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Model Reference Control of Distributed Parameter Systems: Application to the SCOLE Problem

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MODEL REFERENCE CONTROL
OF DISTRIBUTED PARAMETER SYSTEMS
WITH APPLICATION TO THE SCOLE PROBLEM

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OUTLINE

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- MODEL REFERENCE CONTROL OF LUMPED LINEAR SYSTEMS
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- MODEL REFERENCE CONTROL OF DPS
THEORY
SCALE APPLICATION
- CONCLUSIONS AND RECOMMENDATIONS

INTRODUCTION

SCALE MODELS

LUMPED: 16th ORDER WITH 5 FLEXIBLE AND 3 RIGID BODY MODES

DISTRIBUTED: 3 PARTIAL DIFFERENTIAL EQUATIONS FOR ROLL,
PITCH, YAW BEAM BENDING

LUMPED MODEL

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$x^T = (\underline{u}_1^T, \dots, \underline{u}_8^T, \underline{\phi}_{RB}^T, \underline{\theta}_{RB}^T, \underline{\psi}_{RB}^T)$$

$$y_F^T = (\underline{\phi}_S^T, \underline{\theta}_S^T, \underline{\psi}_S^T, \underline{\phi}_r^T, \underline{\theta}_r^T, \underline{\psi}_r^T, \underline{\zeta}_x, \underline{\zeta}_y)$$

$$y^T = y_F^T + (\underline{\phi}_{RB}^T, \underline{\theta}_{RB}^T, \underline{\psi}_{RB}^T, \underline{\phi}_{RB}^T, \underline{\theta}_{RB}^T, \underline{\psi}_{RB}^T, 0, 0)$$

$$\underline{u}^T = (\underline{I}_S, \underline{f}_r, \underline{I}_r)$$

OBJECTIVE: IF $\phi_{RB}(0) = 20^\circ$

$\phi_{RB} \rightarrow 0$ IN ABOUT 10 SEC.

$$|T| \leq 10,000$$

$$|f| \leq 800$$

DISTRIBUTED MODEL

ROLL BEAM BENDING:

$$PA \frac{\partial^2 u_\phi}{\partial t^2} + 2\zeta_\phi \sqrt{PA EI_\phi} \frac{\partial^3 u_\phi}{\partial s^2 \partial t} + EI_\phi \frac{\partial^4 u_\phi}{\partial s^4} = \sum_{n=1}^4 [f_{\phi,n} \delta(s-s_n) + g_{\phi,n} \frac{\partial \delta}{\partial s}(s-s_n)]$$

PITCH BEAM BENDING:

$$PA \frac{\partial^2 u_\theta}{\partial t^2} + 2\zeta_\theta \sqrt{PA EI_\theta} \frac{\partial^3 u_\theta}{\partial s^2 \partial t} + EI_\theta \frac{\partial^4 u_\theta}{\partial s^4} = \sum_{n=1}^4 [f_{\theta,n} \delta(s-s_n) + g_{\theta,n} \frac{\partial \delta}{\partial s}(s-s_n)]$$

YAW BEAM TORSION:

$$PI_\psi \frac{\partial^2 u_\psi}{\partial t^2} + 2\zeta_\psi I_\psi \sqrt{GP} \frac{\partial^3 u_\psi}{\partial s^2 \partial t} + GI_\psi \frac{\partial^2 u_\psi}{\partial s^2} = \sum_{n=1}^4 g_{\psi,n} \delta(s-s_n)$$

MODEL REFERENCE CONTROL OF LUMPED LINEAR SYSTEMS

THEORY

$$\left. \begin{aligned} \dot{x}_p &= A_p x_p + B_p u_p \\ y_p &= C_p x_p \end{aligned} \right\} \text{PROCESS}$$

$$\left. \begin{aligned} \dot{x}_m &= A_m x_m + B_m u_m \\ y_m &= C_m x_m \end{aligned} \right\} \text{REFERENCE MODEL}$$

DESIRE

$$y_p \rightarrow y_m$$

DEFINE IDEAL STATE AND CONTROL

$$\dot{x}_p^* = A_p x_p^* + B_p u_p^*$$

$$y_p^* = C_p x_p^*$$

WHERE $y_p^* = C_m x_m = y_m$

WILL FORCE $x_p \rightarrow x_p^*$

$$\implies y_p \rightarrow y_p^* = y_m$$

ASSUME

$$x_p^* = S_{11} x_M + S_{12} u_M$$

$$u_p^* = S_{21} x_M + S_{22} u_M$$

THEN

$$S_{11} A_M - A_P S_{11} = B_P S_{21}$$

$$S_{11} B_M - A_P S_{12} = B_P S_{22}$$

$$C_P S_{11} = C_M$$

$$C_P S_{12} = 0$$

APPLY

$$u_p = u_p^* + K(y_m - y_p)$$

THEN

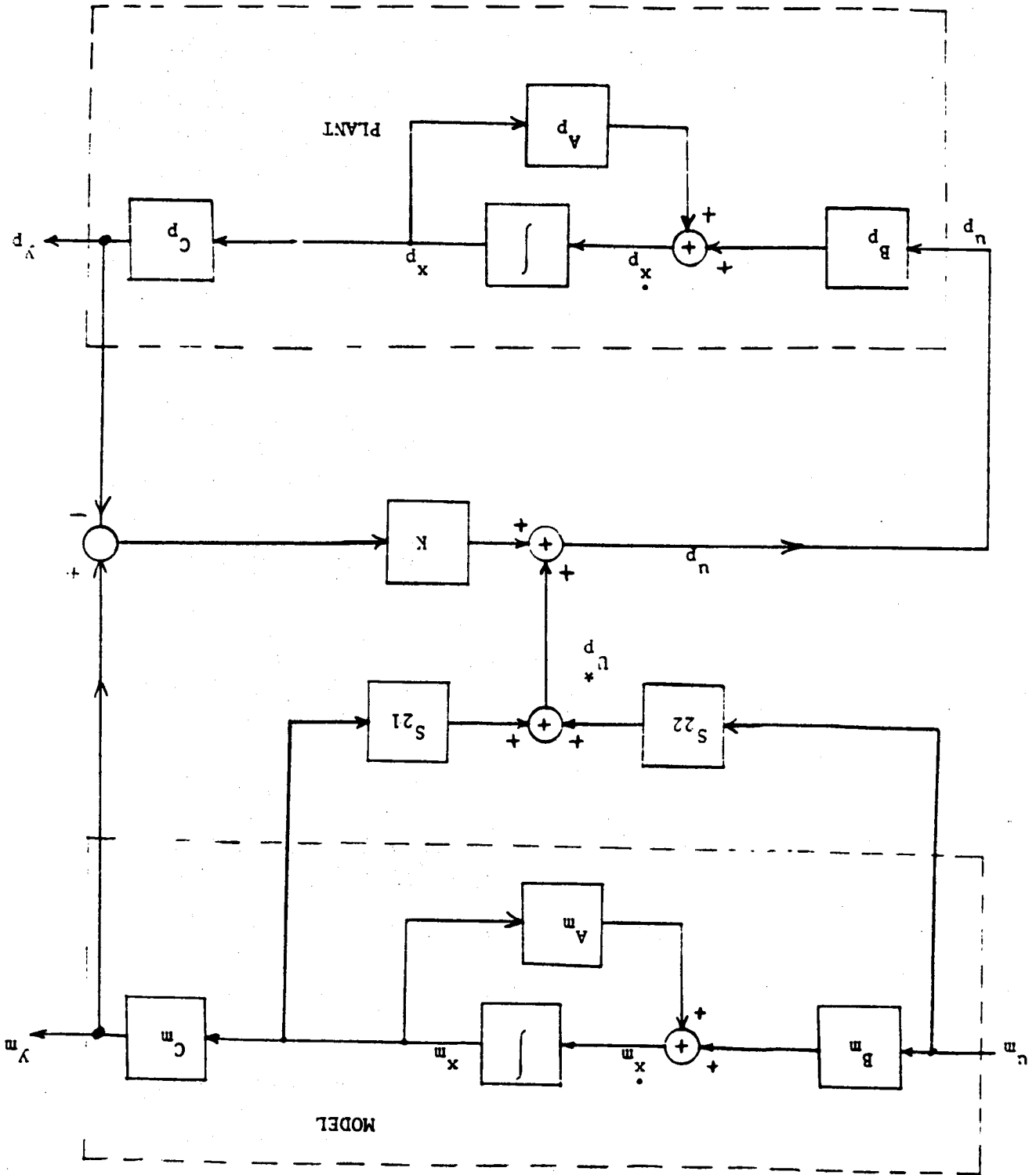
$$\dot{e} = (A_p - B_p K C_p)e$$

WHERE

$$e = x_p^* - x_p$$

∴ CHOOSE K TO STABILIZE $(A_p - B_p K C_p)$

Figure 1: System block diagram.



SPECIAL CASE (PMF)

$$x_p \rightarrow x_m$$

OR $C_p = C_m = I$

$$u_p = S_{21} x_m + S_{22} u_m + K(x_m - x_p)$$

$$B_p S_{21} = A_m - A_p$$

$$B_m = B_p S_{22}$$

$(A_p - B_p K)$ STABLE

SCALE APPLICATION OF LUMPED MODEL FOLLOWING

OBSERVATIONS

- EIGHT CONTROLS
- EIGHT OUTPUT MODES TO BE CONTROLLED

PROCEDURES

- PMF
- $x_p \rightarrow x_m$
- OUTPUT FOLLOWING
- $y_p \rightarrow y_m$

SPECIAL CASES: 8 outputs

CASE I: Consider only positions:

$$Y_P^T = [\theta_S + \theta_{RB}, \theta_S + \theta_{RB}, \psi_S + \psi_{RB}, \xi_X, \xi_Y, \theta_r + \theta_{RB}, \psi_r + \psi_{RB}]^T$$

CASE II: Position and LOS vectors

$$Y_P^T = [\theta_S + \theta_{RB}, \theta_S + \theta_{RB}, \psi_S + \psi_{RB}, \xi_X, \xi_Y, E_1, E_2, E_3]^T$$

E_1, E_2, E_3 - LOS VECTOR COMPONENTS

Note: $R_{LOS} = (R_{LOS})_{NOM} + \Delta$, where $\Delta = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$

$$E_1 = -130 \theta_{RB} - 32.2 \psi_{RB} + \xi_X - 32.2 \psi_r - 260 \theta_r$$

$$E_2 = 130 \theta_{RB} + 18.75 \psi_{RB} + \xi_Y + 18.75 \psi_r - 260 \theta_r$$

$$E_3 = -18.75 \theta_{RB} + 32.2 \theta_{RB} - 18.75 \theta_r - 32.2 \theta_r$$

INITIAL CONDITIONS

Two sets of I.C.'s for each case

CASE I (a):

Choose θ_{RBI} , θ_{RBF} , θ_{RBI} , θ_{RBF} so that $e_{LOS}(0) = 20^\circ$, $e_{LOS}(t_f) = 0^\circ$

$\Rightarrow \theta_{RBI} = 0.1063$ rad ; $\theta_{RBF} = -0.245$ rad

$\theta_{RBI} = -0.1115$ rad ; $\theta_{RBF} = -0.1115$

$t_f = 10$ sec.

$\Rightarrow Y_0^T = [0.0163, -0.1115, 0, 0, 0, 0.1063, -0.1115, 0]^T$

STATE I.C.'s:

$Y_p(0) = C_p X_p(0)$ ----- Solve for $X_p(0)$

Model I.C.'s

$Y_p \rightarrow Y_m, Y_p(0) = Y_m(0)$

$\Rightarrow X_m^T = [0.1063, -0.1115, 0, 0]^T$

COMMAND

II (0) = 1.0, T > 0. i = 1, 2, ..., 8

I.C.'s (cont.)

CASE I (b):

$$\theta_{RB} = 0.34 \text{ rad} = 20^\circ$$

$$\psi_{RB} = \theta_{RB} = 0.0$$

$$\Rightarrow Y_p^T(0) = [0.34, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T$$

Model:

$$X_m^T = [0.34, 0, 0, 0, 0]^T$$

COMMAND:

$$U_{m_i}(0) = 0.0, \quad T \geq 0, \quad i = 1, \dots, 8$$

I. C."s (cont.)

CASE II (a): Same objective as in CASE I (a)

$$Y_P^T(o) = \{0.1063, -0.1115, 0, 0, 0, 14.49, 13.82, 5.51\}^T$$

$$X_M^T(o) = \{0.1063, -0.1115, 0, 0, 1\}$$

$$\Rightarrow Y_M(o) = Y_P(o)$$

$$U_{m_j} = 1.0, \quad j = 1, \dots, 8$$

CASE II(b): Same objective as in CASE I (b)

$$Y_P^T(o) = \{0.34, 0, 0, 0, 0, 44.2, 10.948\}^T$$

$$X_M^T = \{0.34, 0, 0, 0, 1\}^T$$

MODEL

In both cases: 4 states

$$\dot{X}_M = A_m X_M + B_M U_M$$

$$A_M = \text{DIAG } [A_1 \ A_2 \ A_3 \ A_4] = [-.15, -0.10, -0.10, -0.10]^T$$

$$Y_M^T = [Y_{M1} \ Y_{M2} \ \dots \ Y_{M8}]^T$$

CASE I (a, b):

$$C_M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

CASE II (a, b):

$$C_M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & -130 & 0 & 0 \\ 130 & 0 & 0 & 0 \\ 32.2 & -18.75 & 0 & 0 \end{bmatrix}$$

MODEL (cont.)

Matrix BM:

Cases I (a), II (a):

$$B_M = \begin{bmatrix} -3.67 \times 10^{-2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.115 \times 10^{-2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Cases II (b), II (b):

$$B_M = [0]$$

SYSTEM I.C.'s: RESULTS

CASE I (a):

$$Y_P^T(o) = Y_M^T(o) = [0.1063 \quad -0.1115, \quad 0, \quad 0, \quad 0, \quad 0.1063, \quad -0.1115, \quad 0]^T$$

$$X_P^T(o) = [0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0.1063, \quad 0, \quad -0.1115, \quad 0, \quad 0, \quad 0]^T$$

$$X_P^{*T}(o) = [0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0.1063, \quad -0.05269, \quad -0.1115, \quad 0, \quad 0, \quad 0]^T$$

CASE I (b):

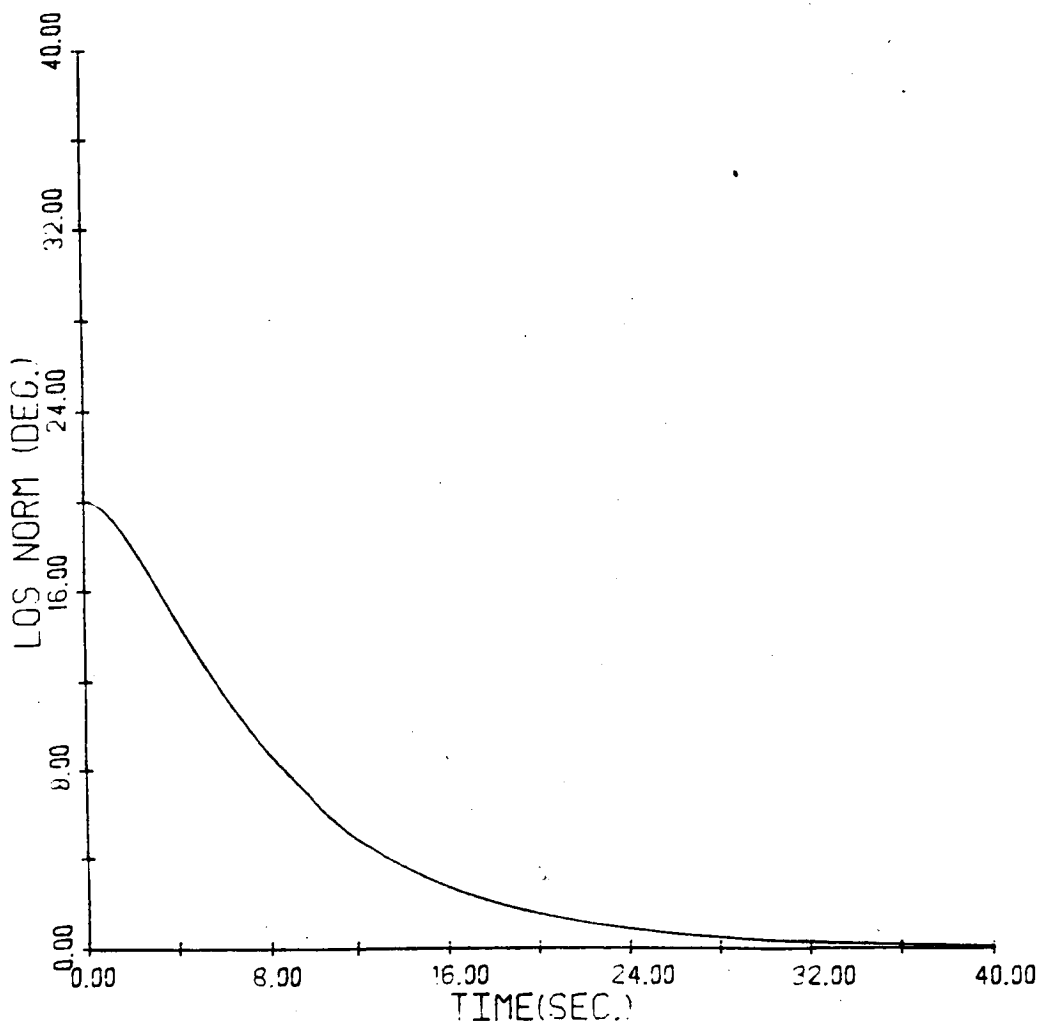
$$Y_P^T(o) = Y_M^T(o) = [0.34, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0.34, \quad 0, \quad 0]^T$$

$$X_P^T(o) = [0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0.34, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0]^T$$

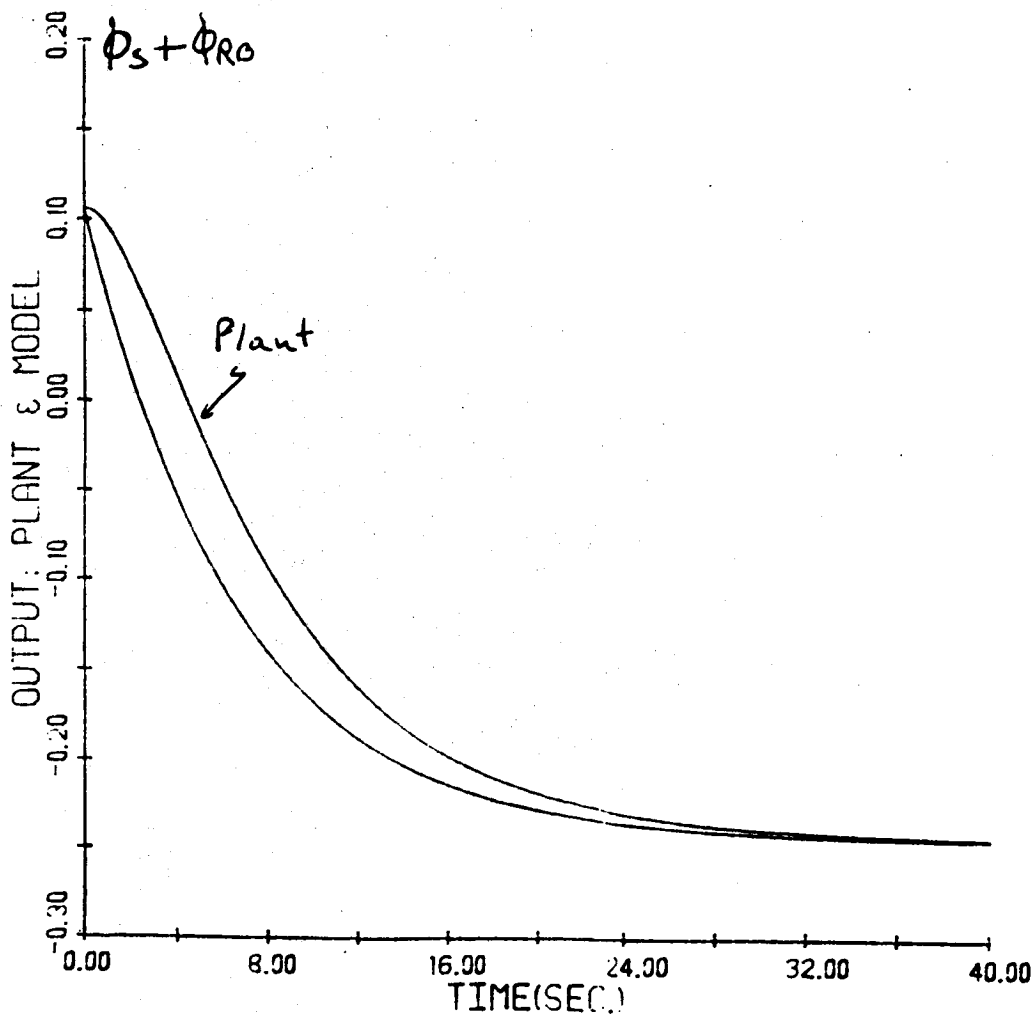
$$X_P^{*T}(o) = [0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0, \quad 0.34, \quad -0.051, \quad 0, \quad 0, \quad 0, \quad 0]^T$$

LOS ERROR : CASE II(a)

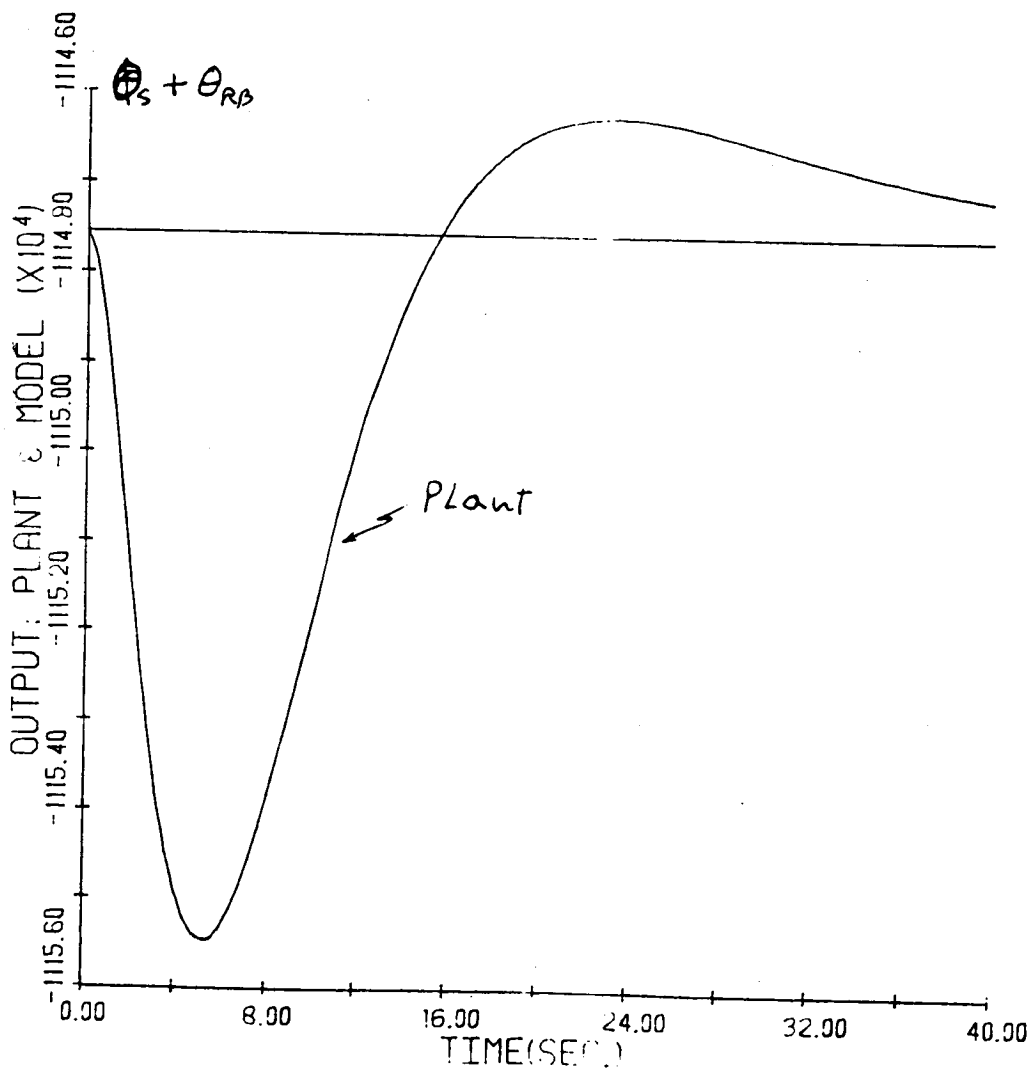
$$e_{LOS} = \sin^{-1} \left[\frac{\| \delta_T \times T_i \bar{R}_{LOS} \|}{\| R_{LOS} \|} \right]$$



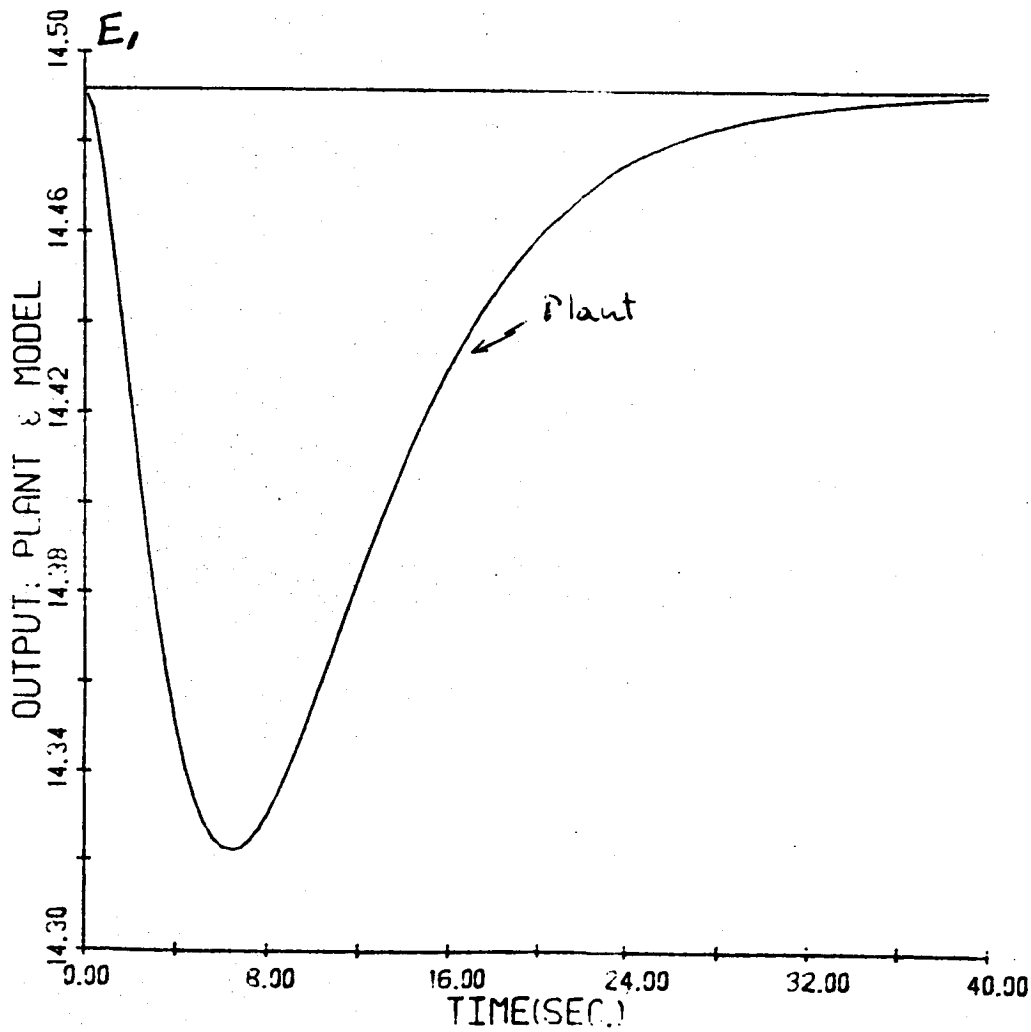
OUTPUT: PLANT and Model
Case II (a)



OUTPUT: PLANT and Model
Case II (a)

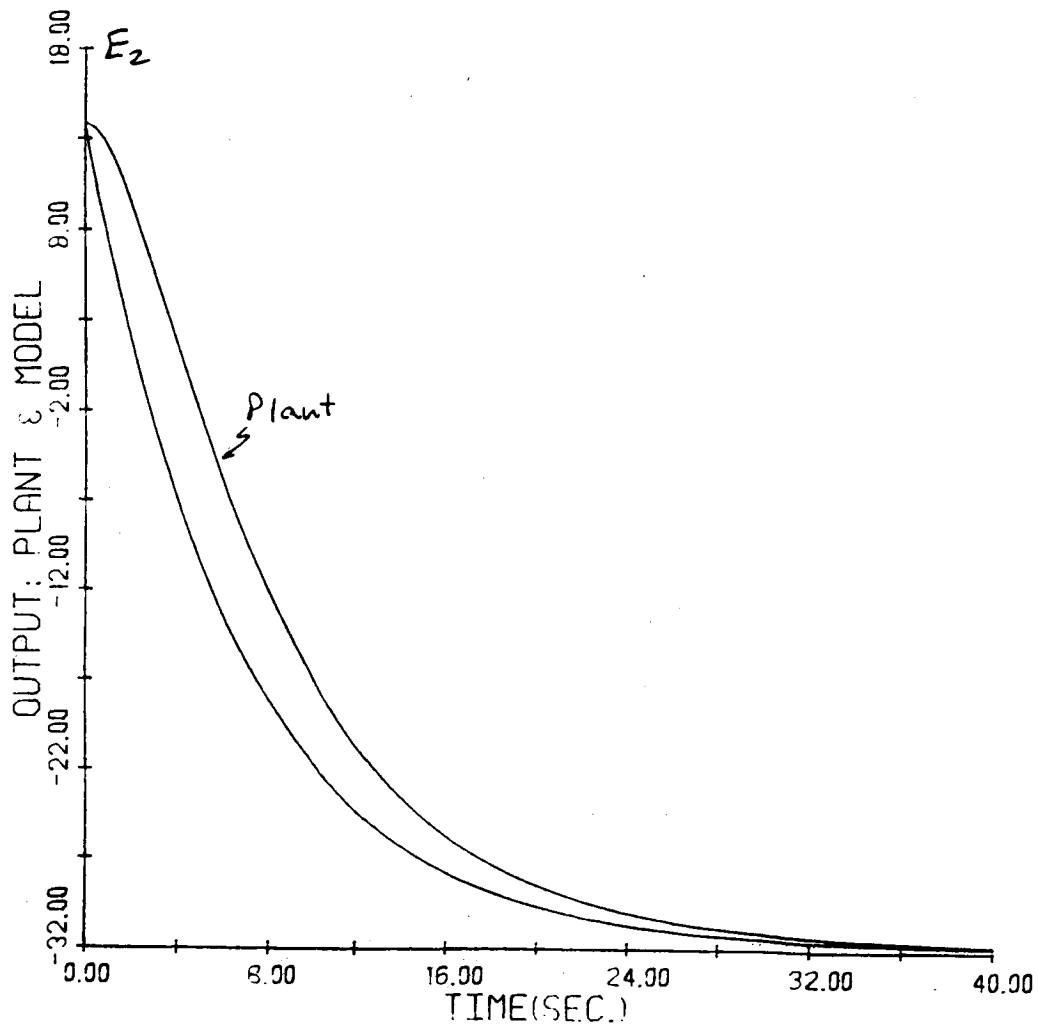


OUTPUT: PLANT and Model
CASE II(a): 1st Component of LOS

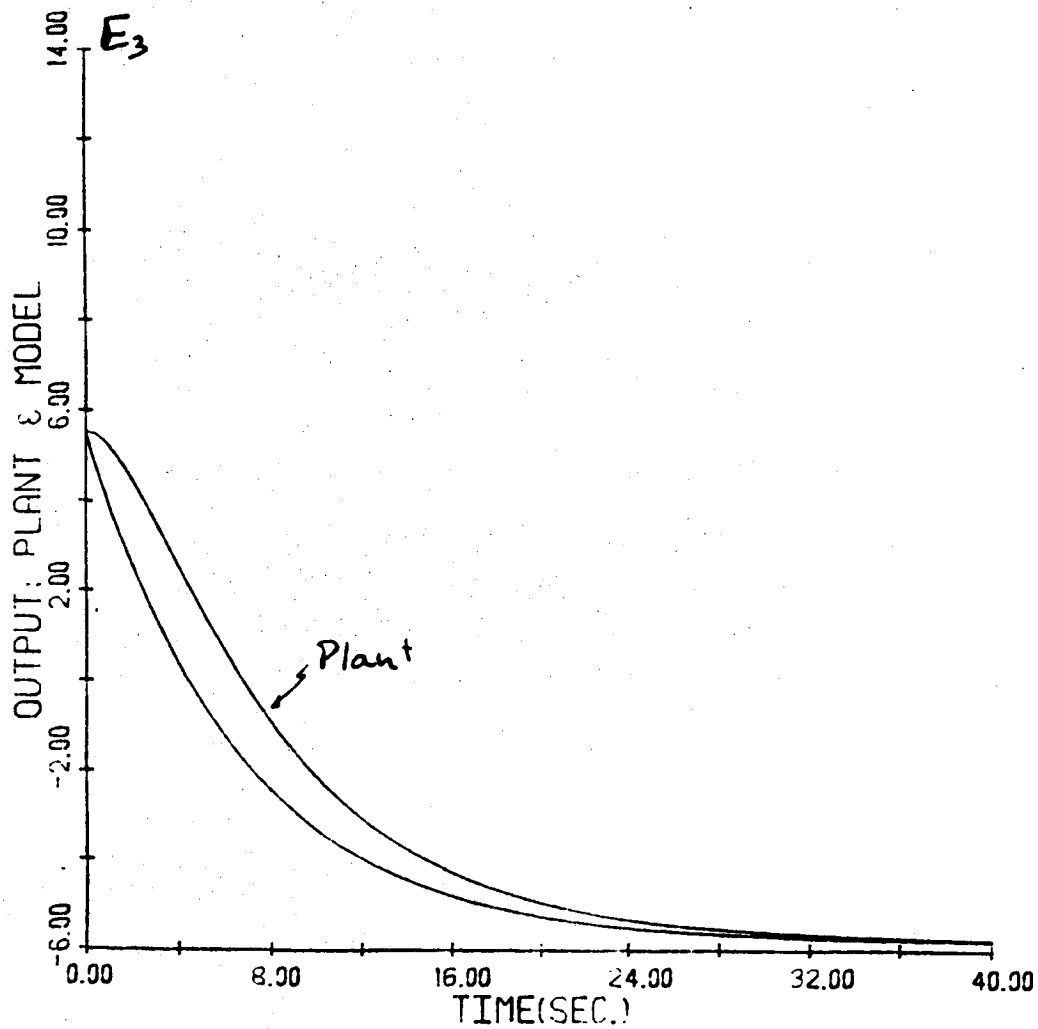


C-3

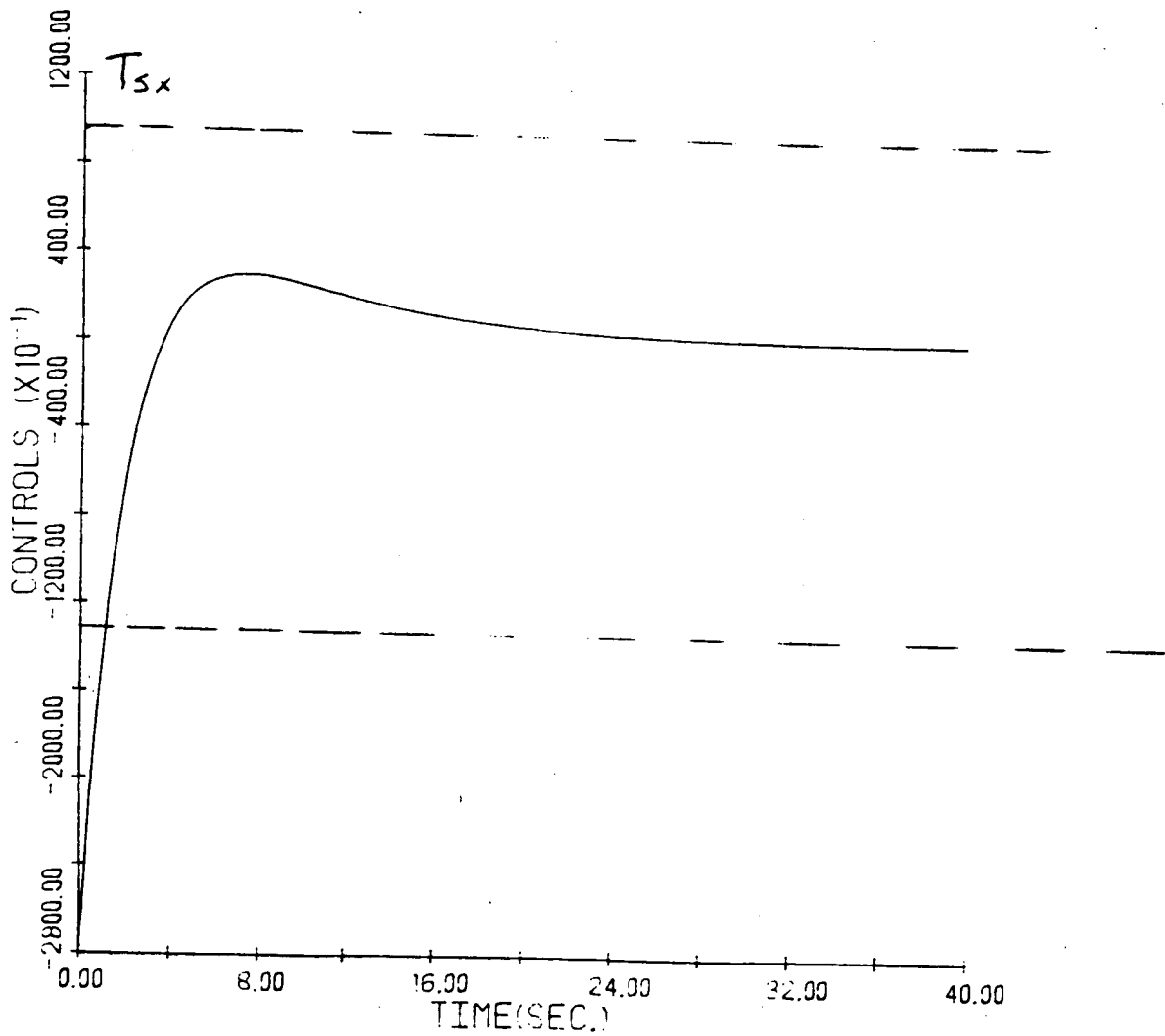
OUTPUT: PLANT and Model
CASE II(a): 2nd Component of LOS



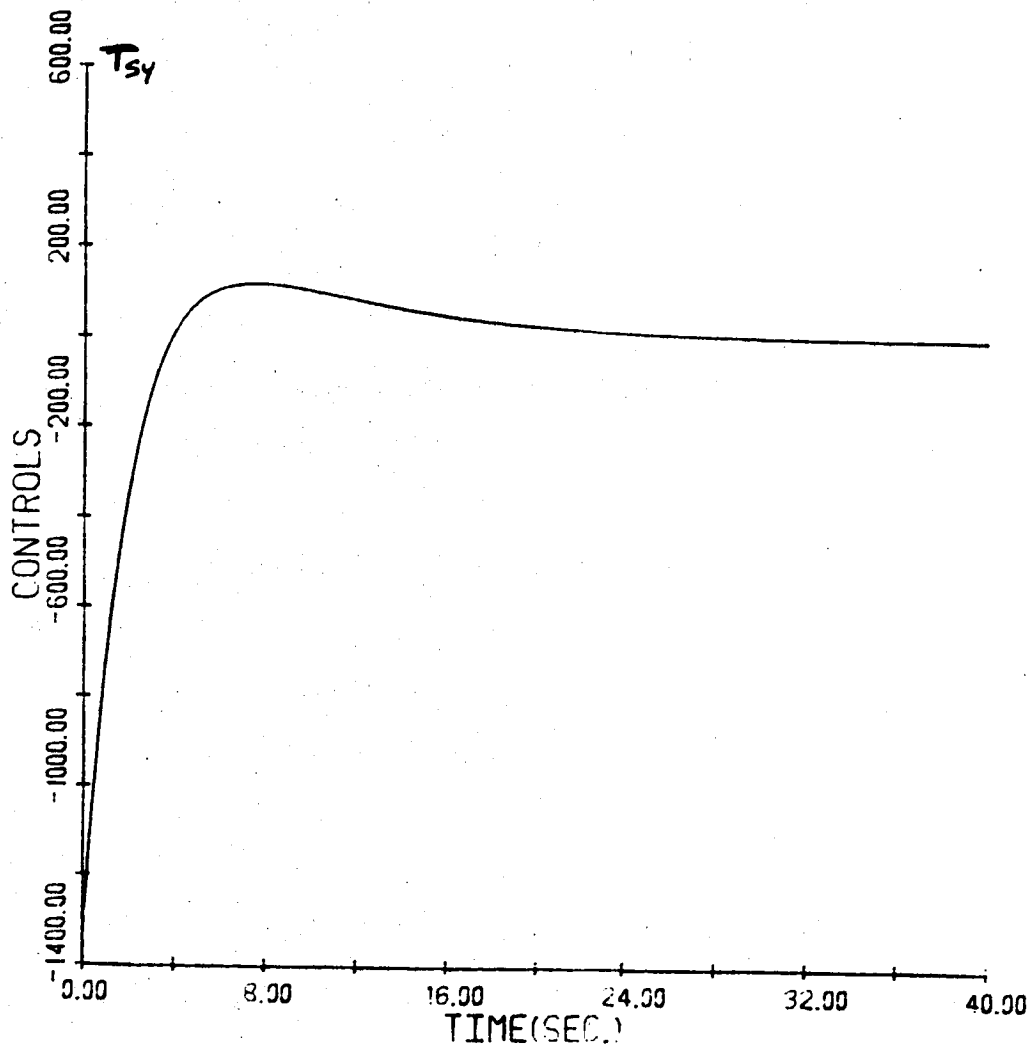
OUTPUT : PLANT and Model
CASE II(a) : 3RD Component of LOS



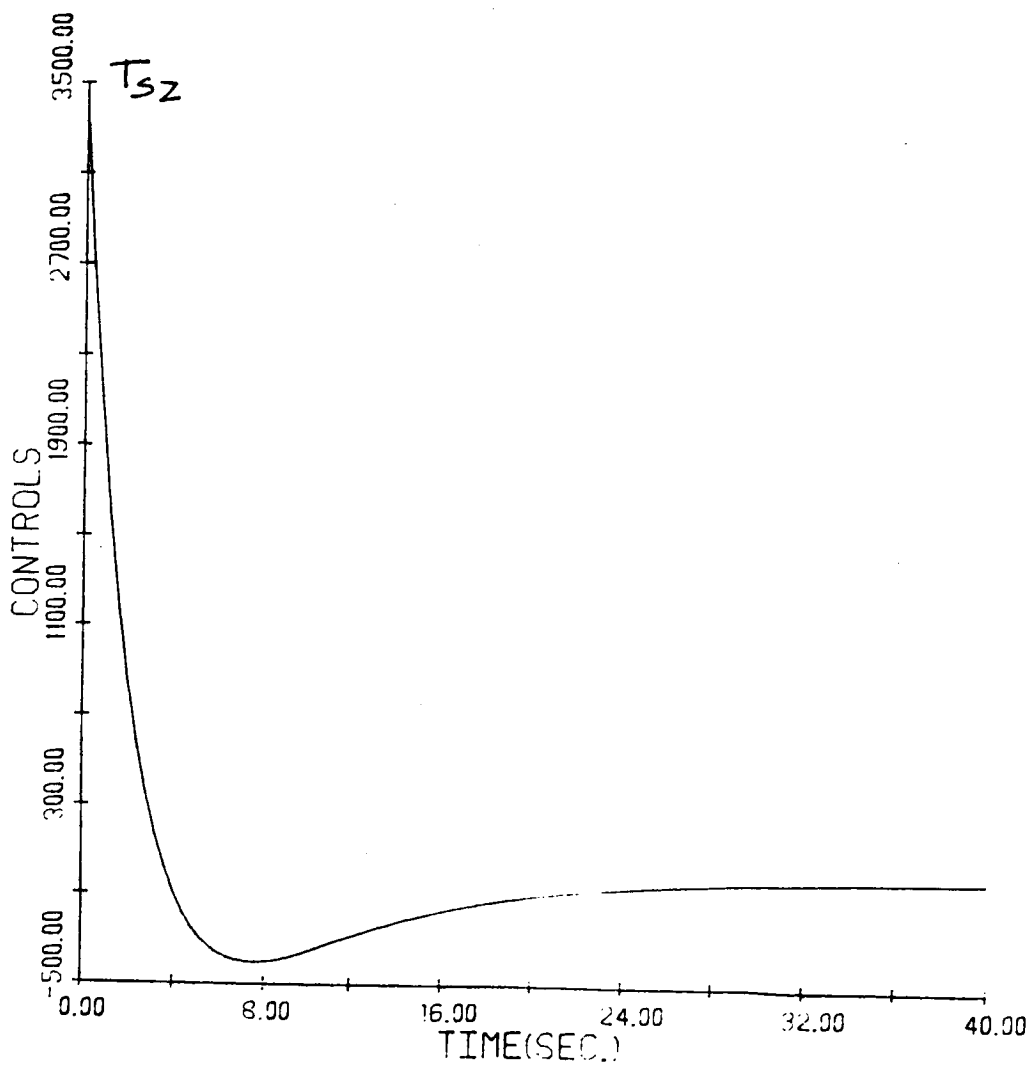
CONTROL : CASA II(a)



CONTROL : CONTROL : CASE II(a)

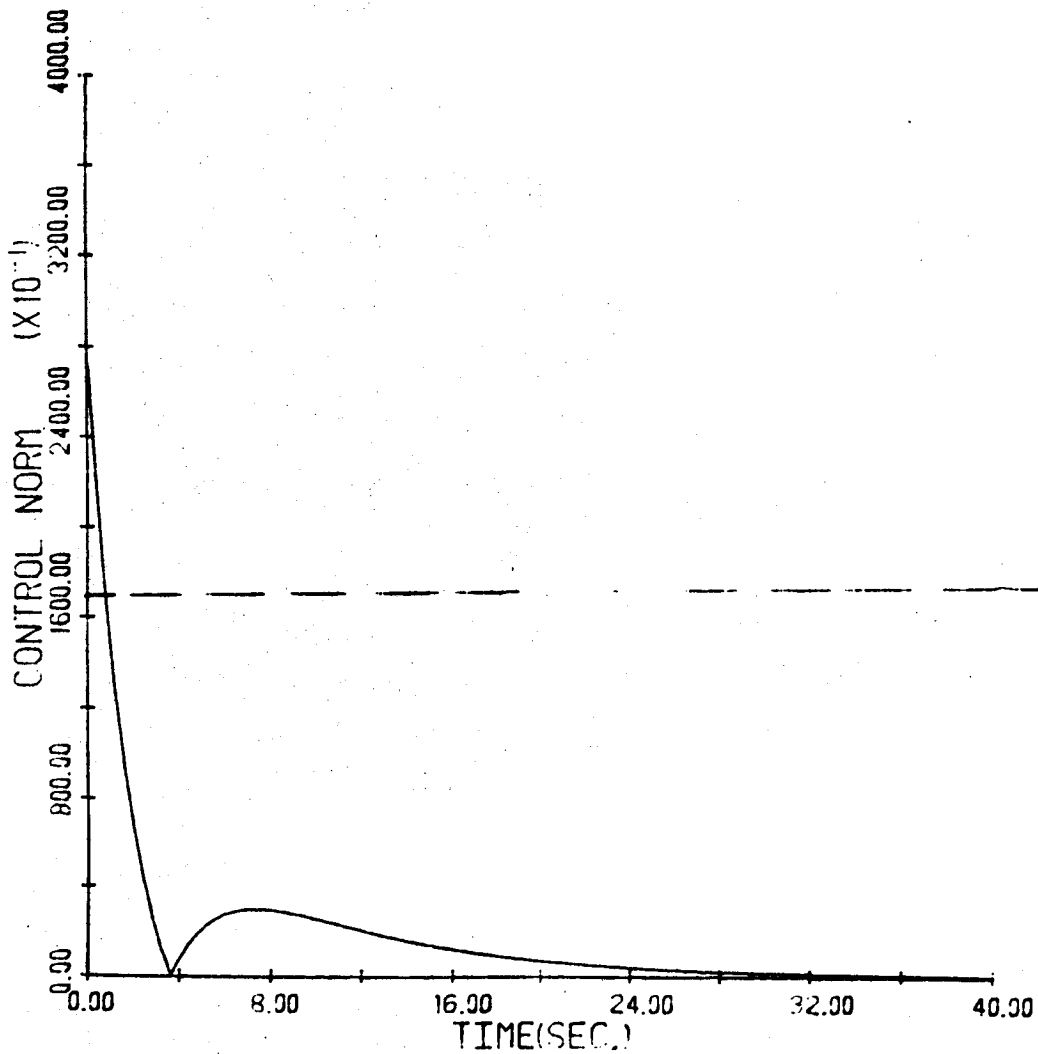


CONTROL: CASE II (a)



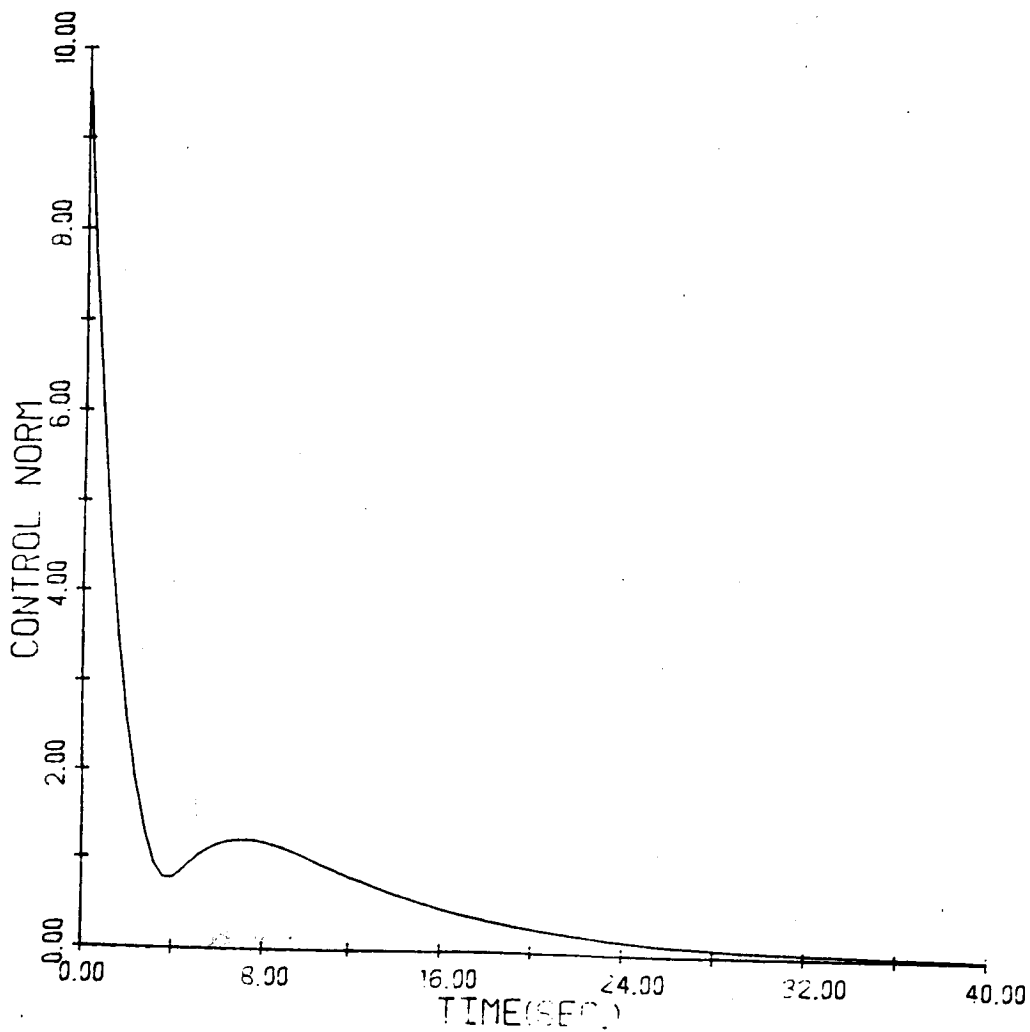
NORM OF MOMENTS AT SHUTTLE : CASE II (a)

$$= (T_{sx}^2 + T_{sy}^2 + T_{sz}^2)^{1/2}$$



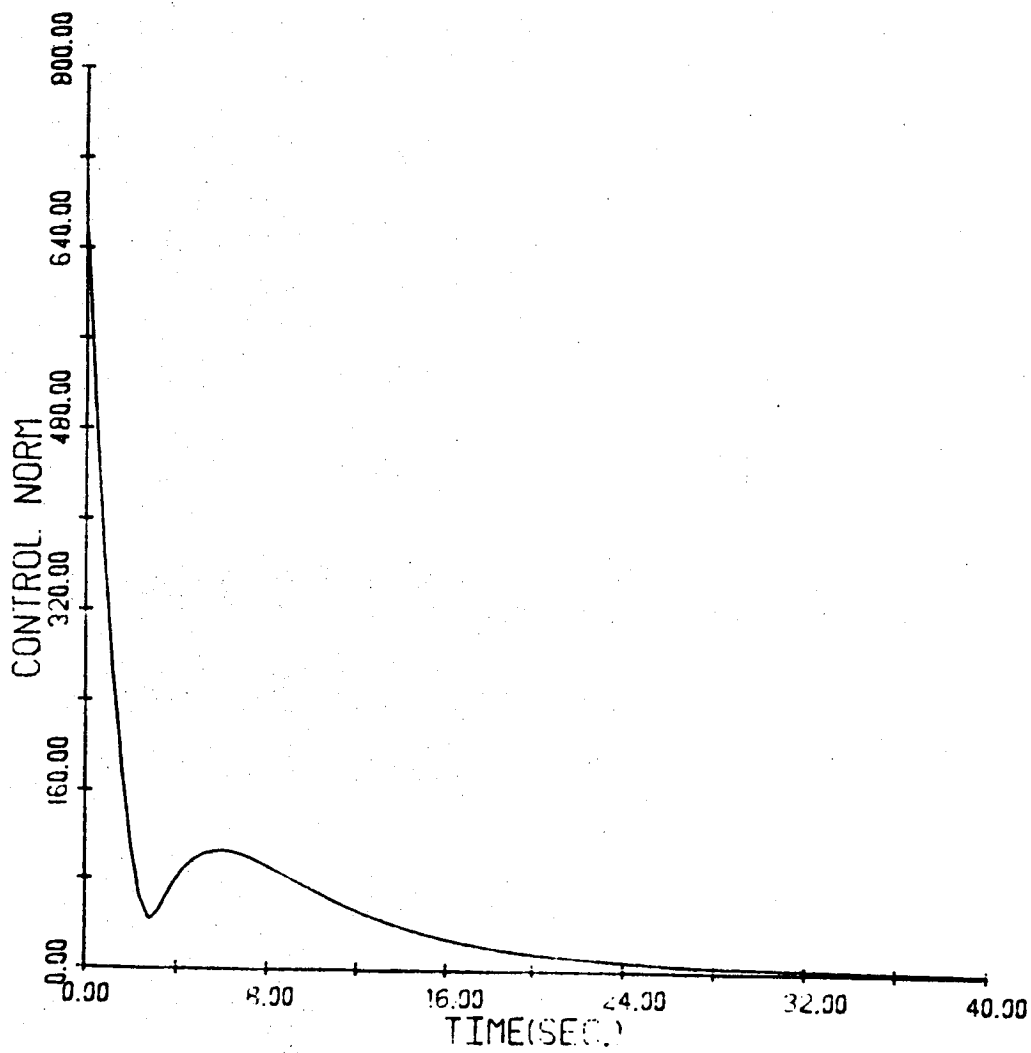
NORM of FORCES AT REFLECTOR : CASE II (a)

$$= (f_{nx}^2 + f_{ny}^2)^{1/2}$$

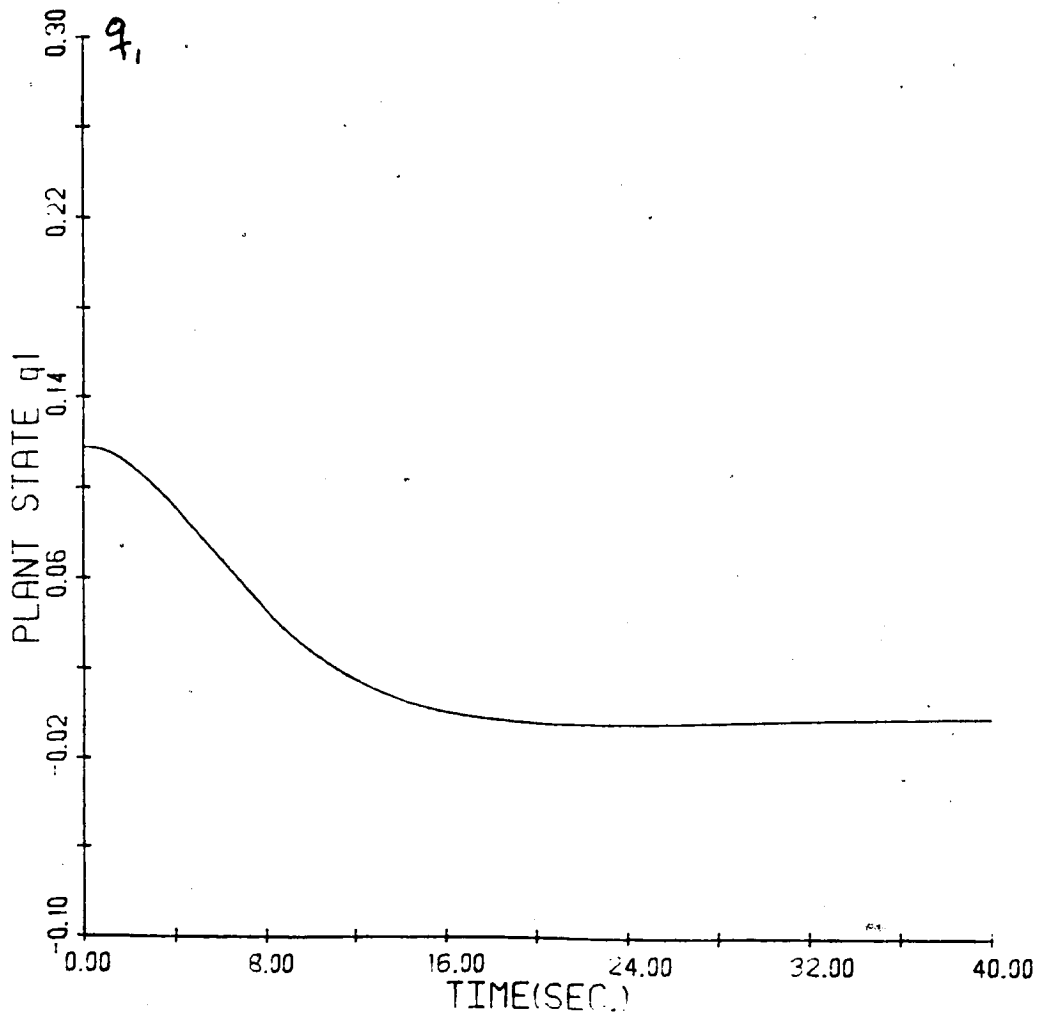


NORM OF MOMENTS AT REFLECTOR: CASE II (a)

$$= (T_{ax}^2 + T_{ay}^2 + T_{az}^2)^{1/2}$$



PLANT STATE : CASE II (a)



MODEL REFERENCE CONTROL OF DISTRIBUTED SYSTEMS

$$m(x)u_{tt}(x,t) + D_0 u_t(x,t) + A_0 u(x,t) = f(x,t)$$

$$v_1 = u(x,t)$$

$$\text{BC } v_1''(0) = v_1''(L) = 0$$

$$v_2 = \frac{\partial}{\partial t} u(x,t)$$

$$\text{and } v_2''(0) = v_2''(L) = 0$$

$$\dot{\underline{v}} = \underline{A}\underline{v} + \underline{B} f(x,t)$$

$$\underline{A} = \begin{bmatrix} 0 & 1 \\ -\frac{A_0}{m(x)} & -\frac{D_0}{m(x)} \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 0 \\ \frac{1}{m(x)} \end{bmatrix}$$

$$y = Cv$$

CONTROL PROBLEM FORMULATION

GIVEN THE DPS, IT IS DESIRED TO FIND A FINITE DIMENSIONAL CONTROLLER

SO THAT THE OUTPUT $y(t)$ "FOLLOWS" A DESIRABLE OUTPUT TRAJECTORY $y_m(t)$.

$$\dot{q} = A_m q + B_m u_m$$

$$y_m = C_m q$$

SOLUTION TO DPS MRC PROBLEM

DEFINE IDEAL STATE AND CONTROL v^* , f^*

$$\frac{\partial v^*(t)}{\partial t} = A v^*(t) + B f^*(t)$$

$$y^*(t) = C v^*(t)$$

$$y^*(t) = y_m(t) = C_m q(t)$$

$$v^*(t) = S_{11}(x) q(t) + S_{12}(x) u_m(t)$$

$$f^*(t) = S_{21} q(t) + S_{22} u_m(t)$$

$$y^*(t) = C S_{11} q + C S_{12} u_m = y_m = C_m q$$

$$S_{11}(x) A_m = A S_{11}(x) + B S_{21}$$

$$S_{11} B_m = A S_{12}(x) + B S_{22}$$

$$C S_{11} = C_m$$

$$C S_{12} = 0$$

$$\dot{e} = \dot{v}^* - \dot{v} =$$

$$A v^* + B f^* - A v - B f =$$

$$A e + B (f^* - f)$$

THIS EQUATION SUGGESTS THAT THE ACTUAL MODEL FOLLOWING

CONTROL (f) BE DEFINED AS:

$$f = f^* + G(y_m - y)$$

$$= f^* + G(C q_m - C v)$$

$$= f^* + G C (v^* - v)$$

$$= f^* + G C e$$

SUBSTITUTION OF (3.14) INTO (3.13) GIVES:

$$\dot{e} = (A - B G C)e$$

FOR ILLUSTRATIVE PURPOSES WE WILL CONTROL

$$\omega_4 = \dot{v}'(t, L) = y_1$$

$$\omega_1 = \dot{v}'(t, 0) = y_2$$

AND

$$y_3 = v(t, s_0) + \alpha \dot{v}(t, s_0)$$

WHERE

$$0 < s_0 < L$$

THUS

$$C = \begin{bmatrix} 0 & \partial/\partial s[\delta(s-L)] \\ 0 & \frac{\partial}{\partial s} \delta(s) \\ \delta(s-s_0) & \alpha \delta(s-s_0) \end{bmatrix}$$

REFERENCE MODEL

$$\dot{q} = a_m q + b_m u_m$$

$$y_{m1} = c_1 q$$

$$y_{m2} = c_2 q$$

$$y_{m3} = c_3 q$$

UNDER THE ASSUMPTION THAT $u_m = 0$,

$$v_1^* = S_{11}^{-1}(s) q(t)$$

$$v_2^* = S_{11}^{-2}(s) q(t)$$

$$f_1^* = S_{21}^{-1} q$$

$$f_2^* = S_{21}^{-2} q$$

$$f_3^* = S_{21}^{-3} q$$

ILLUSTRATIVE APPLICATION TO SCOLE

ROLL BEAM BENDING EQUATION

ASSUMPTIONS: PROOF MASSES AND DAMPING NEGLIGIBLE, REFLECTION MASS NEGLIGIBLE

$$PA \ddot{v}(t,s) + EI v''''(t,s) = f_1 \delta(s-l) + f_2 \delta'(s) + f_3 \delta'(s-l)$$

$$f_1 = F_y$$

$$f_2 = M_l$$

$$f_3 = M_y$$

CASE 1: (ignore shuttle mass)

$$v''(t,0) = v''(t,L) = 0$$

$$v''''(t,0) = v''''(t,L) = 0$$

CASE 2: Simple-free (shuttle mass \rightarrow infinity)

$$v(t,0) = 0 \quad v'(0,t) = 0$$

$$v''(t,L) = 0 \quad v''''(t,L) = 0$$

CASE I Free-Free

$$\begin{aligned}
 S_{11}^1(s) = & \\
 & + S_{21}^1 \sum \frac{X_K(L)}{K} \frac{L}{4} \frac{X_K'(s)}{(PA a_m^2 + EIK^4) X_K^2(L)} \\
 & + S_{21}^2 \sum \frac{-X_K'(0)}{K} \frac{L}{4} \frac{X_K'(s)}{(PA a_m^2 + EIK^4) X_K^2(L)} \\
 & + S_{21}^3 \sum \frac{-X_K'(L)}{K} \frac{L}{4} \frac{X_K'(s)}{(PA a_m^2 + EIK^4) X_K^2(L)}
 \end{aligned}$$

WHERE

$$\begin{aligned}
 X_K(s) = & \left[\left(\frac{\text{Sinh}(KL) - \text{Sin}(KL)}{\text{Cos}(KL) - \text{Cosh}(KL)} (\text{Cosh } Ks + \text{Cos } Ks) \right) \right. \\
 & \left. + \text{Sinh}(Ks) + \text{Sin}(Ks) \right]
 \end{aligned}$$

SOLVING THE EQUATIONS OF MOTION

$$v(t,s) = \sum_{n=1}^{\infty} Q_n(s) y_n(t)$$

$$EI \frac{d^4 Q_n(s)}{ds^4} - W_n^2 P A Q_n(s) = 0 \quad n = 1, 2, \dots, \infty$$

$$Q_n(s) = A \sin k_n s + B \cos k_n s + C \sinh k_n s + D \cosh k_n s$$

CASE 1

$$Q_n(s) = \left[\frac{\sinh k_n L - \sin k_n L}{\cos k_n L - \cosh k_n L} \right] (\cosh k_n s + \cos k_n s) + \sin k_n s + \sinh k_n s$$

CASE

$$Q_n(s) = \frac{1}{N_n} \left[\frac{\sin k_n L + \sinh k_n L}{\cos k_n L + \cosh k_n L} \right] (\cos k_n s - \cosh k_n s) + \sin k_n s + \sinh k_n s$$

CGT GAIN SOLUTION

Output matrix values

$$\text{Note: } S_{11}(0) = \frac{C_1}{A_m}$$

$$S_{11}(L) = \frac{C_2}{A_m}$$

$$S_{11}(s) = \frac{C_3}{1 + \alpha A_m}$$

Want $S_{11}(s)$ to be ideal initial beam shape

Case 1

Case 2

$$C_1 = 0.000251$$

$$C_1 = 0.0002$$

$$C_2 = -0.000251$$

$$C_2 = -0.0001456$$

$$C_3 = 0.009596$$

$$C_3 = 0.0092$$

REFERENCE MODEL SELECTION

PURPOSE: TO DAMP OUT THE STRUCTURAL VIBRATIONS WITHIN TEN SECONDS WITHOUT VIOLATING THE CONTROL MAGNITUDE CONSTRAINTS.

$$\dot{q}(t) = 0.4q(t)$$

$$y_{m1}(t) = C_{m1}q(t)$$

$$y_{m2}(t) = C_{m2}q(t)$$

$$y_{m3}(t) = C_{m3}q(t)$$

$$q(0) = 1$$

RESULTS

0 **FEEDBACK GAINS**

0 **FEEDFORWARD GAINS**

0 **SIMULATIONS**

PARAMETER	VALUE
L = Beam Length	130.0 ft
α = weighting factor	0.25
s = additional beam sensor location	65.0 ft
EI	4.0×10^7 lb-ft ²
PA	0.09556 slugs/ft

TABLE 5.1 : SCOPE BEAM PARAMETERS

MODE #	k_n	ω_n (rad/sec)	f_n (HZ)
1	4.73	27.085	4.311
2	7.853	74.658	11.882
3	10.996	146.378	23.297
4	14.173	243.181	38.703
5	17.274	361.236	57.492

TABLE 5.2 : NATURAL FREQUENCIES FOR CASE I

MODE # n	k_n	ω_n (rad/sec)	f_n (HZ)
1	3.927	18.669	2.971
2	7.069	60.495	9.628
3	10.210	126.199	20.085
4	13.352	215.823	34.349
5	16.493	329.310	52.411

TABLE 5.3 : NATURAL FREQUENCIES FOR CASE II

RECALL THAT FOR TRUE STABILITY WE NEED

$$\underline{f} = \underline{f}^* + G(y_m - y) = \underline{f}^* + GC(v^* - v)$$

THIS SYSTEM WILL BE STABLE FOR

$$G = \begin{bmatrix} G_1 & 0 & 0 \\ 0 & G_{22} & 0 \\ 0 & 0 & G_{33} \end{bmatrix}$$

$$G_{11} > 0$$

FEEDBACK GAINS (NO SENSOR AT s_0)

CASE I

SET A : $G_{11} = 1210.0$ $G_{22} = 1730.0$ $G_{33} = 0.0$

SET B : $G_{11} = 600.0$ $G_{22} = 850.0$ $G_{33} = 0.0$

SET C : $G_{11} = 60.0$ $G_{22} = 85.0$ $G_{33} = 0.0$

CASE II

SET A : $G_{11} = 950.0$ $G_{22} = 300.0$ $G_{33} = 0.0$

SET B : $G_{11} = 475.0$ $G_{22} = 150.0$ $G_{33} = 0.0$

SET C : $G_{11} = 95.0$ $G_{22} = 30.0$ $G_{33} = 0.0$

# modes in series	s_{21}^1	s_{21}^2	s_{21}^3
3	-690.946	-1641.531	-9785.292
4	-690.954	-1044.241	-9188.201
5	-669.460	-1236.139	-8678.003
6	-669.401	-949.532	-8391.477
7	-641.403	-1145.781	-7803.666
8	-641.402	-982.676	-7640.603
9	-631.98	-1066.760	-7320.688
10	-631.916	-959.505	-8279.688
AVERAGE	-658.000	-1127.700	-8279.000

CGT Gains for Case I

# modes in series	S_{21}^1	S_{21}^2	S_{21}^3
3	-355.878	-1141.894	-6309.016
4	-332.129	-805.507	-5354.048
5	-318.384	-1007.015	-4986.424
6	-316.305	-791.389	-4776.447
7	-303.601	-953.259	-4463.880
8	-300.262	-856.221	-4300.704
9	-295.713	-924.748	-4184.084
10	-294.991	-937.512	-4166.822
AVERAGE	-314.235	-926.750	-4817.250

Stavul n rous

Case I : Free-Free

$$G_{11} = 1210$$

$$G_{22} = 1731$$

$$G_{33} = 0$$

$$S_{21} = -669.0$$

$$S_{22} = -949.5$$

$$S_{23} = -8391.5$$

Case II : Simple-Free

$$G_{11} = 950$$

$$G_{22} = 300$$

$$G_{33} = 0$$

$$S_{21} = -316.3$$

$$S_{22} = -791.4$$

$$S_{23} = -4776.4$$

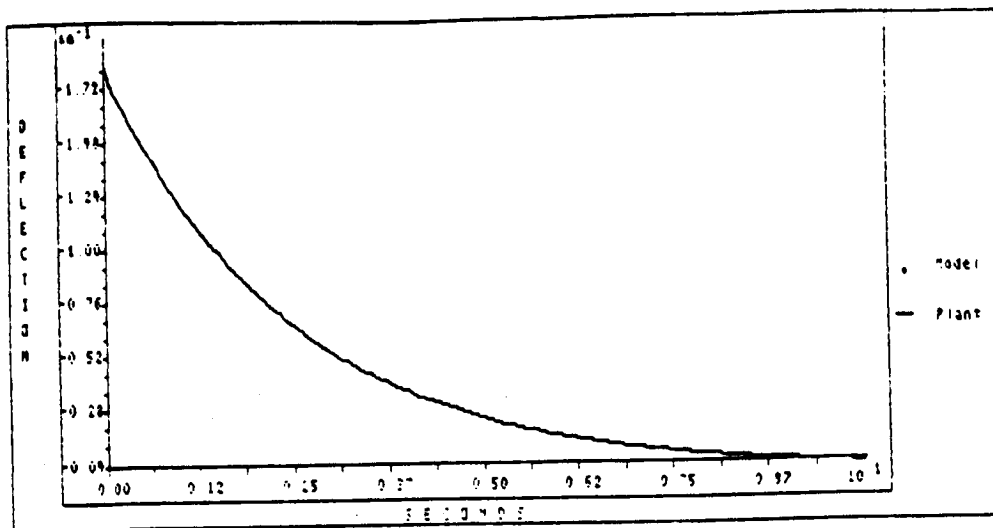


Figure 5.11 Case I : Perfect Model Following -
CGT Gains for 6 Modes

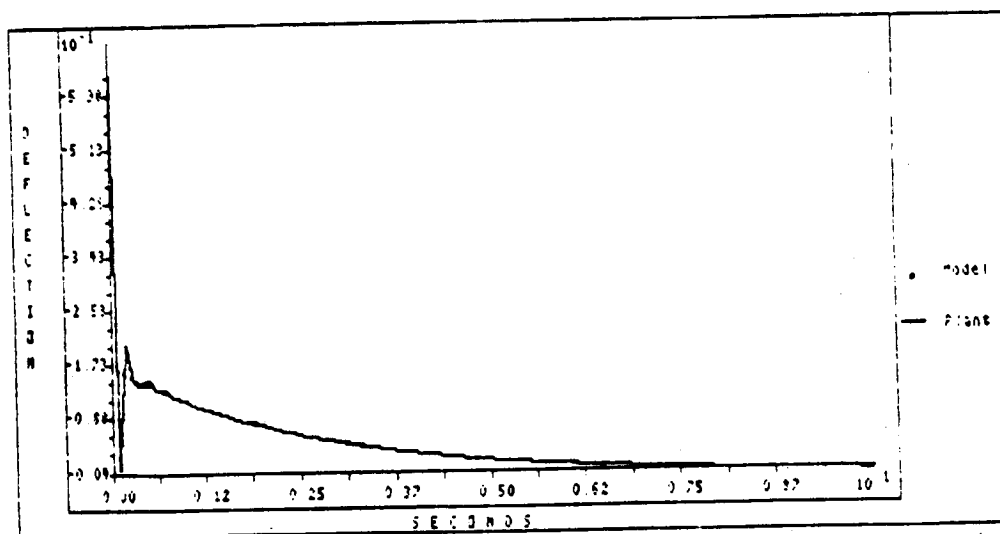


Figure 5.12 Case I : Tracking -
CGT Gains for 4 Modes

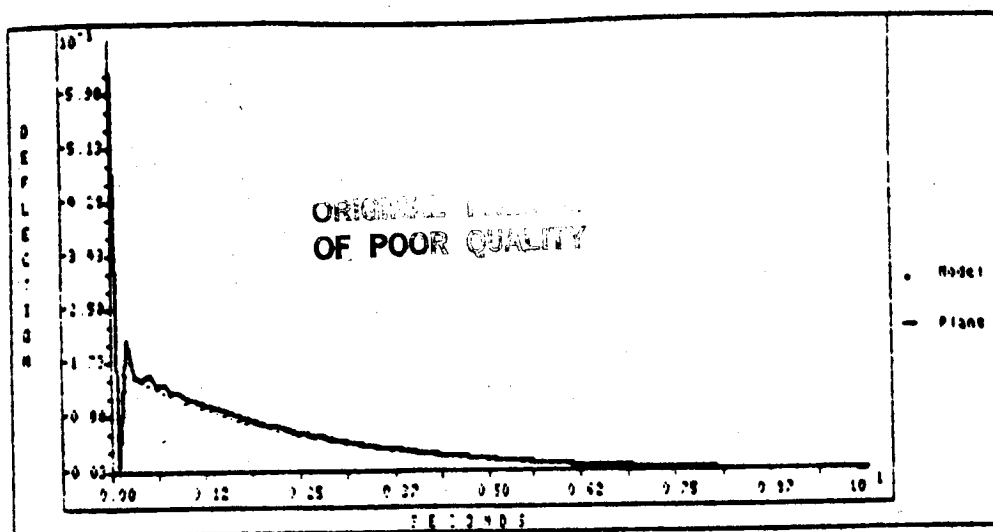


Figure 5.13 Case I : Tracking -
CGT Gains for b Modes

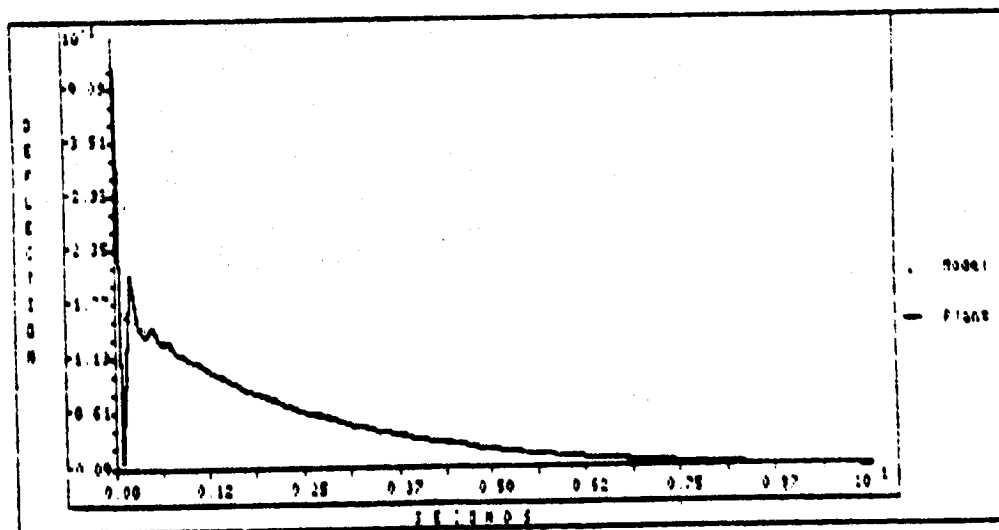


Figure 5.14 Case I : Tracking -
CGT Gains for Ave Modes

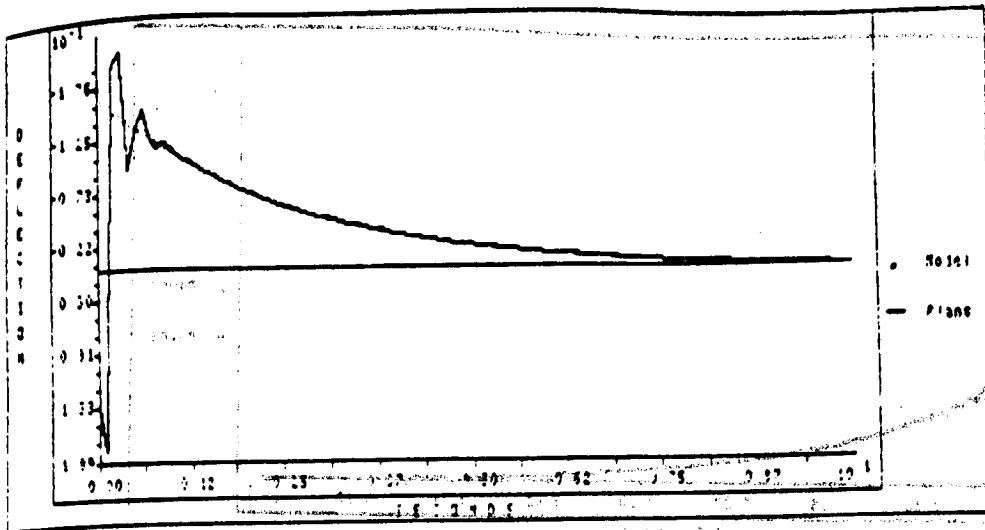


Figure 5.15 Case II : Tracking -
CGT Gains for 4 Modes

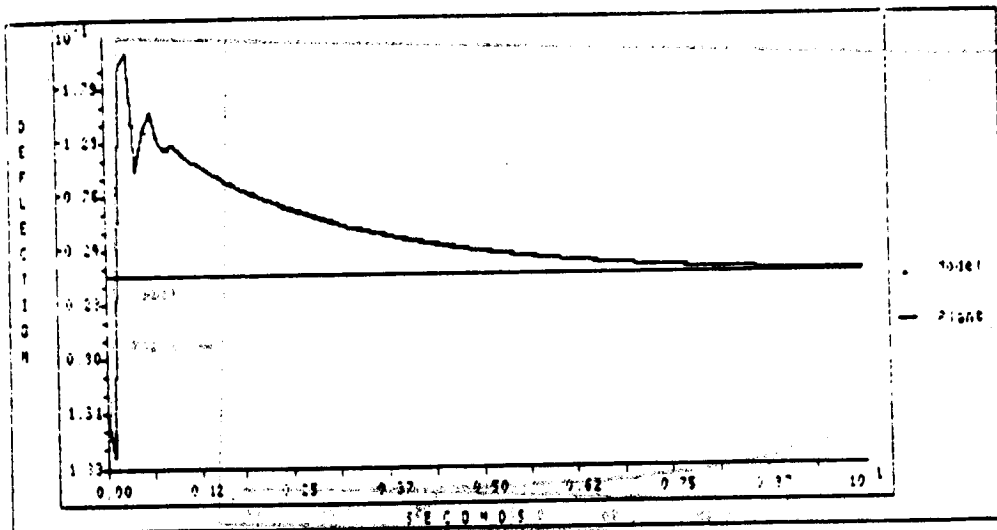


Figure 5.16 Case II : Tracking -
CGT Gains for 16 Modes

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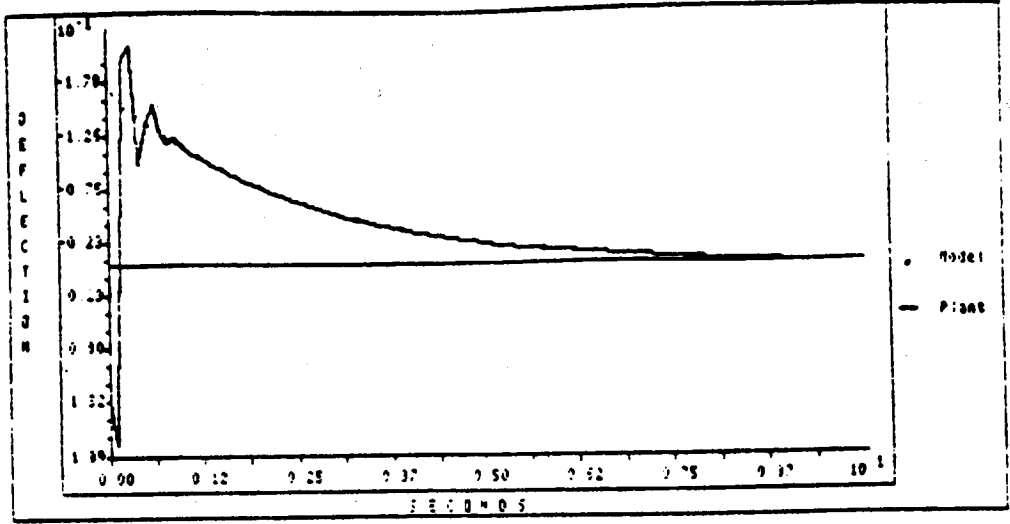


Figure 5.17 Case II : Tracking -
CGT Gains for Ave Modes

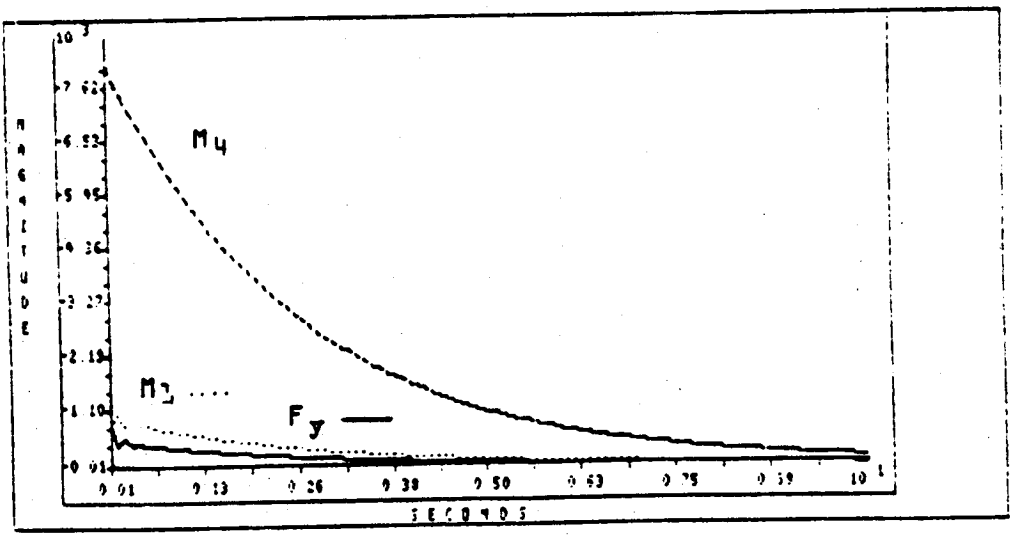


Figure 5.18 Case I : Controls -
CGT Gains for b Modes

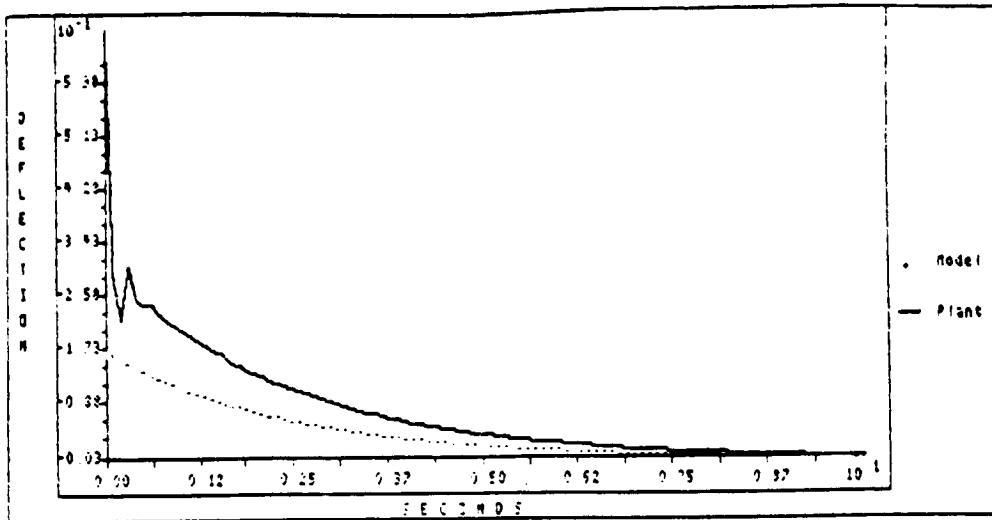


Figure 5.19 Case I : Parameter Variation -
CGT Gains for b Modes

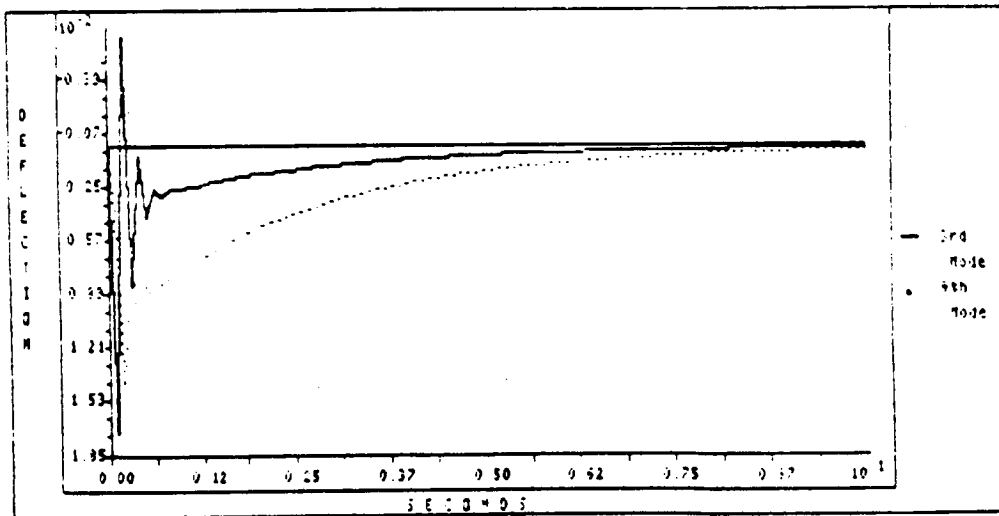


Figure 5.20 Case I : Control Spillover -
Beam Deflection for 3rd and 4th Modes,
CGT Gains for b Modes

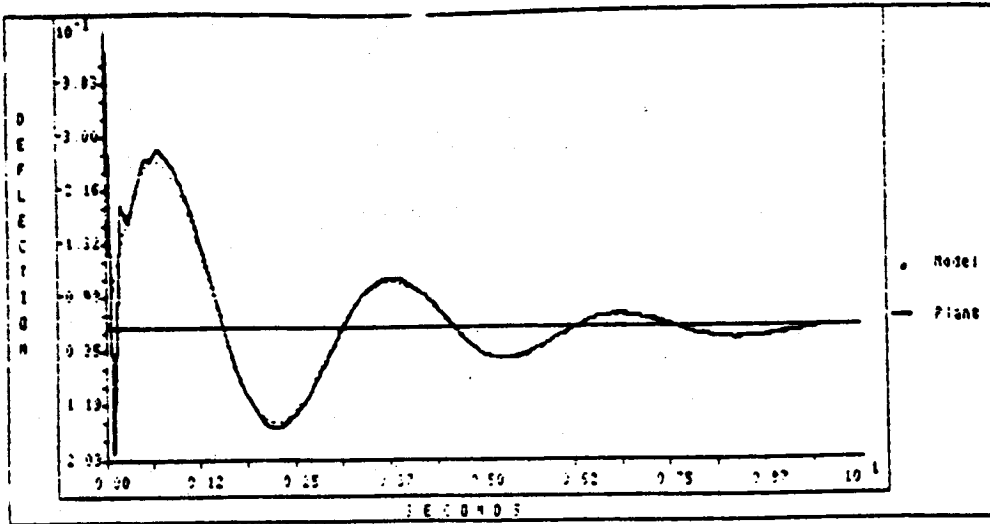


Figure 5.21 Case I : Tracking -
Reference Model $2.04 e^{-.4t} \sin (2.04t)$,
CGT Gains for b Modes

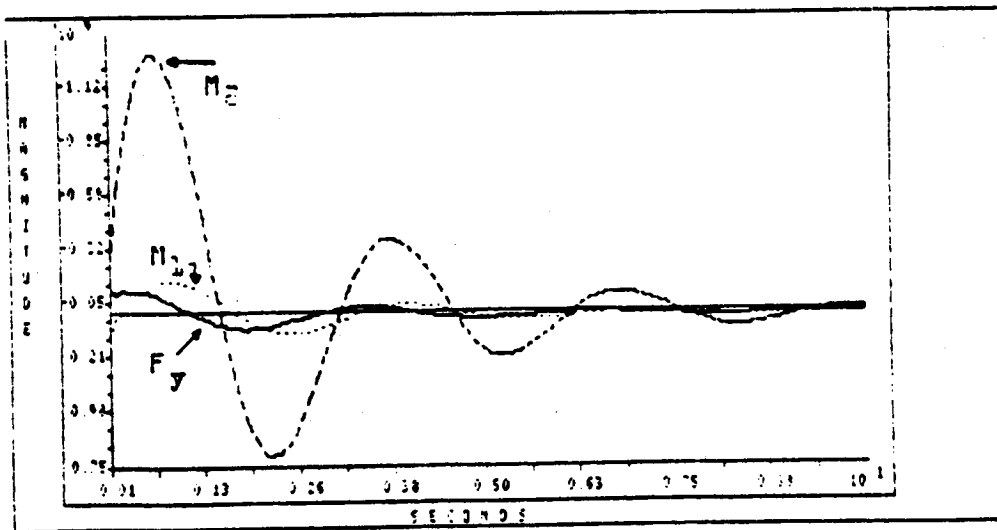


Figure 5.22 Case I : Controls -
Reference Model $2.04 e^{-.4t} \sin (2.04t)$,
CGT Gains for b Modes

CONCLUSIONS

1. A MODEL FOLLOWING PROCEDURE USING CGT THEORY WAS DEVELOPED FOR APPLICATION TO DPS SYSTEMS.
2. THE DESIGN PROCEDURE RESULTS IN A FINITE-DIMENSIONAL CONTROLLER THAT GIVES OUTPUT FOLLOWING AND FULL STATE STABILITY.
3. THE MODEL FOLLOWING CONTROLLER'S APPLICATION TO A MODELS OF THE SCOPE WAS SHOWN.
4. THROUGH SIMULATIONS, IT WAS DEMONSTRATED THAT SATISFACTORY TRACKING OF A DESIRED TRAJECTORY CAN BE ACHIEVED.
5. EXCITATION OF HIGHER ORDER MODES BY THE CONTROLLER AND PARAMETER VARIATIONS DO NOT ADVERSELY AFFECT THE SYSTEM PERFORMANCE.

PLANNED ACTIVITIES

- 0 DEVELOPMENT OF OUTPUT FEEDBACK FOR LUMPED MODEL CONTROLLER
- 0 DEVELOPMENT OF CONTROLS FOR THE PITCH AND YAW TORSION EQUATION
- 0 TESTING OF THE CONTROLLERS, IN THE PRESENCE OF NOISY SENSORS.
- 0 ADAPTIVE CONTROL