NASA Contractor Report 178224

ICASE REPORT NO. 86-79

ICASE

INHERENT ROBUSTNESS OF DISCRETE-TIME ADAPTIVE CONTROL SYSTEMS

(NASA-CR-178224) INHERENT ROBUSTNESS OF DISCRETE-TIME ADAPTIVE CONTROL SYSTEMS Final Report (NASA) 39 p CSCL 01C

N87-18571

Unclas

G3/08 43631

C. C. H. Ma

Contract No. NAS1-18107

December 1986

INSTITUTE FOR COMPUTER APPLICATIONS IN SCIENCE AND ENGINEERING
NASA Langley Research Center, Hampton, Virginia 23665

Operated by the Universities Space Research Association

NASA
National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

INHERENT ROBUSTNESS OF DISCRETE-TIME ADAPTIVE CONTROL SYSTEMS

C. C. H. Ma
Institute for Computer Applications in Science and Engineering

ABSTRACT

Global stability robustness with respect to unmodeled dynamics, arbitrary bounded internal noise, as well as external disturbance is shown to exist for a class of discrete-time adaptive control systems when the regressor vectors of these systems are persistently exciting. Although fast adaptation is definitely undesirable, so far as attaining the greatest amount of global stability robustness is concerned, slow adaptation is shown to be not necessarily beneficial. The entire analysis in this paper holds for systems with slowly varying return difference matrices; the plants in these systems need not be slowly varying.

This research was supported under the National Aeronautics and Space Administration under NASA Contract No. NAS1-18107 while the author was in residence at ICASE, NASA Langley Research Center, Hampton, VA 23665-5225.

1. INTRODUCTION

Persistency of excitation (PE) has been claimed to be essential for guaranteeing some robustness in adaptive control (AC) systems ever since it was proven to ensure robust exponentially fast identification [1,2]. But, to date, no formal proof exists that PE guarantees global stability robustness for discrete time systems in general. In [3] it is shown that if the reference input is PE and the signal-to-disturbance ratio is large, then model reference adaptive control (MRAC) system signals will be globally bounded. In [4] continuous time model reference adaptive control systems are shown to be globally stable with unmodeled dynamics if the reference model is strictly positive real and the regressor vector is persistently exciting. Attempting to extend the same to discrete time MRAC systems failed due to the fact that discrete time MRAC systems do not exactly parallel continuous time MRAC systems [5, Sec. 5.3]. As a result, only local stability is proved and slow adaptation has to be assumed. No direct and explicit proof has yet been shown that persistency of excitation guarantees some global stability robustness against nonzero unmodeled dynamics and bounded internal as well as external (with respect to the plant) noises for an entire class of discrete time AC systems.

Perhaps fueled by the above bold claim together with a lack of any formal proof that PE can guarantee robustness in general, many researchers resorted to showing examples of problems of adaptive control systems and developing ad hoc remedies for these problems [6,7,8]. It is well known in (nonadaptive) robust controls research that a controller design based on the nominal plant properties has only limited robustness against unmodeled dynamics in the plant [9]. Showing robustness problems of AC systems demonstrates that the limit extends to AC systems as well, and one should not expect <u>any</u> AC system to perform satisfactorily under <u>all</u> unpredictable situations. The remedies in [6,7,8] have value in solving the problems specifically exhibited provided their required prior informations (e.g., solution starts within certain bounded set [6], etc.) are verified. However, if an AC system does possess some inherent global stability robustness, due to say PE, whether further incorporating these ad hoc remedies would increase or decrease such basic robustness is a question that remains to be answered.

The contribution of this paper is in proving directly and explicitly that when the regressor vector of a discrete time AC system of a particular class (maybe with a fast time-varying plant, as long as the system <u>return-difference matrix</u> [9] is uniformly bounded and sufficiently slowly varying) is PE, the system global stability is inherently robust against nonzero unmodeled dynamics expressed in terms of stable factor perturbations [9] as well as <u>any</u> bounded noise internal or external to the plant. It is also shown that if the adaptation gain approaches zero (slow adaptation) the margin of such robustness approaches a constant. However, this does not imply that slow adaptation necessarily improves global stability robustness. This is justified by analyzing a special case when the regressor vector is a scalar. The conclusion obtained based on this analysis is that, while increasing the adaptation rate indefinitely reduces the robustness for sure, the maximum robustness of the AC system may be achieved at a finite adaptation rate.

Thus, if the popularity of using the averaging technique in AC system stability analysis and the abundance of results obtained based on slow adaptation, e.g., [5,10], in any way imply a strictly non-negative relationship between slow adaptation and global stability robustness, this result shows the contrary.

This paper is organized as follows. In the following section, some notations and useful facts on robust control and linear time-varying systems are first listed for ease of referencing later. In Section 3, the class of adaptive control systems under consideration is developed to the extent relevant to the derivation of the results of this paper. A general error model is then developed for this class of AC systems in Section 4, followed by the inherent global stability robustness proofs assuming persistency of excitation of the regressor vector. In Section 6, the effects of the adaptation gain on the inherent robustness are analyzed. Finally the results are summarized and the paper is concluded with a discussion.

2. PRELIMINARIES

Let z denote the transfer function of the unit-delay operator. Let $R[z]$ denote the ring of polynomials in z with real coefficients and $R(z)$ denote the field of rational functions assoicated with $R[z]$. Let $Z \varepsilon R(z)$ denote the set of causal stable rational functions of z, i.e., functions with poles strictly outside the closed unit disk. Let $M(Z)$ denote the set of matrices whose elements belong to Z.

The plants considered in this paper are lumped linear causal discrete-time MIMO systems whose transfer matrices (or sequences of transfer matrices if systems are time-varying) belong to $M(R(z))$, where M denotes matrices.

Definition 2.1: Let $U(Z)$ denote the set of unimodular matrices in $M(Z)$. It is defined by

$$U(Z) = \{F \in M(Z): \ F^{-1} \in M(Z)\}.$$

Fact 2.2: $M(R[z]) \subset M(Z)$.

Fact 2.3: Let $E, F \in M(R[z])$. If E, F are left or right coprime in $M(R[z])$, then they are also left or right coprime in $M(Z)$ respectively.

Let a plant $P \in M(R(z))$ and a controller $C \in M(R(z))$ be connected as shown in Figure 1.

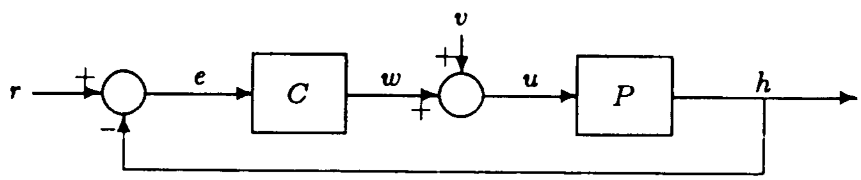

Figure 1. Standard feedback control system

Let $(N,D) \in M(Z)$, $ND^{-1} = P$, be a right coprime (in $M(Z)$) factorization (r.c.f.) of P and $(X,Y) \in M(Z)$, $Y^{-1}X = C$, be a left coprime (in $M(Z)$) factorization (l.c.f.) of C [9].

Fact 2.4: [9] C stabilizes P in the bounded-input bounded-output (BIBO) sense if and only if

$$XN + YD \in U(Z).$$

XN + YD is referred to as the return-difference matrix of the system in Figure 1. BIBO stability is often referred to as internal stability also.

<u>Fact 2.5</u>: The poles of the system in Figure 1 are the zeros of XN + YD.

Let $E \in M(Z)$ be expressed as the infinite sum

$$E(z) = \sum_{i=0}^{\infty} E_i z^i.$$

<u>Definition 2.6</u>: Define the norm $\|\cdot\|_a : M(Z) \to R$ by

$$\|E\|_a = \sum_{j=0}^{\infty} \|E_j\|_{i\infty}$$

where $\|\cdot\|_{i\infty}$ denotes the induced infinity norm [11].

<u>Fact 2.7</u>: Let $E \in M(Z)$. Let $\gamma_\infty\{E\}$ denote the ℓ_∞-gain of E [21, Section 3.2]. Then, for the systems under consideration,

$$\gamma_\infty\{E\} - \|E\|_a.$$

<u>Fact 2.8</u>: [12] Let $U \in U(Z)$ and $V \in M(Z)$. If $\|V - U\|_a < 1/\|U^{-1}\|_a$ then $V \in U(Z)$.

<u>Fact 2.9</u>: [25,19] Let $E, F, G \in M(R[z])$. If E, F are right coprime in $M(R[z])$, then there exists $X, Y \in M(R[z])$ such that

$$XE + YF = G.$$

3. THE ADAPTIVE SYSTEM

The class of AC systems considered in this paper consists of MIMO AC systems with direct signal-feedback connections (as opposed to only parameter-feedback via the parameter adjustment process) from the outputs through the controllers to the inputs of the plants. The primary objective of adaptation for these systems is to satisfy asymptotically the necessary and sufficient condition for internal stability stated in Fact 2.4. An example of an AC system with only parameter-feedback (no explicit signal-feedback) appears in [13]. A general block diagram of this class of AC systems is shown in terms of an r.c.f. (N,D) of the strictly causal plant P and an l.c.f. (X,Y) of the controller C in Figure 2, where d denotes disturbances

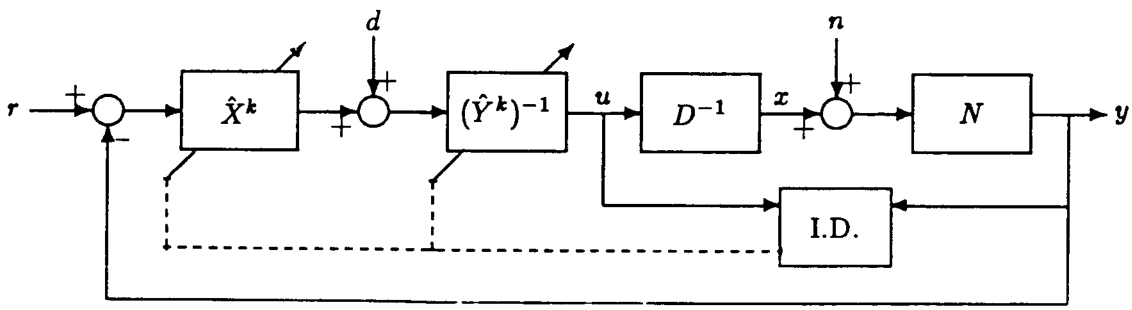

Figure 2. General AC system

external to the plant and n noises internal to the plant. This class of systems includes those studied in [14,15,16,17,12]. A general AC methodology for achieving the asymptotic internal stability objective can be developed roughly as follows. Complete details can be found in [14] for a system in the less general observer-controller structure and in the unit-advance operator representation, and in [16] for a system in exactly the same structure as that in Figure 2 and in the unit-delay operator (z) representation.

Ignoring n, d, the identifier, the time index k and the estimation symbol $\hat{}$, consider the feedback system in Figure 2. Suppose $P \in M(R(z))$ is not precisely known and the objective is to find a $C \in M(R(z))$ such that the feedback system is BIBO stable with poles at the zeroes of

$$U_1 = \text{diag}\{u_i\} \in U(Z),\ u_i \in R[z] \ \forall\ i.$$

Assume $N, D \in M(R[z])$ and are coprime in $M(R[z])$ from now on. By Facts 2.2, 2.3, 2.4, and 2.5, if one can find a pair $(X,Y) \in M(R[z])$ such that

$$XN + YD = U_1, \tag{1}$$

then implementing C by $Y^{-1}X$ will achieve the above stability objective.

To find (X,Y) adaptively through identification, an error function ε needs to be defined. The convergence of this function to zero must result in a solution pair for (1) to be identified. To find such ε, let $x := D^{-1}u$ so that

$$y = ND^{-1}u = Nx,\ u = Dx.$$

Multiplying (1) by x results in

$$Xy + Yu = U_1 x. \tag{2}$$

Since x cannot be measured directly, it must be estimated also. To do this, note that since N and D are right coprime in $M(R[z])$, there exist A_1, $B_1 \varepsilon M(R[z])$ (to be called auxilliary-controller) such that

$$A_1 N + B_1 D = I \tag{3}$$

and, if (1) holds

$$XN + YD = U_1(A_1 N + B_1 D). \tag{4}$$

Now, multiply (4) by x to get

$$Xy + Yu = U_1(A_1 y + B_1 u). \tag{5}$$

Finally, define ε by

$$\varepsilon = U_1(A_1 y + B_1 u) - (Xy + Yu). \tag{6}$$

But, this is not the end to completely defining ε. This is because if we apply a linear estimation algorithm to make $\varepsilon(k)$ converge to zero now, the solution (A_1, B_1) can take a form not necessarily satisfying (3). Fortunately, it can be shown [16] that by imposing degree constraints on X, Y, A_1, B_1, U_1, and restricting the constant term of B_1 to be lower-triangular with 1's in the diagonal, the (A_1, B_1) that solves (6) can be forced to take values such that

$$|A_1N + B_1D| = 1$$

when $\varepsilon(k)$ converges to zero. Therefore, although the to-be-identified solution pair (A_1, B_1) may still not satisfy (3), it will not change the final feedback system poles from the desired values which are the zeros of U_1.

Note that if P were stable to begin with [12], D would be unimodular in U(Z), and (1) can be equated to D instead of U_1, i.e.,

$$XN + YD = D. \tag{7}$$

In this case, multiplying (7) by x yields

$$Xy + Yu = u.$$

Defining ε by

$$\varepsilon = u - (Xy + Yu) \tag{8}$$

a much simpler error function for identifying a controller for the system is obtained. The final poles of such a feedback system would be the same as the poles of the plant.

If instead P were stably invertible with known right interactor matrix $J = z^{-\delta}H$, H a causal polynomial matrix [16], then $JN \varepsilon U(Z)$ and (1) can be equated to JN instead of U_1, i.e.,

$$XN + YD = JN = z^{-\delta}HN. \tag{9}$$

Multiplying (9) by $z^{\delta}x = z^{\delta}D^{-1}u$ yields

$$z^{\delta}\ Xy + z^{\delta}Yu = Hy.$$

Defining ε by

$$\varepsilon = Hy - (Xz^{\delta}y + Yz^{\delta}u) \tag{10}$$

again results in a linear function in terms of the unknowns (X, Y) which can be identified by applying any linear estimation algorithm. If the identified X and Y actually satisfy (9) the feedback system will have all poles at $z = \infty$. If the system is further implemented as shown in Figure 3 instead of

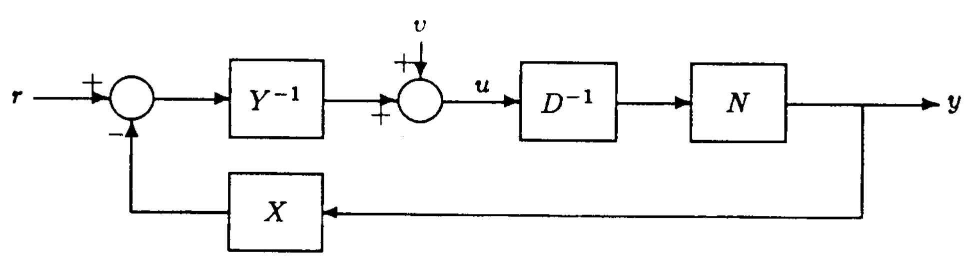

Figure 3. Delayed-tracking system

as in Figure 2, the delayed-tracking scheme of [17] results.

For the rest of this paper, we will consider only the general case when the plant is not necessarily stable or stably invertible and the error function is defined by (6).

To identify the unknowns A_1, B_1, X, and Y using (6), note that since U_1 is diagonal the unknowns can be identified row by row independently. Hence as far as identification is concerned, the MIMO system being considered is no different from a SISO system. For integrity and generality, however, we shall keep the MIMO nature of the system. Let a_i, b_i, x_i, y_i, $i = 1, \cdots, n$ denote the rows of A_1, B_1, X, Y respectively. Let

$$A := \begin{bmatrix} a_1 & & & 0 \\ & a_2 & & \\ & & \ddots & \\ 0 & & & a_n \end{bmatrix}, \quad B := \begin{bmatrix} b_1 & & & 0 \\ & b_2 & & \\ & & \ddots & \\ 0 & & & b_n \end{bmatrix},$$

$$U := \begin{bmatrix} u_1 I \\ u_2 I \\ \vdots \\ u_n I \end{bmatrix}.$$

Then (6) can be rewritten as

$$\begin{aligned} \varepsilon &= [U_1 A_1 \quad U_1 B_1 \quad X \quad Y]\,[y' \quad u' \quad -y' \quad -u']' \\ &= [AU \quad BU \quad X \quad Y]\,[y' \quad u' \quad -y' \quad -u']' \\ &= [A \quad B \quad X \quad Y]\,[(U\,y)' \quad (U\,u)' \quad -y' \quad -u']'. \end{aligned} \tag{11}$$

Let θ_i denote the ordered vector of parameters of $[a_i \quad b_i \quad x_i \quad y_i]$ to be estimated. (Note that usually some of the parameters of $[a_i \quad b_i \quad x_i \quad y_i]$ are assumed known and need not be estimated.) Let $\hat{\theta}_i(k)$ denote the k-th estimate of θ_i and $\phi_i(k)$ denote the k-th ordered regressor vector corresponding to $\hat{\theta}_i(k)$ and made up by the measurements of $-[u_i y' \quad u_i u' \quad -y' \quad -u']'$. Furthermore, let

$$\theta := \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix}, \quad \hat{\theta}(k) := \begin{bmatrix} \hat{\theta}_1(k) \\ \hat{\theta}_2(k) \\ \vdots \\ \hat{\theta}_n(k) \end{bmatrix}, \quad \phi(k) := \begin{bmatrix} \phi_1(k) & & & 0 \\ & \phi_2(k) & & \\ & & \ddots & \\ & & & \phi_3(k) \end{bmatrix},$$

$$\Gamma(k) := \begin{bmatrix} \Gamma_1(k) & & & 0 \\ & \Gamma_2(k) & & \\ & & \ddots & \\ & & & \Gamma_n(k) \end{bmatrix}, \quad \Lambda(k) := \begin{bmatrix} \lambda_1(k) & & & 0 \\ & \lambda_2(k) & & \\ & & \ddots & \\ & & & \lambda_n(k) \end{bmatrix}$$

where $\Gamma_i(k)'s$ are symmetric positive definite gain matrices and $\lambda_i(k)'s$ are positive scalars. The equation error identification algorithm to be used is assumed to be in the form

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \Gamma(k)\,\phi(k)\,(\Lambda(k) + \phi'(k)\,\Gamma(k)\,\phi(k))^{-1}\,\varepsilon(k),$$

$$\varepsilon(k) = \psi(k) - \phi'(k)\,\hat{\theta}(k). \qquad (12)$$

Here $\psi(k)$ denotes the vector of k-th values of signals generated by the <u>not-estimated</u> parameters of [A B X Y] operating on their corresponding signals according to (11). Identification algorithms having precisely this form include at least the recursive least squares and the normalized gradient algorithms [5]. Other algorithms having different forms may also be used.

In case the plant were stably invertible and the simpler error function (10) were used for identifying the controller, the identification algorithm would be assumed to be in the δ-interlaced form

$$\hat{\theta}(k+1) = \hat{\theta}(k-\delta) + \Gamma(k-\delta)\,\phi(k-\delta)(\Lambda(k-\delta) + \phi'(k-\delta)\,\Gamma(k-\delta)\phi(k-\delta))^{-1}\varepsilon(k-\delta)$$

$$\varepsilon(k-\delta) = \psi(k-\delta) - \phi'(k-\delta)\,\hat{\theta}(k-\delta). \tag{13}$$

Note that this is different from the algorithm used in [5, Section 5.3.2] in that $\varepsilon(k-\delta)$ is a function of $\phi'(k-\delta)\,\hat{\theta}(k-\delta)$ instead of $\phi'(k-\delta)\,\hat{\theta}(k)$. This is important because the signal values in $\phi(k-\delta)$ are exactly what the $k-\delta$th estimate $(\hat{x}^{k-\delta}, \hat{y}^{k-\delta})$ of the controller operates on when it is implemented as the controller for one period of time. This is also the key factor in eliminating the slow-adaptation requirement in proving global stability.

(13) is equivalent to

$$\hat{\theta}(k+\zeta) = \hat{\theta}(k) + \Gamma(k)\phi(k)(\Lambda(k) + \phi'(k)\Gamma(k)\phi(k))^{-1}\varepsilon(k), \tag{14}$$

$$\varepsilon(k) = \psi(k) - \phi'(k)\hat{\theta}(k), \quad \zeta \geq 1.$$

Since this is a more general form of identification algorithms than (12), we shall assume that (14) will be used rather than (12) from now on.

The controller C will be assumed to be updated each time (X,Y) is estimated and implemented as $(\hat{Y}^k)^{-1}\,\hat{X}^k$ on line. Hence the implemented AC system will have exactly the structure shown in Figure 2. We are now ready to derive a complete error model for the class of AC systems under consideration.

4. THE COMPLETE ERROR MODEL

Suppose P has been undermodeled so that the pair (N,D) is only a nominal (or tuned [4]) right coprime (in M(R[z])) factorization of P. Correspondingly, (X,Y) and (A,B) are only left coprime (in M(R[z])) factorizations of the nominal controller and nominal auxiliary controller of P respectively. Let $(N_*, D_*) \in M(R[z])$ be a true r.c.f. (coprime in M(R[z])) of P. That is, $N_* D_*^{-1} \equiv P$. Then the stable factor perturbations $(\tilde{N}_*, \tilde{D}_*)$,

$$\tilde{N}_* = N - N_*, \qquad \tilde{D}_* = D - D_*, \tag{15}$$

are obviously due to the unmodeled dynamics in P. The purpose of this paper, as mentioned in the introduction, is to prove that if the adaptive methodology developed in the preceding section for the nominal plant ND^{-1} is applied to the true plant $N_* D_*^{-1}$ as shown in Figure 4, the resulting AC system will stay globally BIBO stable if $\| [\tilde{N}_*', \tilde{D}_*']' \|_a$ is sufficiently small, r, d, n are uniformly bounded, and the regressor matrix $\phi(k)$ is PE. This will be done through error model analysis.

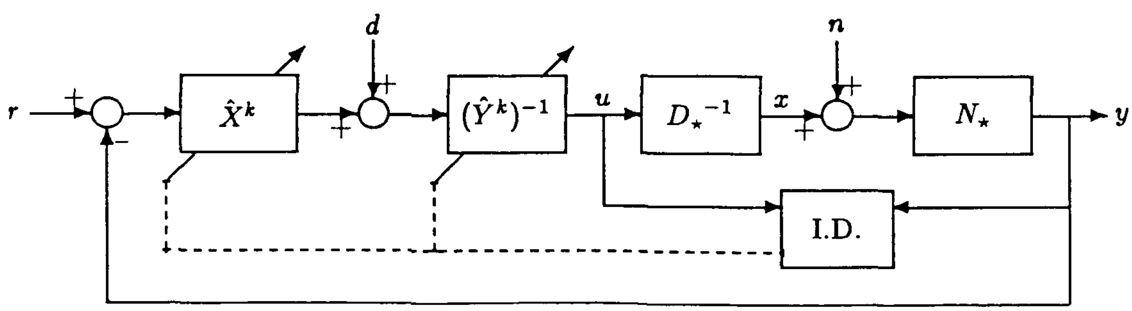

Figure 4. True AC system

To develop an error model for the complete undermodeled AC system, first subtract θ from (14) and let $\tilde{\theta}(k) := \hat{\theta}(k) - \theta$ to yield

$$\tilde{\theta}(k+\zeta) = \tilde{\theta}(k) + \Gamma(k)\phi(k)\left(\Lambda(k) + \phi'(k)\Gamma(k)\phi(k)\right)^{-1}\varepsilon(k),$$
$$\varepsilon(k) = \psi(k) - \phi'(k)\,\hat{\theta}(k). \tag{16}$$

Clearly, $\tilde{\theta}(k)$ must be the ordered vector of coefficients of

$$[\hat{A}^k - A \;\; \hat{B}^k - B \;\; \hat{X}^k - X \;\; \hat{Y}^k - Y] =: [\tilde{A}^k \; \tilde{B}^k \; \tilde{X}^k \; \tilde{Y}^k].$$

corresponding to those of $[\hat{A}^k \; \hat{B}^k \; \hat{X}^k \; \hat{Y}^k]$ estimated. Here it is worth pointing out that coefficients of $[\tilde{A}^k \; \tilde{B}^k \; \tilde{X}^k \; \tilde{Y}^k]$ corresponding to the not-estimated parameters of $[\hat{A}^k \; \hat{B}^k \; \hat{X}^k \; \hat{Y}^k]$ are all zero due to the subtraction. Now note from (14) that

$$\psi(k) - \phi'(k)\,\hat{\theta}(k+\zeta) = \psi(k) - \phi'(k)\hat{\theta}(k)$$
$$- \phi'(k)\Gamma(k)\phi(k)\left(\Lambda(k) + \phi'(k)\Gamma(k)\phi(k)\right)^{-1}\left(\psi(k) - \phi'(k)\hat{\theta}(k)\right)$$
$$= \Lambda(k)\left(\Lambda(k) + \phi'(k)\Gamma(k)\phi(k)\right)^{-1}\left(\psi(k) - \phi'(k)\hat{\theta}(k)\right);$$

therefore

$$\tilde{\theta}(k+\zeta) = \tilde{\theta}(k) - \Gamma(k)\phi(k)\Lambda^{-1}(k)\left(\phi'(k)\hat{\theta}(k+\zeta) - \psi(k)\right) \tag{17}$$

$$\tilde{\theta}(k+\zeta) = -\frac{1}{1 - z^{\zeta}} \Gamma(k)\phi(k)\Lambda^{-1}(k)\left(\phi'(k)\hat{\theta}(k+\zeta) - \psi(k)\right). \tag{18}$$

Let $(A_*, B_*, A_{1*}, B_{1*}, X_*, Y_*) \in M(R[z])$, A_*, B_* in exactly the same block forms as A, B respectively, be a solution of

$$\begin{aligned} X_*N_* + Y_*D_* &= A_*UN_* + B_*UD_* \\ &= U_1A_{1*}N_* + U_1B_{1*}D_* \\ &=: V_* \in U(Z) \end{aligned} \tag{19}$$

closest to (A, B, A_1, B_1, X, Y) in the $\|\cdot\|_a$ norm. Note that a solution of (19) always exists because N_*, D_* are right coprime in $M(R[z])$ (Fact 2.9). By Fact 2.4, this implies that (X_*, Y_*) and (A_{1*}, B_{1*}) are a true controller and a true auxiliary controller of P respectively. Redefine x by $D_*^{-1}u$ then (referring to Figure 4)

$$y = N_*(x + n), \qquad u = D_*x.$$

Multiplying (19) by x yields

$$X_*(y - N_*n) + Y_*u = A_*U(y - N_*n) + B_*Uu = V_*x. \tag{20}$$

Let

$$\tilde{A}_* := A - A_*, \qquad \tilde{B}_* := B - B_*,$$
$$\tilde{X}_* := X - X_*, \qquad \tilde{Y}_* := Y - Y_*;$$

then by the various definitions

$$\psi(k) - \phi'(k)\hat{\theta}(k+\zeta) = \hat{A}^{k+\zeta}Uy(k) + \hat{B}^{k+\zeta}Uu(k) - \hat{X}^{k+\zeta}y(k) - \hat{Y}^{k+\zeta}u(k).$$

$$= (\tilde{A}_* + \tilde{A}^{k+\zeta})\ Uy(k) + (\tilde{B}_* + \tilde{B}^{k+\zeta})Uu(k)$$

$$- (\tilde{x}_* + \tilde{x}^{k+\zeta})y(k) - (\tilde{Y}_* + \tilde{Y}^{k+\zeta})u(k) \tag{21}$$

$$- (X_* - A_*U)N_*n(k).$$

Define V and V_a by

$$V = XN_* + YD_* \in M(Z), \tag{22}$$

$$V_a = AUN_* + BUD_* \in M(Z). \tag{23}$$

<u>Remark 1</u>: Note that if the plant is <u>not</u> undermodeled X, Y, A, and B exist such that $V = V_a$ and the global stability issue can be much more easily addressed. With undermodeling, V and V_a can only be close to each other (in terms of small $\|V - V_a\|_a$) in general if the degree of undermodeling is slight.

Multiplying (22) and (23) by x, one gets

$$Vx = X(y - N_*n) + Yu, \tag{24}$$

$$V_a x = AU(y - N_*n) + BUu. \tag{25}$$

Subtracting (20) from (24) and (25)

$$\tilde{X}_*(y - N_*n) + \tilde{Y}_*u = (V - V_*)x =: \tilde{V}x,$$

$$\tilde{A}_*U(y - N_*n) + \tilde{B}_*Uu = (V_a - V_*)x =: \tilde{V}_a x.$$

Substituting into (21)

$$\psi(k) - \phi'(k)\hat{\theta}(k+\zeta) = \tilde{A}^{k+\zeta}Uy(k) + \tilde{B}^{k+\zeta}Uu(k) + AUN_*n(k) + \tilde{V}_a x(k)$$

$$-\tilde{X}^{k+\zeta}y(k) - \tilde{Y}^{k+\zeta}u(k) - XN_*n(k) - \tilde{V}x(k) \quad (26)$$

$$= -\phi'(k)\tilde{\theta}(k+\zeta) - (\tilde{V} - \tilde{V}_a)x(k) - (X - AU)N_*n(k).$$

Now, from Figure 4 it is obvious that

$$[\hat{X}^k \ \hat{Y}^k] \ [y' \ u']' = [\hat{X}^k \ I] \ [r' \ d']' \ .$$

Therefore,

$$Vx = [X \ Y][y' \ u']' - XN_*n$$

$$= -[\tilde{X}^k \ \tilde{Y}^k][y' \ u']' + \hat{X}^k r + d - XN_*n.$$

Let $\xi(k)$ denote a matrix entirely similar to $\phi(k)$ except made up of the measurements of r and d instead of those of y and u respectively. Let also

$$M_1 := \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & I & \\ & & & I \end{bmatrix} , \qquad T_1 := \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & I & \\ & & & 0 \end{bmatrix} .$$

Then

$$x = V^{-1} \left([\tilde{A}^k \; \tilde{B}^k \; \tilde{X}^k \; \tilde{Y}^k] \left(T_1 \begin{bmatrix} -r \\ -d \\ r \\ d \end{bmatrix} - M_1 \begin{bmatrix} -u_i y \\ -u_i u \\ y \\ u \end{bmatrix} \right) \right.$$

$$\left. + X(r - N_* n) + d \right),$$

$$x(k) = V^{-1}\Big((\xi'(k)T - \phi'(k)M)\tilde{\theta}(k) + X\big(r(k) - N_* n(k)\big) + d(k)\Big) \qquad (27)$$

where T, M are diagonal matrices corresponding to T_1, M_1 and mask out the elements of $\tilde{\theta}(k)$ corresponding to the parameters of $\tilde{A}^k$, $\tilde{B}^k$, $\tilde{Y}^k$ appropriately. Thus, a complete error model is finally developed. It is described by (18), (26), (27) and is shown in Figure 5.

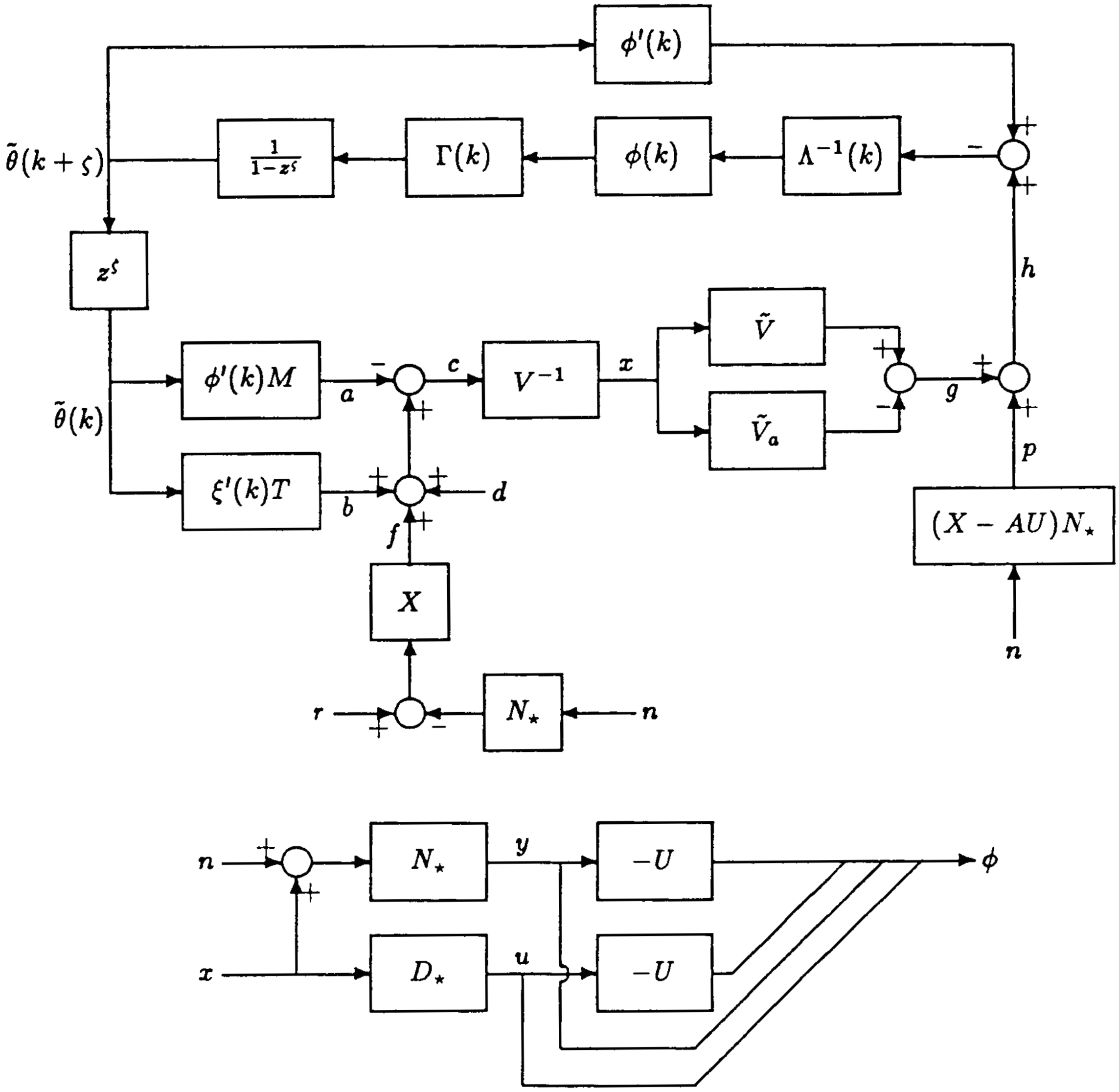

Figure 5. Complete error model

5. ROBUST GLOBAL STABILITY

The global stability proof of the error system in Figure 5 will be analyzed by simply applying the well-known small gain theorem and assuming persistently exciting ϕ; that is, there exists $\alpha, \beta, N, N \geq 1$, $0 < \alpha \leq \beta < \infty$ such that for all $k > (N-1)\zeta$

$$\beta I \geq \sum_{i=0}^{N-1} \phi(k-i\zeta)\phi'(k+i\zeta) \geq \alpha\, I. \tag{28}$$

The error system will be considered as consisting of three subsystems

$$S_1:\ h \mapsto \tilde{\theta}(k-\zeta),$$

$$S_2:\ \tilde{\theta}(k-\zeta) \mapsto (a,b),$$

$$S_3:\ c \mapsto g.$$

We'll begin with the well-posedness issue. Referring to Figure 5, S_1 relates h to $\tilde{\theta}(k-\zeta)$ as follows:

$$\tilde{\theta}(k+\zeta) = -\frac{1}{1-z^{\zeta}}\,\Gamma(k)\phi(k)\Lambda^{-1}(k)\big(h(k) + \phi'(k)\tilde{\theta}(k+\zeta)\big),$$

$$\tilde{\theta}(k+\zeta) = -\Big(I + \frac{1}{1-z^{\zeta}}\,\Gamma(k)\phi(k)\Lambda^{-1}(k)\phi'(k)\Big)^{-1}\frac{1}{1-z^{\zeta}}\,\Gamma(k)\phi(k)\Lambda^{-1}(k)h(k)$$

$$= -\big(I - z^{\zeta}I + \Gamma(k)\phi(k)\Lambda^{-1}(k)\phi'(k)\big)^{-1}\Gamma(k)\phi(k)\Lambda^{-1}(k)h(k).$$

$$\tilde{\theta}(k+\zeta) = \big(I+\Gamma(k)\phi(k)\Lambda^{-1}(k)\phi'(k)\big)^{-1}\big(\theta(k) - \Gamma(k)\phi(k)\Lambda^{-1}(k)h(k)\big)$$

$$= \Gamma^{1/2}(k)\left(I + \Gamma^{1/2}(k)\phi(k)\Lambda^{-1}(k)\phi'(k)\Gamma^{1/2}(k)\right)^{-1}\Gamma^{-1/2}(k)\left(\tilde{\theta}(k) - \Gamma(k)\phi(k)\Lambda^{-1}(k)h(k)\right).$$

S_2 and S_3 can be described by the relations

$$(b - a)(k) = (\xi'(k)T - \phi'(k)M)z^{\zeta}\tilde{\theta}(k+\zeta), \tag{29}$$

$$g(k) = (\tilde{V} - \tilde{V}_a)\, V^{-1}c(k).$$

Signals h and c are related to d, f, and p (which are uniformly bounded if r, n, and d are uniformly bounded) as follows:

$$c = d + f + S_2S_1(p + S_3c)$$

$$c = (I - S_2S_1S_3)^{-1}(d + f + S_2S_1p)$$

$$h = (p + S_3c) = p + S_3(I - S_2S_1S_3)^{-1}(d + f + S_2S_1p).$$

Since $\Lambda(k) > 0$, $I + \Gamma^{1/2}(k)\phi(k)\Lambda^{-1}(k)\phi'(k)\Gamma^{1/2}(k) \geq I$, therefore $\left(I + \Gamma^{1/2}(k)\phi(k)\Lambda^{-1}(k)\phi'(k)\Gamma^{1/2}(k)\right)^{-1}$ is well-defined and S_1 is well-posed as long as $\tilde{\theta}(0)$ is bounded. S_2 is well-posed as long as $\phi(k)$ and $\xi(k)$ (hence r and d) are uniformly bounded. S_3 is well-posed because $\tilde{V}, \tilde{V}_a \in M(Z)$ and the assumption that P is strictly causal (hence N_* is strictly causal) imply that the constant term of $V = XN_* + YD_*$ is non-singular and so $V^{-1} \in M(R(z))$. Therefore the individual subsystems are well-posed. For the entire system, since S_2 consists of at least one unit of delay due to $\zeta \geq 1$, $(I - S_2S_1S_3)^{-1}$ is causal and well-defined.

Therefore, if $d, f, p \in \ell_{\infty e}$ then $k, c \in \ell_{\infty e}$ and the small gain theorem can be applied to the error system.

The homogeneous part of S_1 is made up by the relation

$$\tilde{\theta}(k+\zeta) = \left(I + \Gamma(k)\phi(k)\Lambda^{-1}(k)\phi'(k)\right)^{-1}\tilde{\theta}(k). \qquad (31)$$

This relation has been shown to be exponentially stable for many linear estimation algorithms [18,19] in which $\Gamma(k)$ is updated in certain ways and the elements of $\Lambda(k)$ are constant and initialized between (0,1] provided that the PE condition (28) is satisfied. Assuming one of these algorithms is used and again (28) holds, then the exponential stability of (31) implies that S_1 is BIBO stable [20, Lem. 2.2] and $\gamma_\infty\{S_1\} = \beta_1 < \infty$.

With S_1 being BIBO stable the following is true.

Lemma 2: Provided (28) holds and r, d are both uniformly bounded by say, β_3, S_2 is BIBO stable.

Proof: (28) implies that

$$\|\phi(k)\|_{1\infty} \leq \beta^{1/2} < \infty \quad \forall\ k;$$

that is, $\phi'(k)M$ is BIBO stable. Since $\xi(k)$ is made up by the measurements of the uniformly bounded r and d, $\xi'(k)T$ is also BIBO stable. Therefore, S_2 is BIBO stable.

Hence, S_2S_1 is BIBO stable with

$$\gamma_\infty\{S_2S_1\} \leq (\max\{\beta^{1/2}, \beta_3\})\beta_1 =: \beta_4\beta_1 < \infty.$$

By the small gain theorem [11, Theorem 3.2.1], the entire error system will be BIBO stable if $\gamma_\infty\{S_3\}$ can be made smaller than $1/\gamma_\infty\{S_2S_1\}$. This is always possible due to the following theorem.

Theorem 3: $\gamma_\infty\{S_3\} = \|(\tilde{V} - \tilde{V}_a)V^{-1}\|_a$ is smaller than $1/\gamma_\infty\{S_2S_1\}$ if $\|[\tilde{N}_*' \; \tilde{D}_*']'\|_a$ is sufficiently small.

Proof: By definition,

$$V = XN_* + YD_* = XN + YD - (X\tilde{N}_* + Y\tilde{D}_*) \in M(Z)$$

where $XN + YD \in U(Z)$. Therefore,

$$X\tilde{N}_* + Y\tilde{D}_* = XN + YD - V.$$

By Fact 2.8, if $\|[X \; Y][\tilde{N}_*' \; \tilde{D}_*']'\|_a < 1/\|(XN + YD)^{-1}\|_a$ then $V \in U(Z)$, $V^{-1} \in M(Z)$ and

$$\|V^{-1}\|_a =: \beta_5 < \infty.$$

By definition again,

$$\tilde{V} - \tilde{V}_a = V - V_a$$

$$= XN_* + YD_* - (AUN_* + BUD_*)$$

$$= [X\text{-}AU \quad Y\text{-}BU]\,[N_*' \; D_*']'.$$

Applying (4) and noting that $AU = U_1 A_1$, $BU = U_1 B_1$ yields

$$V - V_a = -[X\text{-}AU \quad Y\text{-}BU][\tilde{N}_*' \; \tilde{D}_*']',$$

$$\|\tilde{V} - \tilde{V}_a\|_a \leq \|[X\text{-}AU \quad Y\text{-}BU]\|_a \; \|[\tilde{N}_*' \; \tilde{D}_*']'\|_a.$$

Since $X, A, U, Y, B \in M(Z)$, $\|[X\text{-}AU \quad Y\text{-}BU]\|_a =: \beta_6 < \infty$. Therefore,

$$\|(\tilde{V} - \tilde{V}_a)V^{-1}\|_a \leq \|\tilde{V} - \tilde{V}_a\|_a \; \|V^{-1}\|_a$$

$$\leq \beta_5 \, \beta_6 \; \|[\tilde{N}_*' \; \tilde{D}_*']'\|_a.$$

Hence if $\|[\tilde{N}_*' \; \tilde{D}_*']'\|_a$ is smaller than

$$\min\{1/(\|(XN + YD)^{-1}\|_a \; \|[X \; Y]\|_a), \qquad 1/(\beta_5 \beta_6 \gamma_\infty\{S_2 S_1\})\}$$

then $\|(\tilde{V} - \tilde{V}_a)V^{-1}\|_a < 1/\gamma_\infty(S_2 S_1)$.

Thus the class of AC systems under consideration is proved to be robustly globally stable against unmodeled dynamics and bounded n and d using the most conservative small gain theorem and assuming persistency of excitation of the regressor vector. It is expected that the margin of such robustness will be less-conservative in practice than that the proof of Theorem 3 has perhaps impressed upon the reader.

Remark 4: Note that the key to the existence of a nonzero global stability robustness margin is the finiteness of $\gamma_\infty(S_2S_1)$. The PE condition only ensures this; it is not a necessary condition for global stability robustness.

The comment preceding Theorem 3 implies that the smaller $\gamma_\infty\{S_2S_1\}$ is the more robust the global stability of the AC systems under consideration will be. In the next section, we offer a qualitative analysis of how the degree of persistency of excitation, i.e., the values of α, β (28), and the gains $\Gamma(k)$, $\Lambda(k)$ affect the smallness of $\gamma_\infty\{S_2S_1\}$.

6. ROBUSTNESS VS. DEGREE OF PE AND ADAPTATION GAINS

Due to the difficulties of analyzing the effects of time-varying matrix gains on the robustness in general, we will consider only SISO AC systems and assume $\Gamma(k)$, $\Lambda(k)$ to be equal to the scalar constants γ, λ respectively in this section.

Recall from Section 5 that S_1 relates h to $\tilde{\theta}(k-\zeta)$ by (28) or

$$\tilde{\theta}(k+\zeta) = \left(I + \frac{\gamma}{\lambda}\phi(k)\phi'(k)\right)^{-1}\left(\tilde{\theta}(k) - \frac{\gamma}{\lambda}\phi(k)h(k)\right)$$

$$= \left(I + \frac{\gamma}{\lambda}\phi(k)\phi'(k)\right)^{-1}\tilde{\theta}(k) - \left(I + \frac{\gamma}{\lambda}\phi(k)\phi'(k)\right)^{-1}\frac{\gamma}{\lambda}\phi(k)h(k)$$

$$= \left(\prod_{i=0}^{m}\left(I + \frac{\gamma}{\lambda}\phi(k-i\zeta)\phi'(k-i\zeta)\right)^{-1}\right)\tilde{\theta}(k-m\zeta) \tag{33}$$

$$- \sum_{j=0}^{m}\left(\prod_{i=0}^{j}\left(I + \frac{\gamma}{\lambda}\phi(k-i\zeta)\phi'(k-i\zeta)\right)^{-1}\right)\frac{\gamma}{\lambda}\phi(k-j\zeta)h(k-j\zeta)$$

where $m := \lceil k/\zeta \rceil$. Let $t := \lfloor j/N \rfloor$, since $\left(I + \frac{\gamma}{\lambda}\phi(k-i\zeta)\phi'(k-i\zeta)\right)^{-1} \leq I$ $\forall k \geq i\zeta$

$$\|\tilde{\theta}(k+\zeta)\|_\infty \leq \|\tilde{\theta}(k-m\zeta)\|_\infty +$$

$$+ \sum_{j=0}^{m}\left(\prod_{\ell=1}^{t}\left\|\prod_{i=(\ell-1)N}^{\ell N-1}\left(I + \frac{\gamma}{\lambda}\phi(k-i\zeta)\phi'(k-i\zeta)\right)^{-1}\right\|_{i\infty}\right)\frac{\gamma}{\lambda}\beta^{1/2}|h(k-j\zeta)|, \tag{34}$$

where if $t < 1$ the term within the parenthesis is defined to be equal to 1. Now,

$$\prod_{i=(\ell-1)N}^{\ell N-1}\left(I + \frac{\gamma}{\lambda}\phi(k-i\zeta)\phi'(k-i\zeta)\right)^{-1} = \left(\prod_{i=\ell N-1}^{(\ell-1)N}\left(I + \frac{\gamma}{\lambda}\phi(k-i\zeta)\phi'(k-i\zeta)\right)\right)^{-1}$$

$$= \left(I + \frac{\gamma}{\lambda}\sum_{i=(\ell-1)N}^{\ell N-1}\phi(k-i\zeta)\phi'(k-i\zeta) + \cdots + \left(\frac{\gamma}{\lambda}\right)^N\prod_{i=(\ell-1)N}^{\ell N-1}\phi(k-i\zeta)\phi'(k-i\zeta)\right)^{-1}. \tag{35}$$

If γ is small and approaches 0, then

$$\prod_{i=(\ell-1)N}^{\ell N-1}\left(I + \frac{\gamma}{\lambda}\phi(k-i\zeta)\phi'(k-i\zeta)\right)^{-1} \to \left(I + \frac{\gamma}{\lambda}\sum_{i=(\ell-1)N}^{\ell N-1}\phi(k-i\zeta)\phi'(k-i\zeta)\right)^{-1}$$

$$\leq (I + \frac{\gamma}{\lambda} \alpha\ I)^{-1} = \frac{\lambda}{\lambda + \gamma\alpha} I,$$

$$\|\tilde{\theta}(k+\zeta)\|_\infty \leq \|\tilde{\theta}(k-m\zeta)\|_\infty + \Big(\sum_{j=0}^{m} (\frac{\lambda}{\lambda + \gamma\alpha})^t \frac{\gamma}{\lambda} \beta^{1/2}\Big) \max_{j=0,m} |h(k-j\zeta)|$$

$$\leq \|\tilde{\theta}(k-m\zeta)\|_\infty + \Big(N \frac{\gamma}{\lambda} \beta^{1/2} \sum_{j=0}^{\infty} (\frac{\lambda}{\lambda + \gamma\alpha})^j\Big) \max_{j=0,m} |h(k-j\zeta)|$$

$$= \|\tilde{\theta}(k-m\zeta)\|_\infty + \frac{N\beta^{1/2}(\lambda + \gamma\alpha)}{\lambda\alpha} \max_{j=0,m} |h(k-j\zeta)|$$

$$\approx \|\tilde{\theta}(k-m\zeta)\|_\infty + \frac{N\beta^{1/2}}{\alpha} \max_{j=0,m} |h(k-j\zeta)|.$$

This implies that $\gamma_\infty\{S_1\} = \frac{N\beta^{1/2}}{\alpha}$ if $\gamma \to 0$. From Figure 5 it is obvious that if $\|r(k)\|_\infty \leq \beta_7 \;\; \forall k$ then

$$\gamma_\infty\{S_2\} = \gamma_\infty\{(\xi'(k)T - \phi'(k)M)z^\zeta\} \leq k_1\beta_7 + k_2\beta^{1/2}$$

where k_1, k_2 denote the ranks of T, M respectively. This leads to

$$\gamma_\infty\{S_2S_1\} \leq \frac{N\beta^{1/2}}{\alpha}(k_1\beta_7 + k_2\beta^{1/2}). \tag{36}$$

Therefore, if $\gamma_\infty\{S_3\} < \alpha/(NB^{1/2}(k_1\beta_7 + k_2\beta^{1/2})) =: \rho$ then the error system, hence the AC system, will be globally BIBO stable for $\gamma \to 0$. In this sense the value ρ may be termed the inherent (due to PE) robustness margin of the AC system when $\gamma \to 0$.

The above result as well as the large number of AC system robustness results obtained based on slow adaptation (or the averaging analysis) [5] does not imply that slow adaptation necessarily increases or decreases the inherent robustness of an AC system. This can be enlightened by analyzing the robustness margin of a SISO system with only <u>one</u> parameter to be estimated.

Consider (35); if $\phi(k)$ is a scalar, then all terms under the inversion sign are nonnegative. Therefore, if (28) is satisfied,

$$\prod_{i=(\ell-1)N}^{\ell N-1} \left(1+\frac{\gamma}{\lambda}\phi(k-i\zeta)\phi'(k-i\zeta)\right)^{-1} \leq \left(1+\frac{\gamma\alpha}{\lambda}+\frac{\gamma^2}{\lambda^2}\Delta(k)\right)^{-1}$$

where $\frac{\gamma^2}{\lambda^2}\Delta(k)$ denotes the nonnegative contribution by the terms in (35) of orders $\frac{\gamma^2}{\lambda^2}$ or higher. Assuming $\Delta(k) > \eta > 0$ for all but finite number of k, then it is reasonable that a $\Delta(\eta) > 0$ exists such that on the average

$$\prod_{i=(\ell-1)N}^{\ell N-1} \left(1+\frac{\gamma}{\lambda}\phi(k-i\zeta)\phi'(k-i\zeta)\right)^{-1} \leq \left(1+\frac{\gamma\alpha}{\lambda}+\frac{\gamma^2}{\lambda^2}\Delta(\eta)\right)^{-1}. \tag{37}$$

Substituting (37) into (34) and go through similar manipulations as for the $\gamma \to 0$ case results in

$$\|\tilde{\theta}(k+\zeta)\|_\infty \leq \|\tilde{\theta}(k-m\zeta)\|_\infty + \frac{N\beta^{1/2}}{\lambda}\left(\gamma+\frac{\lambda^2}{\alpha\lambda+\gamma\Delta(\eta)}\right) \max_{j=0,m} |h(k-j\zeta)|$$

and therefore

$$\gamma_\infty\{S_1\} = \frac{N\beta^{1/2}}{\lambda}\left(\gamma+\frac{\lambda^2}{\alpha\lambda+\gamma\Delta(\eta)}\right) = N\beta^{1/2}\left(\frac{\gamma}{\lambda}+\frac{1}{\alpha+\frac{\gamma}{\lambda}\Delta(\eta)}\right). \tag{38}$$

From (38) it is obvious that

$$\gamma_\infty\{S_1\} \to \frac{N\beta^{1/2}}{\alpha} \quad \text{as } \gamma \to 0,$$

$$\gamma_\infty\{S_1\} \to \infty \qquad \text{as} \quad \gamma \to \infty.$$

But $\frac{N\beta^{1/2}}{\alpha}$ is not necessarily the smallest $\gamma_\infty\{S_1\}$. This can be verified by differentiating (38) with respect to γ and equating to zero to yield

$$2\,\alpha\,\frac{\gamma}{\lambda}\,\Delta(\eta) + \left(\frac{\gamma}{\lambda}\right)^2 \Delta^2(\eta) + (\alpha^2 - \Delta(\eta)) = 0. \tag{39}$$

This equation has only one valid solution at

$$\frac{\gamma}{\lambda} = \frac{-\alpha + \sqrt{\Delta(\eta)}}{\Delta(\eta)} =: \gamma_*$$

if $\Delta(\eta) \geq \alpha^2$. At this solution point

$$\gamma_\infty\{S_1\} = N\beta^{1/2}\left(\frac{2\sqrt{\Delta(\eta)} - \alpha}{\Delta(\eta)}\right)$$

which is smaller than $\frac{N\beta^{1/2}}{\alpha}$ except when $\Delta(\eta) = \alpha^2$. This implies that provided $\Delta(\eta) > \alpha^2$, $\gamma_\infty\{S_1\}$ reaches its lowest value (hence the AC system is the most robust against unmodeled dynamics and disturbances) when the true rate of adaptation $\frac{\gamma}{\lambda}$ is not the slowest possible.

7. CASE OF TIME-VARYING PLANT

Suppose the plant P had been not only undermodeled but also time-varying. The development in Section 4 would go through entirely while the results of Sections 5 and 6 would still hold true almost completely if one simply adds a superscript k to the symbols

N_*, D_*, N, D, X_*, Y_*, X, Y, A_*, B_*, A, B, A_{1*}, B_{1*},

A_1, B_1, $\tilde{N}_*$, $\tilde{D}_*$, $\tilde{X}_*$, $\tilde{Y}_*$, $\tilde{A}_*$, $\tilde{B}_*$, θ, V_*, V, $\tilde{V}$, V_a, and $\tilde{V}_a$.

The results of Sections 5 and 6 will become <u>completely</u> true if one further assumes that the now time-varying subsystem $(V^k)^{-1}$ (see Figure 5) is sufficiently slowly varying while satisfying $(V^k)^{-1} \varepsilon M(Z) \ \forall k$ and $\|N_*^k\|_a$, $\|D_*^k\|_a$, $\|X^k\|_a$, $\|A^k\|_a$ all uniformly bounded over k.

These are required so that $(V^k)^{-1}$ is BIBO stable [28,29] with finite $\gamma_\infty\{(V^k)^{-1}\}$ and f, p, ϕ are uniformly bounded as assumed. Then $\gamma_\infty\{\tilde{V}^k - \tilde{V}_a^k\}$ can be greater than zero and still satisfy the unity-loop-gain condition of the small gain theorem. Thus the global BIBO stability of the AC system with a time-varying plant is also proved to be robust. Note that although assuming the plant varies slowly would make the assumption that $(V^k)^{-1}$ is sufficiently slowly varying more easily satisfied, there is no apparent need to do so, not even for the sake of achieving the desired performance as indicated by our preliminary analysis (not included).

8. RESULTS AND DISCUSSION

It has been shown that, provided the regressor vector is persistently exciting, the class of adaptive control systems with possibly time varying plants developed in Section 3 is robustly globally BIBO stable against not only unmodeled plant dynamics expressed in terms of stable factor perturbations but also uniformly bounded arbitrary internal noises as well as external disturbances. Although the margin of such robustness may seem pretty small from the proofs, its mere existence should be highly useful in terms of enhancing user's confidence on using adaptive methodologies for systems control in practice where a plant is seldom as imprecisely known as has been assumed in the proofs.

Although the analysis of robustness margin as related to the degree of persistency of excitation and the adaptation gain in Section 6 is for the very specialized case when the regressor vector is a scalar, the result casts a doubt on the absolute benefit of slow adaptation. However, notice that our analysis did not take into account of the possible effects of varying the adaptation gain on the uniform upper bound $\beta^{1/2}$. True relationship between slow adaptation and the inherent global stability robustness margin can only be obtained when such effects are properly taken into account.

It is unfortunate that, as in [4,5], the PE condition assumed in this paper presumes a uniform upper bound on the regressor signals before the proof of system stability. A recent result proving stability robustness of an AC methodology without a priori assuming bounded regressor is available in [21]. However, a complex adaptation law needs to be implemented, and stringent yet contradictory conditions on the adaptation gain need to be verified in order for the results to be valid. These conditions also favored slow adaptation which, as we pointed out, is not necessarily nonnegatively related to the inherent global stability robustness. In spite of these shortcomings, though, the solution methods used in [21] may still be highly useful for eliminating the upper-boundedness assumption made via the PE condition.

REFERENCES

[1] Anderson, B. D. O., "Exponential convergence and persistent excitation," Proc. of 21st IEEE CDC, Orlando, Florida, December 1982, pp. 12-17.

[2] Anderson, B. D. O., C. R. Johnson, Jr., "Exponential convergence of adaptive identification and control algorithms," Automatica, Vol. 18, No. 1, 1982.

[3] Narendra, K. S., A. M. Annaswamy, "Robust adaptive control in the presence of bounded disturbances," IEEE Trans. Automat. Contr., Vol. AC-31, April 1986, pp. 301-316.

[4] Kosut, R. L., B. Friedlander, "Robust adaptive control: Conditions for global stability," IEEE Trans. Automat. Contr., Vol. AC-30, No. 7, July 1985.

[5] Anderson, B. D. O., R. R. Bitmead, C. R. Johnson, Jr., P. V. Kokotovic, R. L. Kosut, I. M. Y. Mareels, L. Praly, B. D. Riedle, Stability of Adaptive Systems: Passivity and Averaging Analysis, The MIT Press, Cambridge, Massachusetts, 1986.

[6] Ioannou, P. A., P. V. Kokotovic, "Instability analysis and improvement of robustness of adaptive control," Automatica, Vol. 20, No. 5, September 1984.

[7] Ioannou, P. A., K. Tsakalis, "Instability phenomena in adaptive control independent of persistence of excitation conditions," University of Southern California Technical Report 86-04-1, April 1986.

[8] Sastry, S. S., "Model reference adaptive control-stability, parameter convergence and robustness," IMA J. Math. Control Info., Vol. 1, 1984, pp. 27-66.

[9] Vidyasagar, M., Control System Synthesis - A Factorization Approach, The MIT Press, Cambridge, Massachusetts, 1985.

[10] Riedle, B., P. V. Kokotovic, "A stability-instability boundary for disturbance-free slow adaptation and unmodeled dynamics," Proc. of 23rd CDC, Las Vegas, Nevada, December 1984.

[11] Desoer, C. A., M. Vidyasagar, Feedback Systems: Input-Output Properties, Academic Press, New York, New York, 1975.

[12] Ma, C. C. H., M. Vidyasagar, "Direct globally convergent adaptive regulation of stable multivariable plants," Modeling, Identification and Robust Control, edited by C. I. Byrnes and A. Lindquist, North-Holland, Amsterdam, 1986, pp. 59-69.

[13] Rohrs, C. E., "Stability mechanisms and adaptive control," Proc. of 23rd CDC, Las Vegas, Nevada, December 1984.

[14] Elliott, H., W. A. Wolovich, M. Das, "Arbitrary adaptive pole placement for linear multivariable systems," IEEE Trans. Automat. Contr., Vol. AC-29, No. 3, March 1984.

[15] Kreisselmeier, G., M. C. Smith, "Stable adaptive regulation of arbitrary n-th-order plants," IEEE Trans. Automat. Contr., Vol. AC-31, No. 4, April 1986.

[16] Ma, C. C. H., Discrete Multivariable Adaptive Control - A Stable Coprime Factorization Approach, Thesis dissertation, Electrical Engineering, University of Waterloo, Waterloo, Ontario, Canada, N2L-3G1.

[17] Goodwin, G. C., R. S. Long, "Generalization of results on multivariable adaptive control," IEEE Trans. Automat. Contr., Vol. AC-25, No. 6, December 1980.

[18] Goodwin, G. C., K. S. Sin, Adaptive Filtering Prediction and Control, Prentice-Hall Inc., Englewood Cliffs, New Jersey 07632.

[19] Johnstone, R. M., C. R. Johnson, Jr., R. R. Bitmead, B. D. O. Anderson, "Exponential convergence of recursive least squares with exponential forgetting factor," Proc. of 21st IEEE CDC, December 1982, pp. 994-997.

[20] Anderson, B. D. O., "Internal and external stability of linear time-varying systems," SIAM J. Control Optim., Vol. 20, No. 3, May 1982.

[21] Ortega, R., L. Praly, I. D. Landau, "Robustness of discrete-time direct adaptive controllers," IEEE Trans. Automat. Contr., Vol. AC-30, No. 12, December 1985.

Standard Bibliographic Page

1. Report No. NASA CR-178224 ICASE Report No. 86-79	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle INHERENT ROBUSTNESS OF DISCRETE-TIME ADAPTIVE CONTROL SYSTEMS		5. Report Date December 1986
		6. Performing Organization Code
7. Author(s) C. C. H. Ma		8. Performing Organization Report No. 86-79
9. Performing Organization Name and Address Institute for Computer Applications in Science and Engineering Mail Stop 132C, NASA Langley Research Center Hampton, VA 23665-5225		10. Work Unit No.
		11. Contract or Grant No. NAS1-18107
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Contractor Report
		14. Sponsoring Agency Code 505-90-21-01

15. Supplementary Notes

Langley Technical Monitor:
J. C. South

Submitted to Control Theory and Advanced Technology

Final Report

16. Abstract

Global stability robustness with respect to unmodeled dynamics, arbitrary bounded internal noise, as well as external disturbance is shown to exist for a class of discrete-time adaptive control systems when the regressor vectors of these systems are persistently exciting. Although fast adaptation is definitely undesirable, so far as attaining the greatest amount of global stability robustness is concerned, slow adaptation is shown to be not necessarily beneficial. The entire analysis in this paper holds for systems with slowly varying return difference matrices; the plants in these systems need not be slowly varying.

17. Key Words (Suggested by Authors(s)) adaptive control, robustness, inherent robustness, adaptive system error model	18. Distribution Statement 08 - Aircraft Stability and Control; 18 - Spacecraft Design, Testing and Performance; 71 - Acoustics; 31 - Engineering (General); Unclassified - unlimited

19. Security Classif.(of this report)	20. Security Classif.(of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	38	A03

For sale by the National Technical Information Service, Springfield, Virginia 22161

NASA-Langley, 1987