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Absolute Gain Measurement by the Image Method Under Mismatched Condition

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ABSOLUTE GAIN MEASUREMENT BY THE IMAGE METHOD UNDER
MISMATCHED CONDITION

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Summary

Purcell's image method for measuring the absolute gain of an antenna is particularly attractive for small test antennas. The method is simple to use and utilizes only one antenna with a reflecting plane to provide an image for the receiving antenna. However, the method provides accurate results only if the antenna is matched to its waveguide. In this paper, a waveguide junction analysis is developed to determine the gain of an antenna under mismatched condition. Absolute gain measurements for two standard gain horn antennas have been carried out. Experimental results agree closely with published data.

Introduction

This paper describes an image method for absolute gain measurement under mismatched condition. The image method, first proposed by Purcell, utilizes only one antenna with a metallic reflecting plane to provide an image for the receive antenna as illustrated in figure 1. Energy radiated by the antenna is reflected back to the antenna with the received power related to the transmitted power by the Friis transmission formula.

$$P_r = P_t \frac{G^2 \lambda^2}{(8\pi R)^2} \tag{1}$$

where P_r is the received power; P_t is the transmitted power; G is the gain of the antenna; λ is the free space wavelength and R is the distance from the antenna aperture to the reflecting plane. Under matched condition, power radiated from the antenna is reflected back to the antenna giving rise to a reflected wave in the waveguide. By measuring the reflection coefficient, $|\Gamma|$, as a function of distance, R, the gain can be determined from the mean straight line interpolation of the experimental curve according to

$$\frac{1}{|\Gamma|} = \frac{8\pi}{G\lambda} R \tag{2}$$

where $|\Gamma|^2 = P_r/P_t$ and $8\pi/G\lambda$ is the slope of the line.[1-2] When the waveguide/antenna junction is mismatched, multiple reflections occur at the mismatched junction with the received power partly absorbed and the rest reradiated by the antenna.[3] In this paper, a waveguide junction analysis is applied to calculate the gain of an antenna under mismatched condition. Absolute gain measurements for two standard gain horn antennas have been carried out. Experimental results are found to agree well with published data.

Formulation

In the absence of a reflecting plane, the incident power, P_i, to a mismatched waveguide/antenna junction with a reflection coefficient, Γ_0, can be decomposed into a reflected power component, $P_i|\Gamma_0|^2$, and a transmitted power component, $P_i(1-|\Gamma|_0|^2)$. With a reflecting plane in front of the test antenna as shown in figure 2, power radiated from the antenna sets up multiple reflections between the mismatched junction and the reflecting plane with the amplitude of the received power given by the Friis transmission formula. Assuming the waveguide is matched to the source, the sum of the received power to the waveguide is given by

$$\frac{1}{2} \operatorname{Re} \iint \frac{|e_r|^2}{Z_w} ds = P_{ro}|\Gamma_r|^2\left(1 - |\Gamma_0|^2\right) + P_{r1}|\Gamma_r|^4|\Gamma_0|^2\left(1 - |\Gamma_0|^2\right)$$
$$+ P_{r2}|\Gamma_r|^6|\Gamma_0|^4\left(1 - |\Gamma_0|^2\right) + \ldots \quad (3)$$

where

$$P_{ro} = P_t \frac{G^2\lambda^2}{(8\pi R)^2} \quad , \quad P_{r1} = P_{ro} \frac{G^2\lambda^2}{(8\pi R)^2} \quad , \quad P_{r2} = P_{r1} \frac{G^2\lambda^2}{(8\pi R)^2} \quad \ldots \text{ etc.}$$

$|\Gamma_r|$ is the reflection coefficient of the reflecting plane; $|e_r|$ is the received wave amplitude and Z_w is the guide impedance. By writing the transmitted power, P_t, in terms of the incident wave amplitude, $|e_i|$, equation (3) becomes

$$\frac{1}{2} \operatorname{Re} \iint \frac{|e_r|^2}{Z_w} ds = \left(1 - |\Gamma_0|^2\right)|\Gamma_r|^2\left[1 + y|\Gamma_r|^2|\Gamma_0|^2 + \left(y|\Gamma_r|^2|\Gamma_r|^2\right)^2\right.$$
$$\left. + \left(y|\Gamma_r|^2|\Gamma_0|^2\right)^3 + \ldots\right] \frac{1}{2} \operatorname{Re} \iint \frac{|e_i|^2}{Z_w} ds$$

or

$$|e_r| = \frac{\left(1 - |\Gamma_0|^2\right)|\Gamma_r|\sqrt{y}}{\sqrt{1 - y|\Gamma_r|^2|\Gamma_0|^2}} |e_i| \quad (4)$$

where

$$y = \frac{G^2\lambda^2}{(8\pi R)^2}$$

The total reflected wave amplitude in the waveguide measured by the directional coupler is

$$|e_r'| = |\Gamma_0||e_i| + \frac{\left(1 - |\Gamma_0|^2\right)|\Gamma_r|\sqrt{y}}{\sqrt{1 - y|\Gamma_r|^2|\Gamma_0|^2}} |e_i| \quad (5)$$

2

The ratio $|e_r|/|e_i|$ is the reflection coefficient $|\Gamma|$. In general,

$$y|\Gamma_r|^2|\Gamma_0|^2 < 1,$$

the second term in equation (5) can be expanded in series form. By setting $|\Gamma_r| = 1$, and neglecting all higher order terms, equation (5) can be simplified to

$$\frac{1}{|\Gamma - \Gamma_0|} = \frac{8\pi}{\left(1 - |\Gamma_0|^2\right)G\lambda} \; R \qquad (6)$$

Experimental Results

Absolute gain measurements have been carried out for a Waveline horn (18 to 26 GHz) and a Scientific-Atlanta waveguide horn (12 to 18 A) with known gains of 15.0 db at 22 GHz and 23.9 db at 20 GHz, respectively. The experimental graphs of $1/|\Gamma - \Gamma_0|$ versus R are shown in figures 3 and 4. By interpolating the data points with a straight line through the origin, the experimental results yield a gain of 14.65 db for the Waveline horn and 23.98 db for the Scientific Atlanta horn. In the experiment, the incident and reflected powers are measured through directional coupler using a HP 438A power meter. The reflected power exhibits an approximately sinusoidal variation with R as a result of multiple reflections between the mismatched antenna junction and the reflecting plane. When the antenna is sufficiently far away from the reflecting plane, the measurements become less accurate as demonstrated by wide variations of the data points.

References

1. S. Silver, Microwave Antenna Theory and Design. New York: McGraw-Hill, 1949.

2. J.S. Hollis, T.J. Lyon, and L. Clayton, Jr., Microwave Antenna Measurements. 2nd Rev. ed., Atlanta: Scientific-Atlanta, 1969.

3. Z. Wu, "Effect of Mismatch on Antenna Gain Measurement by Purcell's Method," Electron. Lett., vol. 22, pp. 522-524, 1986.
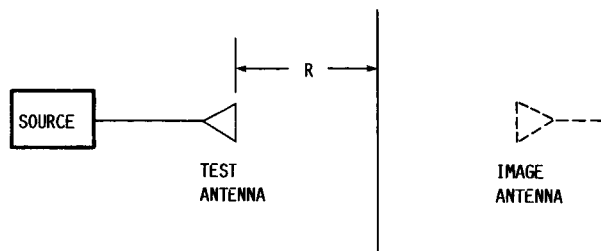
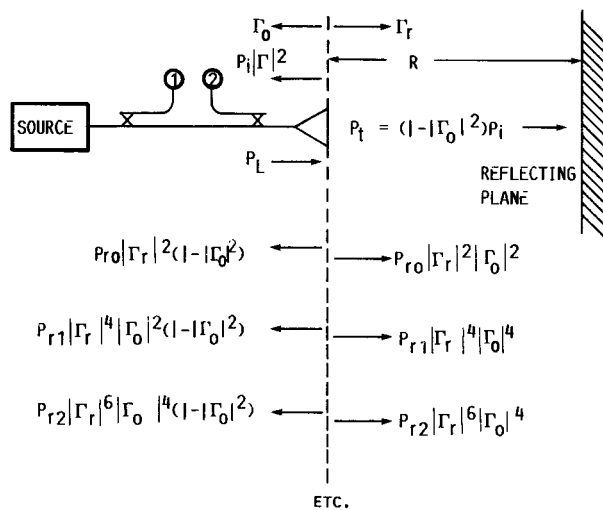FIGURE 1.- THE IMAGE METHOD FOR ABSOLUTE GAIN MEASUREMENT.

$\Gamma_0 \longleftarrow | \longrightarrow \Gamma_r$

$P_i |\Gamma|^2 \longleftarrow$ R

$P_t = (1 - |\Gamma_0|^2) P_i \longrightarrow$

$P_L \longrightarrow$

REFLECTING PLANE

$P_{ro} |\Gamma_r|^2 (1 - |\Gamma_0|^2) \longleftarrow \quad \longrightarrow P_{ro} |\Gamma_r|^2 |\Gamma_0|^2$

$P_{r1} |\Gamma_r|^4 |\Gamma_0|^2 (1 - |\Gamma_0|^2) \longleftarrow \quad \longrightarrow P_{r1} |\Gamma_r|^4 |\Gamma_0|^4$

$P_{r2} |\Gamma_r|^6 |\Gamma_0|^4 (1 - |\Gamma_0|^2) \longleftarrow \quad \longrightarrow P_{r2} |\Gamma_r|^6 |\Gamma_0|^4$

ETC.

FIGURE 2.- REPRESENTATION OF MULTIPLE REFLECTIONS AT A MISMATCHED
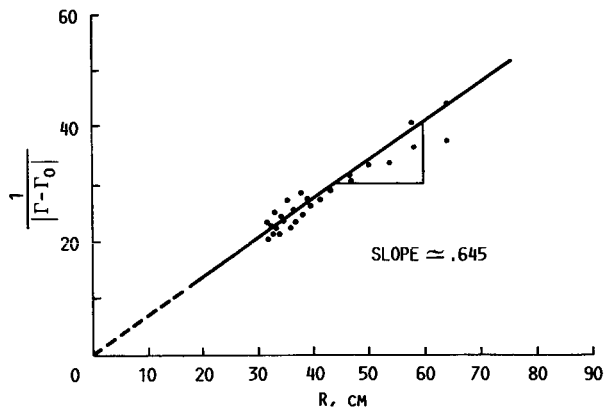WAVEGUIDE/ANTENNA JUNCTION.

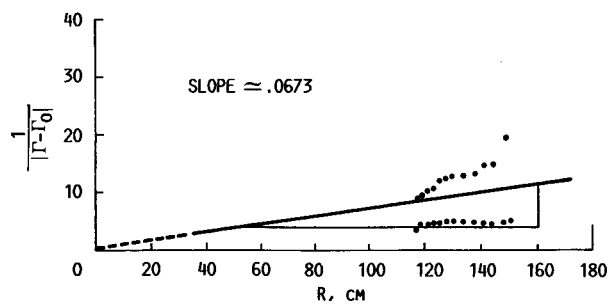FIGURE 3.- GRAPH OF $\dfrac{1}{|\Gamma-\Gamma_0|}$ VERSUS R FOR A WAVELINE HORN

(18-26 GHz).

FIGURE 4.- GRAPH OF $\dfrac{1}{|\Gamma-\Gamma_0|}$ VERSUS R FOR A SCIENTIFIC

ATLANTA WAVEGUIDE HORN (12A-18).

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