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Modeling and Identification of SCOLE

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THIRD ANNUAL SCOLE WORKSHOP

MODELING AND IDENTIFICATION OF SCOLE

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<u>VECTOR DIFFERENTIAL EQUATION FOR DISTRIBUTED STRUCTURES</u>

$$\mathcal{L}\,\underset{\sim}{u}(P,t) + \mathfrak{M}\,\ddot{\underset{\sim}{u}}(P,t) = \underset{\sim}{f}(P,t), \quad P \epsilon D$$

Boundary conditions: $B_i \underset{\sim}{u}(P,t) = \underset{\sim}{0}, \quad i = 1,2,\ldots,p, \quad P \epsilon S$

$\underset{\sim}{u}(P,t)$ = displacement vector at point P in the domain D

$\mathcal{L}$ = stiffness operator matrix with entries of maximum order 2p

$\mathfrak{M}$ = mass density matrix

$\underset{\sim}{f}(P,t)$ = control force density vector

B_i = boundary differential operator matrices with entries of maximum order 2p-1

DISCRETIZATION (IN SPACE) OF THE DISTRIBUTED STRUCTURE

Shuttle and reflector are assumed to be rigid.

Cable is discretized by the Rayleigh-Ritz method (resulting in a small number of degrees of freedom):

$$u_x(z,t) = \sum_{r=1}^{n} \phi_{xr}(t)a_{xr}(t), \quad u_y(z,t) = \sum_{r=1}^{n} \phi_{yr}(t)a_{yr}(t), \quad 0 < z < L_1$$

u_x, u_y = displacements in the x and y direction, respectively.

ϕ_{xr}, ϕ_{yr} = admissible functions

a_{xr}, a_{yr} = generalized coordinates

L_1 = length of cable

(3)

DISCRETIZATION (IN SPACE) OF THE DISTRIBUTED STRUCTURE (CONT'D)

Because the identification and control problems are based on actual displacements of various points, the mast is discretized by the finite element method:

$$\underset{\sim}{u}(z,t) = \begin{bmatrix} u_x(z,t) \\ u_y(z,t) \\ \theta_z(z,t) \end{bmatrix} = L(z)\underset{\sim}{w}_j(t), \quad (j-1)h < z < jh$$

u_x, u_y = bending displacements

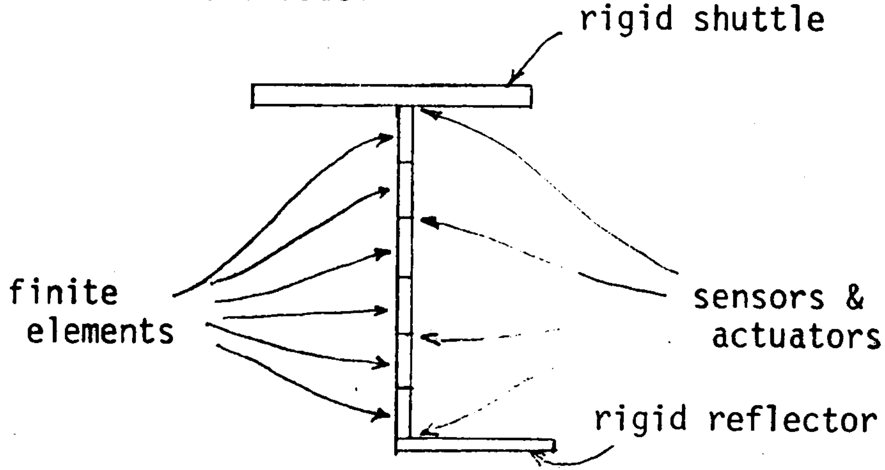
θ_z = torsional displacement

$L(z)$ = matrix of interpolation functions

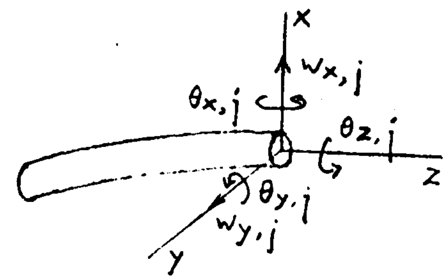
$\underset{\sim}{w}_j(t)$ = vector of nodal coordinates = vector of actual displacements at nodal points

DISCRETIZATION (IN SPACE) OF THE DISTRIBUTED STRUCTURE (CONT'D)

Finite elment model



Nodal coordinates



Element nodal vector

$$\underset{\sim}{w}_j(t) = [w_{x,j-1}\ \theta_{x,j-1}\ w_{y,j-1}\ \theta_{y,j-1}\ \theta_{z,j-1}\ w_{x,j}\ \theta_{x,j}\ w_{y,j}\ \theta_{y,j}\ \theta_{z,j}]^T$$

DISCRETIZATION (IN SPACE) OF THE DISTRIBUTED STRUCTURE (CONT'D)

The cable is represented by four degrees of freedom.

The shuttle has three rotational degrees of freedom.

The mast is divided into six finite elements, each requiring five degrees of freedom. Hence the model has 37 degrees of freedom.

The discretized system equations of motion: $M\ddot{\underset{\sim}{q}}(t) + K\underset{\sim}{q}(t) = \underset{\sim}{F}(t)$

$\underset{\sim}{q}(t)$ = generalized displacement vector

M,K = mass, stiffness matrices

$\underset{\sim}{F}(t)$ = control vector

$$\underset{\sim}{q}(t) = [a_{x1}\ a_{y1}\ a_{x2}\ a_{y2}\ \theta_{x0}\ \theta_{y0}\ \theta_{z0}\ w_{x1}\ \theta_{x1}\ w_{y1}\ \theta_{y1}\ \theta_{z1} \cdots w_{y6}\ \theta_{y6}\ \theta_{z6}]^T$$

PARAMETER IDENTIFICATION

Assume that the shuttle is fixed and identify the mast parameter. Use a perturbation technique in conjunction with frequency response.

Harmonic excitation: $\underset{\sim}{F}(t) = \underset{\sim}{F}^e e^{i\omega^e t}$, $e = 1,2,\ldots,m$

Frequency response: $[K - (\omega^e)^2 M]\underset{\sim}{q}^e = \underset{\sim}{F}^e$

Perturbation technique: $M = M_0 + \Delta M$, $K = K_0 + \Delta K \rightarrow \underset{\sim}{p} = \underset{\sim}{p}_0 + \Delta\underset{\sim}{p}$

$\underset{\sim}{p} = [m_1\ m_2\ \ldots\ m_6\ \ EI_{x1}\ EI_{y1}\ GJ_{z1}\ \ldots\ EI_{x6}\ EI_{y6}\ GJ_{z6}]^T$ = actual parameter vector

$\underset{\sim}{p}_0$ = postulated parameter vector

$\Delta\underset{\sim}{p}$ = parameter perturbation vector

Actual, or perturbed, frequency response:

$$[K_0 + \Delta K - (\omega^e)^2(M_0 + \Delta M)](\underset{\sim}{q}_0^e + \Delta\underset{\sim}{q}^e) = \underset{\sim}{F}^e, \quad e = 1,2,\ldots,m$$

PARAMETER IDENTIFICATION (CONT'D)

$\underset{\sim}{q}_0^e$ = response amplitude computed on the basis of postulated model

$\underset{\sim}{q}^e$ = actual response amplitude

$$\left.\begin{array}{l}\underset{\sim}{q}_0^e \\ \underset{\sim}{q}^e\end{array}\right\} \underset{\sim}{q}^e = \underset{\sim}{q}_0^e + \Delta \underset{\sim}{q}^e$$

Parameter perturbations: $\Delta M = \sum_{\ell=1}^{6} \frac{\partial M}{\partial p_\ell} \Delta p_\ell, \; \Delta K = \sum_{\ell=7}^{24} \frac{\partial K}{\partial p_\ell} \Delta p_\ell$

$$\underset{\sim}{B}_\ell^e = \begin{cases} [-(\omega^e)^2 \frac{\partial M}{\partial p_\ell}]\underset{\sim}{q}_0^e, & \ell = 1,2,\ldots,6 \\ [\frac{\partial K}{\partial p_\ell}]\underset{\sim}{q}_0^e, & \ell = 7,8,\ldots,24 \end{cases} \rightarrow B^e = [\underset{\sim}{B}_1^e \; \underset{\sim}{B}_2^e \; \ldots \; \underset{\sim}{B}_{24}^e]$$

Identification algorithm: $B^e \Delta \underset{\sim}{p} = \underset{\sim}{c}^e, \; \underset{\sim}{c}^e = [K_0 - (\omega^e)^2 M_0] \Delta \underset{\sim}{q}^e, \quad e = 1,2,\ldots,m$

$$B = [B^e], \; \underset{\sim}{c} = [\underset{\sim}{c}^e] \rightarrow B \Delta \underset{\sim}{p} = \underset{\sim}{c} \rightarrow \Delta \underset{\sim}{p} = (B^T B^{-1}) B^T \underset{\sim}{c}$$

PARAMETER IDENTIFICATION (CONT'D)

If the measured output is not the whole state, use Kalman filter to estimate the state. First estimation is based on the postulated model:

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + B[\tilde{F}(t) + \tilde{v}(t)]$$

$$\tilde{x}(t) = [\tilde{q}^T(t) \quad \dot{\tilde{q}}^T(t)]^T, \quad A = \begin{bmatrix} 0 & I \\ -M_0^{-1}K_0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M_0^{-1} \end{bmatrix}$$

$\tilde{v}(t)$ = excitation (actuator) noise vector

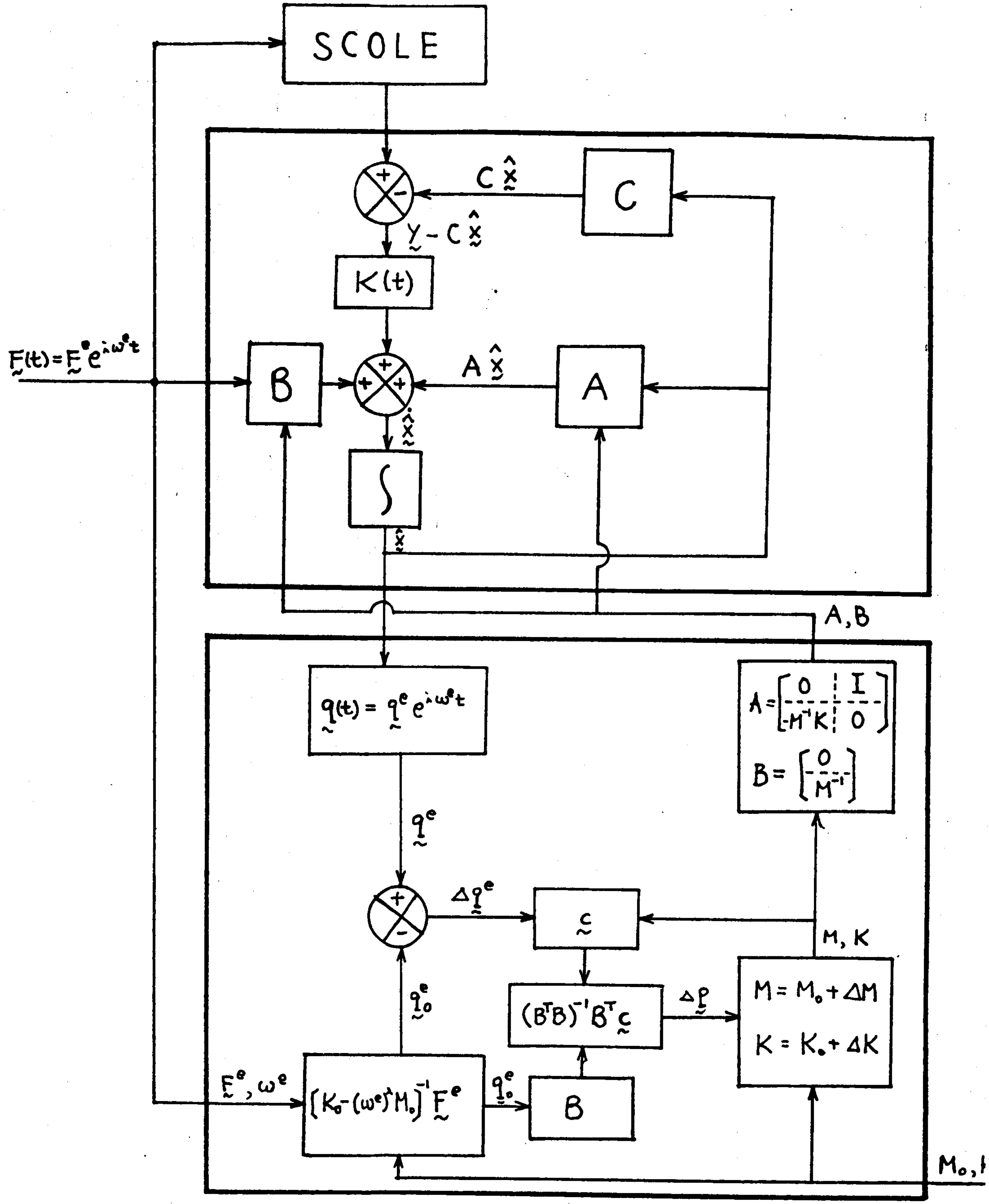
Kalman filter: $\dot{\hat{\tilde{x}}}(t) = A\hat{\tilde{x}}(t) + B\tilde{F}(t) + K(t)[\tilde{y}(t) - C\hat{\tilde{x}}(t)]$

$\tilde{y}(t) = C\tilde{x}(t) + \tilde{w}(t)$ = output vector

$\tilde{w}(t)$ = measurement (sensor) noise vector

The finite-element based identification routine and the Kalman filter work together in a closed-loop fashion.

Identification process is carried out iteratively: identified model becomes postulated model for the new iteration cycle.


SCOLE
C x̂
C
y - C x̂
K(t)
F(t) = F^e e^{iω^e t}
B
A x̂
A
x̂
∫
x̂
A,B
q(t) = q^e e^{iω^e t}
A = [0 I / -M^{-1}K 0]
B = [0 / M^{-1}]
q^e
Δq^e
c
M, K
q_0^e
(B^T B)^{-1} B^T c
Δp
M = M_0 + ΔM
K = K_0 + ΔK
F^e, ω^e
[K_0 - (ω^e)^2 M_0]^{-1} F^e
q_0^e
B
M_0, K_0