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ACTIVE STABILITY AUGMENTATION OF LARGE SPACE STRUCTURES: A STOCHASTIC CONTROL PROBLEM

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1. Introduction

In the 1987-1990 period NASA has planned several ground and flight experiments with the eventual objective of deploying large flexible structures in space. A currently active precursor is the SCOLE experiment [1]. Here the problem is that of slewing an offset antenna on a long (130 ft.) flexible beam-like truss attached to the space shuttle, with rather stringent pointing accuracy requirements (±.02 degrees). This paper examines the relevant methodology aspects in robust feedback-control design for stability augmentation of the beam using on-board sensors. We frame it as a stochastic control problem ~ boundary control of a distributed parameter system described by partial differential equations. While the framework is mathematical, the emphasis is still on an engineering solution.

The fact that the deployment is in space makes model uncertainty the major consideration in control design. Particularly serious in this regard is for instance the modelling of inherent damping in the system long known to be difficult [2], and a still unresolved problem even in theory. Hence robustness becomes a must feature, even at the expense of optimality. Another aspect is the complexity of computation, making any simulation study a costly undertaking.

The overall model involving <u>both</u> slewing and beam stabilization is still not well understood. Hence the two problems -- of slewing and stabilization -- are best studied, at least in initial efforts such as reported here, separately. We attempt stabilization at the termination of the slewing so that in particular the system is essentially linear except for a small nonlinear term contributed by the kinematic nonlinearity. It should be noted that at present we do not have a stochastic time-optimal control theory adequate for optimal slewing based on sensor data.

An abstract mathematical formulation is developed in Section 2 as a nonlinear wave equation in a Hilbert space. We show that the system is controllable and develop a feedback control law that is robust in the sense that it does not require quantitative knowledge of system parameters. The stochastic control problem that arises in instrumenting this law using appropriate sensors is treated in Section 3. Using an Engineering first approximation which is valid for "small" damping, formulas for optimal choice of the control gain are developed.

2. Abstract Formulation

We are concerned with the mast stabilization problem only and the model we use assumes that the angular velocity of the shuttle-antenna system is small enough to be neglected. We model the mast as a thin prismatic beam. There is then the question of whether a finite-element model or a continuum (involving partial differential equations) model should be used. Here we deal only with the latter, the basic governing equations being beam bending and torsion equations with controls at the boundaries.

With reference to Figure 1, the beam of length L is along the Z axis, z being zero at the shuttle end. $u_\phi(\cdot)$, $u_\theta(\cdot)$ will denote the displacements along the Y-Z, X-Z planes and $u_\psi(\cdot)$ the angular deflection about the Z axis. In addition proof-mass controllers are provided at points s_1 and s_2, on the beam, the locations to be chosen optimally. Control moments are applied at both ends as well as control forces at the reflector center. The various moments of inertia and masses are specified in [1], [2].

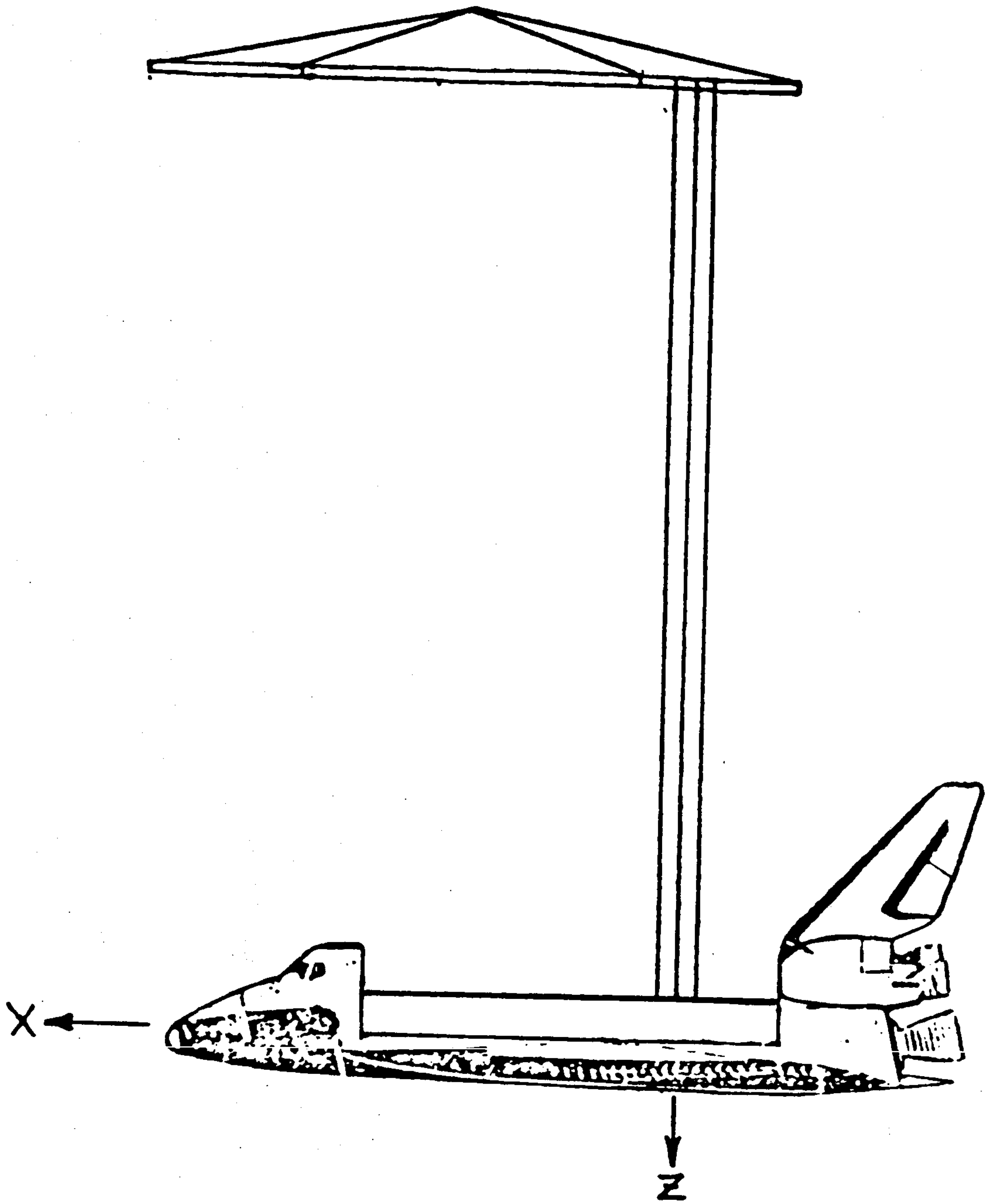

FIGURE 1

SHUTTLE/ANTENNA CONFIGURATION

We first develop an abstract mathematical model. We define

$$H = L_2[0,L]^3 \times R^{14} \qquad 0 < L < \infty$$

with the usual inner-product thereon denoted [,]. We fix the points $0 < s_2 < s_3 < L$ and define a linear operator A into H with domain D in H defined as follows. We use $u_\phi(\cdot)$, $u_\theta(\cdot)$, $u_\psi(\cdot)$ to denote the functions in $L_2[0,L]^3$. Thus an element x in H is denoted

$$\begin{matrix} u_\phi(\cdot) \\ u_\theta(\cdot) \\ u_\psi(\cdot) \\ x_4 \\ \vdots \\ x_{17} \end{matrix}$$

The domain D consists of elements x such that u'_ϕ, u''_θ, $u'''_\psi \in L_2[0,L]$ and $u'''_\phi(\cdot)$ has L_2-derivatives in $[0,s_2]$, $[s_2,s_3]$ and $[s_3,L]$; similarly for $u_\theta(\cdot)$; $u_\psi(\cdot)$ such that $u'_\psi(\cdot)$ and $u''_\psi(\cdot) \in L_2[0,L]$; the remaining components of x are specified as

$$\begin{aligned} x_4 &= u_\phi(0+) & x_{11} &= u'_\phi(L-) \\ x_5 &= u_\theta(0+) & x_{12} &= u'_\theta(L-) \\ x_6 &= u_\phi(L-) & x_{13} &= u_\psi(L-) \\ x_7 &= u_\theta(L-) & x_{14} &= u_\phi(s_2) \\ x_8 &= u'_\phi(0+) & x_{15} &= u_\theta(s_2) \\ x_9 &= u'_\theta(0+) & x_{16} &= u_\phi(s_3) \\ x_{10} &= u_\psi(0+) & x_{17} &= u_\theta(s_3) \end{aligned}$$

Thus at least for x in D, we may identify the finite-dimensional part as the "boundary." The operator A is then defined by

$$y = Ax$$

where the functional part (in $L_2[0,L]^3$) is given by

$$\begin{array}{c} EI_\phi u_\phi''''(\cdot) \\ EI_\theta u_\theta''''(\cdot) \\ -GI_\psi u_\psi''(\cdot) \end{array}$$

and the boundary part by:

$$\begin{aligned} y_4 &= EI_\phi u_\phi'''(0+) & y_{11} &= EI_\phi u_\phi''(L-) \\ y_5 &= EI_\theta u_\theta'''(0+) & y_{12} &= EI_\theta u_\theta''(L-) \\ y_6 &= -EI_\phi u_\phi'''(L-) & y_{13} &= GI_\psi u_\psi'(L-) \\ y_7 &= -EI_\theta u_\theta'''(L-) & y_{14} &= EI_\phi(u_\phi'''(s_2+) - u_\phi'''(s_2-)) \\ y_8 &= -EI_\phi u_\phi''(0+) & y_{15} &= EI_\theta(u_\theta'''(s_2+) - u_\theta'''(s_2-)) \\ y_9 &= -EI_\theta u_\theta''(0+) & y_{16} &= EI_\phi(u_\phi'''(s_3+) - u_\phi'''(s_3-)) \\ y_{10} &= -GI_\psi u_\psi'(0+) & y_{17} &= EI_\theta(u_\theta'''(s_3+) - u_\theta'''(s_3-)) \end{aligned}$$

It may then be verified that D is dense and A is self-adjoint and nonnegative definite. Moreover A has a compact resolvent with a complete orthonormal set of eigenfunctions (modes). Zero is an eigenvalue.

The control system dynamics can then be characterized as a nonlinear wave-equation:

$$M\ddot{x}(t) + Ax(t) + K(\dot{x}(t)) + Bu(t) = 0 \tag{2.1}$$

where M is a 17×17 nonsingular nonnegative definite matrix, and defines self-adjoint positive definite linear operator H onto H. The control $u(\cdot)$ is in R^{12}, and

$$Bu = x$$

$$x = \text{col. } [0, 0, 0, 0, 0, u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}, u_{11}, u_{12}]$$

We have thus only "boundary" control. The nonlinearity is kinematic:

$$K(x) = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \Omega_1 \circledast I_1 \Omega_1 \\ \Omega_4 \circledast I_4 \Omega_4 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

where

$$\Omega_1 = \text{col } (x_8, x_9, x_{10})$$

$$\Omega_4 = \text{col } (x_{11}, x_{12}, x_{13}) \quad .$$

I_1, I_4 are symmetric positive definite (moment) matrices and $\circledast$ denotes vector cross-product.

Two relevant properties of the function $K(\cdot)$ are:

(i) $[K(x), x] = 0$

(ii) $\|K(x)\| \leq \text{const. } \|x\|^2$

We do allow for "state noise" and let

$$N(t) = \begin{vmatrix} N_1(t) \\ N_2(t) \\ N_3(t) \end{vmatrix}$$

$$FN(t) = x(t)$$

where $N(t)$ is white Gaussian with spectral density matrix Λ, and the components of $x(t)$ are defined by

$$x_i(t) = 0 \qquad i = 1,\dots,7$$

$$x_8(t) = N_1(t)$$

$$x_9(t) = N_2(t)$$

$$x_{10}(t) = N_3(t)$$

$$x_i(t) = 0 \qquad i > 10 .$$

Note that the "boundary" values are part of the state.

State-space Form

With

$$Y(t) = \begin{vmatrix} x(t) \\ \dot{x}(t) \end{vmatrix}$$

we go over to the state-space form:

$$\dot{Y}(t) = \mathcal{A}Y(t) + \mathcal{K}(Y(t)) + \mathcal{B}u(t) + \mathcal{F}(N(t)) \qquad (2.2)$$

where

$$\mathcal{A} = \begin{bmatrix} 0 & I \\ -M^{-1}A & 0 \end{bmatrix}$$

$$\mathcal{B}u(t) = \begin{vmatrix} 0 \\ -M^{-1}Bu(t) \end{vmatrix}$$

and in the notation

$$Y = \begin{vmatrix} y_1 \\ y_2 \end{vmatrix}, \qquad Y \in H \times H$$

we have

$$\mathcal{K}(Y) = \begin{vmatrix} 0 \\ -M^{-1}K(y_2) \end{vmatrix}$$

$$\mathcal{F}N(t) = \begin{vmatrix} 0 \\ -M^{-1}FN(t) \end{vmatrix} .$$

As is well known, we can introduce a new inner product, the "energy" inner product

$$[Y,Z]_E = \left[\sqrt{A}\, y_1, \sqrt{A}\, z_1\right] + [My_2, z_2]$$

on $R(A) \times H$. $R(A)$ is the orthogonal complement of the null space of A. We denote the completed space by H_E. We shall from now on consider only H_E. We have:

$$\mathcal{A} + \mathcal{A}^* = 0$$

and of course $\mathcal{A}$ has a compact resolvent and we have an orthogonal decomposition of H_E given by

$$Y = \sum_{1}^{\infty} P_k Y \tag{2.3}$$

where P_k is a two-dimensional projection for each k, $P_k H_E$ spanned by

$$\phi_j^+ = \begin{vmatrix} \phi_k \\ i\omega_k \phi_k \end{vmatrix}$$

$$\phi_k^- = \begin{vmatrix} \phi_k \\ -i\omega_k \phi_k \end{vmatrix}$$

where

$$A\phi_k = \omega_k^2 M\phi_k , \qquad \omega_k^2 > 0, \quad \omega_k^2 \to \infty; \qquad (2.4)$$

$$[M\phi_k, \phi_j] = \delta_j^k .$$

Let $S(t)$ denote the semigroup generated by A. Then we have the representation:

$$S(t)Y = \sum_1^\infty S(t) P_k Y$$

$$P_k S(t) P_k = S(t) P_k .$$

More explicitly, if

$$S(t)Y = \begin{vmatrix} y_1(t) \\ y_2(t) \end{vmatrix} .$$

Then

$$y_2(t) = \dot{y}_1(t)$$

and

$$y_1(t) = \sum_1^\infty [y_1, M\phi_k] \phi_k \cos \omega_k t + \sum_1^\infty [y_2, M\phi_k] \phi_k \frac{\sin \omega_k t}{\omega_k} \qquad (2.5)$$

Note that it is required that y_1 satisfy:

$$\sum_1^\infty [y_1, M\phi_k]^2 \omega_k^2 < \infty .$$

It is easy to establish existence and uniqueness of solution for the integral version of (2.2):

$$Y(t) = S(t)Y(0) + \int_0^t S(t-\sigma) Bu(\sigma)\, d\sigma + \int_0^t S(t-\sigma) FN(\sigma)\, d\sigma$$

$$+ \int_0^t S(t-\sigma) K(Y(\sigma))\, d\sigma , \qquad (2.6)$$

without invoking any nonlinear semigroup theory, by just Picard iteration. See [3].

We can now state the basic result that yields a robust feedback-control law for the deterministic system (seeing $F = 0$).

Theorem 2.1.

Let P be any 12×12 symmetric nonnegative definite nonsingular matrix. Then the feedback control

$$u(t) = -P\,\mathcal{B}^* Y(t) \tag{2.7}$$

is such that the "closed-loop" system

$$\dot{Y}(t) = \mathcal{A}Y(t) - \mathcal{B}P\mathcal{B}^*Y(t) + \mathcal{K}(Y(t)) \tag{2.8}$$

is globally asymptotically stable. That is to say

$$\|Y(t)\|_E \to 0 \qquad \text{as} \quad t \to \infty$$

Proof. We refer to [4] for a proof. The proof exploits the fact that $(\mathcal{A},\mathcal{B})$ is controllable in an essential way. In particular the semigroup $S_B(t)$ generated by $(\mathcal{A} - \mathcal{B}P\mathcal{B}^*)$ is strongly stable: that is to say:

$$\|S_B(t)Y\|_E \to 0 \qquad \text{as} \quad t \to \infty .$$

We also obtain that

$$\int_0^\infty \left(P\mathcal{B}^*S_B(t)Y,\ \mathcal{B}^*S_B(t)Y\right)\,dt = \frac{1}{2}\,\|Y\|_E^2 \quad . \tag{2.9}$$

The control law is also optimal for the quadratic cost functional:

$$\int_0^\infty \|\sqrt{P}\ \mathcal{B}^*Y(t)\|^2\,dt + \int_0^\infty \|u(t)\|^2\,dt$$

for the linear system

$$\dot{Y}(t) = \mathcal{A}Y(t) + \mathcal{B}\sqrt{P}\ u(t)$$

3. Stochastic Control

To instrument the control law

$$u(t) = PB^*\dot{x}(t)$$
$$= P\dot{b}(t) \quad . \tag{3.1}$$

We need to assume co-located (rate) sensors. The sensor output $v(t)$ would then be:

$$v(t) = \dot{b}(t) + N_0(t) \tag{3.2}$$

where $N_0(t)$ represents the sensor noise, modelled as white Gaussian with (12×12) spectral density matrix D. In terms of the state-space representation (2.2), we can rewrite (3.2) as

$$v(t) = CY(t) + N_0(t) \tag{3.2}$$

where

$$C = B^*$$

and C is of course finite-dimensional. If we assume that the separation principle applies, a reasonable choice of control law would be

$$u(t) = P\hat{\dot{b}}(t) \tag{3.3}$$

where, E denoting conditional expectation:

$$\hat{\dot{b}}(t) = E[\dot{b}(t) \mid v(s), \quad s \leq t]$$

and of course

$$\hat{\dot{b}}(t) = C\hat{Y}(t)$$

where $\hat{Y}(t)$ is the Kalman state estimate:

$$\hat{Y}(t) = E[Y(t) \mid v(s), s \leq t] \; .$$

Even if we were to neglect the nonlinear term $\mathcal{K}(\cdot)$, this would require an infinite-dimensional Kalman filter, which even if we could instrument it, would depend on quantitative knowledge of the system parameters. Hence this filter would need to be simplified in considerable measure, in favor of robustness.

The simplest version would be one that did not distort $\dot{b}(t)$ and thus would lead to the control law:

$$u(t) = Pv(t) . \tag{3.4}$$

We are thus introducing a noise input into the system which may excite higher-order modes. Let us therefore study the system response which is now given by the stochastic equation:

$$\dot{Y}(t) = (\mathcal{A} - \mathcal{B}P\mathcal{B}^*)Y(t) - \mathcal{B}PN_0(t) + \mathcal{K}(Y(t)) + \mathcal{F}N(t) \tag{3.5}$$

This can be expressed as an integral equation:

$$Y(t) = Y_o(t) + \int_0^t S_B(t-\sigma)\ \mathcal{K}(Y(\sigma))\ d\sigma \tag{3.6}$$

where

$$Y_o(t) = S_B(t)Y(0) - \int_0^t S_B(t-\sigma)\mathcal{B}PN_0(\sigma)d\sigma + \int_0^t S_B(t-\sigma)\mathcal{F}N(\sigma)\ d\sigma \tag{3.7}$$

We note that because $\mathcal{K}(\cdot)$ is locally Lipschitzian, we may solve (3.6) by Picard iteration:

$$Y_{n+1} = Y_o(t) + \int_0^t S_B(t-\sigma)\mathcal{K}(Y_n(\sigma))\ d\sigma . \tag{3.8}$$

We omit the details; see [3]. More important to us is actually (3.7). We want to show that the process $Y_o(\cdot)$ is asymptotically stationary and evaluate its covariance function. Following [5], since $S_B(\cdot)$ is strongly stable, it is only necessary to show that for Y in H_E:

$$\int_0^\infty [S_B(\sigma)\mathcal{B}PDP\mathcal{B}^*S_B(\sigma)^*Y,\ Y]_E\ d\sigma\ <\ \infty \tag{3.9}$$

and also that

$$\int_0^\infty [S_B(\sigma)F\Lambda F^*S_B(\sigma)^*Y,\ Y]_E\ d\sigma\ <\ \infty\ . \tag{3.10}$$

For this purpose we note that $S_B(t)^*$ is strongly stable with generator

$$A^* - \mathcal{B}P\mathcal{B}^*$$

and analogous to (2.9) we have that

$$\int_0^\infty \|\sqrt{P}\,\mathcal{B}^*S_B(t)^*Y\|^2\ dt\ =\ \frac{1}{2}\ \|Y\|_E^2\ .$$

Hence

$$\int_0^\infty \|\sqrt{D}\,P\mathcal{B}^*S_B(t)^*Y\|^2\ dt\ \leq\ \frac{\|D\|}{2}\ \|P\|\ \|Y\|_E^2\ <\ \infty\ .$$

Since

$$\|F^*S_B(t)^*Y\|\ \leq\ \|\mathcal{B}^*S_B(t)^*Y\|$$

we also obtain (3.10). For Y, Z in H_E let

$$[R(t,s)Y,\ Z]\ =\ E([Y_0(t),\ Y][Y_0(s),Z])\ .$$

Then we have that

$$R(t,s)\ =\ S_B(t-s)\ R(s,s)\ , \qquad t \geq s$$

and hence it follows that

$$\lim_{L\to\infty} R(t+L,\ s+L)\ =\ S_B(t-s)\ R_\infty\ , \qquad t \geq s \tag{3.11}$$

where

$$[R_\infty Y,\ Y]\ =\ \int_0^\infty \|\sqrt{D}\,P\mathcal{B}^*S_B(t)^*Y\|^2\ dt\ +\ \int_0^\infty \|\sqrt{\Lambda}\,F^*S_B(t)^*Y\|^2\ dt\ . \tag{3.12}$$

The process $Y_o(\cdot)$ is thus asymptotically stationary with covariance operator

$$S_B(t-s)R_\infty \qquad t \geq s$$

We note that R_∞ is not necessarily nuclear, even though $R(t,t)$ will be if $R(0,0)$ is. Indeed taking

$$D = dI \; ; \qquad P = I$$

we obtain that

$$\int_0^\infty \|\sqrt{D}\, P\mathcal{B}^* S_B(t)^* Y\|^2 \, dt = \frac{d}{2} \|Y\|_E^2 \,.$$

From (3.8) we can show that the process $Y(t)$ is asymptotically stationary, since $Y_n(\cdot)$ will have this property for each n. Since it would appear that the nonlinearity is small, we shall now concentrate our attention on the linear approximation $Y_o(\cdot)$.

The eigenfunctions of $(\mathcal{A} - \mathcal{B}P\mathcal{B}^*)$ are approximately the same as that of $\mathcal{A}$ and the eigenvalues are

$$\sigma_k \pm i\omega_k \; ; \qquad \frac{\sigma_k}{\omega_k} \ll 1 \,.$$

where

$$2\sigma_k = [Pb_k, b_k] \,. \tag{3.13}$$

Hence

$$[R_\infty \phi_k^+, \phi_k^+]_E = \omega_k^2 \frac{([DPb_k, Pb_k] + [F^*\phi_k, F^*\phi_k])}{[Pb_k, b_k]} \tag{3.14}$$

which is thus the noise energy in the kth mode. We see that increasing P increases the damping but also increases the noise excitation. In practice one would want a compromise between increasing damping at selected low order modes but keeping the noise excitation at higher order modes within bound. Clearly further work is needed before any attempt at control design.

We may also mention one point of purely theoretical interest. To characterize the distributions of the noise response of a nonlinear system described by ordinary differential equations one uses the Fokker-Planck-Kolmogorov equations which are partial differential equations. In (3.5) we have a nonlinear partial differential equation; it would be of interest to develop a corresponding tool to study the distributions.

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