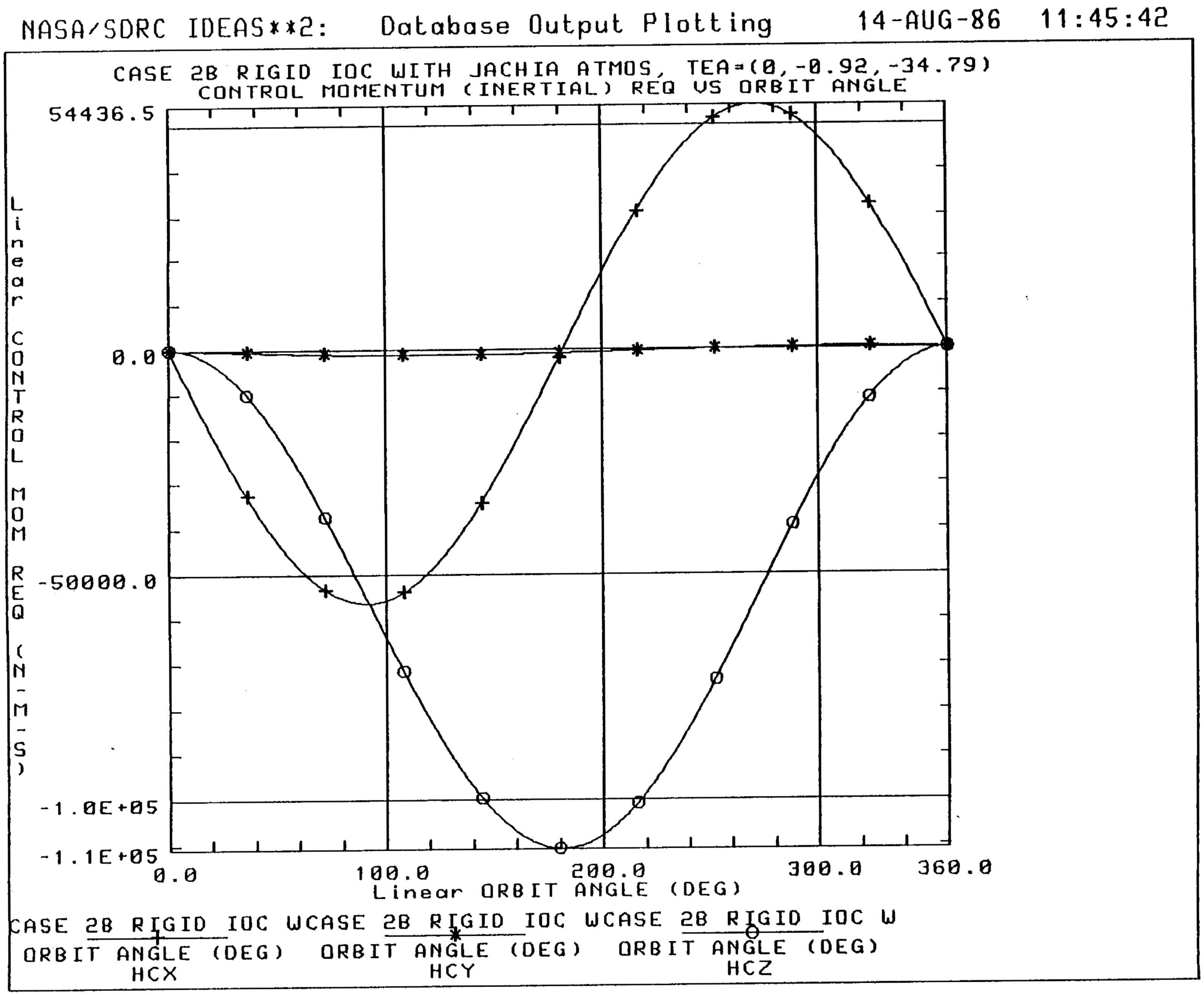

NASA/SDRC IDEAS**2: Database Output Plotting
14-AUG-86 11:45:42
CASE 2B RIGID IOC WITH JACHIA ATMOS, TEA=(0,-0.92,-34.79)
CONTROL MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
54436.5
0.0
-50000.0
-1.0E+05
-1.1E+05
Linear CONTROL MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 2B RIGID IOC W
ORBIT ANGLE (DEG)
HCX
CASE 2B RIGID IOC W
ORBIT ANGLE (DEG)
HCY
CASE 2B RIGID IOC W
ORBIT ANGLE (DEG)
HCZ

Fig. 10

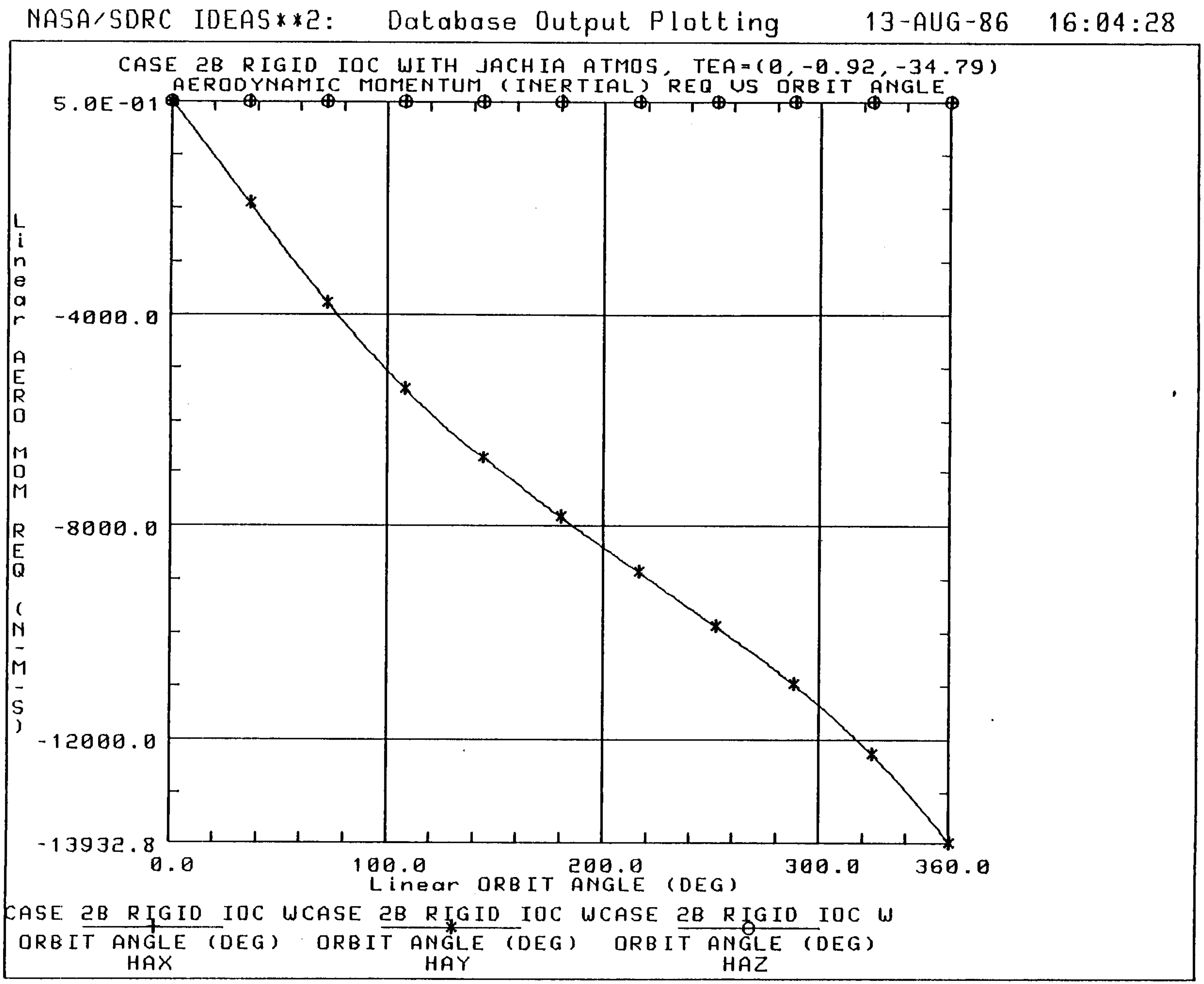
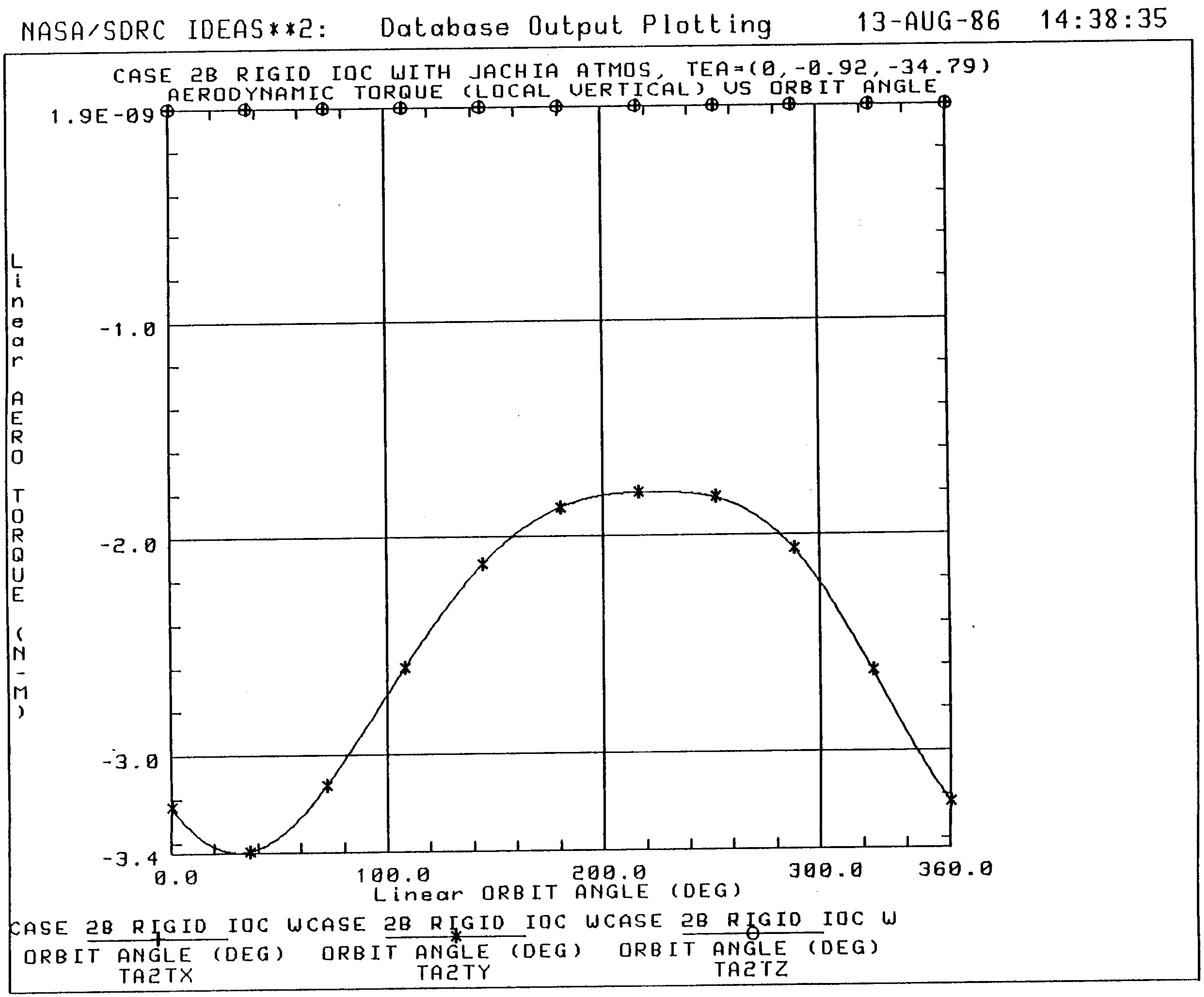

NASA/SDRC IDEAS**2: Database Output Plotting 13-AUG-86 16:04:28
CASE 2B RIGID IOC WITH JACHIA ATMOS, TEA=(0,-0.92,-34.79)
AERODYNAMIC MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
5.0E-01
-4000.0
-8000.0
-12000.0
-13932.8
Linear AERO MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 2B RIGID IOC WCASE 2B RIGID IOC WCASE 2B RIGID IOC W
ORBIT ANGLE (DEG) ORBIT ANGLE (DEG) ORBIT ANGLE (DEG)
HAX HAY HAZ

Fig. 11



NASA/SDRC IDEAS**2: Database Output Plotting 13-AUG-86 14:38:35
CASE 2B RIGID IOC WITH JACHIA ATMOS, TEA=(0,-0.92,-34.79)
AERODYNAMIC TORQUE (LOCAL VERTICAL) VS ORBIT ANGLE
1.9E-09
-1.0
-2.0
-3.0
-3.4
Linear AERO TORQUE (N-M)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 2B RIGID IOC W
ORBIT ANGLE (DEG)
TA2TX
CASE 2B RIGID IOC W
ORBIT ANGLE (DEG)
TA2TY
CASE 2B RIGID IOC W
ORBIT ANGLE (DEG)
TA2TZ

Fig. 12

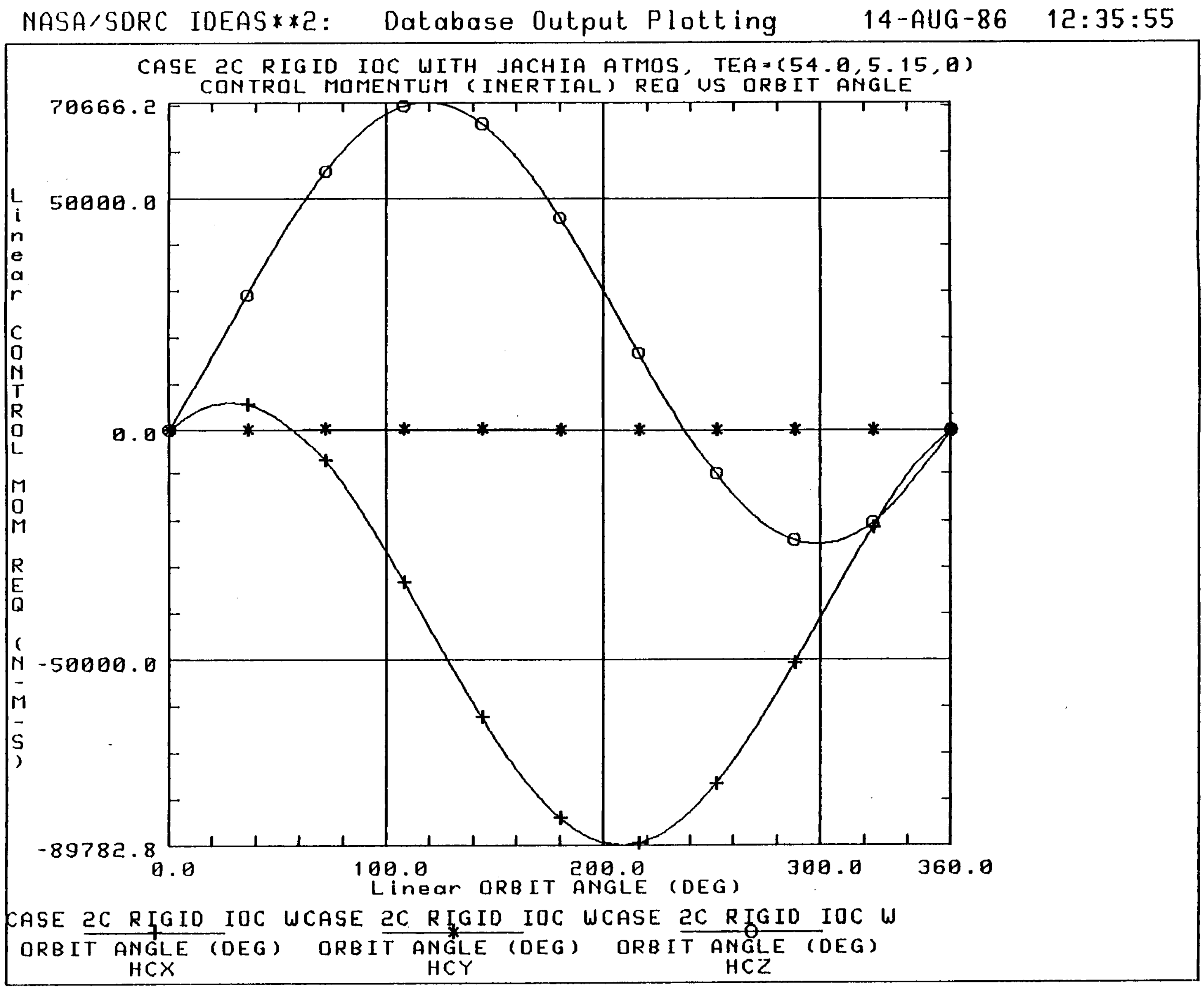

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 12:35:55
CASE 2C RIGID IOC WITH JACHIA ATMOS, TEA=(54.0,5.15,0)
CONTROL MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
Linear CONTROL MOM REQ (N-M-S)
70666.2
50000.0
0.0
-50000.0
-89782.8
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 2C RIGID IOC WCASE 2C RIGID IOC WCASE 2C RIGID IOC W
ORBIT ANGLE (DEG) ORBIT ANGLE (DEG) ORBIT ANGLE (DEG)
HCX HCY HCZ

Fig. 13

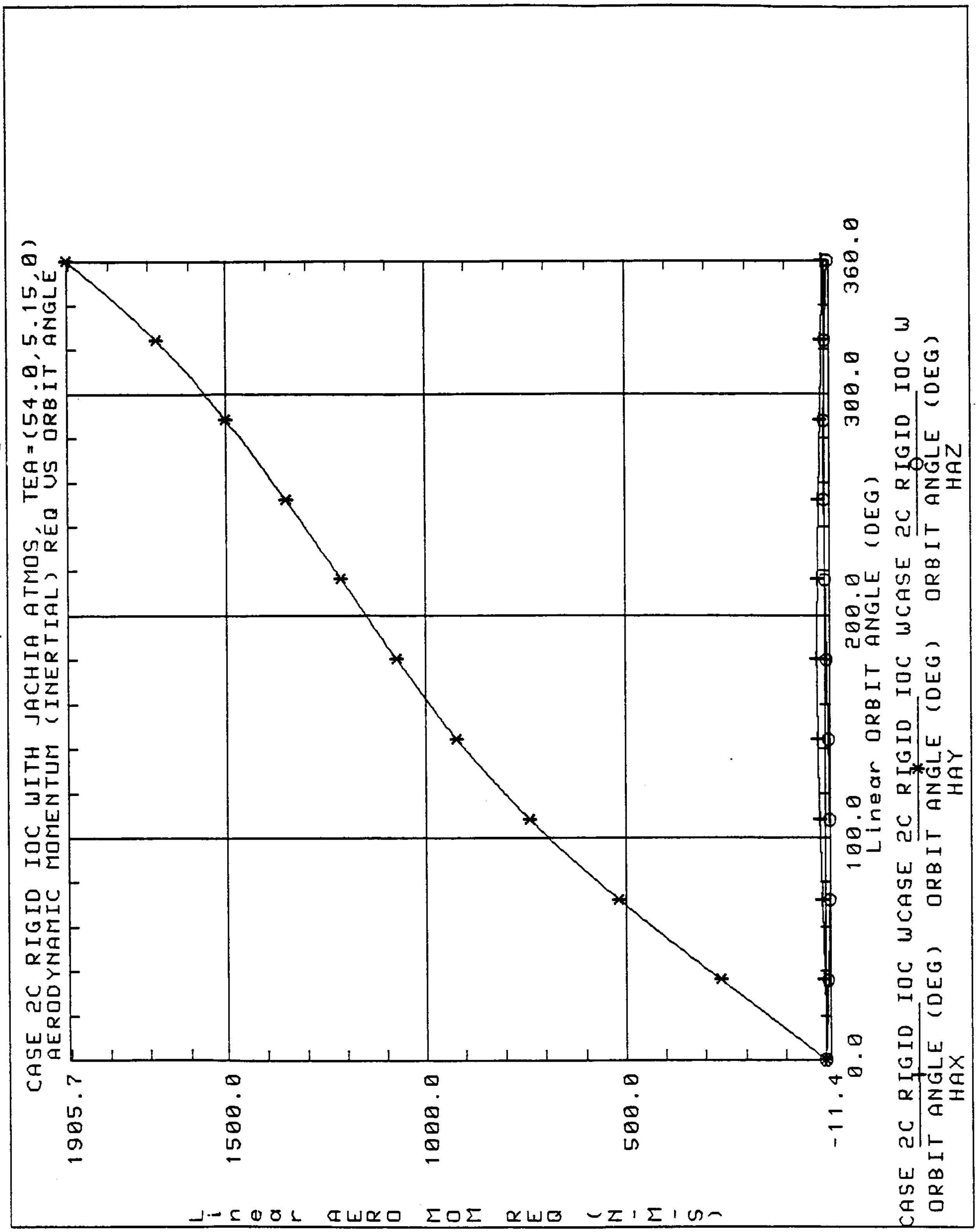

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 12:40:40
CASE 2C RIGID IOC WITH JACHIA ATMOS, TEA=(54.0,5.15,0)
AERODYNAMIC MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
1905.7
1500.0
1000.0
500.0
-11.4
Linear AERO MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 2C RIGID IOC WCASE 2C RIGID IOC WCASE 2C RIGID IOC W
ORBIT ANGLE (DEG) ORBIT ANGLE (DEG) ORBIT ANGLE (DEG)
HAX
HAY
HAZ

Fig. 14

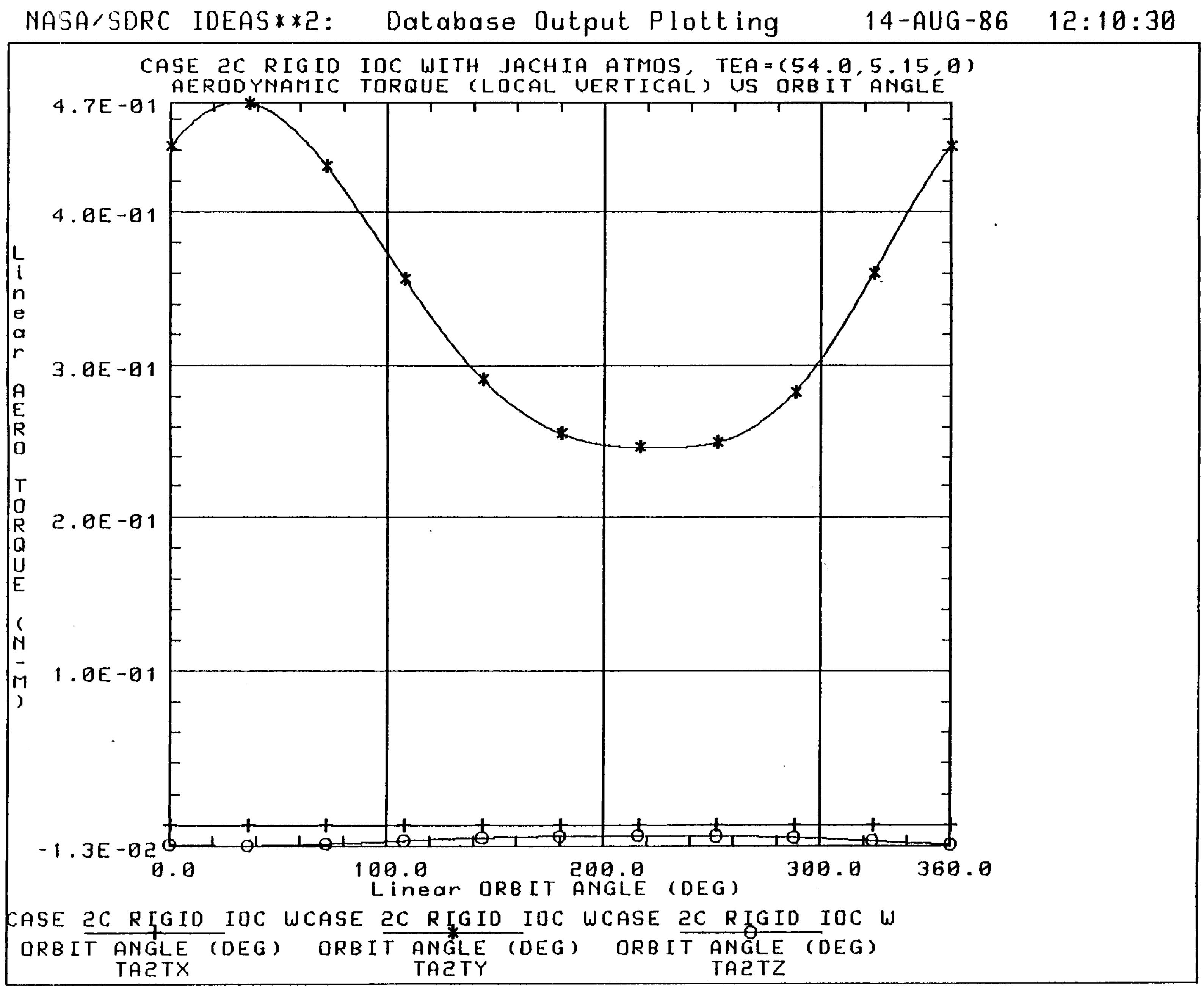

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 12:10:30
CASE 2C RIGID IOC WITH JACHIA ATMOS, TEA=(54.0,5.15,0)
AERODYNAMIC TORQUE (LOCAL VERTICAL) VS ORBIT ANGLE
Linear AERO TORQUE (N-M)
4.7E-01
4.0E-01
3.0E-01
2.0E-01
1.0E-01
-1.3E-02
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 2C RIGID IOC W
ORBIT ANGLE (DEG)
TA2TX
CASE 2C RIGID IOC W
ORBIT ANGLE (DEG)
TA2TY
CASE 2C RIGID IOC W
ORBIT ANGLE (DEG)
TA2TZ

Fig. 15

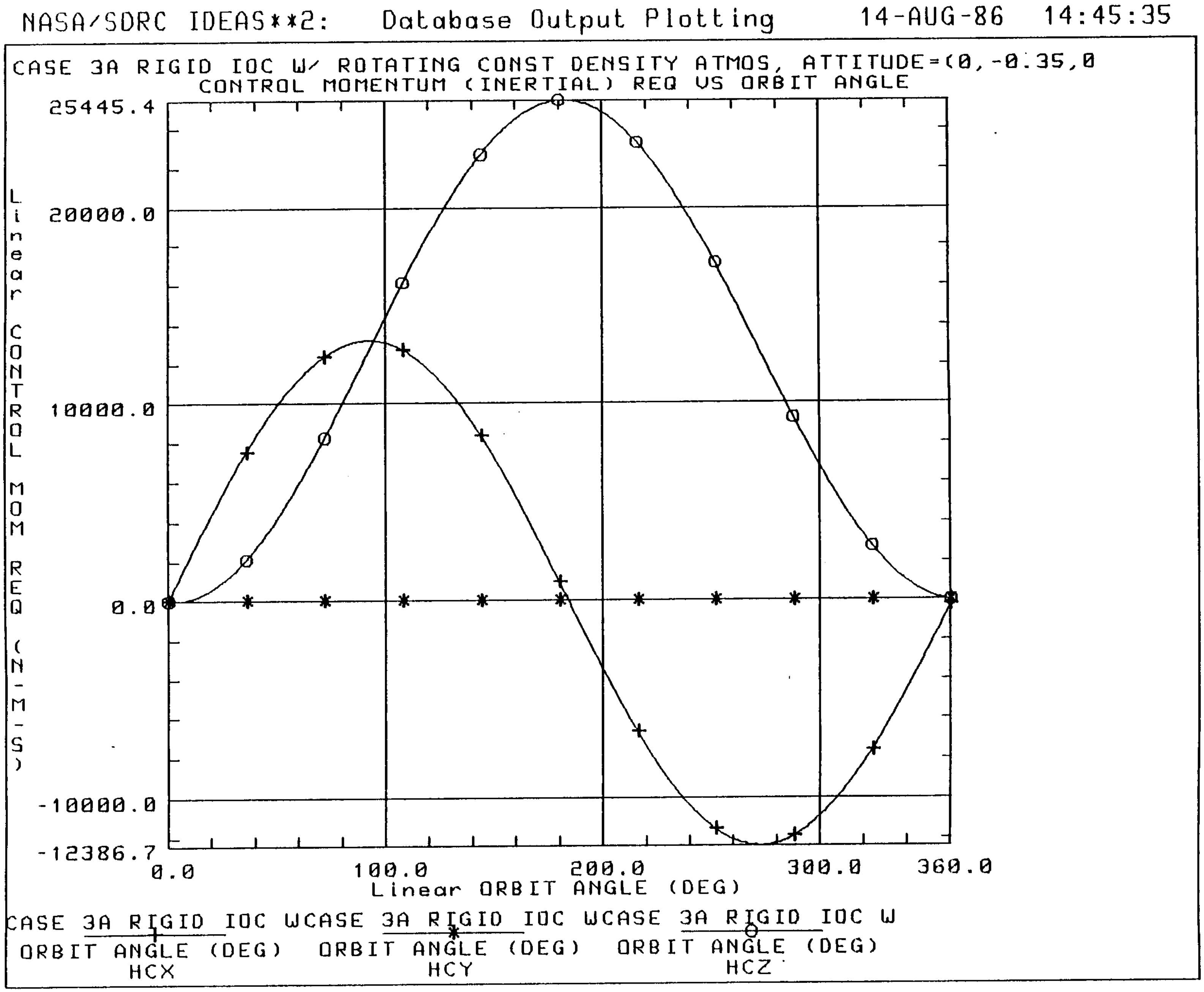

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 14:45:35
CASE 3A RIGID IOC W/ ROTATING CONST DENSITY ATMOS, ATTITUDE=(0,-0.35,0
CONTROL MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
Linear CONTROL MOM REQ (N-M-S)
25445.4
20000.0
10000.0
0.0
-10000.0
-12386.7
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 3A RIGID IOC W ORBIT ANGLE (DEG) HCX
CASE 3A RIGID IOC W ORBIT ANGLE (DEG) HCY
CASE 3A RIGID IOC W ORBIT ANGLE (DEG) HCZ

Fig. 16

vehicle, as show in Fig. 17. An inspection of Fig. 18 shows that the secular IOP aerodynamic momentum contribution results primarily from the time-dependent torque a_x due to the effect of the rotating atmosphere, since a_z is nearly constant for this case.

An attitude was determined using ARCD which produced cyclic control momentum requirements by eliminating a_x and not allowing a_z to contribute secular aerodynamic momentum. In the case of pure roll (ϕ) motion only, the IOP aerodynamic torques in LVLH coordinates may be obtained from Eq. (22a) and Eq. (22c).

$$a_x = \tfrac{1}{2}\rho C_D V_y \Big\{ A_1 |V_x| \Big[-r_{1_y} s\phi - r_{1_z} c\phi \Big] + A_2 |V_y c\phi| \Big[-r_{2_z} c\phi \Big]$$

$$+ A_3 |V_y s\phi| \Big[-r_{3_y} s\phi \Big] \Big\} \tag{27a}$$

$$a_z = \tfrac{1}{2}\rho C_D \Big\{ A_1 |V_x| \Big[(-r_{1_y} c\phi + r_{1_z} s\phi) V_x \Big] + A_2 |V_y c\phi| \Big[r_{2_x} V_y + r_{2_z} V_x s\phi \Big]$$

$$+ A_3 |V_y s\phi| \Big[r_{3_x} V_y - r_{3_y} V_x c\phi \Big] \Big\} \tag{27b}$$

Since $V_x \gg V_y$, the V_y^2 terms may be neglected in Eqs. (27) to give

$$a_x \simeq \tfrac{1}{2}\rho C_D V_y A_1 |V_x| (-r_{1_y} s\phi - r_{1_z} c\phi) \tag{28a}$$

$$a_z \simeq \tfrac{1}{2}\rho C_D V_x \Big\{ A_1 |V_x| (-r_{1_y} c\phi + r_{1_z} s\phi) + A_2 |V_y c\phi| r_{2_z} s\phi - A_3 |V_y s\phi| r_{3_y} c\phi \Big\} \tag{28b}$$

By setting $a_x = 0$ in Eq. (28a), it can be seen that

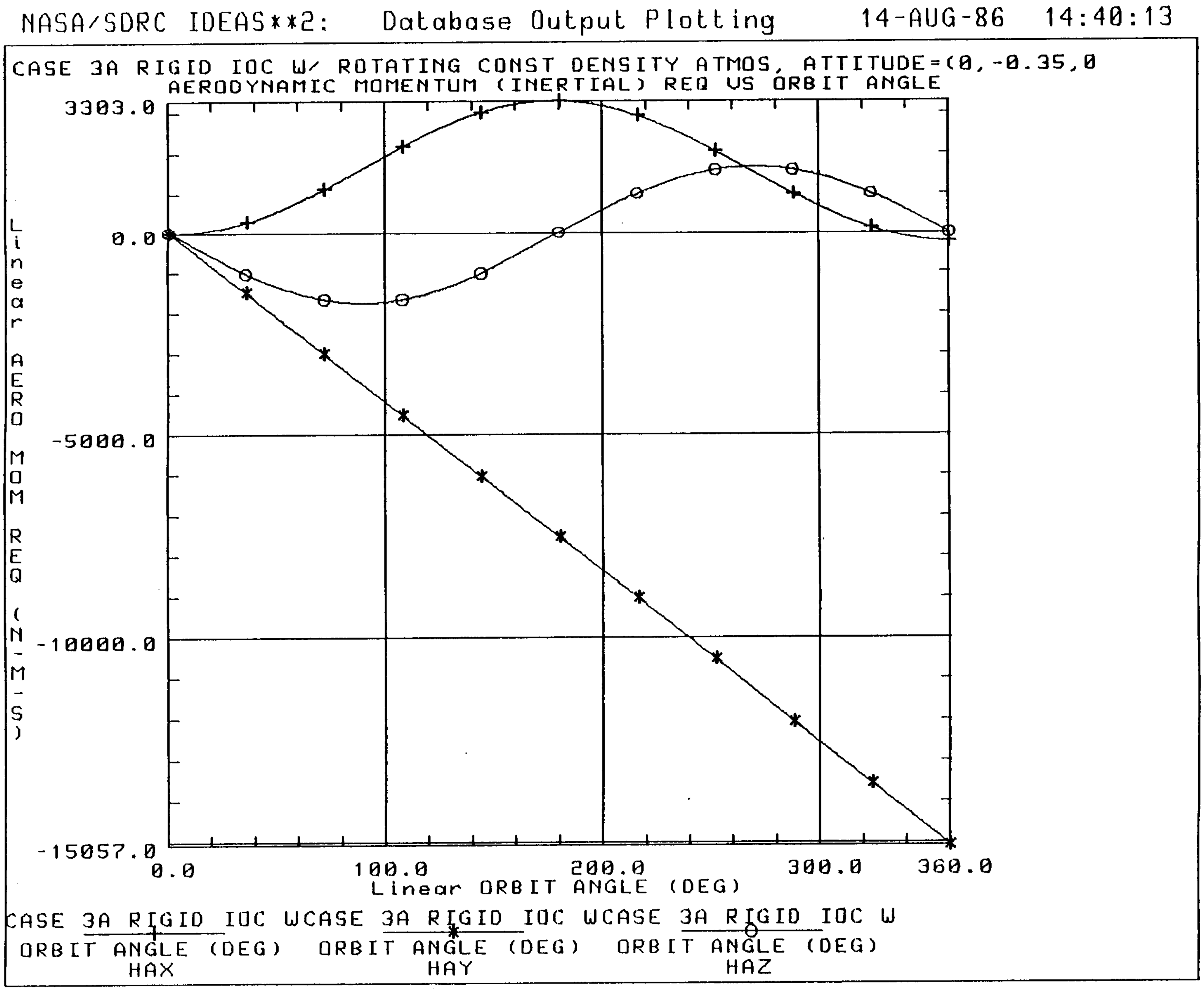

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 14:40:13
CASE 3A RIGID IOC W/ ROTATING CONST DENSITY ATMOS, ATTITUDE=(0,-0.35,0
AERODYNAMIC MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
3303.0
0.0
-5000.0
-10000.0
-15057.0
Linear AERO MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 3A RIGID IOC W
ORBIT ANGLE (DEG)
HAX
CASE 3A RIGID IOC W
ORBIT ANGLE (DEG)
HAY
CASE 3A RIGID IOC W
ORBIT ANGLE (DEG)
HAZ

Fig. 17

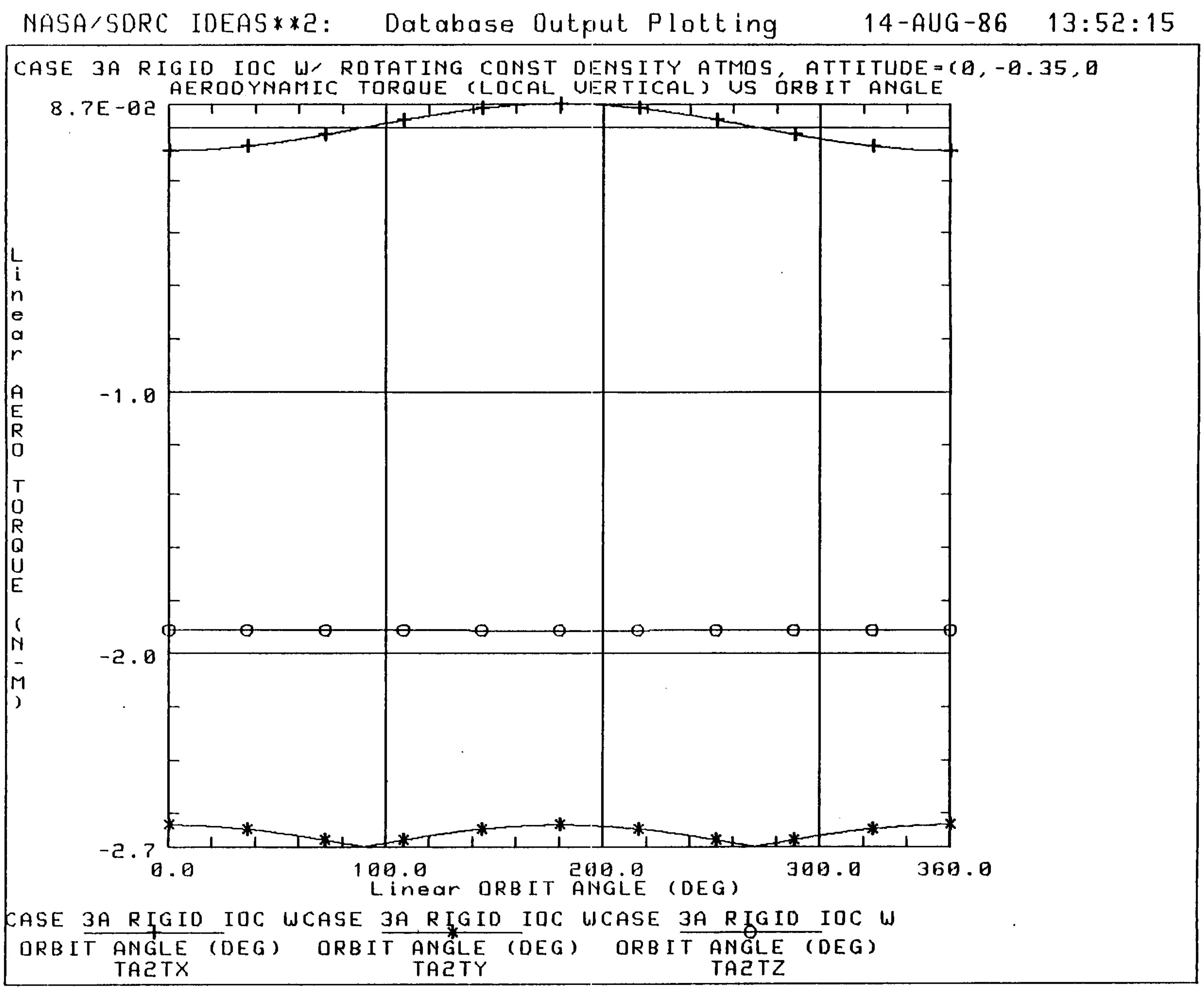

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 13:52:15
CASE 3A RIGID IOC W/ ROTATING CONST DENSITY ATMOS, ATTITUDE=(0,-0.35,0
AERODYNAMIC TORQUE (LOCAL VERTICAL) VS ORBIT ANGLE
8.7E-02
-1.0
-2.0
-2.7
Linear AERO TORQUE (N-M)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 3A RIGID IOC WCASE 3A RIGID IOC WCASE 3A RIGID IOC W
ORBIT ANGLE (DEG) ORBIT ANGLE (DEG) ORBIT ANGLE (DEG)
TA2TX
TA2TY
TA2TZ

Fig. 18

$$\tan\phi \quad -\frac{r_{1_z}}{r_{1_y}} \tag{29}$$

An examination of Eq. (28b) reveals that the only time-dependent term affecting a_z is $|V_y|$. It can be shown that a spacecraft in a circular orbit has

$$V_y(\tau) = K \cos \omega_0\tau \tag{30}$$

so a_z can be written as

$$a_z(\tau) = C_1 + C_2 \, | \cos \omega_0\tau | \tag{31}$$

where C_1 and C_2 are constants. For this case, the secular IOP momentum contributed by a_z is given by

$$H_{A_x}(2\pi/\omega_0) = \int_0^{2\pi/\omega} - (C_1 + C_2 | \cos \omega_0\tau |) \sin \omega_0\tau \, d\tau \tag{32a}$$

$$H_{A_z}(2\pi/\omega_0) = \int_0^{2\pi/\omega_0} (C_1 + C_2 | \cos \omega_0\tau |) \cos \omega_0\tau \, d\tau \tag{32b}$$

Since the integrals in Eqs. (32) are identically zero for constant C_1 and C_2, the $(0,0,\phi)$ solution given by Eq. (29) will result in purely cyclic IOP control momentum requirements. For the Space Station under study a $\phi = 55^0$ was calculated using Eq. (29), while an attitude for Case 3B of $(\psi,\theta,\phi) = (0,0.81,53.0)$ was determined using the program ARCD. Plots of control momentum requirements, aerodynamic momentum, and aerodynamic torques are shown in Figs. 19, 20, and 21, respectively, for this case. Note that although a_z is not constant, the IOP aerodynamic induced momentum contribution is zero.

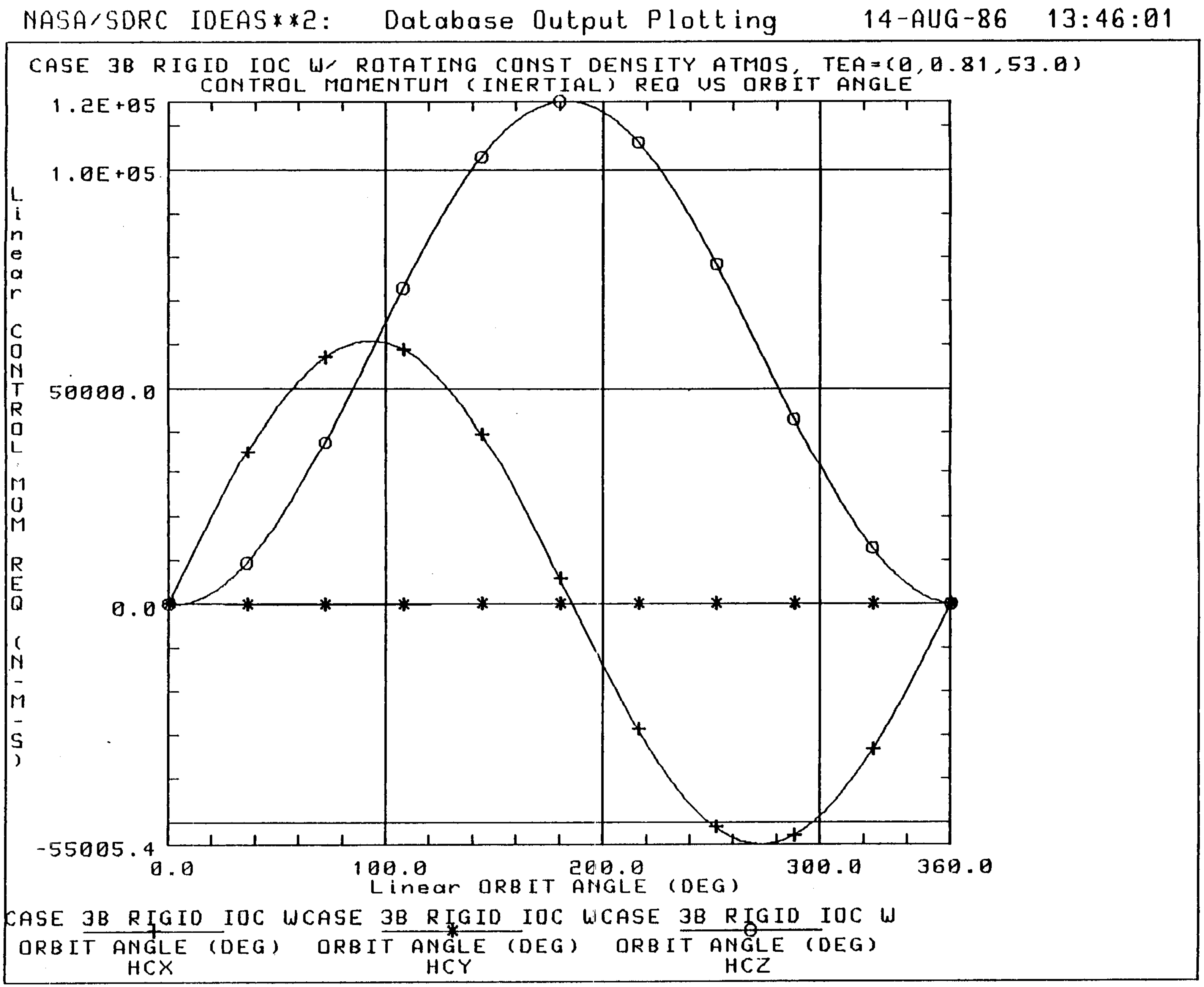

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 13:46:01
CASE 3B RIGID IOC W/ ROTATING CONST DENSITY ATMOS, TEA=(0,0.81,53.0)
CONTROL MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
1.2E+05
1.0E+05
50000.0
0.0
-55005.4
Linear CONTROL MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 3B RIGID IOC W
ORBIT ANGLE (DEG)
HCX
CASE 3B RIGID IOC W
ORBIT ANGLE (DEG)
HCY
CASE 3B RIGID IOC W
ORBIT ANGLE (DEG)
HCZ

Fig. 19

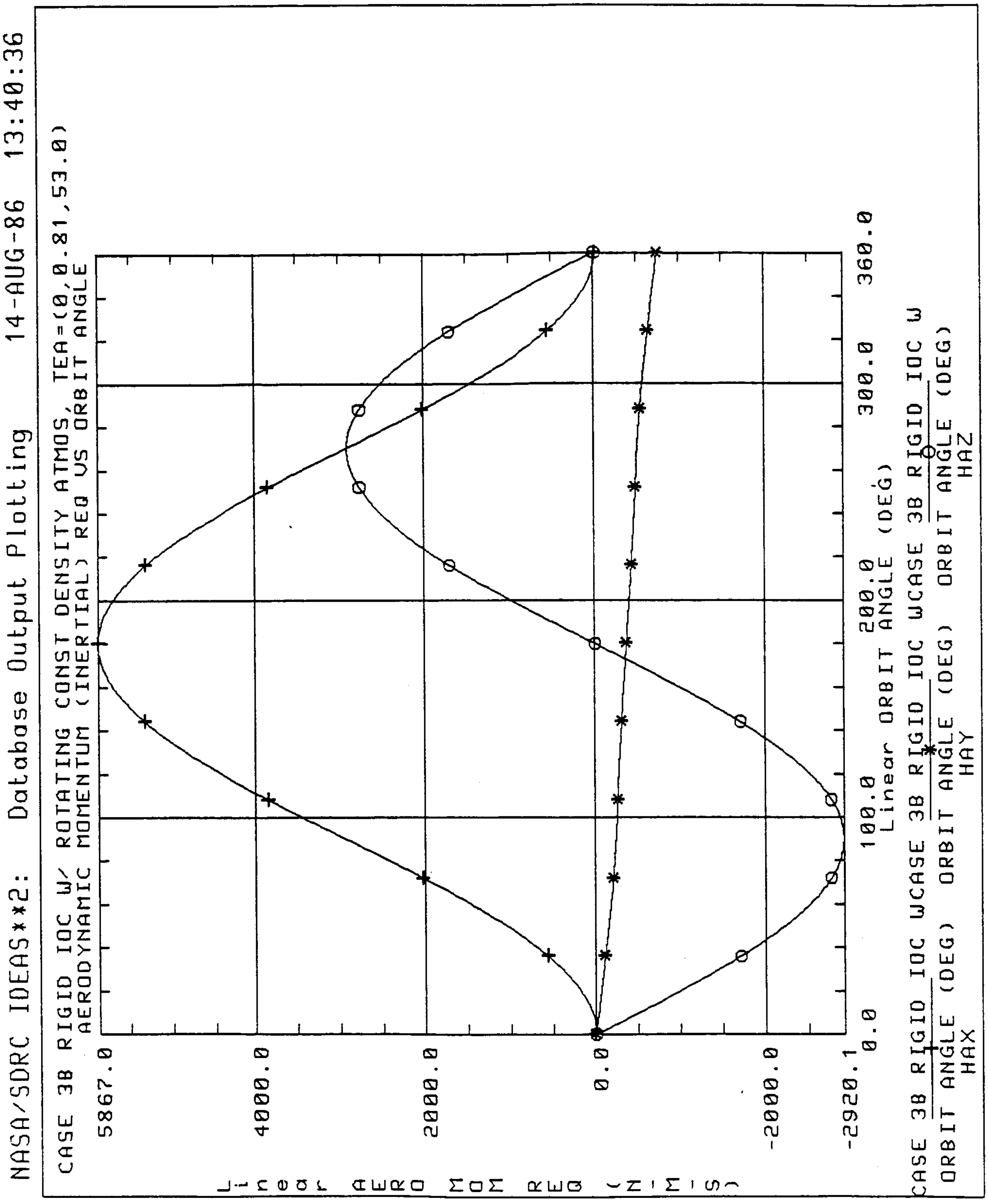

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 13:40:36
CASE 3B RIGID IOC W/ ROTATING CONST DENSITY ATMOS, TEA=(0,0.81,53.0)
AERODYNAMIC MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
Linear AERO MOM REQ (N-M-S)
5867.0
4000.0
2000.0
0.0
-2000.0
-2920.1
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 3B RIGID IOC W
ORBIT ANGLE (DEG)
HAX
CASE 3B RIGID IOC W
ORBIT ANGLE (DEG)
HAY
CASE 3B RIGID IOC W
ORBIT ANGLE (DEG)
HAZ

Fig. 20

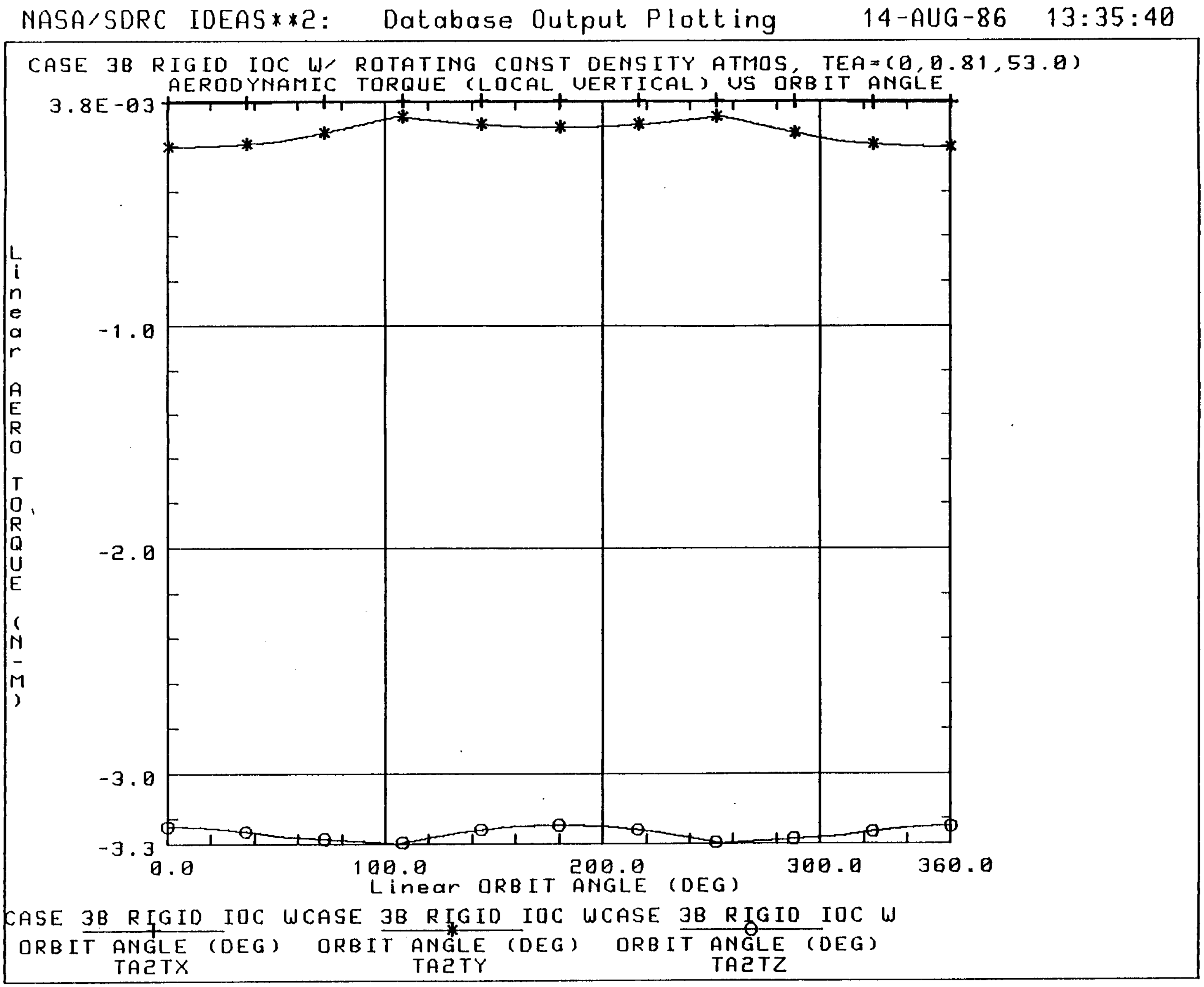

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 13:35:40
CASE 3B RIGID IOC W/ ROTATING CONST DENSITY ATMOS, TEA=(0,0.81,53.0)
AERODYNAMIC TORQUE (LOCAL VERTICAL) VS ORBIT ANGLE
3.8E-03
-1.0
-2.0
-3.0
-3.3
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
Linear AERO TORQUE (N-M)
CASE 3B RIGID IOC W
ORBIT ANGLE (DEG)
TA2TX
CASE 3B RIGID IOC W
ORBIT ANGLE (DEG)
TA2TY
CASE 3B RIGID IOC W
ORBIT ANGLE (DEG)
TA2TZ

Fig. 21

5.4 Case 4 Non-Articulating; Variable Density, Rotating Atmosphere

For a fourth case, a rigid Space Station was simulated flying through a rotating atmosphere with a Jachia 1970 (J70) atmosphere density model and no solar pressure. Figure 22 shows a time history of the control momentum required to maintain the attitude $(\psi,\theta,\phi) = (0,-0.92,-34.79)$ from Case 2B [Case 4A]. Secular IOP control momentum requirements are due to the aerodynamic momentum contribution as shown in Fig. 23. Figure 24 shows that the secular IOP aerodynamic momentum contribution results from a_x and a_z, which are now both time-dependent in LVLH coordinates.

The program ARCD was used to determine a unique 3 axis attitude solution of $(\psi,\theta,\phi) = (-39.2,8.09,-27.2)$ for Case 4B such that control momentum requirements were purely cyclic. This attitude balances the secular aerodynamic momentum contributed by a_x with the aerodynamic momentum contributed by a_z to achieve zero secular IOP control momentum requirements. In addition, the POP gravity gradient momentum balances the POP aerodynamic momentum to achieve zero secular POP control momentum requirements. Figures 25 through 30 show the control momentum requirements, control torque requirements, aerodynamic momentum, aerodynamic torques, gravity gradient momentum, and gyroscopic momentum, respectively, for Case 4B.

In all of the previous cases, no attempt was made to reduce the control momentum cyclic peaks which influence the AMCD sizing. It can be seen from Case 4B that although large angle attitude solutions may give rise to zero secular control momentum requirements, they also tend to result in large IOP control momentum and control torque cyclic peaks. These huge peaks are caused by the gravity gradient and gyroscopic effects, which in general become large whenever the principal axes are not aligned with LVLH.

5.5 Case 5 - Non-Articulating; Principal Axis Attitudes

For a fifth case, the Space Station was simulated flying through a rotating Jachia atmosphere at a principal axis attitude of $(\psi,\theta,\phi) = (-0.77,0.33,-5.66)$ [Case 5A]. No gravity gradient or gyroscopic torques can be generated at this attitude, so the control momentum requirements shown in Fig. 31 are due solely to the aerodynamic momentum contribution. For this attitude, relatively small peak control momentum requirements exist; however, a large secular control momentum build-up must be incurred in order to maintain this orientation.

The program ARCD was used to determine an attitude of $(\psi,\theta,\phi) = (-0.77,-0.32,-5.66)$ [Case 5B] which minimized the secular momentum without substantially increasing the peak momentum. In section 3.1 it was shown that a DOF in pitch about the principal axis solution existed such that the IOP gravity gradient momentum and the gyroscopic momentum remained zero. This DOF was used by ARCD to balance the POP gravity gradient momentum shown in Fig. 32 with the POP aerodynamic momentum shown in Fig. 33, to give zero secular POP control momentum requirements, as shown in Fig. 34. It was not possible to reduce the secular IOP control momentum requirements resulting from aerodynamics with small yaw and roll angles.

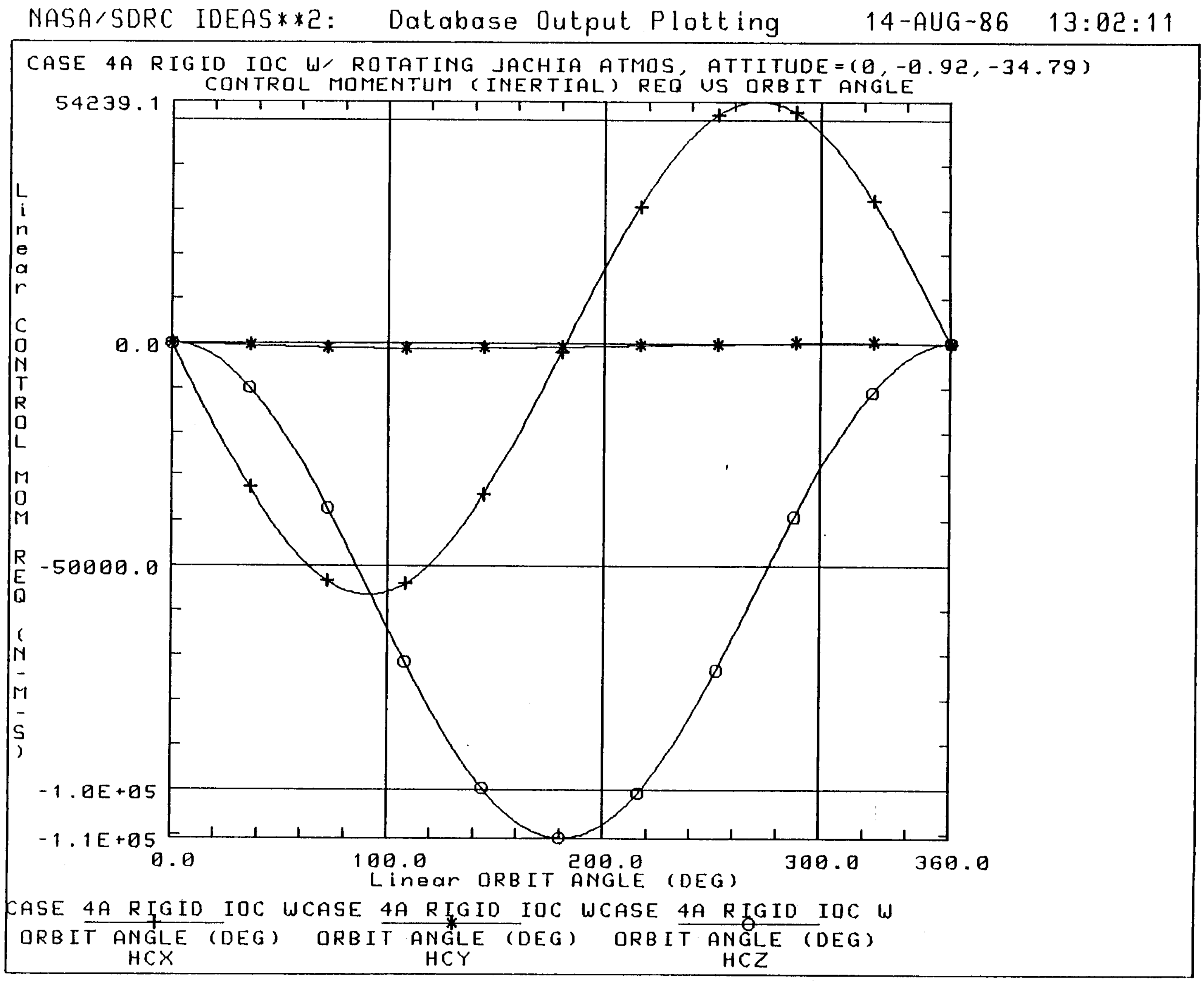

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 13:02:11
CASE 4A RIGID IOC W/ ROTATING JACHIA ATMOS, ATTITUDE=(0,-0.92,-34.79)
CONTROL MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
54239.1
0.0
-50000.0
-1.0E+05
-1.1E+05
Linear CONTROL MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 4A RIGID IOC W
ORBIT ANGLE (DEG)
HCX
CASE 4A RIGID IOC W
ORBIT ANGLE (DEG)
HCY
CASE 4A RIGID IOC W
ORBIT ANGLE (DEG)
HCZ

Fig. 22

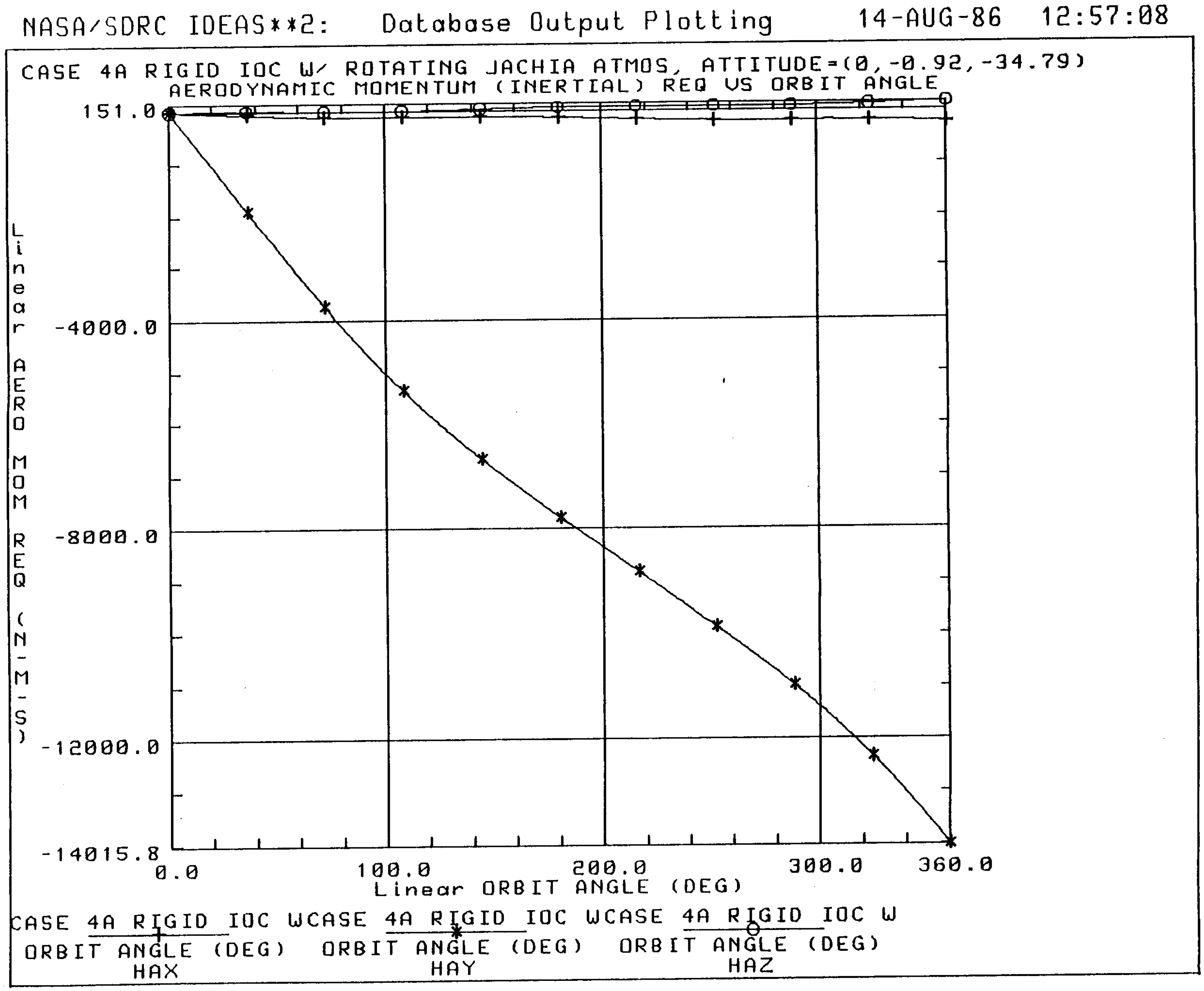

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 12:57:08
CASE 4A RIGID IOC W/ ROTATING JACHIA ATMOS, ATTITUDE=(0,-0.92,-34.79)
AERODYNAMIC MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
Linear AERO MOM REQ (N-M-S)
151.0
-4000.0
-8000.0
-12000.0
-14015.8
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 4A RIGID IOC W
ORBIT ANGLE (DEG)
HAX
CASE 4A RIGID IOC W
ORBIT ANGLE (DEG)
HAY
CASE 4A RIGID IOC W
ORBIT ANGLE (DEG)
HAZ

Fig. 23

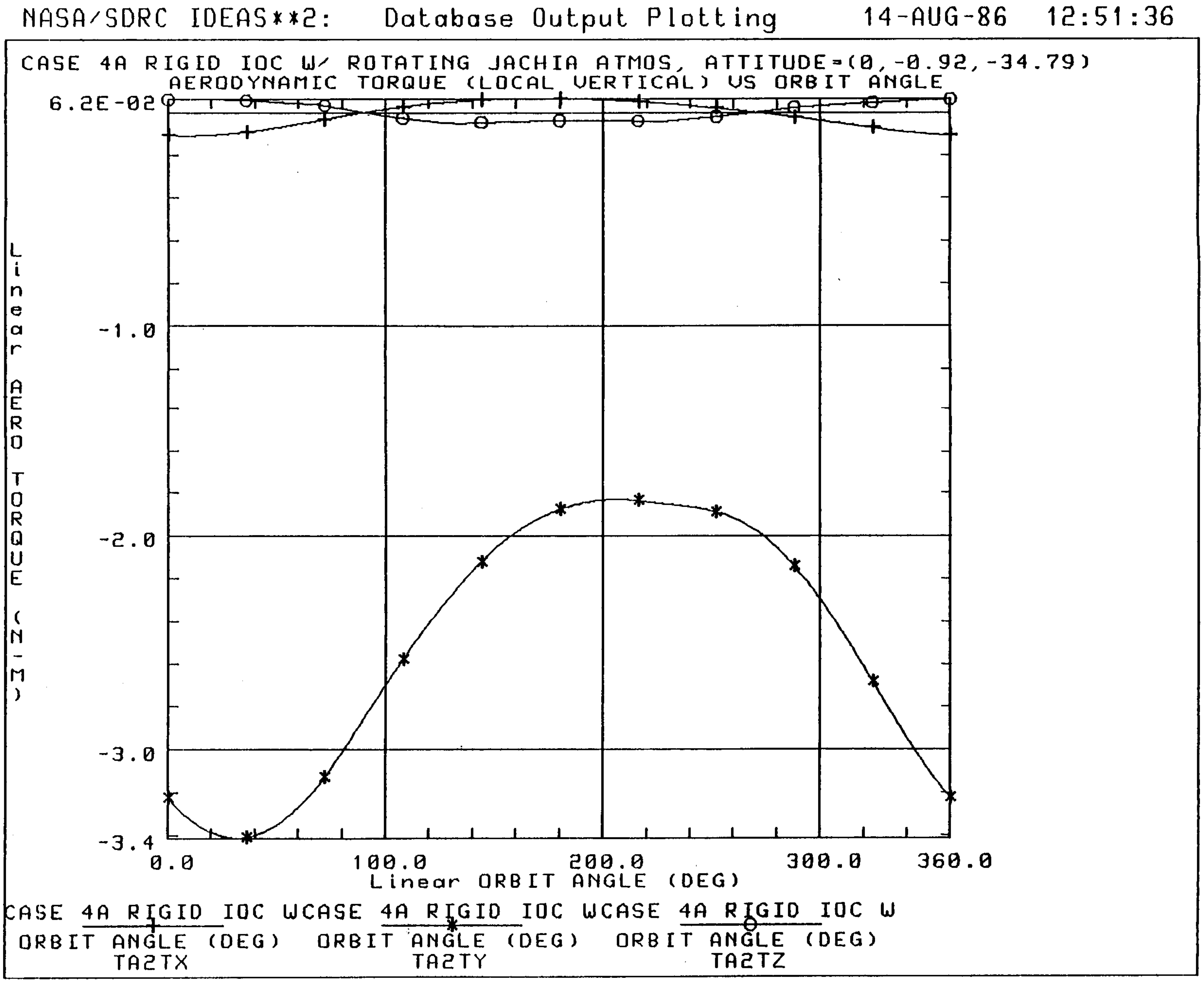
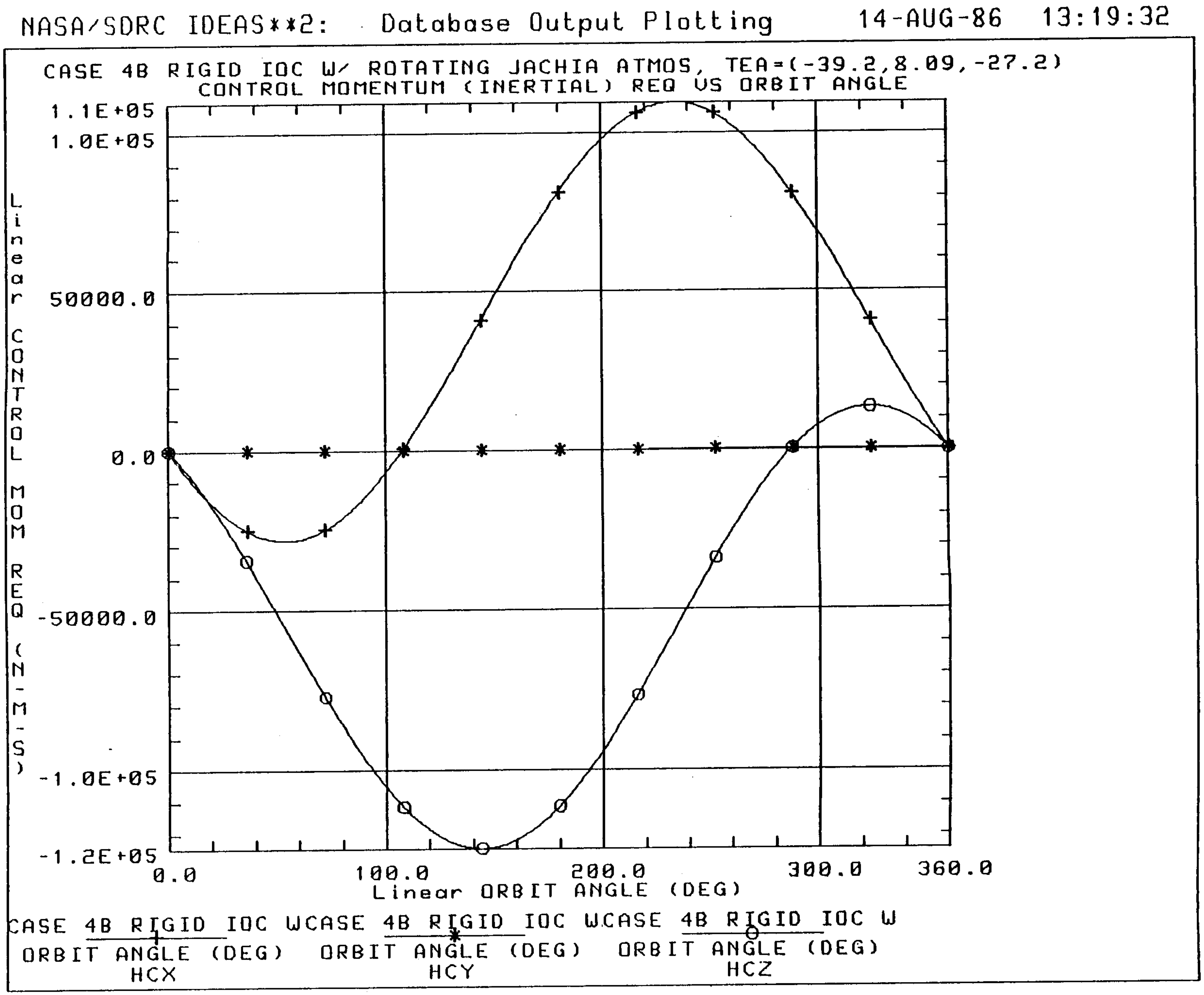

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 12:51:36
CASE 4A RIGID IOC W/ ROTATING JACHIA ATMOS, ATTITUDE=(0,-0.92,-34.79)
AERODYNAMIC TORQUE (LOCAL VERTICAL) VS ORBIT ANGLE
6.2E-02
-1.0
-2.0
-3.0
-3.4
Linear AERO TORQUE (N-M)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 4A RIGID IOC W
ORBIT ANGLE (DEG)
TA2TX
CASE 4A RIGID IOC W
ORBIT ANGLE (DEG)
TA2TY
CASE 4A RIGID IOC W
ORBIT ANGLE (DEG)
TA2TZ

Fig. 24



NASA/SDRC IDEAS**2: Database Output Plotting
14-AUG-86 13:19:32
CASE 4B RIGID IOC W/ ROTATING JACHIA ATMOS, TEA=(-39.2,8.09,-27.2)
CONTROL MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
1.1E+05
1.0E+05
50000.0
0.0
-50000.0
-1.0E+05
-1.2E+05
Linear CONTROL MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 4B RIGID IOC W
ORBIT ANGLE (DEG)
HCX
CASE 4B RIGID IOC W
ORBIT ANGLE (DEG)
HCY
CASE 4B RIGID IOC W
ORBIT ANGLE (DEG)
HCZ

Fig. 25

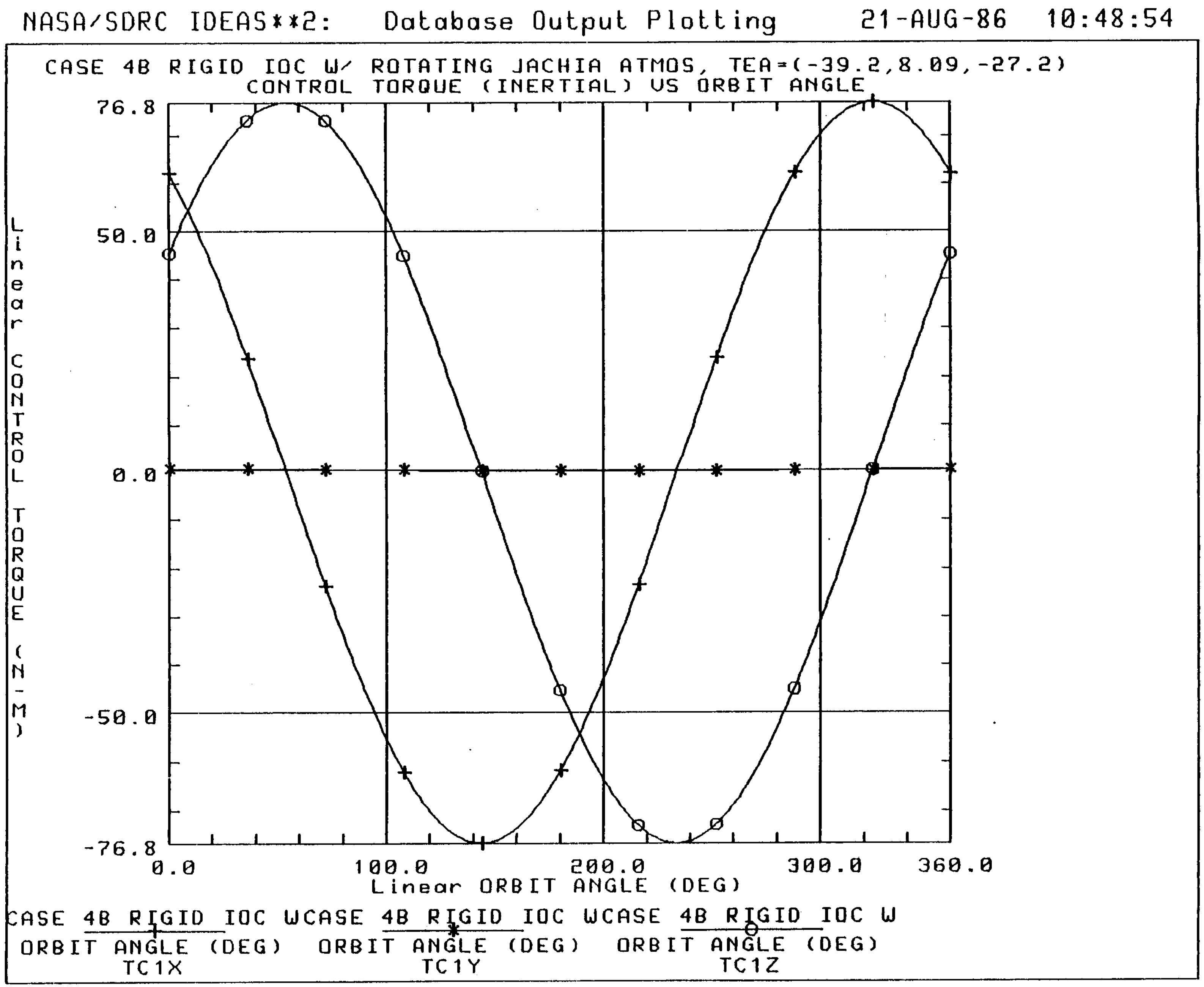

NASA/SDRC IDEAS**2: Database Output Plotting 21-AUG-86 10:48:54
CASE 4B RIGID IOC W/ ROTATING JACHIA ATMOS, TEA=(-39.2,8.09,-27.2)
CONTROL TORQUE (INERTIAL) VS ORBIT ANGLE
Linear CONTROL TORQUE (N-M)
76.8
50.0
0.0
-50.0
-76.8
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 4B RIGID IOC W
ORBIT ANGLE (DEG)
TC1X
CASE 4B RIGID IOC W
ORBIT ANGLE (DEG)
TC1Y
CASE 4B RIGID IOC W
ORBIT ANGLE (DEG)
TC1Z

Fig. 26

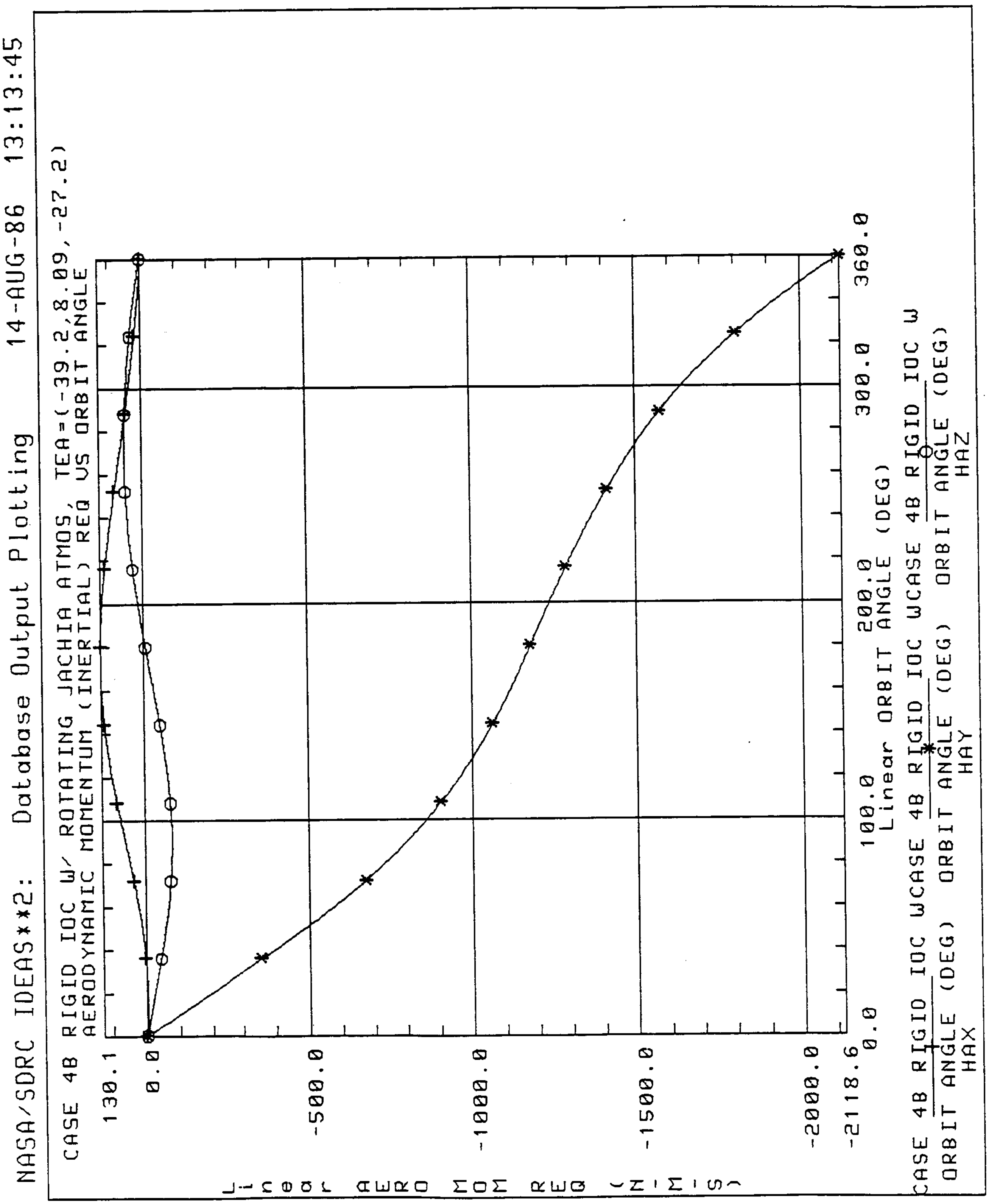

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 13:13:45
CASE 4B RIGID IOC W/ ROTATING JACHIA ATMOS, TEA=(-39.2,8.09,-27.2)
AERODYNAMIC MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
Linear AERO MOM REQ (N-M-S)
130.1
0.0
-500.0
-1000.0
-1500.0
-2000.0
-2118.6
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 4B RIGID IOC WCASE 4B RIGID IOC WCASE 4B RIGID IOC W
ORBIT ANGLE (DEG) ORBIT ANGLE (DEG) ORBIT ANGLE (DEG)
HAX
HAY
HAZ

Fig. 27

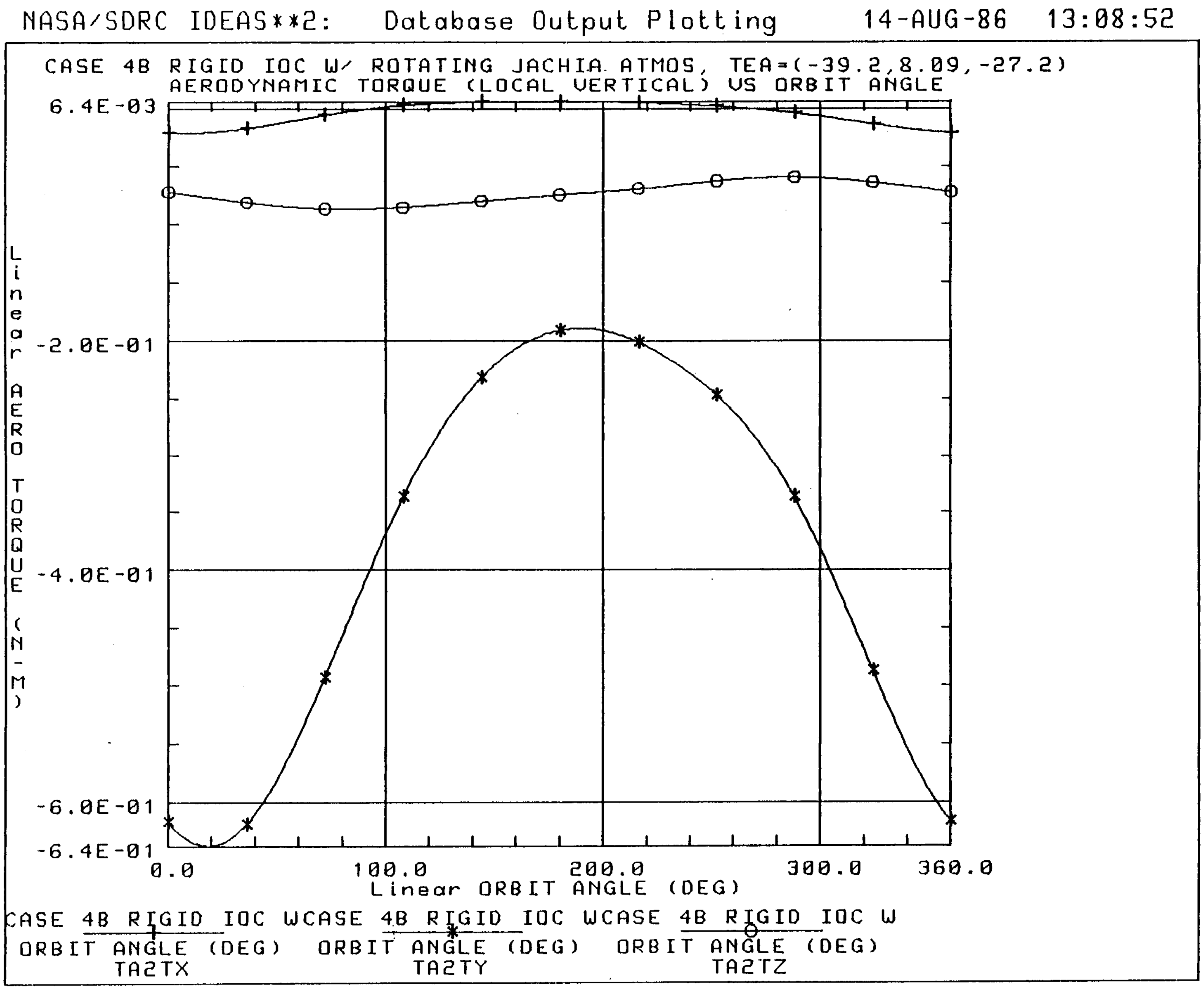
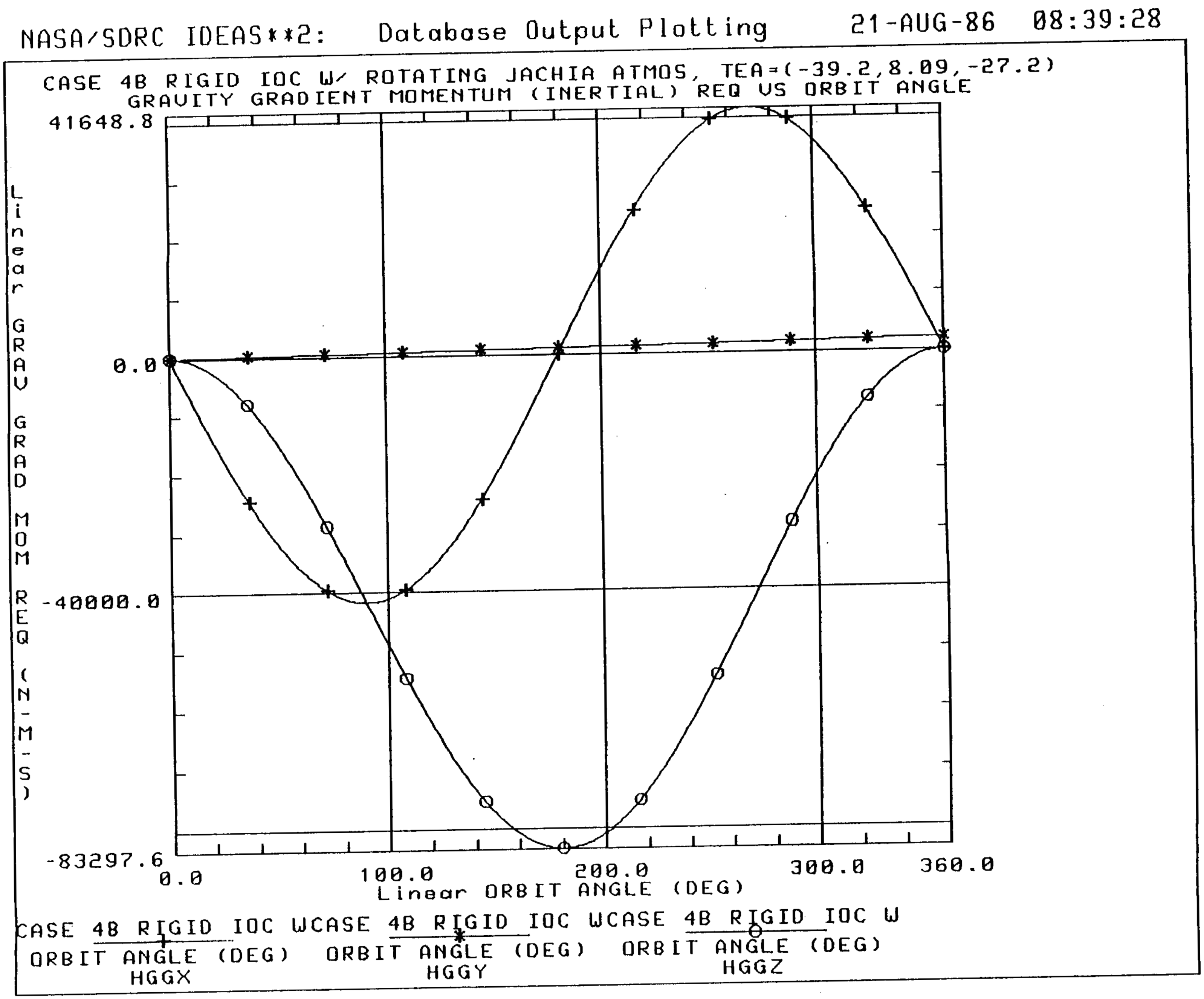

NASA/SDRC IDEAS**2: Database Output Plotting 14-AUG-86 13:08:52
CASE 4B RIGID IOC W/ ROTATING JACHIA ATMOS, TEA=(-39.2,8.09,-27.2)
AERODYNAMIC TORQUE (LOCAL VERTICAL) VS ORBIT ANGLE
6.4E-03
-2.0E-01
-4.0E-01
-6.0E-01
-6.4E-01
Linear AERO TORQUE (N-M)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 4B RIGID IOC W
ORBIT ANGLE (DEG)
TA2TX
CASE 4B RIGID IOC W
ORBIT ANGLE (DEG)
TA2TY
CASE 4B RIGID IOC W
ORBIT ANGLE (DEG)
TA2TZ

Fig. 28



NASA/SDRC IDEAS**2: Database Output Plotting 21-AUG-86 08:39:28
CASE 4B RIGID IOC W/ ROTATING JACHIA ATMOS, TEA=(-39.2,8.09,-27.2)
GRAVITY GRADIENT MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
41648.8
0.0
-40000.0
-83297.6
Linear GRAV GRAD MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 4B RIGID IOC W
ORBIT ANGLE (DEG)
HGGX
CASE 4B RIGID IOC W
ORBIT ANGLE (DEG)
HGGY
CASE 4B RIGID IOC W
ORBIT ANGLE (DEG)
HGGZ

Fig. 29

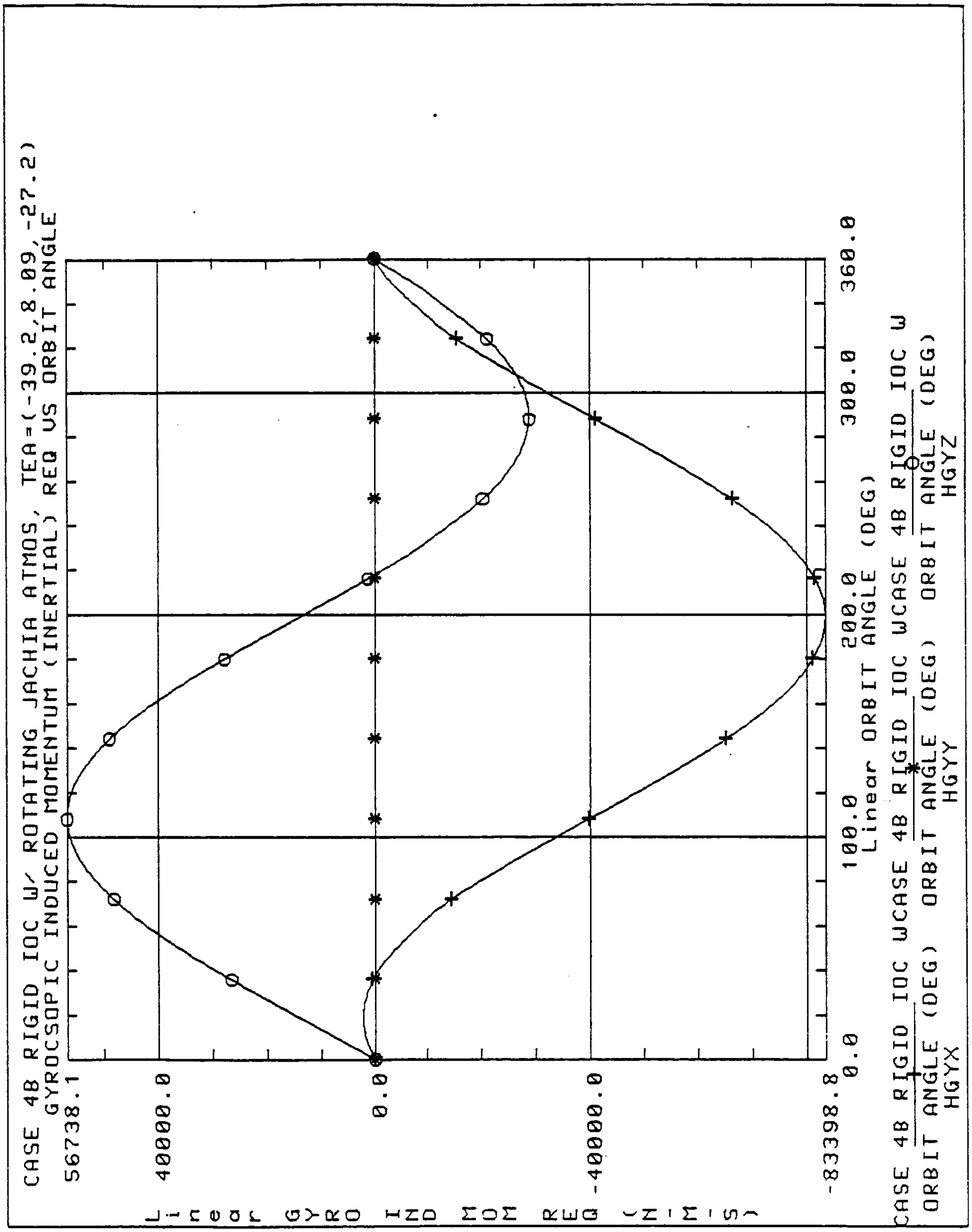

NASA/SDRC IDEAS**2: Database Output Plotting 21-AUG-86 08:42:50
CASE 4B RIGID IOC W/ ROTATING JACHIA ATMOS, TEA=(-39.2,8.09,-27.2)
GYROCSOPIC INDUCED MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
56738.1
40000.0
0.0
-40000.0
-83398.8
Linear GYRO IND MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 4B RIGID IOC WCASE 4B RIGID IOC WCASE 4B RIGID IOC W
ORBIT ANGLE (DEG) ORBIT ANGLE (DEG) ORBIT ANGLE (DEG)
HGYX HGYY HGYZ

Fig. 30

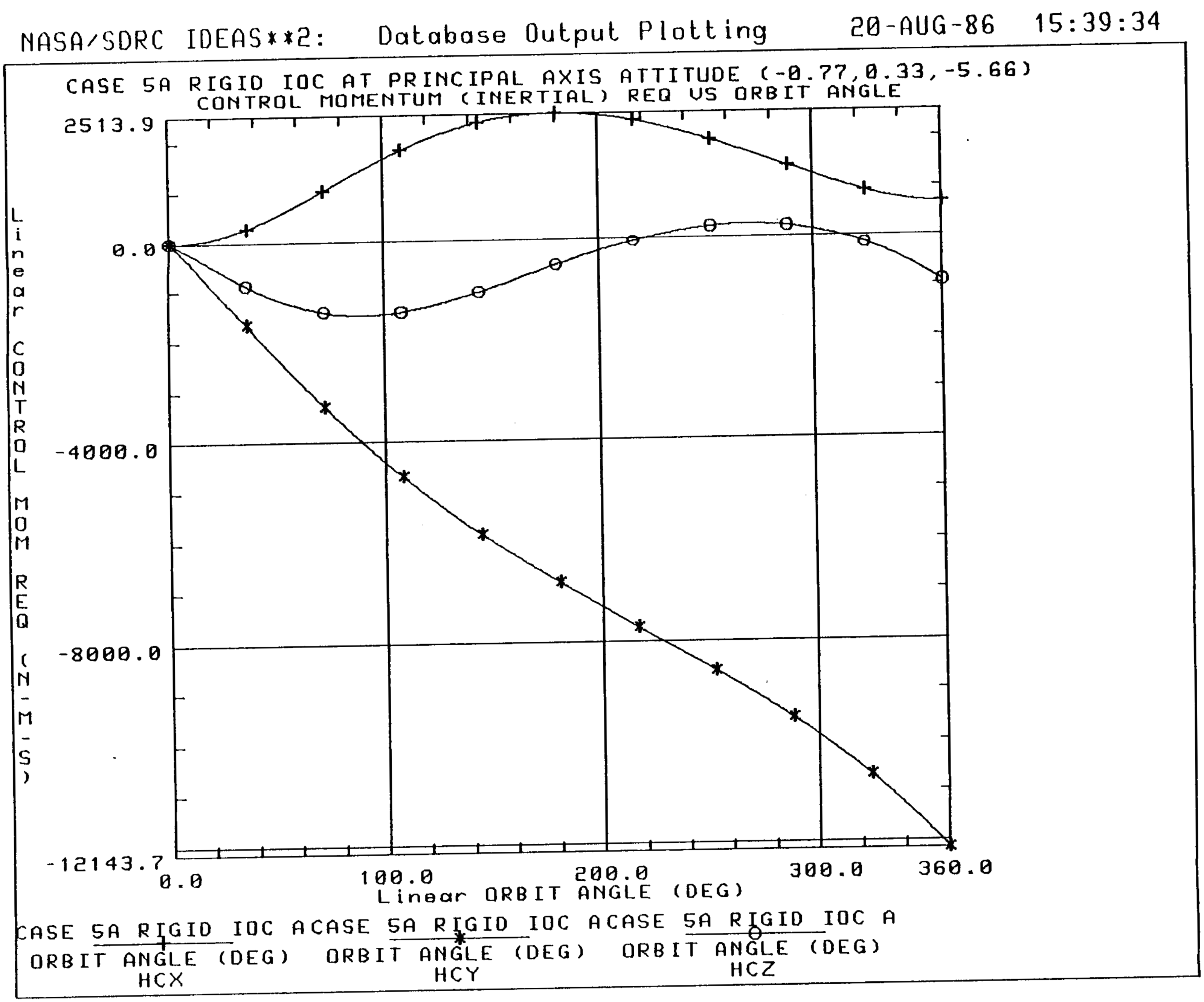
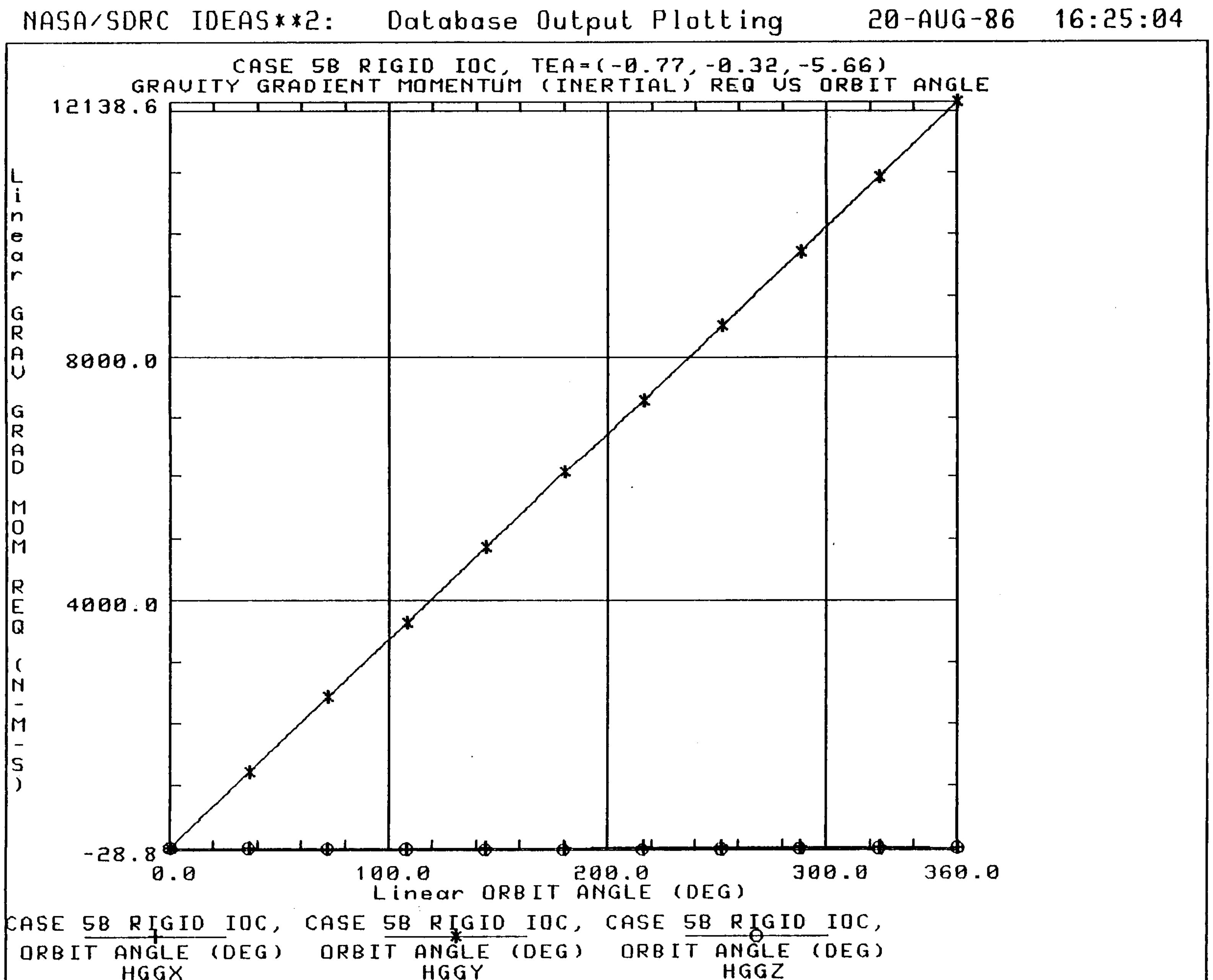

NASA/SDRC IDEAS**2: Database Output Plotting 20-AUG-86 15:39:34
CASE 5A RIGID IOC AT PRINCIPAL AXIS ATTITUDE (-0.77,0.33,-5.66)
CONTROL MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
2513.9
0.0
-4000.0
-8000.0
-12143.7
Linear CONTROL MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 5A RIGID IOC A
ORBIT ANGLE (DEG)
HCX
CASE 5A RIGID IOC A
ORBIT ANGLE (DEG)
HCY
CASE 5A RIGID IOC A
ORBIT ANGLE (DEG)
HCZ

Fig. 31



NASA/SDRC IDEAS**2: Database Output Plotting 20-AUG-86 16:25:04
CASE 5B RIGID IOC, TEA=(-0.77,-0.32,-5.66)
GRAVITY GRADIENT MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
12138.6
8000.0
4000.0
-28.8
Linear GRAV GRAD MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 5B RIGID IOC, ORBIT ANGLE (DEG) HGGX
CASE 5B RIGID IOC, ORBIT ANGLE (DEG) HGGY
CASE 5B RIGID IOC, ORBIT ANGLE (DEG) HGGZ

Fig. 32

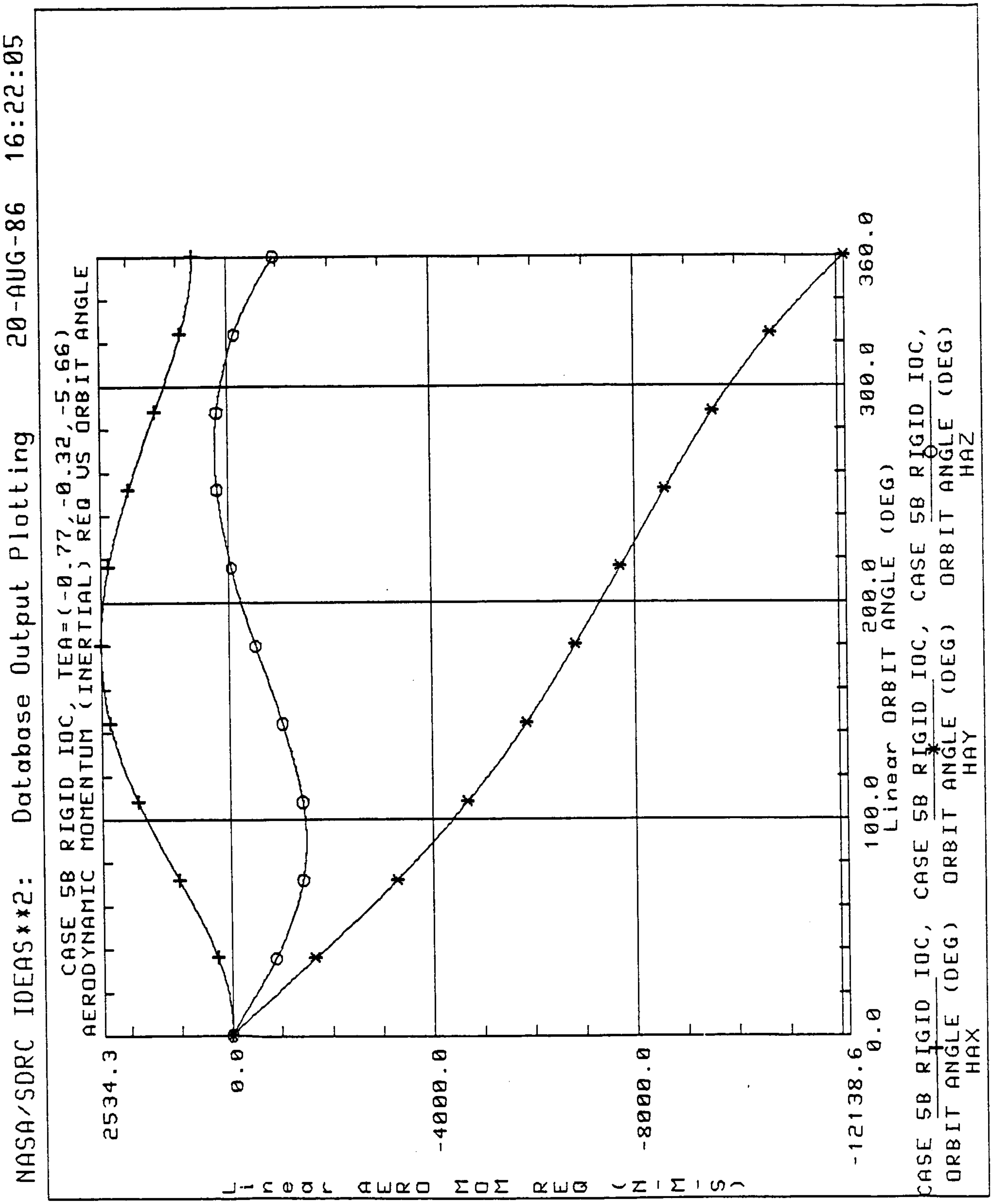

NASA/SDRC IDEAS**2: Database Output Plotting 20-AUG-86 16:22:05
CASE 5B RIGID IOC, TEA=(-0.77,-0.32,-5.66)
AERODYNAMIC MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
2534.3
0.0
-4000.0
-8000.0
-12138.6
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
Linear AERO MOM REQ (N-M-S)
CASE 5B RIGID IOC, ORBIT ANGLE (DEG) HAX
CASE 5B RIGID IOC, ORBIT ANGLE (DEG) HAY
CASE 5B RIGID IOC, ORBIT ANGLE (DEG) HAZ

Fig. 33

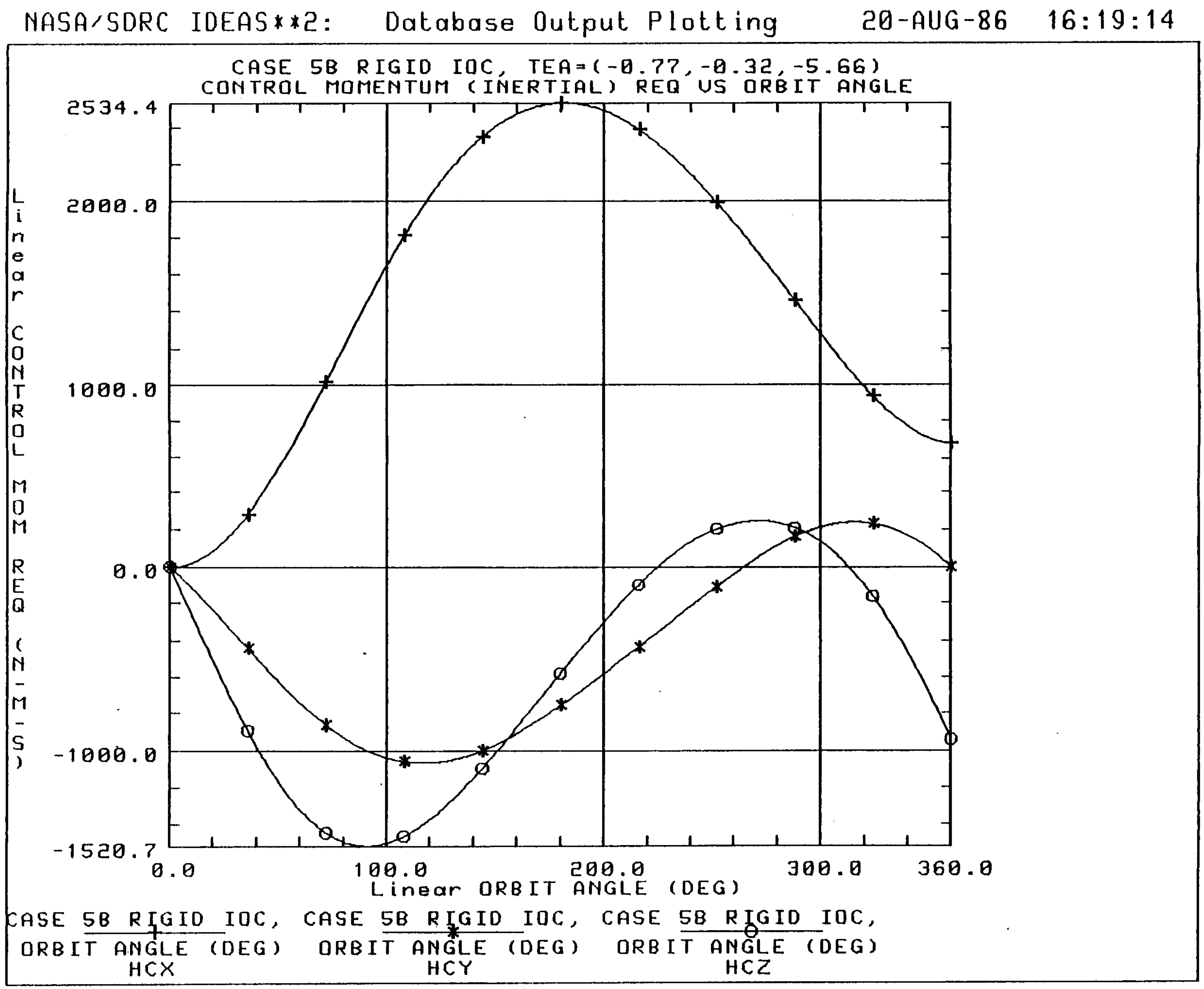

NASA/SDRC IDEAS**2: Database Output Plotting 20-AUG-86 16:19:14
CASE 5B RIGID IOC, TEA=(-0.77,-0.32,-5.66)
CONTROL MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
Linear CONTROL MOM REQ (N-M-S)
2534.4
2000.0
1000.0
0.0
-1000.0
-1520.7
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 5B RIGID IOC, ORBIT ANGLE (DEG) HCX
CASE 5B RIGID IOC, ORBIT ANGLE (DEG) HCY
CASE 5B RIGID IOC, ORBIT ANGLE (DEG) HCZ

Fig. 34

In summary, Case 4B had zero secular momentum requirements, but large attitude angles with corresponding large cyclic momentum requirements, while Case 5A (principal axis attitude) has small cyclic momentum requirements, but non-zero secular momentum requirements. Case 5B maintained the small cyclic momentum requirements utilizing the degree of freedom to reduce secular momentum requirements. Figures 34 and 35 show that the peak control momentum and torque requirements are relatively small at an attitude close to the principal axis attitude for a rigid spacecraft.

5.6 Case 6 - Articulating; Gravity Gradient and Gyroscopic Torques Only

In order to illustrate the effects of part articulation on the gyroscopic and gravity gradient momentum contributions, a sixth case was simulated where an articulating Space Station was flown with all aerodynamic and solar torques zero. For Case 6A, the special case of a Space Station with articular parts which rotate about their respective c.g's at orbit rate and have their principal axes aligned with the articular part axes was simulated. Since gyroscopic momentum is always cyclic if articular parts rotate at orbit rate (see Section 4.1), and for this special case gravity gradient momentum is cyclic IOP (see Section 4.2), a single DOF is sufficient to ensure that control momentum requirements are cyclic by eliminating the secular POP gravity gradient momentum contribution. Figures 36 and 37 show time histories of the gravity gradient and gyroscopic momentum contribution, respectively for an attitude of $(\psi,\theta,\phi) = (0,0.33,0)$ [Case 6A]. Note in Fig. 38 that the POP gravity gradient torque is not constant due to part articulation, even though the spacecraft maintains a constant attitude.

For Case 6B, a Space Station with actual articular parts (which do not rotate about their c.g. and do not have their principal axes aligned with the articular part axes) was simulated. Figures 39 and 40 show time histories of the gravity gradient and gyroscopic momentum contributions, respectively, at the same attitude of $(\psi,\theta,\phi) = (0,0.33,0)$ used in Case 6A. This attitude still eliminates the secular POP gravity gradient momentum; however, the IOP gravity gradient momentum is now secular and equal to approximately 250 Nms.

5.7 Case 7 - Articulating; Full Environment

For a final case, an articulating Space Station was simulated flying through a "full" environment comprised of a rotating Jachia atmosphere and solar pressure. In Case 7A, the program ARCD was used to determine an attitude $(\psi,\theta,\phi) = (-16.3,-2.34,15.8)$ which eliminated secular control momentum requirements, as shown in Fig. 41. This attitude balances the secular gravity gradient momentum contribution, shown in Fig. 42, with the secular aerodynamic and solar momentum contributions, shown in Figs. 43 and 44, respectively. Since the articular parts rotate at orbit rate, the gyroscopic momentum contribution shown in Fig. 45 is purely cyclic. Figure 46 shows that relatively large peak control torques (60Nm) and hence large peak control momentum (99,300 Nms) requirements are necessary to maintain attitude.

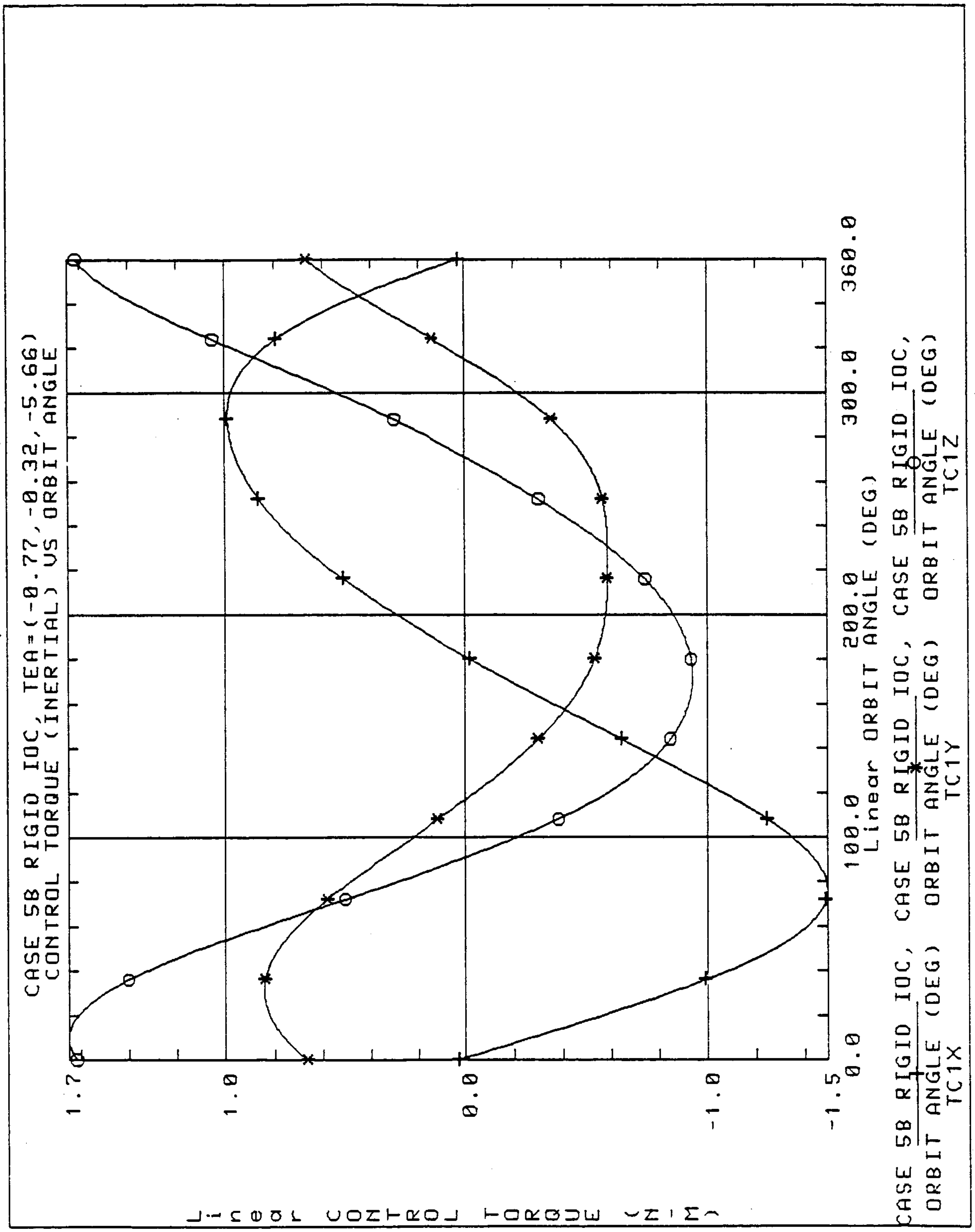
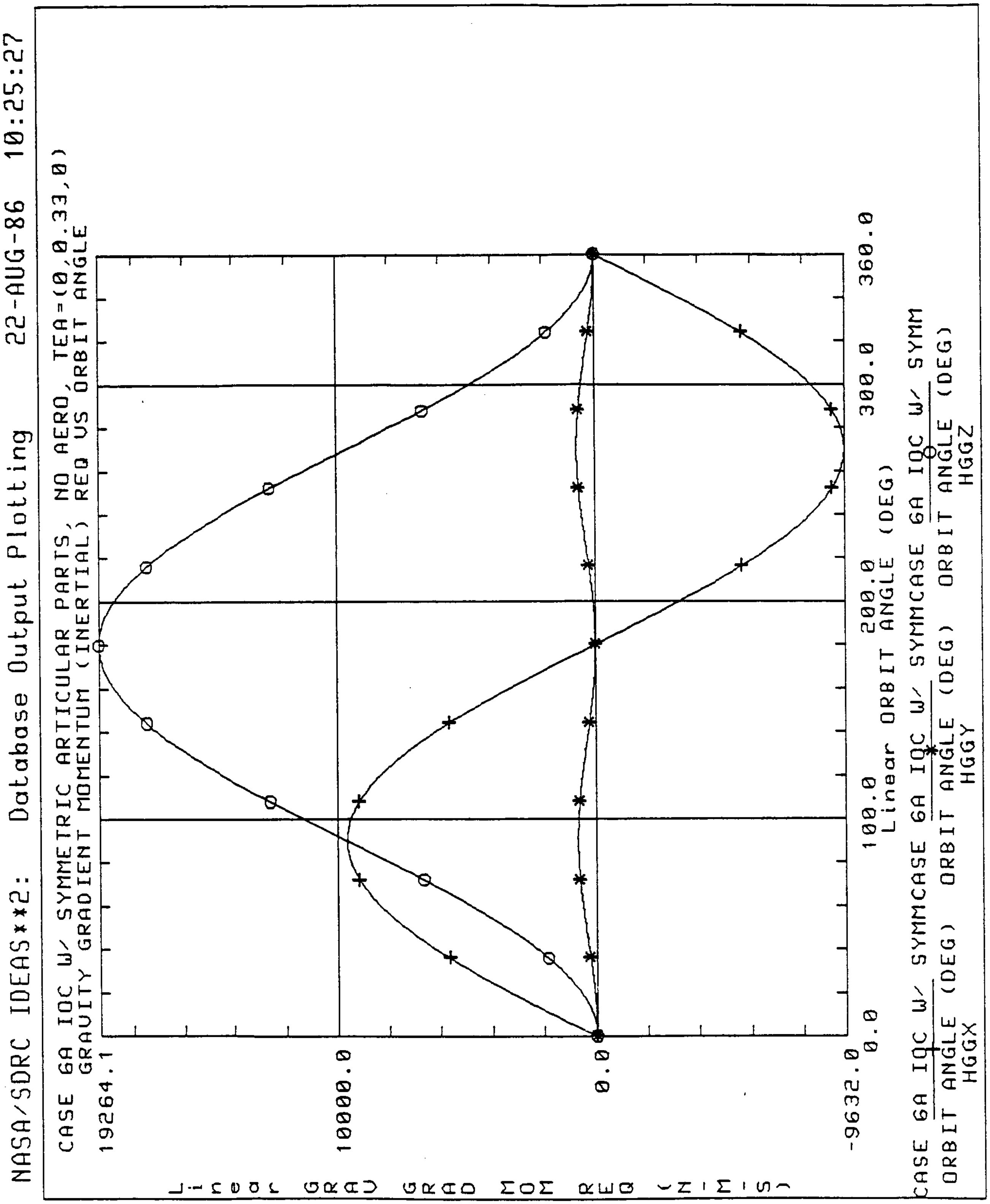

NASA/SDRC IDEAS**2: Database Output Plotting 21-AUG-86 11:43:52
CASE 5B RIGID IOC, TEA=(-0.77,-0.32,-5.66)
CONTROL TORQUE (INERTIAL) VS ORBIT ANGLE
1.7
1.0
0.0
-1.0
-1.5
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
Linear CONTROL TORQUE (N-M)
CASE 5B RIGID IOC, ORBIT ANGLE (DEG) TC1X
CASE 5B RIGID IOC, ORBIT ANGLE (DEG) TC1Y
CASE 5B RIGID IOC, ORBIT ANGLE (DEG) TC1Z

Fig. 35



NASA/SDRC IDEAS**2: Database Output Plotting 22-AUG-86 10:25:27
CASE 6A IQC W/ SYMMETRIC ARTICULAR PARTS, NO AERO, TEA=(0,0.33,0)
GRAVITY GRADIENT MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
19264.1
10000.0
0.0
-9632.0
Linear GRAD GRAD MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 6A IQC W/ SYMMCASE 6A IQC W/ SYMMCASE 6A IQC W/ SYMM
ORBIT ANGLE (DEG) ORBIT ANGLE (DEG) ORBIT ANGLE (DEG)
HGGX
HGGY
HGGZ

Fig. 36

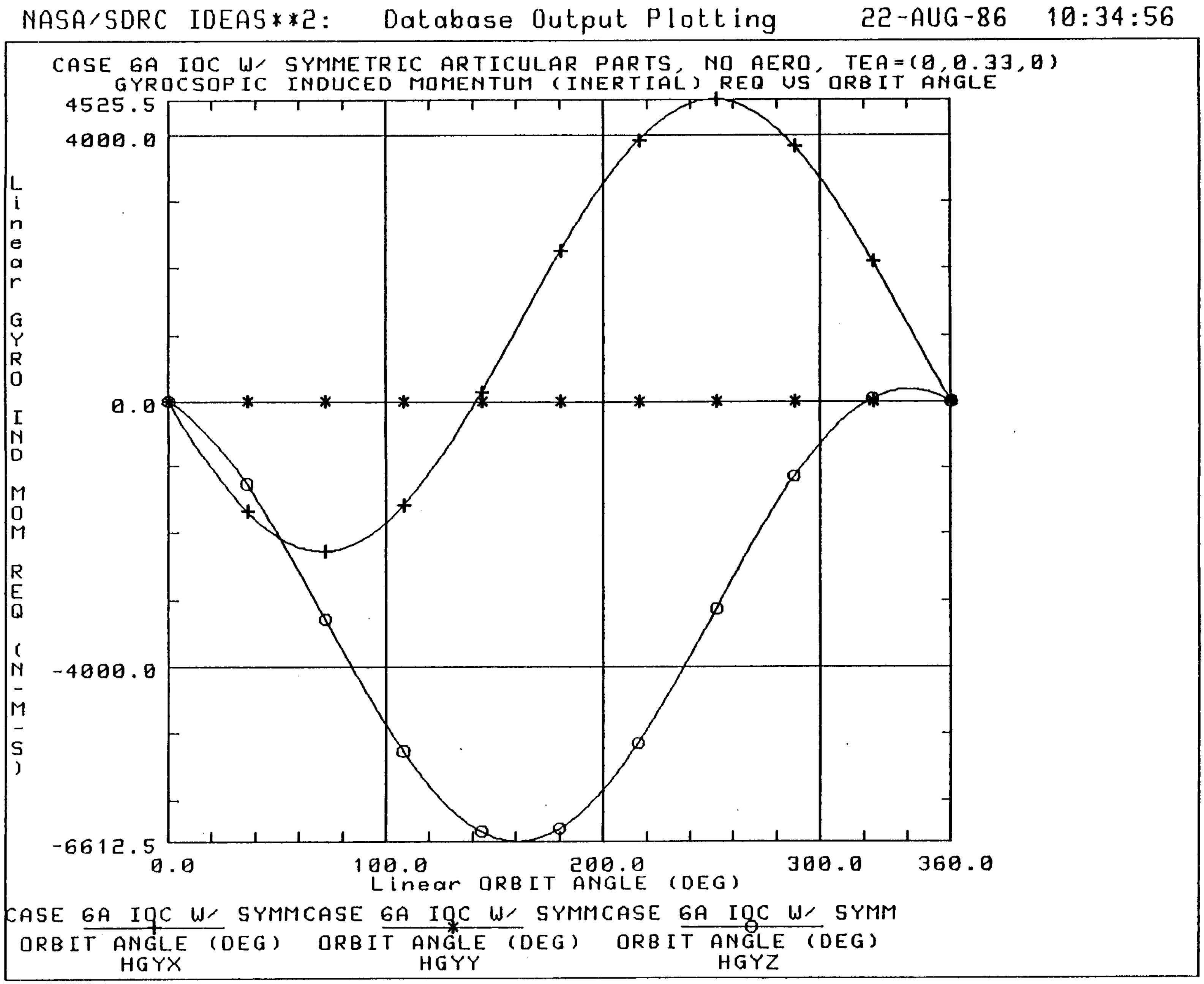
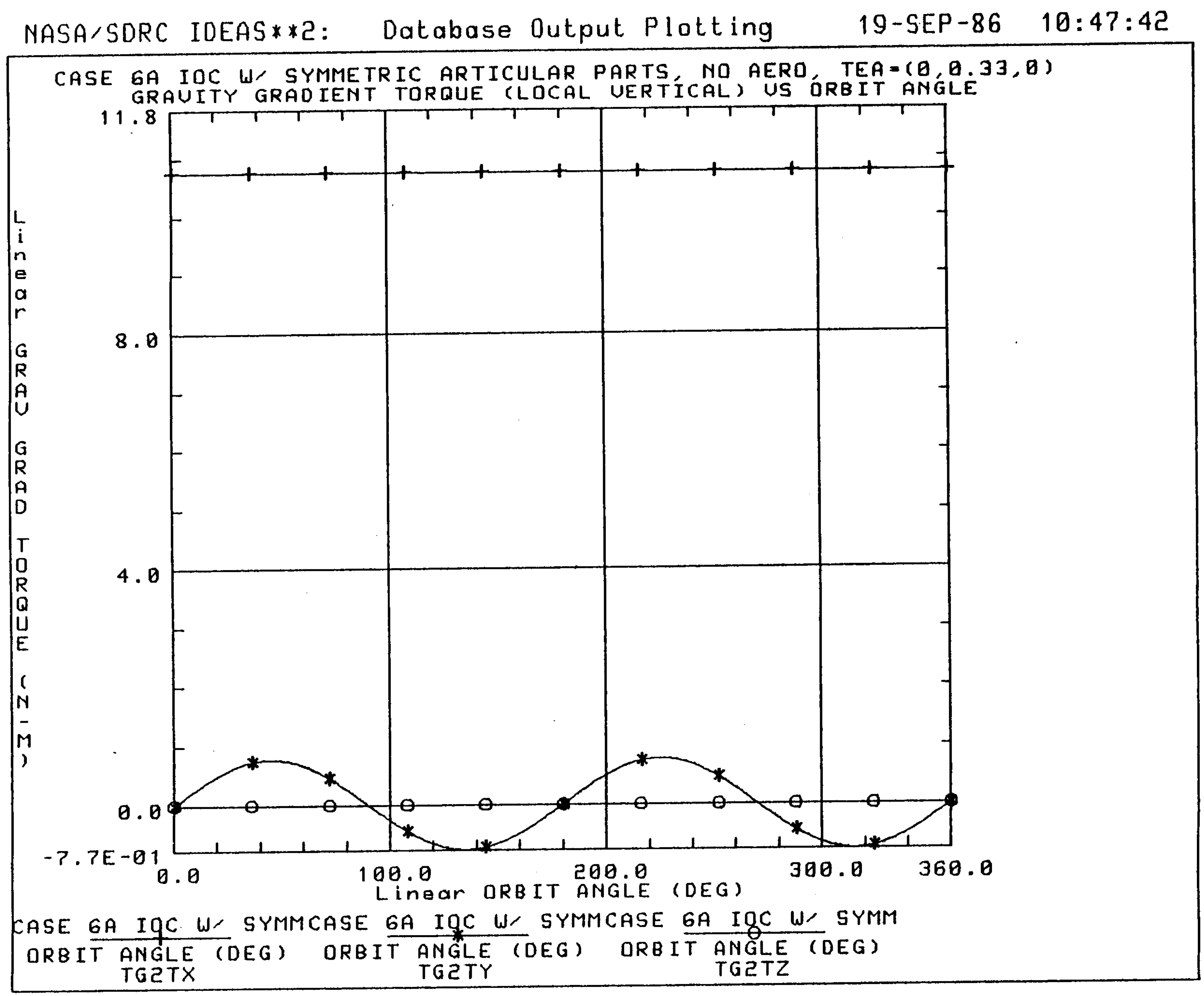

NASA/SDRC IDEAS**2: Database Output Plotting 22-AUG-86 10:34:56
CASE 6A IOC W/ SYMMETRIC ARTICULAR PARTS, NO AERO, TEA=(0,0.33,0)
GYROCSOPIC INDUCED MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
4525.5
4000.0
0.0
-4000.0
-6612.5
Linear GYRO IND MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 6A IOC W/ SYMM ORBIT ANGLE (DEG) HGYX
CASE 6A IOC W/ SYMM ORBIT ANGLE (DEG) HGYY
CASE 6A IOC W/ SYMM ORBIT ANGLE (DEG) HGYZ

Fig. 37



NASA/SDRC IDEAS**2: Database Output Plotting 19-SEP-86 10:47:42
CASE 6A IOC W/ SYMMETRIC ARTICULAR PARTS, NO AERO, TEA=(0,0.33,0)
GRAVITY GRADIENT TORQUE (LOCAL VERTICAL) VS ORBIT ANGLE
11.8
8.0
4.0
0.0
-7.7E-01
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
Linear GRAV GRAD TORQUE (N-M)
CASE 6A IOC W/ SYMM
ORBIT ANGLE (DEG)
TG2TX
CASE 6A IOC W/ SYMM
ORBIT ANGLE (DEG)
TG2TY
CASE 6A IOC W/ SYMM
ORBIT ANGLE (DEG)
TG2TZ

Fig. 38

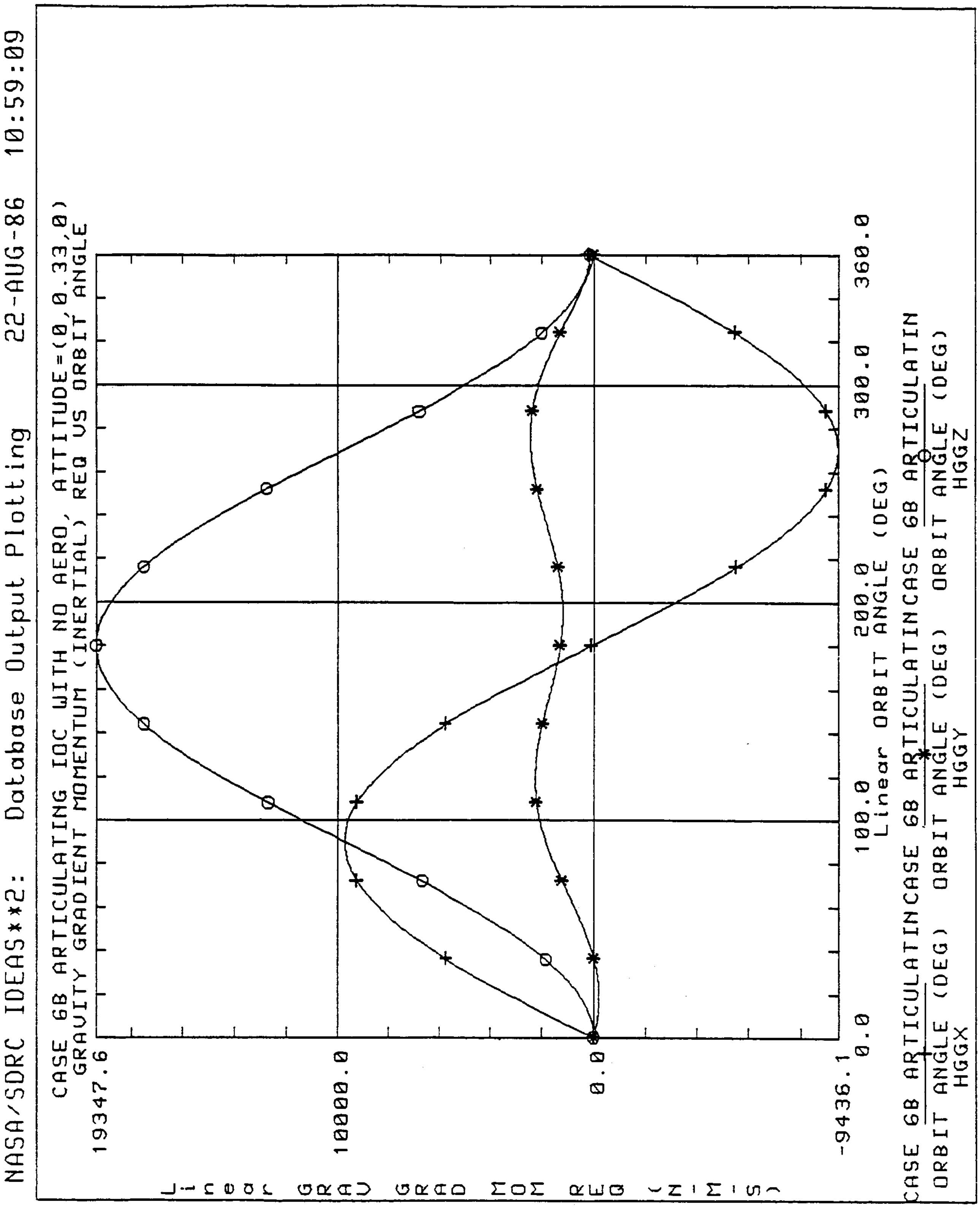

NASA/SDRC IDEAS**2: Database Output Plotting 22-AUG-86 10:59:09
CASE 6B ARTICULATING IOC WITH NO AERO, ATTITUDE=(0,0.33,0)
GRAVITY GRADIENT MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
Linear GRAV GRAD MOM REQ (N-M-S)
19347.6
10000.0
0.0
-9436.1
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 6B ARTICULATINCASE 6B ARTICULATINCASE 6B ARTICULATIN
ORBIT ANGLE (DEG) ORBIT ANGLE (DEG) ORBIT ANGLE (DEG)
HGGX
HGGY
HGGZ

Fig. 39

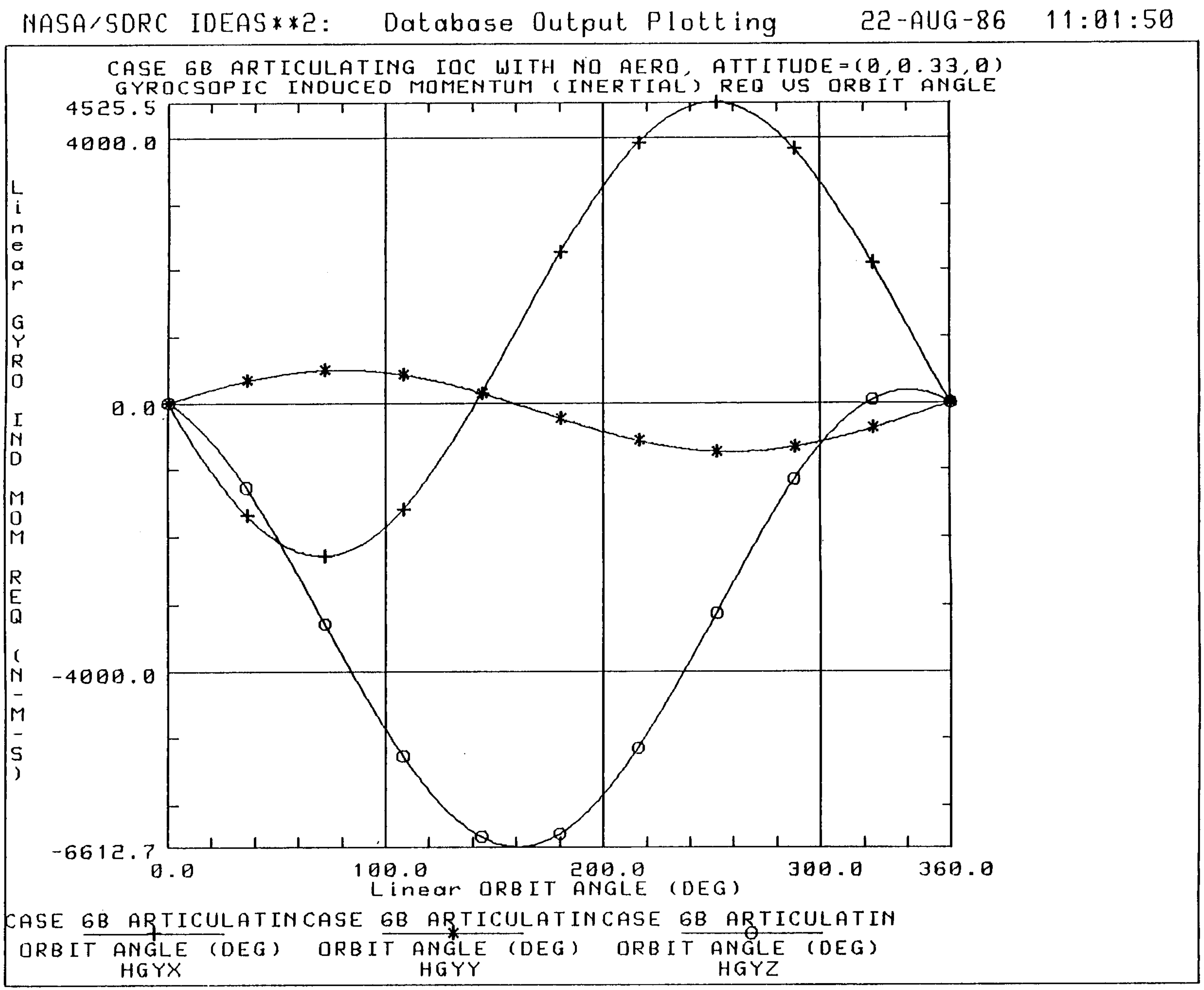

NASA/SDRC IDEAS**2: Database Output Plotting 22-AUG-86 11:01:50
CASE 6B ARTICULATING IOC WITH NO AERO, ATTITUDE=(0,0.33,0)
GYROCSOPIC INDUCED MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
4525.5
4000.0
0.0
-4000.0
-6612.7
Linear GYRO IND MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 6B ARTICULATIN ORBIT ANGLE (DEG) HGYX
CASE 6B ARTICULATIN ORBIT ANGLE (DEG) HGYY
CASE 6B ARTICULATIN ORBIT ANGLE (DEG) HGYZ

Fig. 40

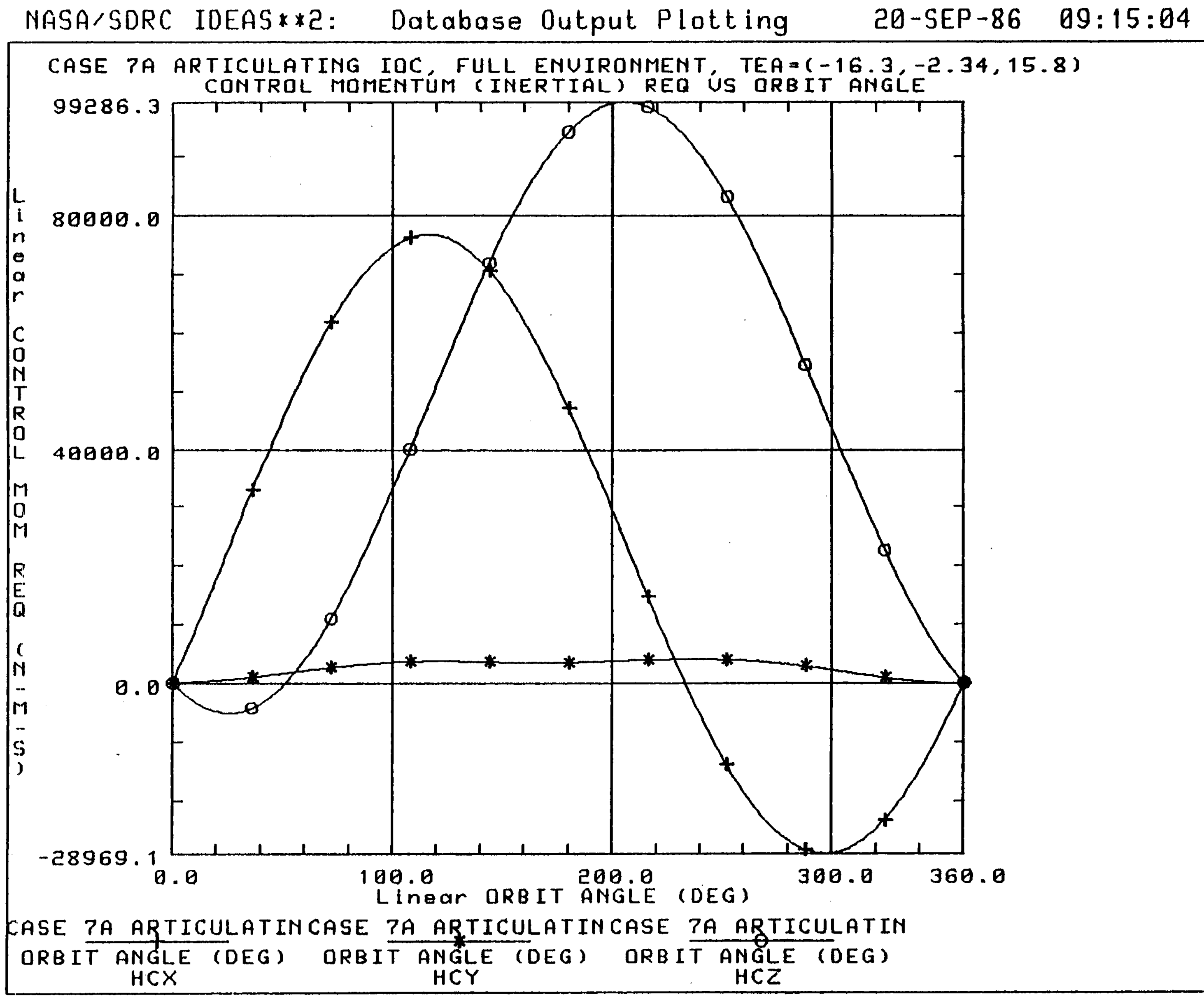

NASA/SDRC IDEAS**2: Database Output Plotting 20-SEP-86 09:15:04
CASE 7A ARTICULATING IOC, FULL ENVIRONMENT, TEA=(-16.3,-2.34,15.8)
CONTROL MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
99286.3
80000.0
40000.0
0.0
-28969.1
Linear CONTROL MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 7A ARTICULATIN
ORBIT ANGLE (DEG)
HCX
CASE 7A ARTICULATIN
ORBIT ANGLE (DEG)
HCY
CASE 7A ARTICULATIN
ORBIT ANGLE (DEG)
HCZ

Fig. 41

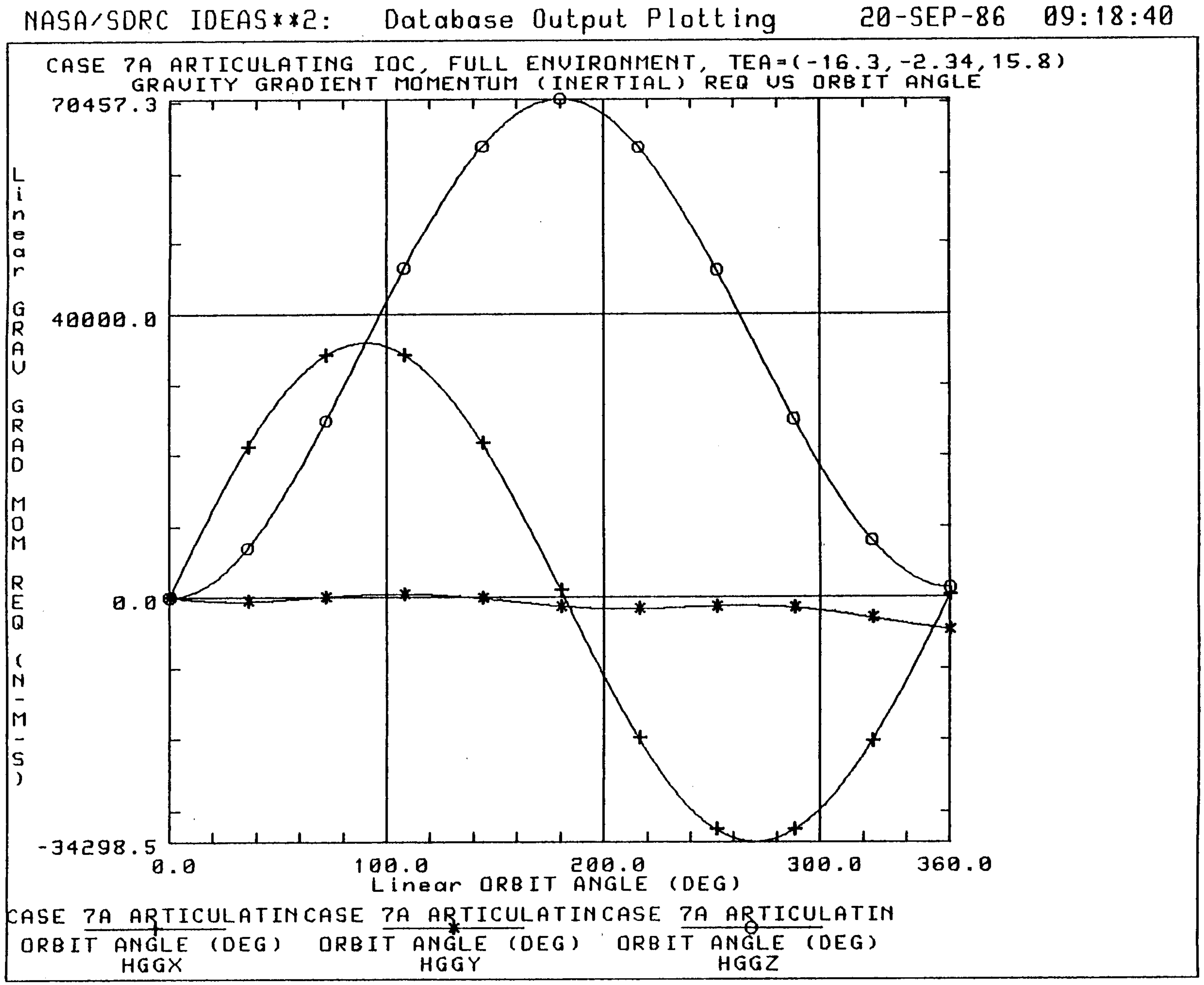
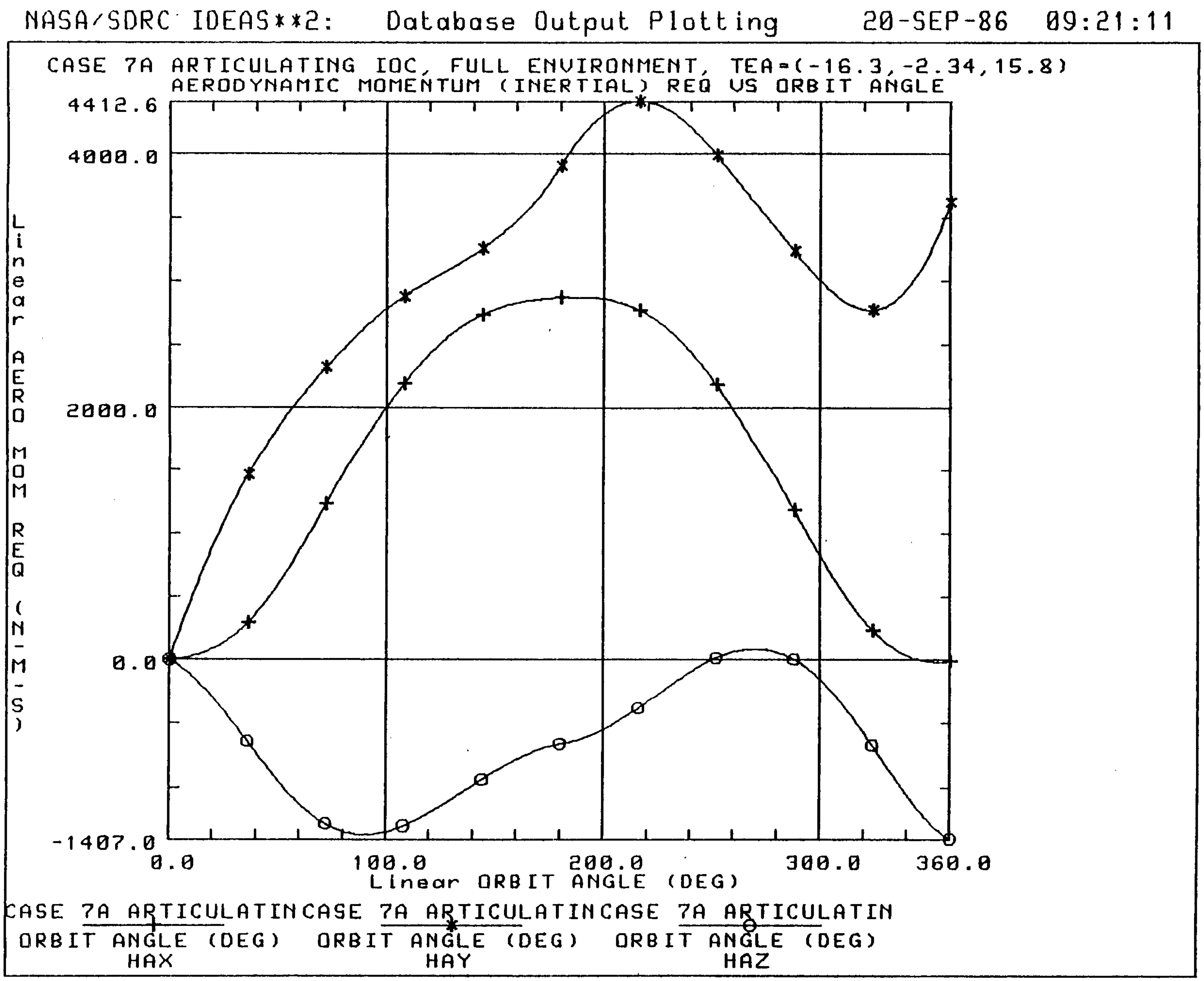

NASA/SDRC IDEAS**2: Database Output Plotting 20-SEP-86 09:18:40
CASE 7A ARTICULATING IOC, FULL ENVIRONMENT, TEA=(-16.3,-2.34,15.8)
GRAVITY GRADIENT MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
70457.3
40000.0
0.0
-34298.5
Linear GRAV GRAD MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 7A ARTICULATIN
ORBIT ANGLE (DEG)
HGGX
CASE 7A ARTICULATIN
ORBIT ANGLE (DEG)
HGGY
CASE 7A ARTICULATIN
ORBIT ANGLE (DEG)
HGGZ

Fig. 42



NASA/SDRC IDEAS**2: Database Output Plotting 20-SEP-86 09:21:11
CASE 7A ARTICULATING IOC, FULL ENVIRONMENT, TEA=(-16.3,-2.34,15.8)
AERODYNAMIC MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
4412.6
4000.0
2000.0
0.0
-1407.0
Linear AERO MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 7A ARTICULATIN
ORBIT ANGLE (DEG)
HAX
CASE 7A ARTICULATIN
ORBIT ANGLE (DEG)
HAY
CASE 7A ARTICULATIN
ORBIT ANGLE (DEG)
HAZ

Fig. 43

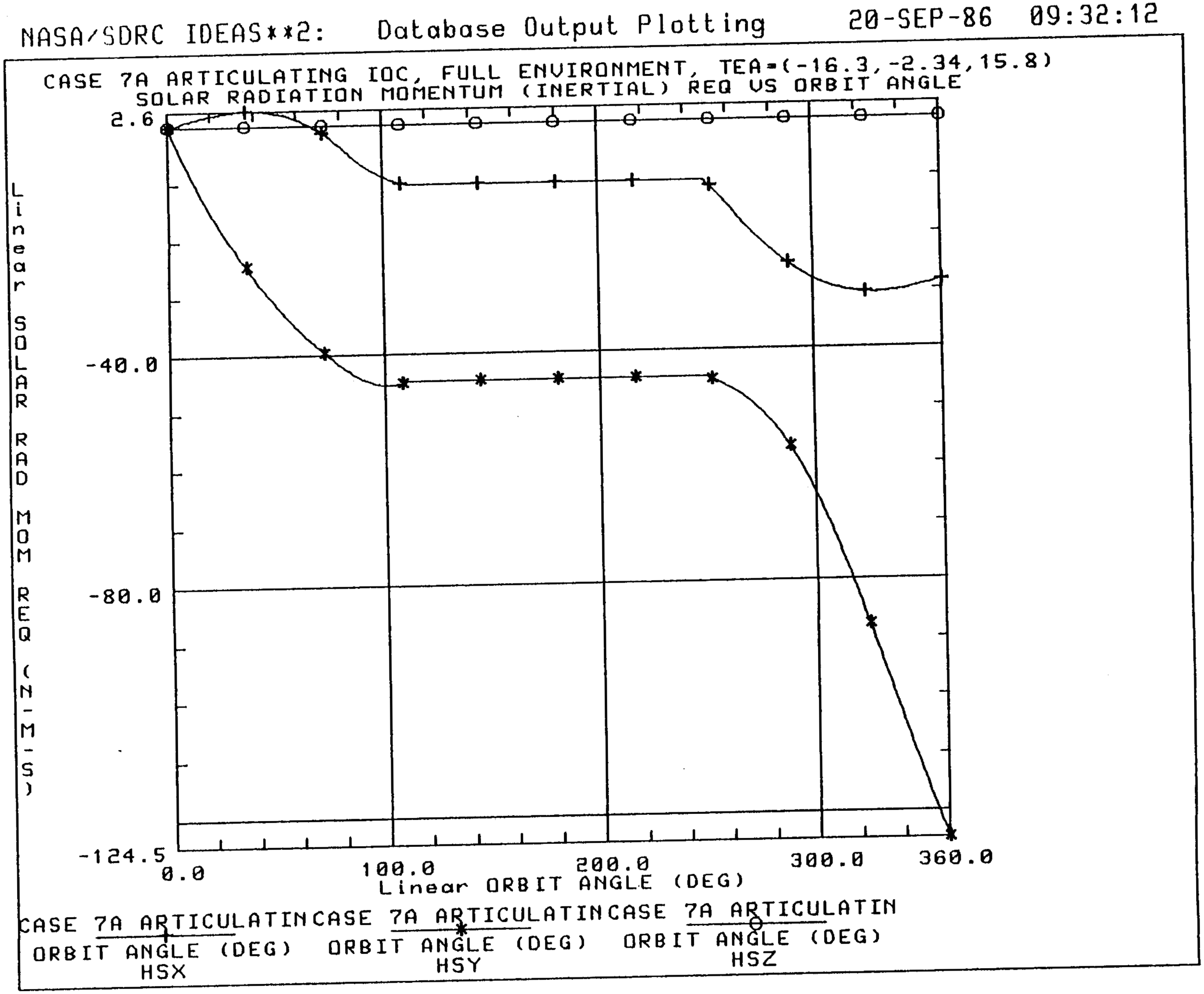
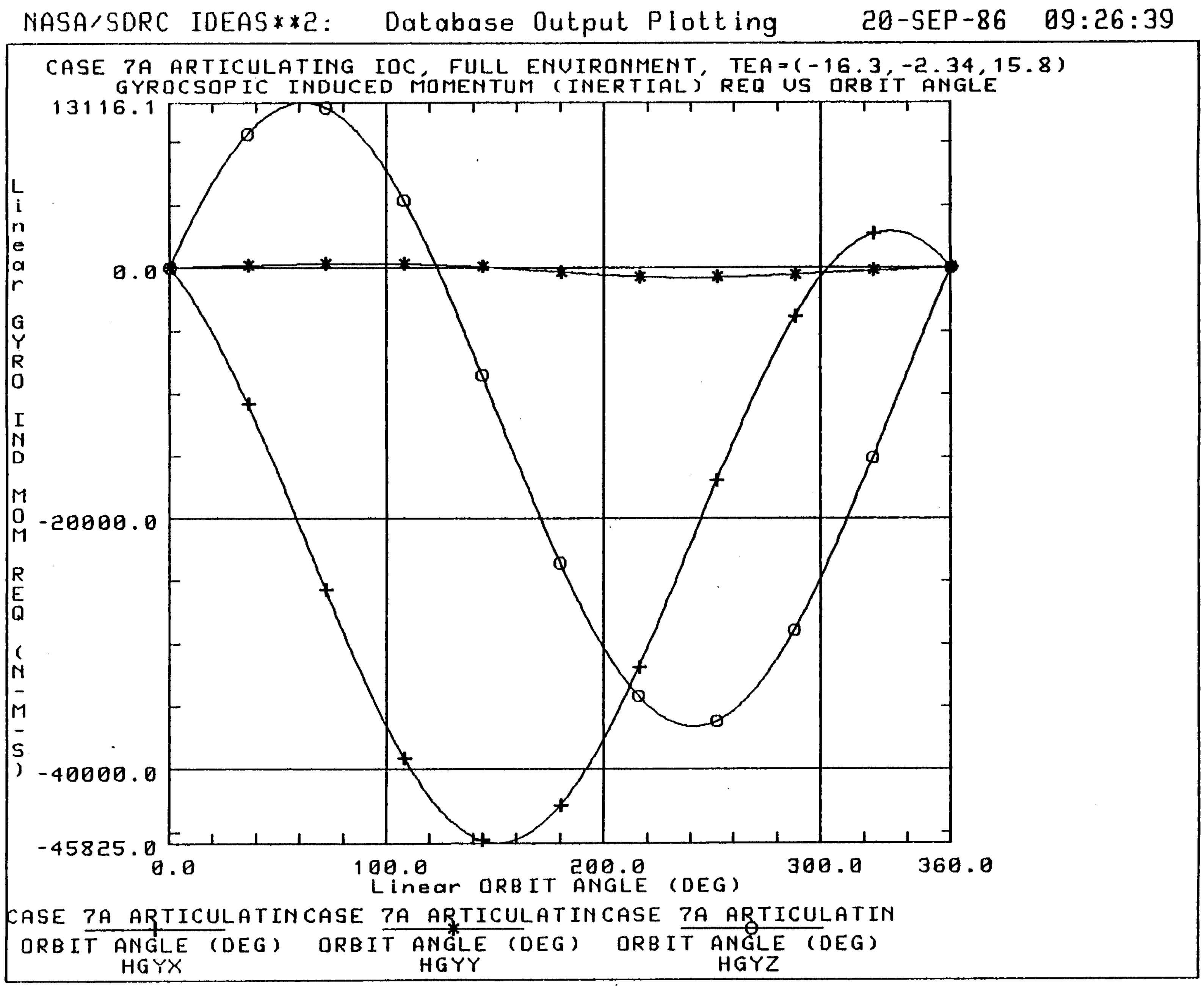

NASA/SDRC IDEAS**2: Database Output Plotting 20-SEP-86 09:32:12
CASE 7A ARTICULATING IOC, FULL ENVIRONMENT, TEA=(-16.3,-2.34,15.8)
SOLAR RADIATION MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
Linear SOLAR RAD MOM REQ (N-M-S)
2.6
-40.0
-80.0
-124.5
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 7A ARTICULATIN ORBIT ANGLE (DEG) HSX
CASE 7A ARTICULATIN ORBIT ANGLE (DEG) HSY
CASE 7A ARTICULATIN ORBIT ANGLE (DEG) HSZ

Fig. 44



NASA/SDRC IDEAS**2: Database Output Plotting 20-SEP-86 09:26:39
CASE 7A ARTICULATING IOC, FULL ENVIRONMENT, TEA=(-16.3,-2.34,15.8)
GYROCSOPIC INDUCED MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
13116.1
0.0
-20000.0
-40000.0
-45825.0
Linear GYRO IND MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 7A ARTICULATIN
ORBIT ANGLE (DEG)
HGYX
CASE 7A ARTICULATIN
ORBIT ANGLE (DEG)
HGYY
CASE 7A ARTICULATIN
ORBIT ANGLE (DEG)
HGYZ

Fig. 45

NASA/SDRC IDEAS**2: Database Output Plotting 20-SEP-86 09:23:44

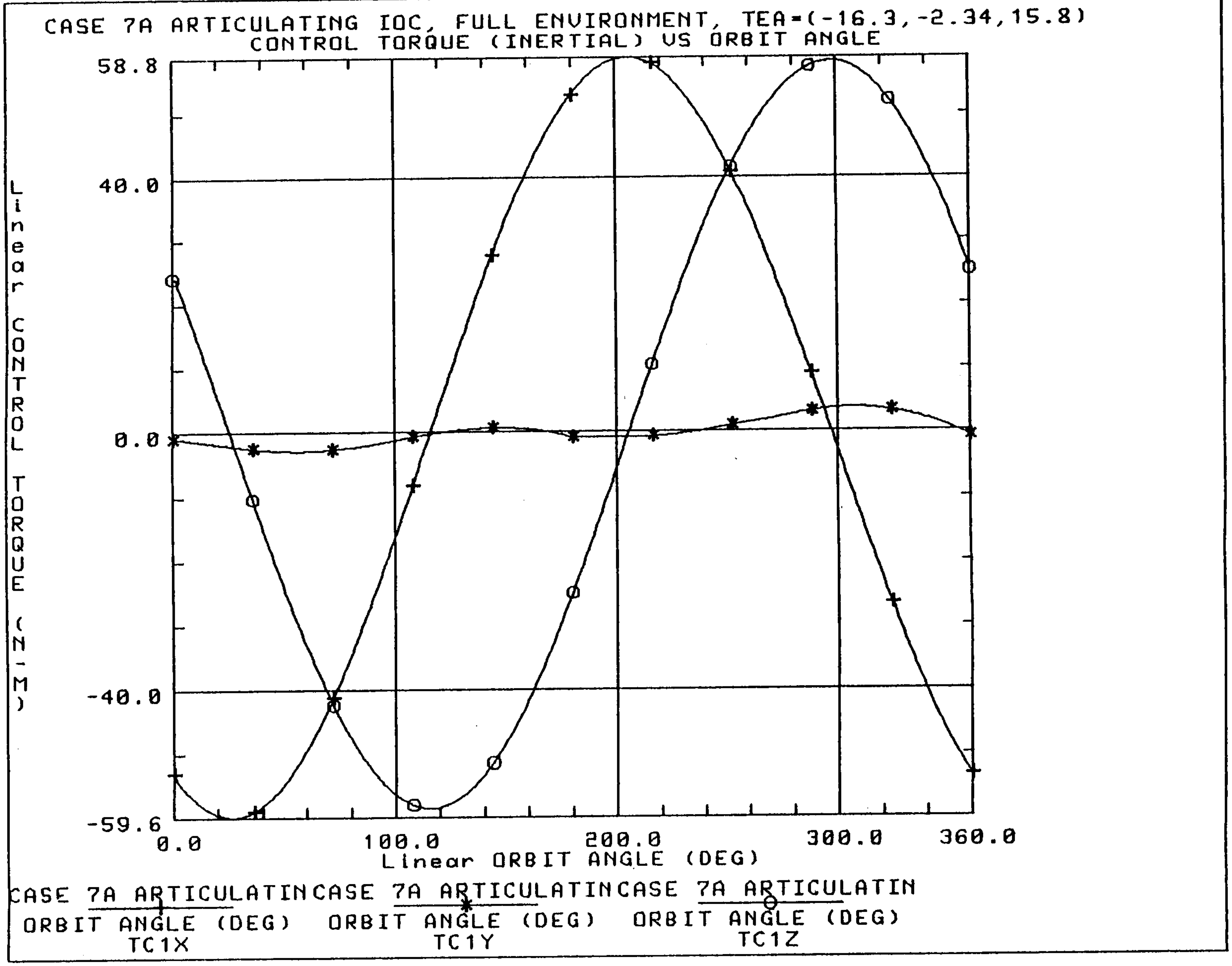


Fig. 46

In order to minimize the peak control torque and momentum requirements, the program ARCD was used to determine an attitude (ψ,θ,ϕ) = (-0.1,0.29,-4.80) for Case 7B close to the principal axis solution (Case 5). Figures 47 through 52 show the control momentum requirements, gravity gradient momentum, aerodynamic momentum, solar momentum, gyroscopic momentum, and control torque requirements for Case 7B, respectively. The peak control torque (4Nm) and peak control momentum (4100 Nms) requirements have been reduced significantly for this attitude. In addition, the secular POP momentum has been eliminated with a small bias pitch angle offset; however, the secular IOP control momentum is non-zero and has a magnitude of 1600 Nms. The secular control momentum requirements result from the secular IOP aerodynamic momentum contribution, which cannot be eliminated or counterbalanced with secular IOP gravity gradient momentum resulting from part articulation at an attitude near the principal axis solution. This indicates that periodic roll and/or yaw dumping would be required to reduce the accumulated secular momentum build-up.

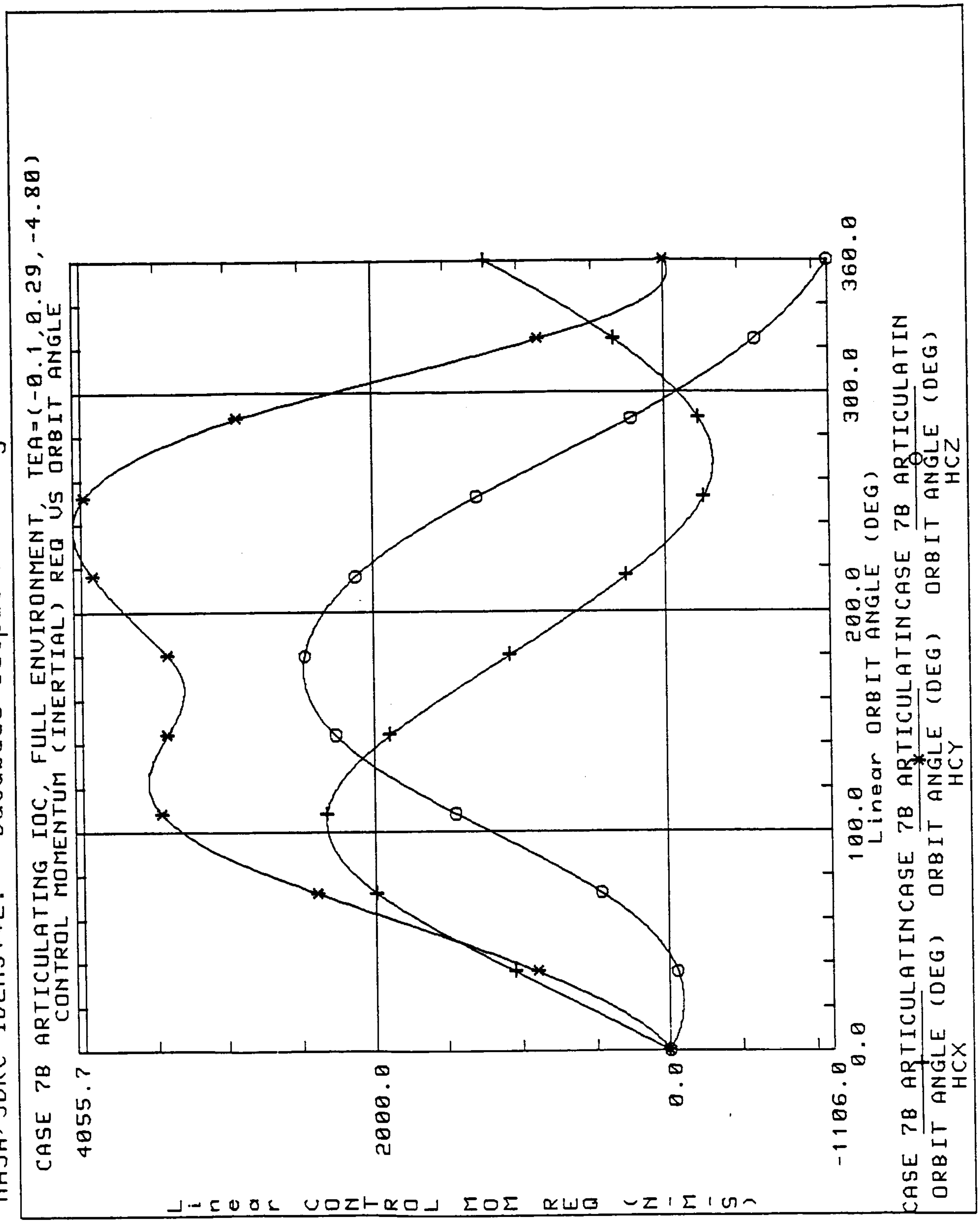

NASA/SDRC IDEAS**2: Database Output Plotting 20-SEP-86 11:15:31
CASE 7B ARTICULATING IOC, FULL ENVIRONMENT, TEA=(-0.1,0.29,-4.80)
CONTROL MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
4055.7
2000.0
0.0
-1106.0
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
Linear CONTROL MOM REQ (N-M-S)
CASE 7B ARTICULATINCASE 7B ARTICULATINCASE 7B ARTICULATIN
ORBIT ANGLE (DEG) ORBIT ANGLE (DEG) ORBIT ANGLE (DEG)
HCX HCY HCZ

Fig. 47

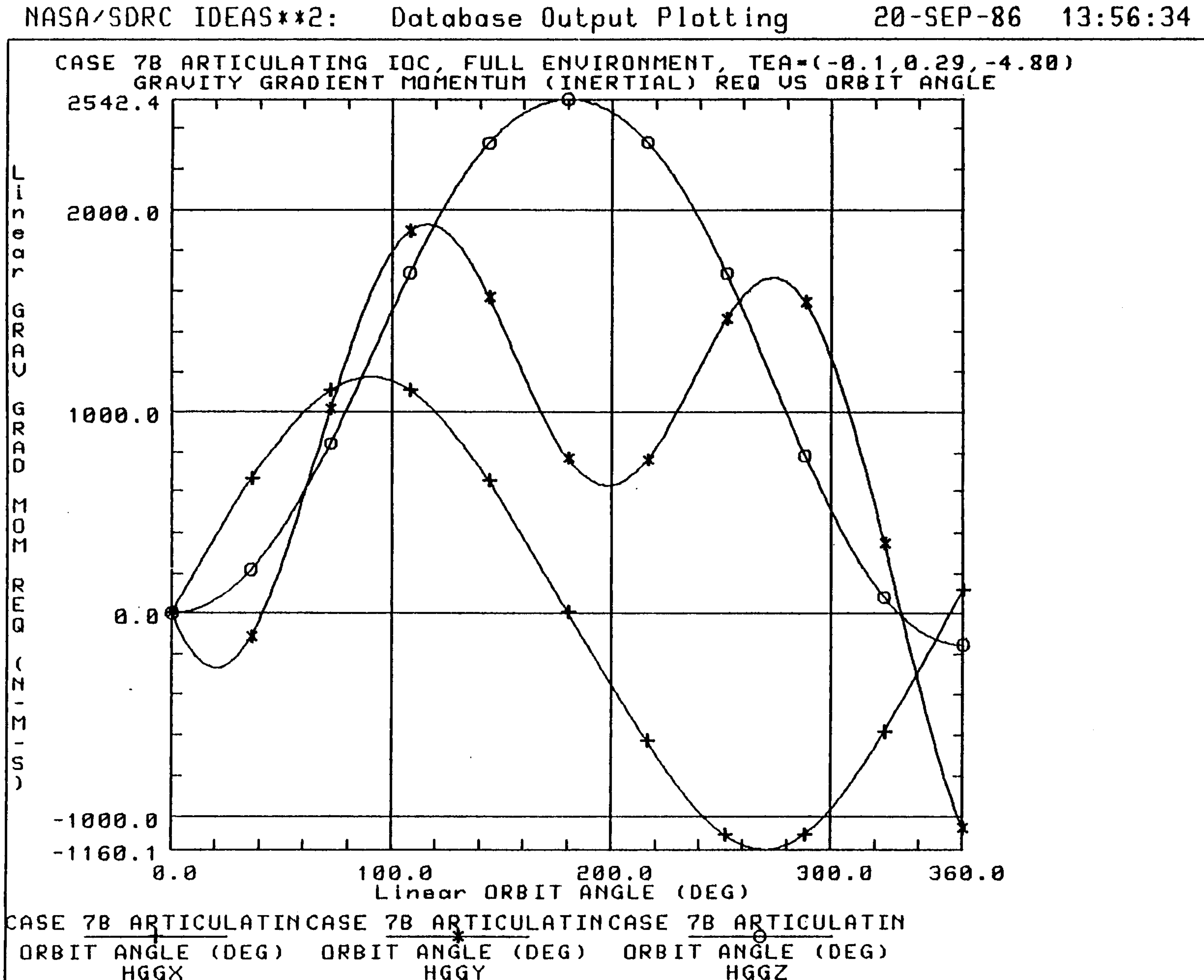

NASA/SDRC IDEAS**2: Database Output Plotting 20-SEP-86 13:56:34
CASE 7B ARTICULATING IOC, FULL ENVIRONMENT, TEA=(-0.1,0.29,-4.80)
GRAVITY GRADIENT MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
2542.4
2000.0
1000.0
0.0
-1000.0
-1160.1
Linear GRAV GRAD MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 7B ARTICULATIN ORBIT ANGLE (DEG) HGGX
CASE 7B ARTICULATIN ORBIT ANGLE (DEG) HGGY
CASE 7B ARTICULATIN ORBIT ANGLE (DEG) HGGZ

Fig. 48

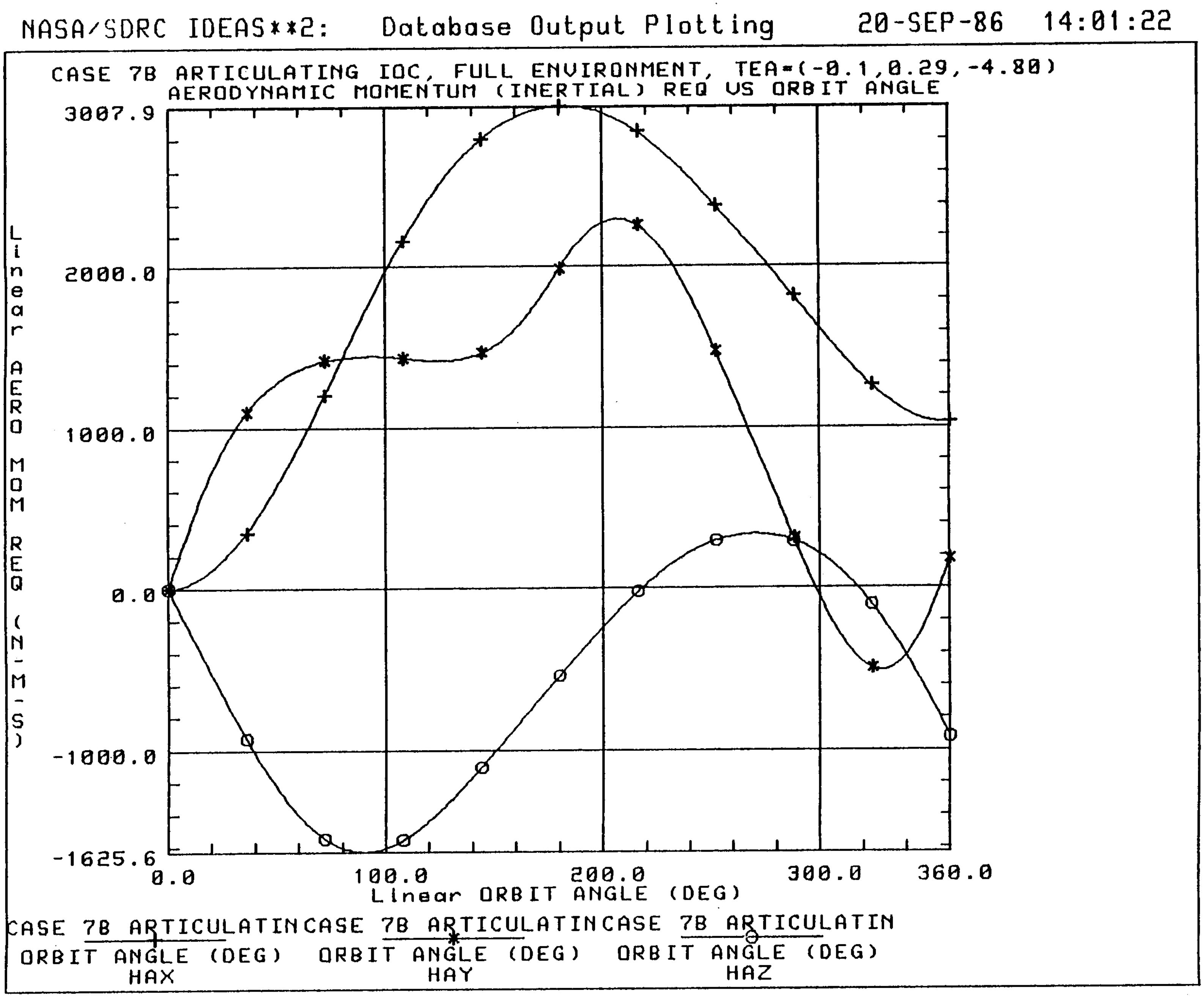

NASA/SDRC IDEAS**2: Database Output Plotting 20-SEP-86 14:01:22
CASE 7B ARTICULATING IOC, FULL ENVIRONMENT, TEA=(-0.1,0.29,-4.80)
AERODYNAMIC MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
3007.9
2000.0
1000.0
0.0
-1000.0
-1625.6
Linear AERO MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 7B ARTICULATIN ORBIT ANGLE (DEG) HAX
CASE 7B ARTICULATIN ORBIT ANGLE (DEG) HAY
CASE 7B ARTICULATIN ORBIT ANGLE (DEG) HAZ

Fig. 49

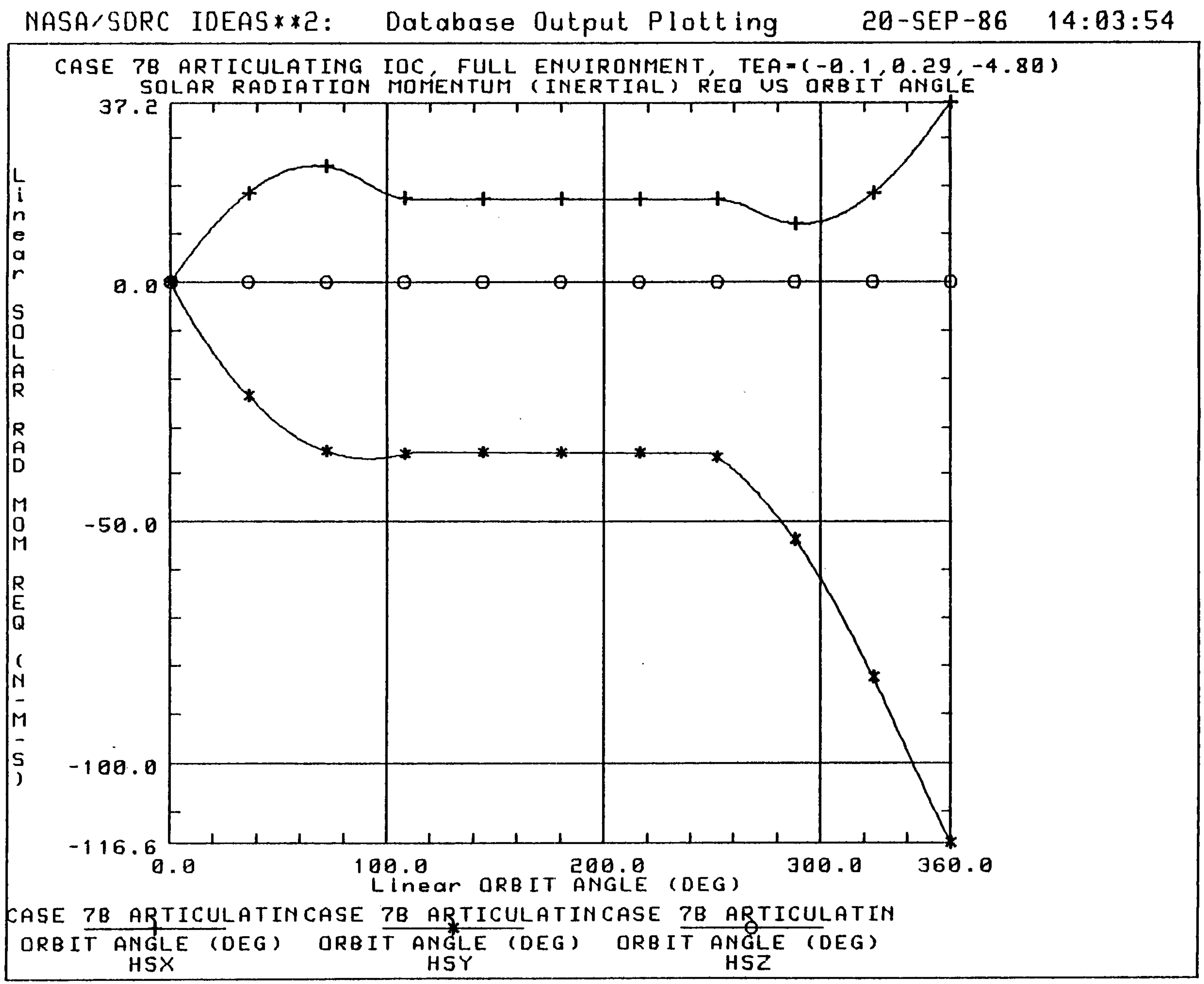

NASA/SDRC IDEAS**2: Database Output Plotting 20-SEP-86 14:03:54
CASE 7B ARTICULATING IOC, FULL ENVIRONMENT, TEA=(-0.1,0.29,-4.80)
SOLAR RADIATION MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
Linear SOLAR RAD MOM REQ (N-M-S)
37.2
0.0
-50.0
-100.0
-116.6
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 7B ARTICULATIN ORBIT ANGLE (DEG) HSX
CASE 7B ARTICULATIN ORBIT ANGLE (DEG) HSY
CASE 7B ARTICULATIN ORBIT ANGLE (DEG) HSZ

Fig. 50

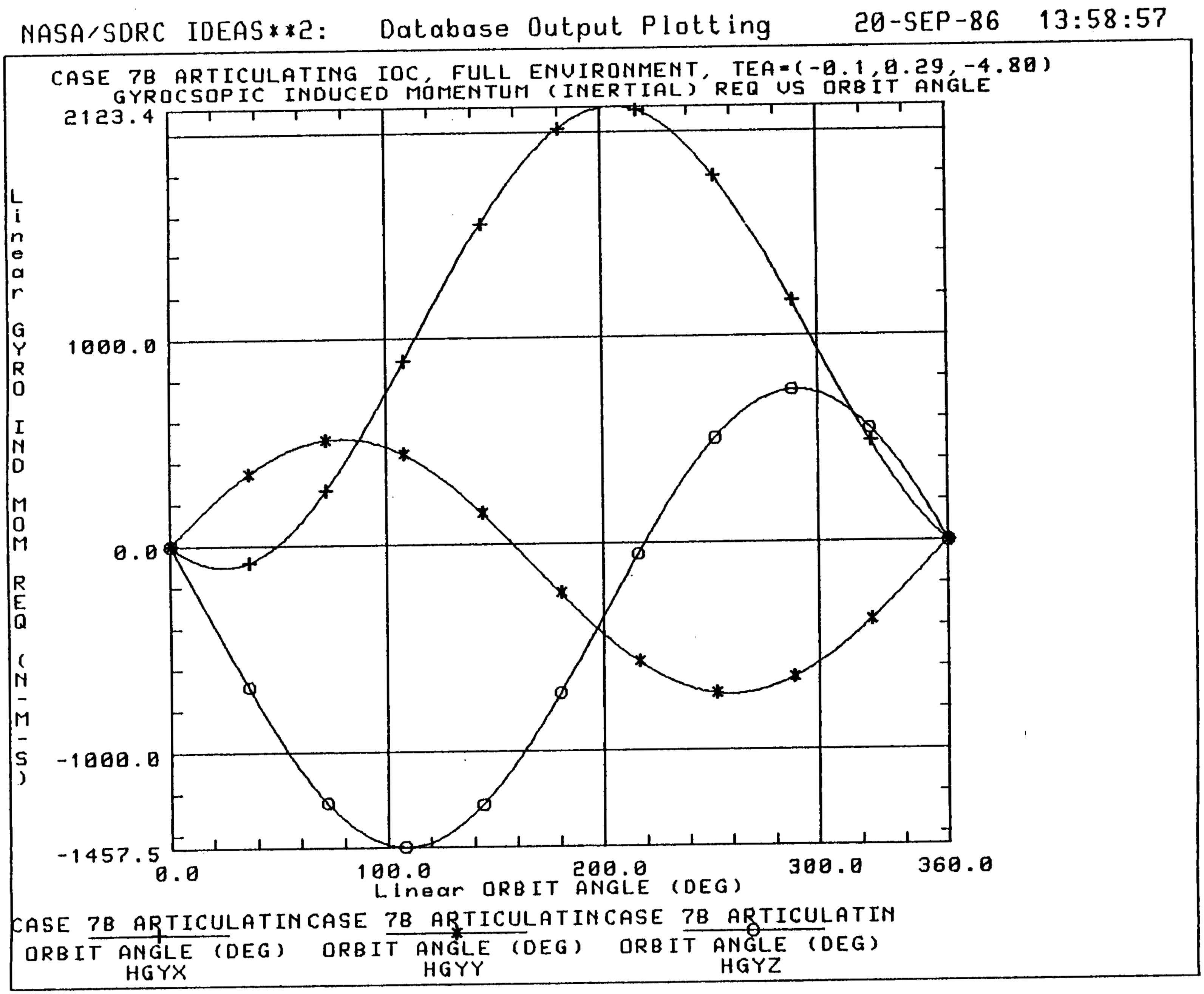

NASA/SDRC IDEAS**2: Database Output Plotting 20-SEP-86 13:58:57
CASE 7B ARTICULATING IOC, FULL ENVIRONMENT, TEA=(-0.1,0.29,-4.80)
GYROCSOPIC INDUCED MOMENTUM (INERTIAL) REQ VS ORBIT ANGLE
2123.4
1000.0
0.0
-1000.0
-1457.5
Linear GYRO IND MOM REQ (N-M-S)
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
CASE 7B ARTICULATIN
ORBIT ANGLE (DEG)
HGYX
CASE 7B ARTICULATIN
ORBIT ANGLE (DEG)
HGYY
CASE 7B ARTICULATIN
ORBIT ANGLE (DEG)
HGYZ

Fig. 51

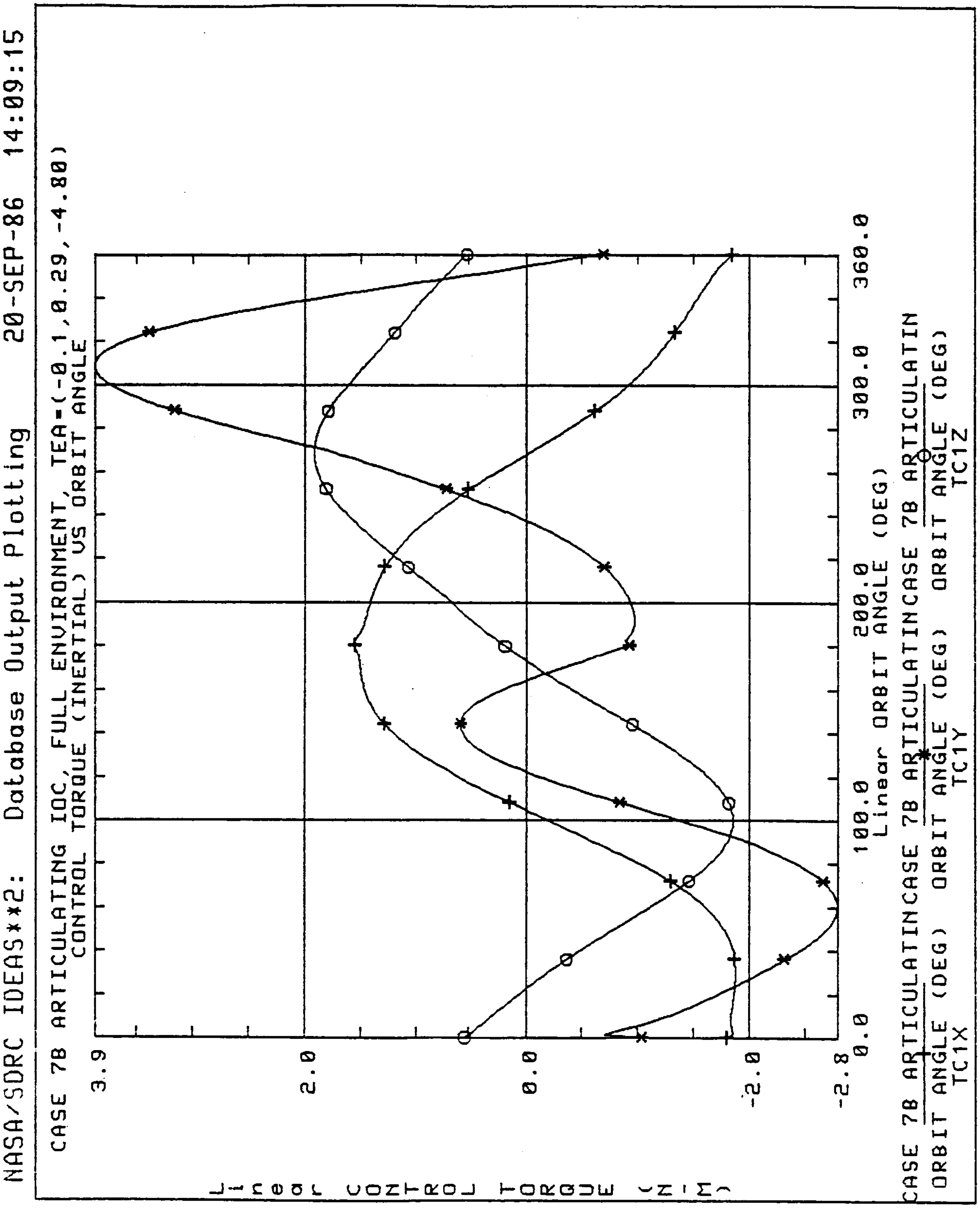

NASA/SDRC IDEAS**2: Database Output Plotting 20-SEP-86 14:09:15
CASE 7B ARTICULATING IOC, FULL ENVIRONMENT, TEA=(-0.1,0.29,-4.80)
CONTROL TORQUE (INERTIAL) VS ORBIT ANGLE
3.9
2.0
0.0
-2.0
-2.8
0.0
100.0
200.0
300.0
360.0
Linear ORBIT ANGLE (DEG)
Linear CONTROL TORQUE (N-M)
CASE 7B ARTICULATIN
ORBIT ANGLE (DEG)
TC1X
CASE 7B ARTICULATIN
ORBIT ANGLE (DEG)
TC1Y
CASE 7B ARTICULATIN
ORBIT ANGLE (DEG)
TC1Z

Fig. 52

CHAPTER 6

SUMMARY

This study represents a comprehensive analysis of the relationship between spacecraft attitude and attitude control angular momentum requirements. The sources of attitude control torque requirements were examined, specifically, gyroscopic, gravity gradient, aerodynamic, and solar radiation pressure. Analytic equations were derived which showed the attitude/momentum relationship, and were used to determine attitude solutions to eliminate secular angular momentum build-up and/or minimize cyclic angular momentum peaks for non-articulating spacecraft. The effects of aerodynamic induced disturbance torques were examined by studying the individual contributions due to a variable density atmosphere, a rotating atmosphere, and spacecraft part articulation. Gravity gradient and gyroscopic contributions to the cyclic and secular angular momentum requirements were examined for both fixed and articulating spacecraft. Peak cyclic momentum values due to gyroscopic or gravity gradient effects were as large as 110,000 Nms for the Space Station configuration studied. The effects of the dynamic atmosphere contributed up to 1300 Nms of secular angular momentum in the orbit plane (IOP) after one orbit, while non-symmetric articulation gravity gradient induced secular momentum contributed up to 250 Nms IOP after one orbit. For the configurations studied, the secular momentum perpendicular to the orbit plane (POP) could always utilize a pitch angle bias to offset the average aerodynamic and gravity gradient sources. Gyroscopic effects, even with part articulation, did not contribute to secular momentum accumulation.

A series of seven test case sets were designed and solved numerically to demonstrate the validity of the analytic results, complete with selected computer generated plots of the torque and momentum requirements and contributions.

For purposes of quick reference, Table 1 lists the sources of secular angular momentum build-up for both fixed and articulating near Earth orbiting spacecraft.

TABLE 1 - SOURCES OF SECULAR ANGULAR MOMENTUM BUILD-UP

	IOP (in orbit plane)	POP (perpendicular to orbit plane)
FIXED (Non-Articulating S/C)		
Gravity Gradient	NO	YES[1]
Gyroscopic	NO	NO
Aerodynamic	YES[2]	YES[3]
Solar	YES[4]	YES[4]
ARTICULATING		
Gravity Gradient	YES[5]	YES[1]
Gyroscopic	NO[6]	NO[6]
Aerodynamic	YES	YES
Solar	YES[4]	YES[4]

1. Can be eliminated with principal axis attitudes or pitch bias aero offsets
2. Due to atmospheric dynamics, can be eliminated with "large" angle attitudes (roll, yaw)
3. Can be eliminated using pitch bias gravity gradient offsets
4. Small compared to aerodynamics for Space Station altitudes
5. Zero if all articular parts rotate at orbit rate about an axis coincident with an articular part principal axis and passing through the articular part cg.
6. Assumes final and initial configurations are identical (e.g., articular parts rotate at orbit rate)

CHAPTER 7

CONCLUSION

The angular momentum control requirements for Earth orbiting spacecraft result from a variety of environmental effects. For the most general, and realistic, simulation of articulating spacecraft, three rotational degrees of freedom were utilized to determine attitudes which resulted in zero secular momentum build-up; however, this required large angle attitudes for the Space Station configuration studied and large cyclic angular momentum peak values. Three axis solutions were found, however, which resulted in acceptably small cyclic and (non-zero) secular momentum requirements. This, of course, implies the need for periodic desaturation of the angular momentum control devices. Deviations from this attitude resulted in large cyclic momentum penalties for relatively modest reductions in secular momentum build-up. Since the primary contributor of the secular momentum is aerodynamics, appropriate mass and area adjustments to the spacecraft configuration might be considered to reduce composite center of pressure - center of gravity offsets, thus minimizing torques resulting from the aerodynamic forces. Additionally, symmetric articular components designed to rotate about their center of gravity would eliminate gravity gradient induced secular momentum build-up due to part articulation.

CHAPTER 8

REFERENCES

1. Heck, M.L., Orr, Lynne H., DeRyder, L.J., "Articulated Space Station Controllability Assessment," AIAA Paper No. 85-0024, Jan., 1985, Reno, Nevada.

2. "IDEAS^2 Users Manual," Structural Dynamics Research Corporation, Dec., 1985, San Diego, CA.

3. "Space Station Reference Configuration Description," Systems Engineering and Integration Space Station Program Office, NASA Johnson Space Center, JSC-19989, Aug., 1984 (updated 1986).

Appendix A - Space Station Physical Properties

total weight = 192,000 kg

location of cg (x, y, z) = (-3.82, 0.91, -7.72) m

(origin located at center of transverse boom)

inertia tensor elements

$$I_{xx} = 1.52\ \text{E}8\ \ \text{kg}\cdot\text{m}^2$$

$$I_{yy} = 7.24\ \text{E}7\ \ \text{kg}\cdot\text{m}^2$$

$$I_{zz} = 1.01\ \text{E}8\ \ \text{kg}\cdot\text{m}^2$$

$$I_{xy} = 1.05\ \text{E}6\ \ \text{kg}\cdot\text{m}^2$$

$$I_{xz} = 4.00\ \text{E}5\ \ \text{kg}\cdot\text{m}^2$$

$$I_{yz} = -2.86\ \text{E}6\ \ \text{kg}\cdot\text{m}^2$$

projected areas in body

x : $A_1 = 2110\ \text{m}^2$

y : $A_2 = 884\ \text{m}^2$

z : $A_3 = 995\ \text{m}^2$

cp - cg offset looking in body x : $r_{1_y} = -1.82$ m, $r_{1_z} = 2.56$ m

y : $r_{2_x} = 1.67$ m, $r_{2_z} = -5.50$ m

z : $r_{3_x} = 2.55$ m, $r_{3_y} = 9.22$ m

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16. Abstract

The relationship between attitude and angular momentum control requirements is derived for a fixed attitude, Earth orbiting spacecraft with large area articulating appendages. Environmental effects such as gravity gradient, solar radiation pressure, and aerodynamic forces arising from a dynamic, rotating atmosphere are examined. It is shown that, in general, each environmental effect contributes to both cyclic and secular momentum requirements both within and perpendicular to the orbit plane. The gyroscopic contribution to the angular momentum control requirements resulting from a rotating, Earth oriented spacecraft is also discussed. Special conditions are described where one or more components of the angular momentum can be made to vanish, or become purely cyclic. Computer generated plots for a candidate Space Station configuration are presented to supplement the analytically derived results.

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