# INVESTIGATION OF PLASMA CONTACTOR 

FOR USE WITH ORBITING WIRES

Grant NAG9-126

## Semiannual Report \#1

For the period 1 January 1986 through 30 June 1986

Principal Investigator
Dr. Robert D. Estes
$\qquad$

February 1987

Prepared for
National Aeronautics and Space Administration Lyndon B. Johnson Space Center Houston, Texas 77058

## Smithsonian Institution <br> Astrophysical Observatory

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The NASA Technical Officer for this Grant is Dr. James M. McCoy, Code SN3, Lyndon B. Johnson Space Center, Houston, Texas 77058

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\subsection*{1.0 INTRODUCTION}

The SAO effort is mainly geared toward providing useful input into the planning of the upcoming first experiments in space on short electrodynamic tethers equipped with hollow cathode devices. In addition, we see it as our role to suggest and participate in the design of plasma chamber experiments that might provide results useful for this primary task. The Challenger disaster has postponed the shuttle-borne experiments. We will continue to focus on these experiments even though sounding rocket experiments may occur first. Of course, much of the basic physics remains the same, but the operation of hollow cathodes at the ends of an orbiting tether 200 m long will differ in significant ways from that of hollow cathodes necessarily placed much closer together on a sounding rocket that is continually changing altitude.

Given that these first experiments, by their nature, will last for only a few minutes, one of the major decisions to be made in planning them is their timing, i.e. the position on the orbital path and the local time, both of which strongly influence the ambient electron density, which is probably the single most important variable in the experiments. To emphasize the importance of the choice of the timing of the experiments we have included in this report the results of a number of orbital simulations for a 300 km orbital height and \(28^{\circ}\) orbital inclination. We are not prepared to make recommendations about the most desirable electron density, but we want to make the point that variations in electron density of over two orders of magnitude in a single revolution are not uncommon. The motioninduced electromotive force also varies with orbital position, and this factor should also be taken into account. A sample of the induced voltages encountered in simulated orbits are also included to graphically make this point. These
simulations are briefly discussed below.

Determining the range of electron densities encountered is the easy part. Deciding whether high or low values of electron density are to be preferred is more difficult and depends upon what the aims of the experiment are and what our best theory of hollow cathode operation tells us. It is questions such as these that we hope to help provide answers to. Once it has been decided what electron densities are desirable, then we have to turn our attention to the question of how we can choose the initiation point of the experiment to match the desired electron density and induced voltage.

The timing of the experiments in the other sense, i.e. the sequence and duration of the various phases of the experiment during the deployment (or tether extension period), is the other critical aspect of the experiments to which we mean to make useful contributions. This will involve estimating the relevant time constants for plasma processes taking place.

The bulk of our theoretical effort so far has gone into trying to determine the shape, size, and other properties of the plasma clouds that will be emitted by the hollow cathodes. Prof. Robert Hohlfeld of Boston University, a Visiting Scientist at SAO, has applied his experience in plasma boundary value problems and space physics to some of the fundamental aspects of this problem. He has summarized the results of his initial researches for this report. These are but the first steps in the projected analysis, and we are still debating some of the points. We are including this work as a major part of our progress report to indicate the direction in which we are headed. Perhaps the most significant conclusion reached so far is that plasma processes with time constants in the 10 msec range will be important. Thus a higher data samping rate than the one presently planned is
highly desirable.

One of the questions to which we would like to have an answer is how far the tether will have to be extended for the plasma clouds of the respective hollow cathodes to be considered as separate, in the electrical sense. If the clouds overlap, i.e. if the regions in which they maintain low impedance paths to their respective terminals remain in contact with each other, then the functioning of the system in the motion-induced current mode would seem unlikely. Since the two clouds would be experiencing the same \(\vec{v} \times \vec{B}\) force, the circuit would be shorted and no current would flow, just as no current would flow in a closed metal loop moving through a constant field. This is one of the areas in which the question of optimal electron density might come into play. Given the presence in the experimental apparatus of batteries which can be added in series with the tether, there is in principle a way to determine when the overlap has ended, since a current could be drawn through the plasma clouds when the battery was included in the circuit. Thus obtaining current with the battery but not obtaining current without the battery would imply overlap of the hollow cathode plasma clouds. The phasing and duration of modes such as these are among the aspects of the experiments we should be able to provide some guidance on, although real-time control of the experiment sequence is definitely desirable, since the complexity of the phenomena involved preclude reliable predictions with much accuracy. Whether real-time control is practical or not remains to be determined. Alternatively, a simple feedback system could be programmed to control the experiment sequence.

The next period of our activity will see us extending our effort to describe the plasma cloud emitted by hollow cathode devices in the environment of the planned experiments. In addition we will attempt to answer the challenges to the
feasibility of electrodynamic tethers (including the experiments we are concerned with) that have arisen recently from two separate sources. First, there are the experimental results reported by Urrutia and Stenzel [1986(a) and (b)] of UCLA. These investigators claim that nonlinear instabilities will prevent substantial current values from being attained. A completely different theoretical analysis of radiation from an electrodynamic tethered satellite system carried out by Barnett and Olbert [1986] of MIT has found that high wave impedances ( 10,000 ohms) will restrict tether current values. We will deal with each of these investigations in our next report.

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1. Barnett, A. and S. Olbert, 1986. "Radiation of Plamsa Waves by a Conducting Body Moving Through a Magnetized Plasma." Jounral Geophys. Research 91, 10117.
2. Stenzel, R.L. and J.M. Urrutia, 1986(a). "Laboratory Model of a Tethered Balloon - Electron Beam Current System," Geophysical Research Letters, 13, 797-800.
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\subsection*{2.0 VARIATIONS IN THE ENVIRONMENT ALONG THE ORBIT}

The operation of an electrodynamic tethered satellite system depends upon there being a \(\vec{v} \times \vec{B}\) force to drive the current and sufficient charge in the ionospheric plasma to feed the tether current across the charge-exchanging interfaces of the system with the ionosphere. Just what plasma density is sufficient depends on how well the hollow cathode devices (or other chargeexchange mechanisms) are able to fulfill their role as plasma contactors as a function of ionospheric plasma density and on what tether current is desired.

Clearly, if a certain minimum current were required at all times, then the hollow cathode system would have to be designed to attain that level under the least favorable conditions encountered in its orbit. Our present task is rather to choose the plasma density most likely to give both a demonstration of the system's ability to draw a substantial current and to maximize the scientific return of an experiment lasting only five minutes.

Understanding of hollow cathode devices is insufficient at present for us to be able to describe hollow cathode performance as a function of ionospheric plasma density. We hope to have made some progress in this area before our study is completed. For the present, it would seem that high plasma densities are desirable, just to be on the safe side from the standpoint of drawing a substantial current. It is important for us to know how the ionospheric plasma density encountered by the system varies.

The \(\vec{v} \times \vec{B}\) force experienced by the system also varies along the orbital path. Since the vertical component of this force drives the tether current
( \(\vec{v} \times \vec{B} \cdot \vec{L}\) is the equivalent voltage across the tether, where \(L\) is the vector parallel to the tether with magnitude \(L\), the tether length), it is the quantity whose variation needs to be determined.

The variations in plasma density and induced voltage have been examined in the following way. The SKYHOOK computer program previously developed at SAO to study tethered satellite system dynamics already included a model of the terrestrial magnetic field and ionospheric plasma. Since the tether dynamics were not of primary interest at this point, we modified the SKYHOOK code to advance the system in its orbit by an analytical formula, while obtaining values of the induced tether voltage and ionospheric plasma density at points along the orbital path.

The ionospheric model included in SKYHOOK was the Jones-Stewart [1970] model. This model is based on a trigonometric expansion fit to a large number of measurements made worldwide during the month of November in 1966 (a year of moderate solar activity). The obvious weakness of the model is that its strict applicability is limited to that month or other periods with similar solar activity levels, etc. It may, however, be a better picture of such periods than what can be obtained by a model that attempts to model the physical processes that cause the variations in ionospheric parameters.

SAO has obtained the International Reference Ionosphere computer code from the World Data Center in Boulder. This model, however, is least accurate for lower latitudes, the very region we are most interested in at present. Comparisons with SLIM [Anderson, 1985], the ionospheric model soon to be incorporated into IRI for low latitudes, showed that the Jones-Stewart model, with its large variations in plasma density encountered in a circular orbit, probably

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gives a more believable picture of the range of plasma densities encountered, although this range will depend upon the season and the solar activity level.

We consider a 300 km orbital height circular orbit. The first two plots (Figures 2.1 and 2.2) show the latitude and longitude versus elapsed time. These can be used to get an idea of the geographical co-ordinates that correspond to the features seen in the other plots of quantities versus time. Since the orbit shown has an inclination of \(28^{\circ}\), the latitude varies between \(\pm 28^{\circ}\). The local time is plotted versus elapsed time in Figure 2.3.

The electron density (in units of electrons \(/ \mathrm{m}^{3}\) ) is plotted versus elapsed time in Figure 2.4. This plot shows some well-known features of the electron density distribution. The most obvious of these is the big decrease in electron density at night due to recombination in the absence of ionizing solar radiation. These are the deep troughs that occur in each orbit (of which roughly \(11 \frac{1}{2}\) are displayed). A sharp spike is seen to emerge from each of these nighttime troughs, in some cases rising above the peak daytime value encountered. The daytime values encountered shown in some revolutions (most prominently in the last three) two peaks on the left side (morning side) of the daytime distribution. The trough between these peaks is the Appleton anomaly or equatorial trough.

The electron density is translated into random electron current collected by a sphere with radius two meters in Figure 2.5 which displays the current versus local time. A sphere with radius 20 meters would collect 100 times as much current, and so on. For a 20 m radius the current collected would vary all the way from 60 A (at the maximum peak in electron density encountered, where \(n_{e}>\) \(2 \times 10^{12} / \mathrm{m}^{3}\) ) down to less than 0.3A. This obviously is relevant to the experiments we are considering, even if the dependence of current collected on plasma density
is not linear. The deep troughs in electron density are seen to occur between 1800 and 2000 local time. The Appleton anomaly occurs between 0900 and 1200 local time. Other low values of electron density are seen just before sunrise. The nighttime peaks occur between 2000 and 2200 local time.

The tether voltage due to the \(\vec{v} \times \vec{B}\) force is plotted in Figure 2.6 for a 20 km tether. Since the voltage is linear in the tether length, obtaining results for other lengths is simple. The variation in the voltage encountered in the first few revolutions is relatively small, but in one of the later revolutions the voltage is seen to vary all the way from 1750 V to 4500 V . For the fully extended 200 m cable length in the planned experiment this corresponds to a variation between 17.5 and 45.0 volts.

References to Section 2
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\subsection*{3.0 HOLLOW CATHODE THEORY}

\subsection*{3.1 Introduction And General Overview .}

Several problems of great significance for understanding plasma contactors and the physics of plasma contactor clouds have been identified in the first portion of this research. Work done on these problems will be of immediate utility for determining the efficacy of plasma contactors for maintaining spacecraft electrical neutrality during experiments involving electrodynamic tethers, and for the design of experiments relating to understanding the operations of plasma contactors in the lower ionosphere. Preliminary results of theoretical calculations undertaken on these problems are reported here.

We consider the plasma contactor cloud as a conducting object embedded in an ionspheric medium flowing past the Shuttle at orbital velocity. This viewpoint is consistent with the qualitative picture of the mechanism by which a plasma contactor operates as being due to its larger collecting area available for collection of charge from the ambient plasma. It also allows us to make a direct connection with the body of literature pertaining to the charging of spacecraft in general. This qualitative mechanism of the plasma contactor operation suggests the crucial importance of determining the characteristic size and the detailed geometry of the boundary of the plasma contactor cloud. These questions have an immediate bearing on the value of the current drawn through the plasma contactor as a function of applied voltage, and on the possible overlap of the two plasma contactor clouds in the upcoming Shuttle experiment.

To the best of our knowledge, up until the present, plasma contactors have been operated either from sounding rockets or from satellites in geosynchronous orbit. In these situations the relative motion of the plasma contactor and the ambient plasma is comparatively slow. The proposed Shuttle experiments introduce the novel feature of significant motion of the ambient plasma with respect to the plasma contactor. Our preliminary
results indicate that this changes the basic physics describing the plasma contactor cloud in several significant ways.

Motivated by the requirement of collecting charge to maintain spacecraft neutrality during electrodynamic experiments, we have considered the trajectories of charged particles in the neighborhood of a charged satellite (such as the plasma contactor when gas flow is turned off) and in the neighborhood of the conducting plasma contactor cloud. We have determined that even under very modest applied voltages, the guiding center approximation, as applied to the trajectories of particles in the ionosphere outside the plasma contactor cloud, breaks down. This has the effect of increasing the effective cross section of the plasma contactor cloud for collecting charge from the ionosphere. Further research will be able to derive an improved estimate of the gain in effective collecting area obtained. Experiments are suggested to be performed in plasma chambers which could illuminate this question.

To begin to investigate the effects of the Shuttle's orbital velocity on the collection of charge, we have computed the relevant dimensionless ratios, notably the magnetic Reynolds' number, which characterize the flow in the neighborhood of the plasma contactor cloud. These calculations are also important for characterizing the geometry of the geomagnetic field near the plasma contactor cloud, particularly the diffusion of magnetic field lines into the plasma contactor cloud. If significant diffusion occurs, the access of charged particles into the cloud is much enhanced.

Attempts have been made (and are continuing) to obtain physically meaningful bounds on the dimensions of the plasma contactor cloud and on its characteristic shape. We have computed a fluid dynamic estimate of the size of the plasma contactor cloud using a technique analogous to those used by workers investigating the interactions of comets with the solar wind. If it is assumed that the mean free path for plasma contactor cloud particles and ionospheric particles is sufficiently small that a fluid dynamic description is valid, the growth of the plasma contactor cloud is limited by the ram pressure due to the
motion of the ionosphere with respect to the Shuttle. Given the assumptions made in this calculation, it is apparent that this calculation yields a lower bound on the characteristic size of the plasma contactor cloud.

The analogy we have made with comets and the plasma contactor cloud suggests the possiblity of the existence of a standing shock wave in the ionsophere and shock-heated plasma surrounding the plasma contactor cloud, which would be bounded by a tangential discontinuity. There would also be expected to be a substantial elongation of the plasma contactor cloud along the direction of the line of flight (though not a dramatic as a comet tail). Further investigations of the applicability of this model are underway.

Plasma kinetic calculations have been formulated which will provide an upper bound on the plasma contactor cloud size. The general character of these calculations will be discussed below. The intention is, using calculations with differing physical assumptions to bound the plasma contactor cloud dimensions above and below.

An immediate result of the theoretical calculations described here is a set of estimates of relevant time scales for the evolution of plasma contactor clouds. We have found that almost all relevant physical time scales are of the order of tens of milliseconds. If it is desired to sample the rise times of the current trace when voltage is applied, faster data acquisiiton rates will be required. In view of the information contained in the transient response, such data is highly desirable. Experiments in plasma chambers are suggested which will provide insight into possible breakdown of the guiding center approximation in the neighborhood of the plasma contactor cloud. Geometric considerations of the plasma contactor cloud suggest experiments which can be tried in the Shuttle experiment in which two plasma contactors with separately definable bias voltages will be deployed.

\subsection*{3.2 Magnetically Limited Flow In A Plasma Contactor Experiment - Breakdown Of Guiding Center Motion}

Typical electron gyroradii in the ionosphere are of the order of 1 centimeter. This is much smaller than other relevant scale lengths for the collection of current by a plasma contactor cloud, or by a metallic collecting surface. Consequently, we may consider electrons as being effectively "tied" to magnetic field lines and will treat their motion in a guiding center approximation. Current will only be collected from magnetic field lines which intersect with the collection surface, and so the magnetic field will act to limit the total current which may be collected by such devices. We shall begin with a treatment of the limits of validity of a guiding center approximation treatment of electron trajectories.

The mathematical treatment here will be based on the results of Parker and Murphy [1967], who attempted to calculate the current collected by a conductor biased positive with respect to the ambient plasma. Since electrons may be collected only if the magnetic field lines which determine their gyro-orbits intersect the current collector, the relevant scale for current collection is the cross-sectional area of the current collector projected normal to the magnetic field. The current collecting surface for this experiment is a cylinder \(143 / 4\) inches in diameter and 10 inches long. Since the ratio of diameter to length of this cylinder is near unity, we can approximate it as a sphere with a diameter the geometric mean of these two dimensions, i.e. 30.8 centimeters. (The principal motivation for considering a spherical collector is to eliminate the orientation of the collector with respect to the direction of the magnetic field vector as a relevant physical parameter.)

We shall work in a cylindrical \((r, \theta, z)\) coordinate system centered on the current collector. Electrons being collected by the system will be tightly bound to geomagnetic field lines, but will experience a radial drift velocity due to the potential, \(\Phi\). This radial drift
velocity is given by,
\[
\begin{equation*}
v_{r}=-\left(v_{x} / m \omega^{2}\right) \frac{\partial^{2} \Phi}{\partial r \partial z}, \tag{1}
\end{equation*}
\]
with \(\omega=e B / m c\), the gyrofrequency. If a form for the potential is adopted of a strictly Coulombic field,
\[
\begin{equation*}
\Phi=-\Phi_{0} a / \sqrt{r^{2}+z^{2}} \tag{2}
\end{equation*}
\]
where \(a\) is the radius of the current collector, then from equation (1)
\[
\begin{equation*}
\frac{d r}{d z}=-\frac{1}{m \omega^{2}} \frac{\partial^{2} \Phi}{\partial r \partial z}=\alpha a^{3} r z /\left(r^{2}+a^{2}\right)^{5 / 2} \tag{3}
\end{equation*}
\]
where
\[
\alpha \equiv 3 \Phi_{0} /\left(m \omega^{2} a^{2}\right)=-1.71 \times 10^{-3} \frac{V[\text { volts }]}{(a[\text { meters }] B[\text { gauss }])^{2}}
\]

Taking \(a=0.154\) meter and \(B=0.45\) gauss, we find that \(\alpha=(0.356) V[\) volts \(]\). This would give \(\alpha=17.8\), even for a bias voltage of only 50 volts, as currently contemplated for the plasma contactor experiment. The value we have chosen for \(a\) would be appropriate for a description of current collection when the gas flow through the plasma contactor is turned off and current collection occurs only due to the bias voltage applied to the contactor. A larger value of \(a\) would be appropriate if the gas flow is on, generating a conducting plasma cloud around the contactor.

Parker and Murphy have derived that values of \(\alpha<7.2\) are required for the validity of the drift approximation of electron motion in the vicinity of the current collector. On the basis of the calculation given above, we can see that the regime of conditions in which the guiding center approximation breaks down is easily accessible in this experiment when plasma is not being generated by the plasma contactor.

We may adopt a simple model to describe current collection in the case when the guiding center approximation breaks down. We shall assume that all electrons whose trajectories depart from guiding center motion will eventually impinge on the collector. This is probably not a bad approximation, since these electrons are not well confined
to magnetic field lines, although not all such trajectories can be expected necessarily to intersect the collector surface. On this basis we can define an effective current collection radius, \(a_{e f f}\), by
\[
\begin{align*}
& 7.2=3 \Phi_{0} /\left(m \omega^{2} a_{e f f}^{2}\right) \\
\Rightarrow & a_{e f f}=\frac{0.0154}{B[\mathrm{gauss}] \sqrt{V[\text { volts }]}} \tag{4}
\end{align*}
\]
where \(a_{\text {eff }}\) in equation (4) is measured in meters. When \(B=0.45\) gauss, we have that \(a_{\text {eff }}\) meters for a bias voltage of 100 volts.

It is apparent that current collection with an applied bias, and with gas flow through the plasma contactor turned off, will almost certainly be in a regime in which electron trajctories deviate significantly from the guiding center approximation in the neighborhood of the current collector. However, for laboratory experiments in which we can control \(B\), we may recover a regime of guiding center electron trajectories, for the purposes of comparison with theoretical limits on current collection. For example, if we take \(B=10\) gauss and \(V=100\) volts, we find that \(\alpha=0.072\) which is still definitely in the guiding center regime.

\subsection*{3.3 Magnetic Diffusion, Magnetic Reynolds Numbers, And Access Of Electrons To A Plasma Contactor}

As electrons in the earth's ionosphere are effectively tied to geomagnetic field lines (since typical gyroradii are on the order of \(\mathbf{1}\) centimeter), in order for current collection to occur by a conductor orbiting through the ionosphere, it is necessary for magnetic field lines to diffuse through some conducting surface. This is true whether the conductor in question is a metallic conductor, or the plasma cloud generated by a plasma contactor. The time available for diffusion of magnetic field lines through conducting surfaces will be limited by the orbital motion of the spacecraft, amounting to approximately 8 kilometers per second in low earth orbit. (Note that this simple picture of accessibility of electrons along magnetic field lines is applicable as long as the guiding center approximation holds. The previous calculation demonstrated that that this breakdown may occur at comparatively modest potential differences with respect to the local plasma potential, if the collector is of a sufficiently small size.)

The diffusion time for magnetic field to fully penetrate a conductor of scale length \(\ell\), and conductivity \(\sigma\) is (in Gaussian cgs units),
\[
\begin{equation*}
\tau=4 \pi \sigma \ell^{2} / c^{2} \tag{5}
\end{equation*}
\]
and the magnetic Reynolds number if that conductor is moving at a velocity, \(v\) is,
\[
\begin{equation*}
R_{M}=v \tau / L \tag{6}
\end{equation*}
\]
where \(L\) is a scale length. Note that while \(\ell\) and \(L\) are both scale lengths, they may not be equal; \(\ell\) refers to a scale length in which shielding currents may flow in the conductor, while \(L\) is the overall scale of the conducting object. \(L\) and \(\ell\) may be different, for example, as for a sphere of a thickness of order \(\ell\) and a radius of order \(L\).

When \(R_{M} \ll 1\), magnetic field can fully diffuse into the conductor in the time in which the objects orbital motion carries it past the magnetic field line. On the other hand, when
\(R_{M} \gg 1\) magnetic field exterior to the object does not substantially enter the conducting object, etiher due to its orbital velocity or high conductivity. Magnetic field then "piles up" in front of the conducting surface, but magnetic field lines do not intersect the conducting surface. The situation is analogous to that which occurs when the solar wind encounters a conducting ionosphere of a planet in a high magnetic Reynolds number flow.

We now consider some characteristic numbers to attempt to characterize the flow regime for magnetized plasma around the plasma contactor experiment. First we shall consider the conducting metal components, independently of the presence of the plasma cloud. Say that the relevant scale length for the thickness of conductors is \(\ell \approx 1 \mathrm{~cm}\). The resistivity of aluminum is \(2.824 \times 10^{-6} \Omega-\mathrm{cm}\). This implies a conductivity of \(3.54 \times 10^{7} \mathrm{mho} / \mathrm{m}\), or \(3.19 \times 10^{17} \mathrm{sec}^{-1}\) in cgs units. Calculating the magnetic diffusion time for these parameters yields \(\tau=4.45 \times 10^{-3}\) sec. The magnetic Reynolds number is determined by the length scale of the overall dimensions of the collector, \(L \approx 15 \mathrm{~cm}\), for the present case. We make take \(v\) as the orbital velocity of the Shuttle, i.e. \(v \approx 8 \times 10^{5} \mathrm{~cm} / \mathrm{sec}\). These values will yield \(R_{M} \approx 237.0\), a surprisingly large value, which has significant implications for the collection of current by the plasma contactor when gas flow is turned off. \(R_{M} \gg 1\) implies that the ionospheric field lines passing by the plasma contactor will not significantly penetrate the contactor collecting surface, and so as long as electrons are effectively tied to magnetic field lines, current collection will be very inefficient. In fact, the breakdown of the guiding center approximation, as considered in the calculation above, will be required to obtain any significant current collection.

It is interesting to note that the theories for current collection of conductors in the ionosphere of Parker and Murphy, and other workers, have had their greatest successes either for geosynchronous satellites, or for sounding rockets launched at high latitudes. These are cases for which velocities transverse to the magnetic field are small and which have correspondingly small values of \(R_{M}\).

If gas is flowing from the plasma contactor, there will be a sphere of some characteristic size, \(a\), with a characteristic electron number density, \(n_{e}\), and a characteristic neutral number density, \(n_{0}\). We need to consider the resistivity of this plasma sphere in order to compute a characteristic magnetic diffusion time and a magnetic Reynolds number.

We will consider two limits, the first in which the ionization of the plasma generated by the plasma contactor cloud is nearly complete, and the second in which the plasma is weakly ionized, either due to the ionization fraction of the plasma produced being low, or due to dilution by ambient ionospheric neutral particles streaming into the plasma contactor cloud.

For the first case which the ionization fraction, \(f \approx 1\), the electrical conductivity of the plasma may be expressed in terms of the collision frequency, \(\nu_{c}\) and the plasma frequency, \(\omega_{p}\) by,
\[
\begin{equation*}
\sigma=\omega_{p}^{2} / 4 \pi \nu_{c} \tag{7}
\end{equation*}
\]
i.e.
\[
\begin{equation*}
\sigma=n_{e} e^{2} / m_{e} \nu_{c} . \tag{8}
\end{equation*}
\]
[Krall and Trivelpiece, 1973]. This may be shown in the weak (electric) field limit to be
\[
\begin{equation*}
\sigma=\frac{3 m_{e}}{(16 \sqrt{\pi}) Z e^{2} \ln \Lambda}\left(\frac{2 k T_{e}}{m_{e}}\right)^{3 / 2}, \tag{9}
\end{equation*}
\]
which is valid when the electric field satisfies
\[
\begin{equation*}
E<\frac{n_{e} e}{\sigma} \sqrt{\frac{k T_{e}}{m_{e}}} . \tag{10}
\end{equation*}
\]

Note that this conductivity is independent of \(n_{e}\). The number of charge carriers will increase as \(n_{e}\) increases, but the number of scattering centers also increases proportionately, and so the conductivity is unchanged. We will, for purposes of estimation, take \(\ln \Lambda \approx\) 10, which is certainly correct within a factor of 2 or better. The plasma produced by the plasma contactor is assumed to be only singly-ionized, and so we take \(Z=1\). The temperature inside the contactor is \(T_{e} \approx 1000 \mathrm{~K}\). The temperature inside the plasma cloud
will almost certainly be lower due to adiabatic expansion of the plasma as it expands away from the plasma contactor. We note that this implies an upper bound on the plasma conductivity, since \(\sigma \propto T_{e}^{3 / 2}\). Substituting numerical values into equation (10), we find that \(\sigma \leq 2.2 \times 10^{11} s e c^{-1}\).

If we attempt to estimate the magnetic diffusion time for the plasma cloud, taking a scale length of 10 meters, we find that \(\tau \leq 3.1 \times 10^{-3} s e c\), and that the magnetic Reynolds number is \(R_{M} \leq 2.5\). A magnetic Reynolds number of order unity suggests that the penetration of the magnetic field into the plasma contactor cloud will not be complete and that some reduction of the estimated current collection by the plasma cloud may be in order. However, the sensitive dependence of this result on the value of the electron temperature should be noted. We have used an estimated maximum value for the electron temperature here, and hence we have almost certainly significantly overestimated the conductivity of the plasma contactor cloud and the magnetic Reynolds number. A modestly reduced value of \(T_{e}\), owing to adiabatic expansion of the plasma contactor cloud would put the system into a physical regime with \(R_{M} \ll 1\).

Processes which will raise the electron temperature in the plasma contactor cloud must be carefully considered, as they will raise \(R_{M}\) and complicate treatment of mathematical models of current collection. In particular, plasma instabilities or plasma turbulence in the plasma contactor cloud may heat electron significantly. This possibility will require careful consideration.

We shall now consider crudely the conductivity of a plasma contactor cloud when the ionization fraction is small. The conductivity in such a situation is given be equation (8), where \(\nu_{c}\) is interpreted as an inverse time-scale for momentum exchange between electrons and some other species, in this case neutral atoms emitted by the plasma contactor cloud, as well as ionospheric neutral atoms streaming through the plasma contactor cloud. We take then \(\nu_{c} \sim 10^{9} \mathrm{sec}^{-1}\) and \(n_{e} \sim 10^{7} \mathrm{~cm}^{-3}\), which implies \(\sigma=5.0 \times 10^{8} \mathrm{sec}^{-1}\). This value is approximately 440 times less than that in the high ionization limit. Accordingly, \(\tau\) will
be less than \(7.0 \times 10^{-6} \sec\) and \(R_{M} \leq 5.7 \times 10^{-3}\). In this regime, penetration of ionospheric magnetic field lines inot the plasma cloud will be essentially complete.

\subsection*{3.4 Fluid-Dynamic Estimation Of Plasma Contactor Characteristic Scales}

It is desirable to get a range of realistic estimates of the characteristic size and evolutionary time scales of plasma contactor clouds as a necessary step in planning experiments for testing the efficacy of plasma contactors for exchanging charge between the Shuttle and the ionosphere.

One extreme limit in modeling such a system is to assume the plasma cloud behaves as a fluid medium flowing out of the plasma contactor. This may be justified as long as the mean free path within the cloud is very small. The plasma cloud then exhibits a ram pressure determined by the expansion velocity of the cloud and its density (which is a function of radius from the plasma contactor). The ionosphere is also flowing past the plasma contactor cloud and thus exhibits its own dynamic ram pressure. A characteristic length scale of the plasma contactor cloud, effectively a "stand-off distance", may be obtained by finding the radius at which the dynamic pressure of the plasma contactor cloud is balanced by the dynamic pressure of the ionosphere (as viewed in a reference frame co-moving with the Shuttle).

The similarities of this physical description with the interaction of a comet with the solar wind should be noted. The possibility of the existence of a standing bow shock wave and a contact discontinuity in the flow around the plasma contactor must also be carefully considered. (See Figure 3.1)

Let \(\dot{m}\) denote the mass flow rate from the plasma contactor. For the purposes of this crude estimate, assume that the contactor is effectively a point source of adiabatically expanding gas. Sufficiently far from the plasma contactor, the gas flow will be effectively a free expansion, and will thus be characterized by an expansion velocity,
\[
\begin{equation*}
\left.\left.v_{e x} \approx \frac{1}{\sqrt{3}} c_{0}\right|_{\text {contactor }} \approx(k T / m)^{1 / 2}\right|_{\text {contactor }} \tag{11}
\end{equation*}
\]

Here \(c\), denotes the sound speed, \(k\) is Boltzmann's constant, \(m\) is the mass of the gas particles (atoms or ions), and \(T\) is the temperature at the exit aperture of the plasma contactor. It can be seen that the characteristic expansion velocity of the plasma contactor cloud is determined by the temperature of the gas emitted by the plasma conactor and by the mass of the species released. If \(T\) (contactor) \(\approx 10^{3}{ }^{\circ} \mathrm{K}\) and the gas released is xenon, then \(m \approx 131 m_{p}=\left(1.67 \times 10^{-24} \operatorname{grams}\right)(131) \approx 2.19 \times 10^{-22} \mathrm{gram}\). This yields \(v_{e x} \approx 2.5 \times 10^{4} \mathrm{~cm} / \mathrm{s}=250 \mathrm{~m} / \mathrm{s}\) Now define \(\phi(r)\) as the mass flux from the contactor. Then,
\[
\begin{equation*}
\dot{m}=4 \pi r^{2} \phi(r) \Rightarrow \phi(r) \frac{\dot{m}}{4 \pi r^{2}} \tag{12}
\end{equation*}
\]

We want to determine the mass density as a function of radius in the outflow, \(\rho(r)\). Since \(\phi(r)=\rho(r) v_{e x}\), then
\[
\begin{equation*}
\rho(r)=\frac{\dot{m}}{4 \pi r^{2} v_{e x}} \tag{13}
\end{equation*}
\]

As the gas is expanding adiabatically, the gas pressure will fall off very rapidly with radius; the contribution from the gas pressure adding to the dynamic pressure of the expanding gas cloud should be insignificant. This may be verified easily. Adiabatic expansion implies that
\[
P \propto \rho^{\gamma} \Rightarrow P \propto\left(\dot{m} / 4 \pi r^{2} v_{e x}\right)^{\gamma} \propto r^{-2 \gamma},
\]
where \(P\) denotes the gas pressure and \(\gamma\) the ratio of specific heats (adiabatic exponent). For inert gases such as xenon and argon, \(\gamma=5 / 3 \Rightarrow P \propto r^{-10 / 3}\). It might be reasonably expected that the expansion factor for the gas might be at least several orders of magnitude (compared to the aperture of the plasma contactor), the gas pressure will drop by at least 10 orders of magnitude from its value at the aperture of the contactor. This of course neglects sources of heat for the the plasma contactor cloud which will certainly be important in the actual experiment, but should not be important for this crude estimate.

We can now balance the pressures and obtain an estimate for the scale size of the plasma contactor cloud. Let \(v_{\text {orb }}\) be the orbital velocity of the Shuttle, and \(\rho_{i o n}\), the mass
density of the ionosphere. In the reference frame of the Shuttle, the ram pressure of the plasma contactor cloud is
\[
\begin{equation*}
\rho(r) v_{e x}^{2}=\frac{\dot{m}}{4 \pi r^{2} v_{e x}} v_{e x}^{2}=\frac{\dot{m} v_{e x}}{4 \pi r^{2}} \tag{14}
\end{equation*}
\]

Now solving for \(r\) such that \(\rho(r) v_{e x}^{2}=\rho_{i o n} v_{o r b}^{2}\), we find that
\[
\begin{equation*}
r=\frac{1}{v_{o r b}} \sqrt{\frac{\dot{m} v_{e x}}{4 \pi \rho_{i o n}}} \tag{15}
\end{equation*}
\]

This then is the desired "stand-off distance" for the flow from the plasma contactor cloud. Experimentally, it is controlled by the release rate of the gas and the expansion velocity (determined by \(T\) and \(m\) ). There is also a significant dependence on the ambient plasma density. Substituting appropriate numerical values into equation (15), \(\rho_{i o n}=2.7 \times 10^{-14} \mathrm{gm} / \mathrm{cm}^{3}\), \(\dot{m}=3.0^{-3} \mathrm{gm} / \mathrm{sec}=1 / 2\) standard cubic centimeter per second, and \(v_{\text {orb }}=8.0 \times 10^{5} \mathrm{~cm} / \mathrm{sec}\) we obtain a value of \(r \approx 19 \mathrm{~cm}\). This value is remarkably small. Given the assumptions made in the fluid dynamic approximation to the dynamics of the expansion of the plasma contactor cloud, this must be regarded as a lower bound on the size of the cloud. Certainly in the limit of a more collisionless plasma cloud, atoms of the cloud may travel a somewhat larger distance before experiencing collisions with ionospheric particles.

One minor correction which must be considered relates to the adiabatic expansion of the plasma contactor cloud from the aperture of the plasma contactor. For the numbers chosen above, the expansion ratio may not be sufficient to drive the gas pressure to very low values. Nonetheless, the basic conclusion of an unexpectedly small contactor cloud can still be expected to hold and should be considered seriously pending the results of a more detailed plasma kinetic calculation.

The characteristic length scales that have been computed here allow us to estimate characteristic time scale for the establishment and decay of the plasma contactor flow. A rough estimate of the time required to establish the flow field around the contactor is
\[
\begin{equation*}
r / v_{e x} \approx \frac{19 \mathrm{~cm}}{2.5 \times 10^{4} \mathrm{~cm} / \mathrm{sec}} \approx 7.6 \times 10^{-4} \mathrm{sec} . \tag{16}
\end{equation*}
\]
i.e. about a millisecond. If it is considered experimentally desirable to measure the electrodynamic behavior of the plasma contactor cloud as the plasm flow is turned on, data rates as high as \(10^{4} \mathrm{samples} / \mathrm{sec}\). (at least for short periods of time) would be required.

If the flow around the contactor is drawn out into a long "comet tail" as this model calculation permits, we might expect that much of the surface area over which charge transfer with the ionosphere takes place is in this "comet tail". (One possible approach for modeling this system is to consider this plasma stream as a lossy transmission line.) The time scale for the current flow through the contactor to diminish once the mass flow is cut off will be approximately \(2 r / v_{e} x \approx 1.6 \times 10^{-3} \mathrm{sec}\), at which time a high conductivity path to the tail of the plasma contactor cloud will no longer be available. The possiblility exists that the cutoff in the current flow through the plasma contactor cloud may be rather abrupt.

\section*{References for Section 3}
1. Krall, N.A. and A.W. Trivelpiece, 1973. Principles of Plasma Physics. McGraw-Hill.
2. Parker, L.W. and B.L. Murphy, 1967. "Potential Buildup on an ElectronEmitting Ionospheric Satellite,"Journal Geophys. Research 72, 1631-1636.


Figure 3.1 - Hollow cathode in low earth orbit.```

