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Abstract

As spacecraft structural concepts increase in size and flexibility, the vibration frequencies become more closely-spaced. The identification and control of such closely-spaced frequencies present a significant challenge. To validate system identification and control methods prior to actual flight, simpler space structures will be flown. To challenge the above technologies it will be necessary to design these structures with closely-spaced or coupled vibration modes. Thus there exists a need to develop a systematic method to design a structure which has closely-spaced vibration frequencies. This paper describes an optimization procedure which is used to design a large flexible structure to have closely-spaced vibration frequencies. The procedure uses a general-purpose finite-element analysis program for the vibration and sensitivity analyses and a general-purpose optimization program. Results are presented from two studies. The first study uses a detailed model of a large flexible structure to design a structure with one pair of closely-spaced frequencies. The second study uses a simple equivalent beam model of a large flexible structure to obtain a design with two pairs of closely-spaced frequencies.

Nomenclature

A boom cross-sectional area  
 $f_B$  second bending frequency, COFS-I  
 $f_D$  first natural frequency of diagonal, COFS-I  
 $f_T$  first torsion frequency, COFS-I  
 $f_1$  first natural frequency, COFS-I  
 $f_i$   $i^{\text{th}}$  frequency  
F objective function  
 $F(V)$  objective function  
g set of constraints  
 $g_i$   $i^{\text{th}}$  constraint  
[I] identity matrix  
 $I_{yy}$  boom moment of inertia, COFS-II  
 $I_{zz}$  boom moment of inertia, COFS-II  
[K] stiffness matrix  
 $L_1$  mast length, COFS-II  
 $L_2$  boom length, COFS-II  
m tip mass, COFS-II  
[M] mass matrix

NDV number of design variables  
 $R_I$  inner radius of diagonal, COFS-I  
 $R_O$  outer radius of diagonal, COFS-I  
 $R_S$  "strong" longeron inner radius, COFS-I  
 $R_W$  "weak" longeron inner radius, COFS-I  
V set of design variables  
 $V_k$   $k^{\text{th}}$  design variable  
 $\frac{\partial}{\partial V_k}$  derivative with respect to  $V_k$   
 $\epsilon_i$  frequency spacing tolerance  
 $\{\theta\}_j$   $j^{\text{th}}$  eigenvector  
 $\omega_j^2$   $j^{\text{th}}$  eigenvalue

Introduction

As spacecraft structural concepts increase in size and flexibility, the vibration frequencies become more closely-spaced. Since the identification and control of such closely-spaced frequencies present a significant challenge, a need exists to develop and validate analytical methods to design and assess the performance of such structures. A NASA Langley research program is underway to investigate the control of large flexible space structures. This program denoted COFS (Control of Flexible Structures)<sup>1-7</sup> involves both ground and flight tests. One aspect of the COFS program is to determine the dynamic characteristics and control requirements of a candidate structure. The structure is to be designed to have closely-coupled vibration modes. This is contrary to the normal process in which the designer tries to control rigid body motions and avoid control/structures interactions. However, the COFS program requires a structure which has closely-spaced frequencies in order to challenge control law and system identification methodology. The close-spacing of frequencies does not necessarily mean mode coupling, but it is felt if the frequencies are closely-spaced there is a better chance of mode coupling. This paper describes an optimization procedure to systematically design a large flexible structure to have closely-spaced vibration frequencies. The procedure uses the general-purpose finite-element analysis program EAL (Engineering Analysis Language System)<sup>8</sup> and the general-purpose optimization program CONMIN<sup>9</sup>.

Results will be presented for the COFS-I configuration<sup>1-5</sup> and a candidate COFS-II

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configuration<sup>6</sup>. The first study uses a detailed model of the COFS-I configuration to obtain a design which has one pair of closely-spaced frequencies. The second study uses a simple model of the COFS-II configuration to develop the methodology for systematically obtaining two pairs of closely-spaced frequencies.

### Optimization Procedure

#### Formulation - General

The use of formal mathematical programming to obtain an optimum design requires the specification of an objective function  $F(V)$  (the quantity to be minimized), a set of inequality constraints  $g$  (requirements which must be satisfied), and a set of design variables  $V$  (the quantities which are changed to reach an optimum design). The optimization problem can be stated as follows:

Find the set of variables  $V$  such that

$$F(V) \rightarrow \text{Minimum}$$

subject to

$$g \leq 0$$

Specific forms of  $F$ ,  $g$ , and  $V$  will be given in later sections of the paper.

A flowchart of the optimization procedure is shown in figure 1. Each design cycle consists of eigenvalue analysis, sensitivity derivative calculation, and the optimization and the approximate analysis block. Each will be discussed briefly in the following sections.

#### Analysis

The matrix equation for a free vibration eigenvalue problem is

$$([K] - \omega_j^2 [M]) \{\theta\}_j = 0 \quad (1)$$

where  $\{\theta\}_j$  and  $\omega_j^2$  are the eigenvector and the eigenvalue of the  $j^{\text{th}}$  mode, respectively,  $[K]$  is the stiffness matrix, and  $[M]$  is the mass matrix. Normalizing the modes with respect to the mass matrix results in the following equation

$$\{\theta\}_j^T [M] \{\theta\}_j = [I] \quad (2)$$

where  $[I]$  is the identity matrix.

Differentiating equation 1 with respect to the  $k^{\text{th}}$  design variable,  $V_k$ , results in the following equation

$$([K] - \omega_j^2 [M]) \frac{\partial \{\theta\}_j}{\partial V_k} - \frac{\partial \omega_j^2}{\partial V_k} [M] \{\theta\}_j - \frac{\partial [K]}{\partial V_k} \{\theta\}_j + \omega_j^2 \frac{\partial [M]}{\partial V_k} \{\theta\}_j = 0 \quad (3)$$

Premultiplying equation 3 by  $\{\theta\}_j^T$  and using equations 1 and 2 gives the following expression for the derivative of the  $j^{\text{th}}$  eigenvalue with respect to the  $k^{\text{th}}$  design variable

$$\frac{\partial \omega_j^2}{\partial V_k} = \{\theta\}_j^T \left( \frac{\partial [K]}{\partial V_k} - \omega_j^2 \frac{\partial [M]}{\partial V_k} \right) \{\theta\}_j \quad (4)$$

#### Eigenvalue and Sensitivity Analysis Implementation

The eigenvalue analysis is performed using the Engineering Analysis Language (EAL) System<sup>8</sup> which is a general purpose commercial finite-element analysis program. The EAL system contains individual processors that communicate through a data base containing data sets. The data sets typically contain data describing the finite-element model of the structure (such as geometry) as well as response information that is accumulated during the execution of the processors. The processors can be executed in any appropriate sequence, and a sequence of processor executions is denoted as a "runstream". The EAL system also uses a set of flexible FORTRAN-like statements called executive control system (ECS) commands. These commands allow branching, testing data, looping, and calling runstreams (similar to calling FORTRAN subroutines). The EAL processors, with the appropriate ECS commands organized as runstreams are used to calculate the eigenvalues and eigenvectors (eqns. 1 and 2), and derivatives of eigenvalues (eqn. 4).

#### Optimization Algorithm

The basic optimization algorithm to be used in this study is a combination of the general-purpose optimization program CONMIN<sup>9</sup> and piecewise linear approximate analyses for computing the objective function and constraints. Since the optimization process requires many evaluations of the objective function and constraints before an optimum design is obtained, the process can be very expensive if full analyses are made for each function evaluation. However, as Miura<sup>10</sup> points out, the optimization process primarily uses analysis results to move in the direction of the optimum design; therefore, a full analysis needs to be made only occasionally during the design process and always at the end to check the final design. Thus, various approximation techniques can be used during the optimization to reduce costs. In the present work, the objective function and constraints are approximated using a piecewise linear analysis that consists of linear Taylor series expansions for the objective function and the constraints based on the values for the design variables from CONMIN and the eigenvalues and sensitivity derivatives from EAL. Details of this algorithm are contained in reference 11.

CONMIN. - CONMIN is a general-purpose optimization program that performs constrained

minimization using a usable-feasible directions search algorithm. In the search for new design variable values, CONMIN requires derivatives of the objective function and constraints. The user has the option of either letting CONMIN determine the derivative by finite differences or supplying derivatives to CONMIN. The latter method will be used herein.

Piecewise linear approximation. - In the approximate analysis method, previously calculated derivatives of the objective function and constraint functions with respect to the design variables are used for linear extrapolation of these functions. The assumption of linearity is valid over a suitably small change in the design variable values and will not introduce a large error into the analysis provided the changes are small. This approximate analysis will be referred to as a "piecewise linear approximation."

Specifically, the objective function  $F$ , the constraints  $g$ , and their respective derivatives are calculated for the design variables  $V$  using a full analysis. The linear Taylor series approximations for the new objective function and the constraint values are as follows:

$$F = F_0 + \sum_{k=1}^{NDV} \frac{\partial F}{\partial V_k} (V_k - V_{0,k}) \quad (5)$$

and

$$g = g_0 + \sum_{k=1}^{NDV} \frac{\partial g}{\partial V_k} (V_k - V_{0,k}) \quad (6)$$

where  $NDV$  is the number of design variables,  $F$  is the extrapolated value of the objective function,  $g$  is the extrapolated value of the constraint, and  $V_k$  is the design variable value obtained from CONMIN. The symbols  $F_0$ ,  $g_0$ , and  $V_{0,k}$  are the values for the objective function, constraints, and design variables, respectively, from the full analysis.

Errors which may be introduced by use of the piecewise linear approach are controlled by imposing "move limits" on each design variable during a design cycle. A move limit which is specified as a fractional change,  $\delta$ , of each design variable value (for this work,  $\delta=0.1$ ) is imposed as an upper and lower design variable bound on each cycle. These move limits must not exceed the absolute design variable values.

### Applications

The optimization procedure has been applied in two studies. The first study addresses a two-dimensional vibration problem using a detailed model of the COFS-I design in which one pair of frequencies is to be closely spaced. The second study is a more general three-dimensional vibration problem and uses an equivalent beam model of a COFS-II conceptual design in which two pairs of frequencies are to be closely spaced.

### COFS-I MODEL

The COFS-I flight mast shown fully deployed from the Space Shuttle in figure 2 is approximately 60 meters long and consists of 54 bays of single-laced latticed beams with unequal area longerons (two "weak" longerons and one "strong" longeron). The "strong" longeron is located on the centerline of the Shuttle. The longerons have different cross-sectional areas to promote the coupling between modes. Further details of the COFS-I flight mast can be found in references 4, 5, and 12.

The mast was originally designed using parametric studies to have one pair of closely-spaced frequencies (the first torsion and the second bending frequencies). It was subsequently determined that there were some deficiencies with the original design. In particular, the diagonal members of the original COFS-I design might buckle during deployment. There was also a concern that individual member frequencies might interact with global frequencies of the mast. An in-house redesign team was formed to address these concerns. As part of this effort, an optimization procedure based on the previous section was formulated and applied using a detailed model of the original COFS-I configuration to determine if it was possible to meet additional requirements and maintain the close-spacing of the frequencies. This study will not address the buckling problem per se but will consider the individual member frequency concern. However, it is felt that addressing the individual member frequencies will also help the buckling concern.

A finite-element model of the entire COFS-I mast and Shuttle consisting of 360 joints is used in this study. The Shuttle is modeled as a stick model with very stiff beam elements. The battens, longerons, and diagonals of the mast are modeled by tubes which have bending, torsional, and axial stiffnesses. The model includes lumped masses to represent hinges, deployer retractor assembly, sensor and actuator platforms, etc. Further details of the finite-element model can be found in reference 12. A typical two-bay segment of the mast is shown in figure 3.

### COFS-I Design Variables

In order to have minimal impact on the original design, a limited number of quantities are allowed to vary. The number of bays, all lengths of individual members (battens, longerons, and diagonals), and all physical properties of the battens are held constant. The outer radii of the longerons are also held constant to permit the mast to fold into a canister in the Shuttle without redesigning the hinges. The inner radii ( $R_S$  and  $R_W$ ) of the longerons and the inner and outer radii of the diagonals ( $R_I$  and  $R_O$ , respectively) are allowed to vary in order to meet the design requirements discussed above. The four design variables are shown in figure 3. The upper and lower design variable values are given in table 1 and are based on manufacturing limitations.

## COFS-I Optimization Formulation

The COFS-I optimization formulation can be stated as follows: minimize the total mass while meeting the following design requirements. The first requirement is that the first natural frequency ( $f_1$ ) of the mast be greater than 0.18 Hz. This requirement assures that the frequencies of the mast do not couple with those of the Shuttle control system. The second requirement is to preserve the close-spacing of the first torsion and second bending frequencies denoted by  $f_T$  and  $f_B$ , respectively. The third requirement is that the first natural frequency ( $f_D$ ) of the diagonal be greater than 15 Hz. The diagonal frequency is calculated from a simple formula based on assumptions of simply-supported ends with the mass of the hinge concentrated at the center of the diagonal.<sup>13</sup> This requirement is a stiffness constraint to ensure that individual member frequencies of the diagonals are outside the mast frequency range in which frequencies are to be closely-spaced (to preclude interaction of member frequency upon the global frequency). Although individual member frequencies of the longerons and battens are also of concern, it is felt that the diagonals because of their length and their large hinge masses are more likely to have individual member frequencies inside the mast frequency range. The fourth requirement is that the inner radius  $R_W$  of the weak longeron be at least 0.254 mm larger than the inner radius  $R_S$  of the strong longeron. The fifth requirement is a minimum gage requirement on the wall thickness ( $R_O - R_I$ ) of the diagonal members (the minimum wall thickness must be greater than 0.56 mm). For convenience, the constraints involving frequencies are represented in terms of eigenvalues  $\omega_1^2$ , where for example  $\omega_1 = 2\pi f_1$ , etc. Thus the COFS-I optimization formulation can be stated as follows:

Find the values for  $R_W$ ,  $R_S$ ,  $R_O$ , and  $R_I$  such that

$$F = \text{total mass} \rightarrow \text{minimum}$$

while meeting the following constraints

$$\begin{aligned} g_1 &= 1 - \frac{\omega_1^2}{[2\pi(0.18)]^2} \leq 0 \\ g_2 &= \left| \frac{\omega_T^2 - \omega_B^2}{\omega_T^2} \right| - \epsilon \leq 0 \\ g_3 &= 1 - \frac{\omega_D^2}{[2\pi(15)]^2} \leq 0 \\ g_4 &= 0.000254 - (R_W - R_S) \leq 0 \\ g_5 &= 0.00056 - (R_O - R_I) \leq 0 \end{aligned} \quad (7)$$

where  $\epsilon=0.01$  and the radii are in meters.

## COFS-I Optimization Results

The initial and final values for the design variables and objective function (includes mass of the Shuttle) are given in table 2. The initial values are the nominal COFS-I design values. Plots showing convergence of the COFS-I design as a function of design cycle are shown in figure 4. The optimization procedure begins with four satisfied design requirements (two of which are active). As shown in figure 4a, initially the frequencies  $f_B$  and  $f_T$  are closely-spaced and the Shuttle requirement on the first natural frequency  $f_1$  of the mast is active ( $f_1=0.188$  Hz). It can be seen from figure 4b that the requirement on the weak and strong longerons ( $R_W - R_S$ ) and the diagonal wall thickness ( $R_O - R_I$ ) are satisfied with the latter requirement being active. However, from figure 4c, it is seen that initially the diagonal frequency ( $f_D=11.5$  Hz) is lower than the required value of 15 Hz. As stated earlier, the diagonal frequency requirement was not considered in the original design. As the optimization process proceeds, the values of the design variables are changed systematically until the diagonal frequency requirement is satisfied (fig. 4c). The two frequencies ( $f_B$  and  $f_T$ , fig. 4a) are not as close as they were initially since the diagonal frequency works against this requirement. Specifically, when the diagonal frequency  $f_D$  is increased by an increase in stiffness, the first torsion frequency  $f_T$  is also increased.

The "dips" in the diagonal frequency and the frequency pairs at cycles 9, 13, and 20 are partly due to the optimizer which attempts to satisfy all constraints even at the expense of increasing the objective function and partly due to the linearization of the problem. The optimizer concentrates on satisfying the diagonal frequency constraint until cycle 8. Then the optimizer tries to satisfy the frequency spacing requirement. The optimizer chooses values for the four radii which closely-space the frequencies (see cycle 9, fig. 4a), but those choices lower the diagonal frequency (see cycle 9, fig. 4c). Now the optimizer tries to satisfy this diagonal frequency constraint which as mentioned previously works against the frequency spacing requirement (see fig. 4, cycles 10-12). This same process occurs again at cycles 13 and 20. The spacing of the two frequencies ( $f_B$  and  $f_T$ ) cannot be made closer than 0.18 Hz (fig. 4a). The "dips" are also due to the linearization of the problem. During the optimization process, mode switching occurs at cycles 9, 13, and 20. As the optimizer chooses values for the radii to satisfy the constraints, these values can cause the modes to switch. For example, if at the beginning of the cycle, the second bending mode is associated with  $f_{10}$  and the first torsion mode is associated with  $f_{11}$ , changes in the radii can cause the second bending mode to be associated

with  $f_9$  and the torsion mode with  $f_{11}$ . However, the optimizer is choosing values for the design variables based on derivative information at the start of the cycle (i.e. which mode is torsion and which mode is second bending). This is rectified when a full analysis is performed. The design process is also being limited by the minimum gage requirements - namely,  $R_I$  and  $R_W$  are at their upper and lower bounds (fig. 5), respectively. The inner radius,  $R_W$ , is within 0.25 mm of minimum gage (limited by the fourth design requirement fig. 4b).

A plot of the mass (objective function which includes mass of the Shuttle) as a function of design cycle is shown in figure 6. The optimization procedure obtains a design for the mast which better satisfies the design requirements at the expense of an additional 40 kg of mass. This increase in mass from the original design is mainly due to the diagonal frequency requirement.

From the study, it is concluded that no feasible design exists which can be obtained by simply varying longeron radii and diagonal tube thickness within the prescribed limits. Therefore, there is a need for more design freedom in the optimization procedure in order to achieve a fully satisfactory design.

#### COFS-II Model

While the first study is basically a two-dimensional vibration problem and used a detailed model, the second study is aimed at a three-dimensional vibration problem and involves preliminary concepts. A conceptual design of a candidate COFS-II configuration such as the one shown in figure 7 is used. The configuration consists of a mast, a boom, and a structure attached to the tip (such as an antenna). The properties of the mast are fixed except for the length  $L_1$ . In the beam segment from the top of the mast to the tip of the boom, none of the properties are fixed. At this stage a detailed model is not justified, so the simple model shown in figure 8 is used in this study. The model which is based on the geometry derived from reference 14 is modeled as an equivalent beam with 17 joints. Table 3 contains the properties of the COFS-II model. Earlier parametric studies using this model indicate the most suitable frequency pairs for close-spacing are: the third frequency  $f_3$  with the fourth frequency  $f_4$  and the fifth frequency  $f_5$  with the sixth frequency  $f_6$ . The third mode is characterized by bending and twisting of the mast and rigid body movement of the boom. The fourth mode is characterized by first in-plane bending of the mast and first in-plane bending of the boom. The fifth mode is characterized as second in-plane bending of the mast and second in-plane bending of the boom. The sixth mode is characterized by second out-of-plane bending coupled with torsion of the mast and first out-of-plane bending of the boom.

#### COFS-II Design Variables

The six design variables are shown in figure 8: the mast length ( $L_1$ ), the boom length ( $L_2$ ), the boom cross-sectional area ( $A$ ), the two boom area moments of inertia ( $I_{yy}$  and  $I_{zz}$ ), and since the attachment at the tip of the beam has not yet been defined, it is modeled as a concentrate mass ( $m$ ). The upper and lower design variable values are given in table 4. Since the mast is to be deployable to an essentially arbitrary length in increments of two-bay lengths and must fold inside a canister on the Shuttle, the mast length  $L_1$  is allowed to vary between 40 and 60 meters and the boom length  $L_2$  between 1 and 25 meters. The tip mass representative of an attachment such as an antenna is allowed to vary between 10 and 30 kg. The range of values for  $A$ ,  $I_{yy}$ , and  $I_{zz}$  are chosen to prevent mode switching (i.e. want to ensure  $f_3$  is paired with  $f_4$  and  $f_5$  is paired with  $f_6$ ).

#### COFS-II Optimization Formulation

The objective function is the total mass of the structure. The design requirements are that two pairs of adjacent frequencies are closely-spaced - i.e.,  $f_3$  and  $f_4$  are within a specified arbitrarily small  $\epsilon_1$  while  $f_5$  and  $f_6$  are within a specified arbitrarily small  $\epsilon_2$ . These latter conditions are modeled as constraints in the optimization. (Again, for convenience  $\omega_1$  is used for  $f_1$  in the constraints). Thus the COFS-II optimization problem is formulated as follows:

Find the values for  $L_1$ ,  $L_2$ ,  $A$ ,  $I_{yy}$ ,  $I_{zz}$ , and  $m$  such that

$$F = \text{total mass} \rightarrow \text{minimum}$$

while satisfying the following requirements

$$g_1 = \left| \frac{\omega_4^2 - \omega_3^2}{\omega_4} \right| - \epsilon_1 \leq 0$$

and

$$g_2 = \left| \frac{\omega_6^2 - \omega_5^2}{\omega_6} \right| - \epsilon_2 \leq 0$$

where  $\epsilon_1 = \epsilon_2 = 0.01$  and  $\omega_1 = 2\pi f_1$ .

#### COFS-II Optimization Results

The initial and final values for the design variables and the objective function (mass) are given in table 5. Plots of vibration frequency as a function of design cycle are shown in figure 9. The first pair of frequencies ( $f_3$  and  $f_4$ ) are

closely-spaced after 5 design cycles. After about 16 design cycles, both pairs of frequencies are closely-spaced. The reason the optimization procedure is able to closely-space the first pair of frequencies ( $f_3$  and  $f_4$ ) so quickly but

requires 11 more cycles to closely-space the second pair of frequencies ( $f_5$  and  $f_6$ ) can be determined from examining sensitivity derivative (gradient) information. Table 6 contains the values for the derivatives of the mass (objective function), the constraint on ( $f_3-f_4$ ) frequency spacing, and the constraint on the ( $f_5-f_6$ )

frequency spacing with respect to each design variable  $V_k$  (denoted by  $\partial F/\partial V_k$ ,  $\partial g_1/\partial V_k$ , and  $\partial g_2/\partial V_k$ , respectively) for design cycles 0, 6, 7, 16, and 18. The magnitude and sign of the derivative are important. A negative value means that an increase in the design variable value will cause a decrease in the objective function (or constraint). A positive value means that an increase in the design variable value will cause an increase in the objective function (or constraint). The magnitude of the derivative also indicates to the optimizer which design variable it is more effective to change. For example, in table 6 (design cycle 0) the derivatives of the objective function and constraints can be interpreted as follows. A decrease in mast length  $L_1$  will cause a decrease in the mass, a decrease in the ( $f_3-f_4$ ) frequency spacing, and a slight increase in the ( $f_5-f_6$ ) frequency spacing. Similarly, an increase in the boom length  $L_2$  will cause an increase in the mass, a decrease in the ( $f_3-f_4$ ) frequency spacing, and a slight increase in the ( $f_5-f_6$ ) frequency spacing. From the magnitude of the derivatives of the ( $f_3-f_4$ ) frequency spacing compared to the magnitude of the derivatives of the ( $f_5-f_6$ ) frequency spacing, the optimizer will choose design variable values which closely-space the  $f_3$  and  $f_4$  frequencies and only slightly affect the ( $f_5-f_6$ ) frequency spacing (as shown in fig. 9). After 5 cycles the first pair of frequencies ( $f_3$  and  $f_4$ ) are closely-spaced. The design variable values at the end of cycle 5 are given in table 5. The optimizer decreases the mast length  $L_1$  to its lower bound ( $L_1=40$  m), increases the boom length  $L_2$  to approximately 22 meters, increases the area, decreases  $I_{yy}$  and  $I_{zz}$ , and only slightly changes the tip mass  $m$ .

With the first pair of frequencies ( $f_3$  and  $f_4$ ) closely-spaced (cycle 5, fig. 9), the sensitivity derivatives have changed to the values shown in tables 6b-e. Now the magnitude of the derivative of the ( $f_5-f_6$ ) frequency spacing with respect to  $I_{zz}$  (indicated by  $\partial g_2/\partial I_{zz}$ ) is larger compared to the magnitude of the derivative of the ( $f_3-f_4$ ) frequency spacing

(indicated by  $\partial g_1/\partial I_{zz}$ ). Thus the optimizer chooses values for the design variables which will closely-space the second pair of frequencies ( $f_5$  and  $f_6$ ) while at the same time preserving the close-spacing of the first pair of frequencies ( $f_3$  and  $f_4$ ). After 16 cycles both pairs of frequencies are closely-spaced. The final values for the design variables are given in table 5. The optimizer increases the boom length  $L_2$  to its upper bound, increases both the cross-sectional area  $A$  and  $I_{yy}$  of the boom. The design variable most effective in closely-spacing the second frequency pair ( $f_5$  and  $f_6$ ) is  $I_{zz}$ . The tip mass decreases slightly. The results are consistent with parametric studies of the design variables done at the initial and cycle 5 design values.

A plot of the mass as a function of design cycle is shown in figure 10. The optimization procedure obtains a design for a conceptual COFS-I configuration which closely-spaces two pairs of adjacent frequencies and provides some reduction in total mass (approximately 11 kg).

#### Concluding Remarks

Optimization procedures have been developed to systematically provide closely-spaced vibration frequencies for large flexible spacecraft. The optimization procedures combine a general-purpose finite-element program for eigenvalue and sensitivity analyses with formal mathematical programming techniques. The formal mathematical programming technique combines a general-purpose optimization program and approximate analyses. Analytical derivatives of the eigenvalues are used. The procedure is formulated with minimum mass as the objective function and the frequency spacing as constraints. Results are presented for two studies. The first study uses a detailed model of a large flexible spacecraft. The structure is to be designed so that it will have one pair of closely-spaced frequencies while satisfying additional requirements on local member frequencies and manufacturing tolerances. No feasible design solution existed which satisfies all the design requirements for the choices of design variables and the upper and lower design variable values used. Therefore, there is a need for more design freedom in the optimization procedure in order to achieve a full satisfactory design. The second study uses a simple model of a large flexible structure to obtain a design with more than one pair of closely-spaced frequencies. Application of the procedure produced a design which had two pairs of closely-spaced frequencies.

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Table 1 - Upper and lower design variable values for COFS-I flight mast

	Upper (mm)	Lower (mm)
$R_s$	9.4	3.175
$R_w$	9.4	3.175
$R_I$	9.4	3.175
$R_o$	11.0	3.175

Table 2 - Initial and final design values for COFS-I flight mast

	Initial	Final
$R_s$ (mm)	5.20	3.176
$R_w$ (mm)	7.35	3.433
$R_I$ (mm)	8.95	9.399
$R_o$ (mm)	9.55	10.320
Mass (kg)	93726	93765

Table 3 - COFS-II properties

	Mast	Boom
$E$ (GN/m <sup>2</sup> )	137.9	72.14
$A$ (m <sup>2</sup> )	1.305E-3	D.V.*
$J$ (m <sup>4</sup> )	1.385E-5	1.25E-6
$G$ (GN/m <sup>2</sup> )	68.94	36.2
$\rho$ (kg/m <sup>3</sup> )	2758.7	2760
$I_{yy}$ (m <sup>4</sup> )	1.552E-4	D.V.*
$I_{zz}$ (m <sup>4</sup> )	1.821E-4	D.V.*

\* D.V. denotes a design variable varied to obtain optimum design.

Table 4 - Upper and lower design variable values for COFS-II model

	Upper	Lower
$L_1$ (m)	60.0	40.0
$L_2$ (m)	25.0	1.0
$A$ (m <sup>2</sup> )	2.0E-4	2.0E-6
$I_{yy}$ (m <sup>4</sup> )	5.0E-5	1.0E-5
$I_{zz}$ (m <sup>4</sup> )	5.0E-5	1.0E-5
$m$ (kg)	30.0	10.0



Table 5 - Initial, intermediate, and final design values for COFS-II model

	Initial	Cycle 5	Final
L <sub>1</sub> (m)	45.0	40.0	40.0
L <sub>2</sub> (m)	18.0	21.9	25.0
A (m <sup>2</sup> )	9.426E-5	1.244E-4	1.806E-4
I <sub>yy</sub> (m <sup>4</sup> )	3.713E-5	3.078E-5	3.673E-5
I <sub>zz</sub> (m <sup>4</sup> )	3.713E-5	3.314E-5	1.919E-5
m (kg)	18.0	18.4	17.3
Mass (kg)	184.7	170.5	173.7

Table 6 - Sensitivity information for COFS-II model

a) Cycle 0

Design Variable	$\frac{\partial F}{\partial v_k}$	$\frac{\partial g_1}{\partial v_k}$	$\frac{\partial g_2}{\partial v_k}$
L <sub>1</sub>	3.6	0.23	-0.006
L <sub>2</sub>	0.26	-0.52	0.008
A	4680.	-98.8	-20.1
I <sub>yy</sub>	0.	7013.	-2883.
I <sub>zz</sub>	0.	-48.2	38.5
m	1.	-0.16	.007

b) Cycle 6

Design Variable	$\frac{\partial F}{\partial v_k}$	$\frac{\partial g_1}{\partial v_k}$	$\frac{\partial g_2}{\partial v_k}$
L <sub>1</sub>	3.6	0.047	-0.003
L <sub>2</sub>	0.34	-0.72	0.002
A	60872.	-221.8	-137.4
I <sub>yy</sub>	0.	18704.	-2172.
I <sub>zz</sub>	0.	-124.4	446.1
m	1.	-0.03	.005

c) Cycle 7

Design Variable	$\frac{\partial F}{\partial v_k}$	$\frac{\partial g_1}{\partial v_k}$	$\frac{\partial g_2}{\partial v_k}$
L <sub>1</sub>	3.6	-0.057	-0.0006
L <sub>2</sub>	0.36	0.07	-0.003
A	63916.	284.4	-385.5
I <sub>yy</sub>	0.	-20803.	2036.
I <sub>zz</sub>	0.	174.2	1319.5
m	1.	0.03	.005

Table 6 - concluded

d) Cycle 16

Design Variable	$\frac{\partial F}{\partial v_k}$	$\frac{\partial g_1}{\partial v_k}$	$\frac{\partial g_2}{\partial v_k}$
L <sub>1</sub>	3.6	0.049	0.062
L <sub>2</sub>	0.55	-0.061	-0.099
A	69000.	-292.2	-2886.
I <sub>yy</sub>	0.	16776.	-2943.
I <sub>zz</sub>	0.	-540.	33315.
m	1.	-0.03	.001

e) Cycle 18

Design Variable	$\frac{\partial F}{\partial v_1}$	$\frac{\partial g_1}{\partial v_1}$	$\frac{\partial g_2}{\partial v_1}$
L <sub>1</sub>	3.6	-0.048	0.063
L <sub>2</sub>	0.50	0.06	-0.101
A	69000.	298.	-3311.
I <sub>yy</sub>	0.	-17248.	-2908.
I <sub>zz</sub>	0.	591.	36702.
m	1.	0.03	.002

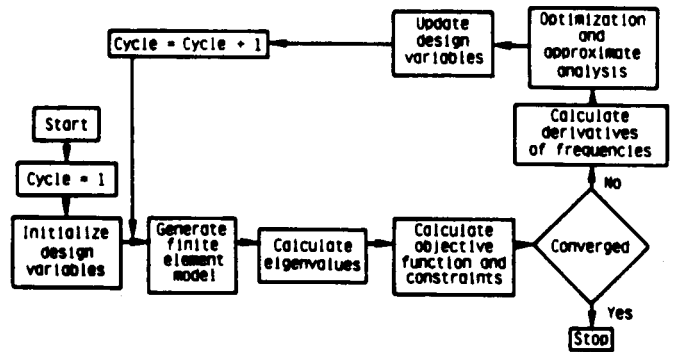


Fig. 1 Optimization procedure flowchart.

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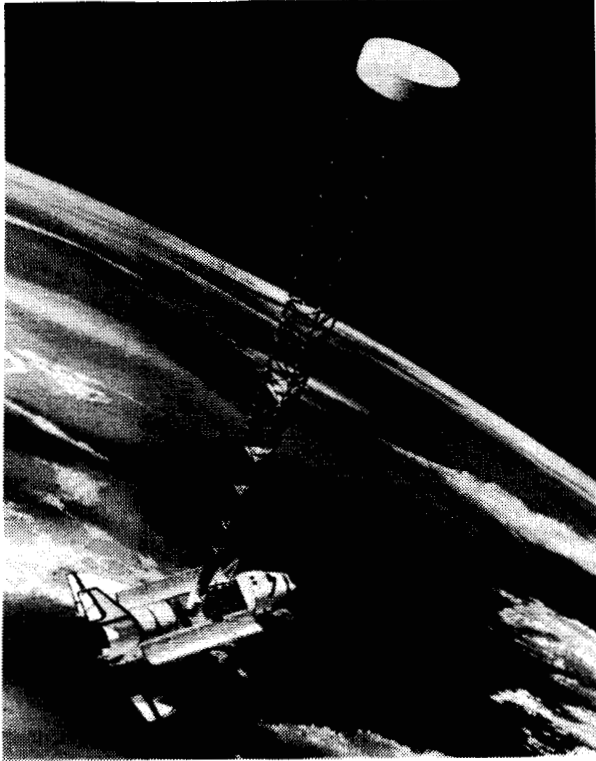
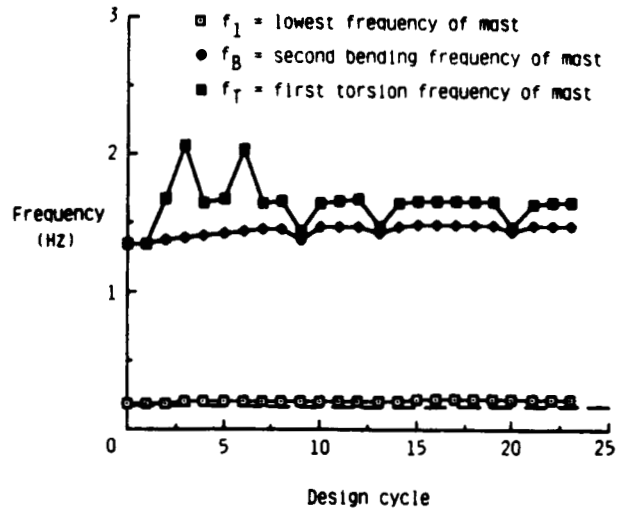
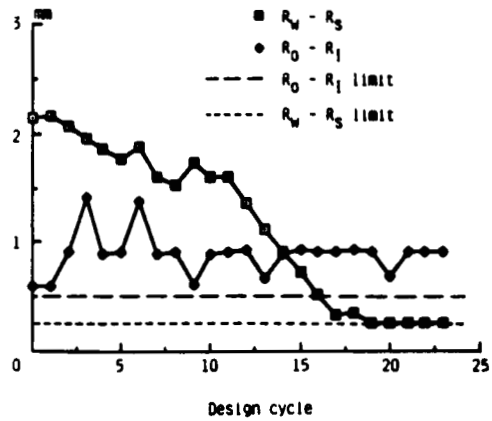


Fig. 2 COFS-I flight mast.



4a) First natural, second bending, and first torsion frequencies.



4b) Diagonal wall thickness and [\"strong\" - \"weak\"] longeron requirements.

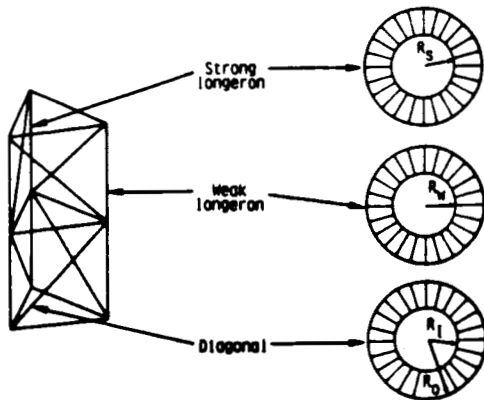
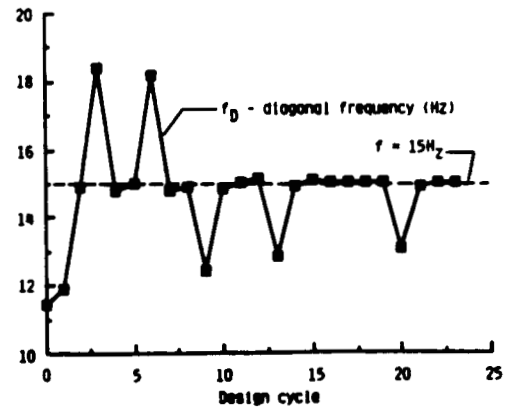


Fig. 3 Finite-element model of typical 2-bay segment of COFS-I flight mast.



4c) Diagonal frequency.

Fig. 4 Convergence of COFS-I flight mast design as a function of design cycle.

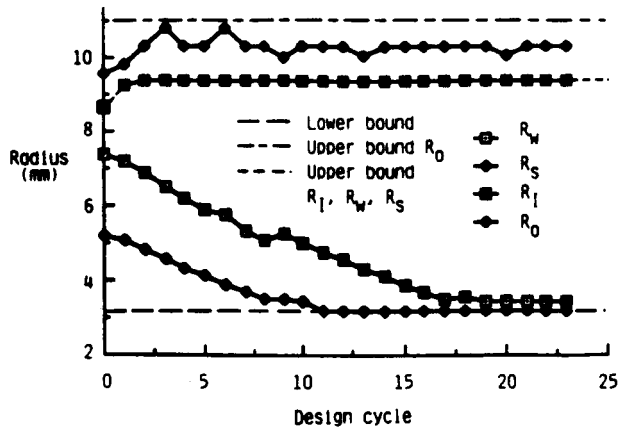


Fig. 5 Design variable history for COFS-I flight mast.

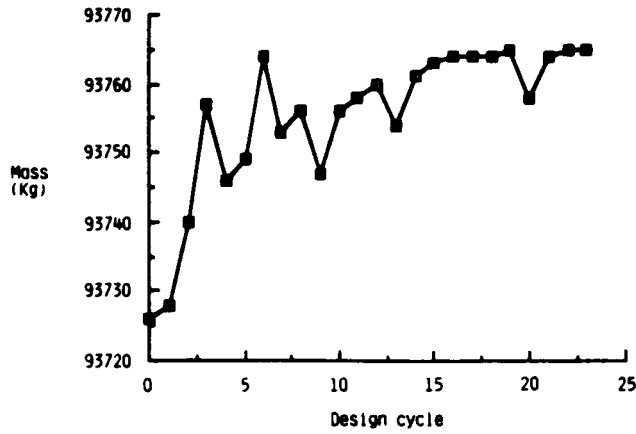


Fig. 6 Convergence of mass for COFS-I flight mast (includes Shuttle mass of 92389 kg).

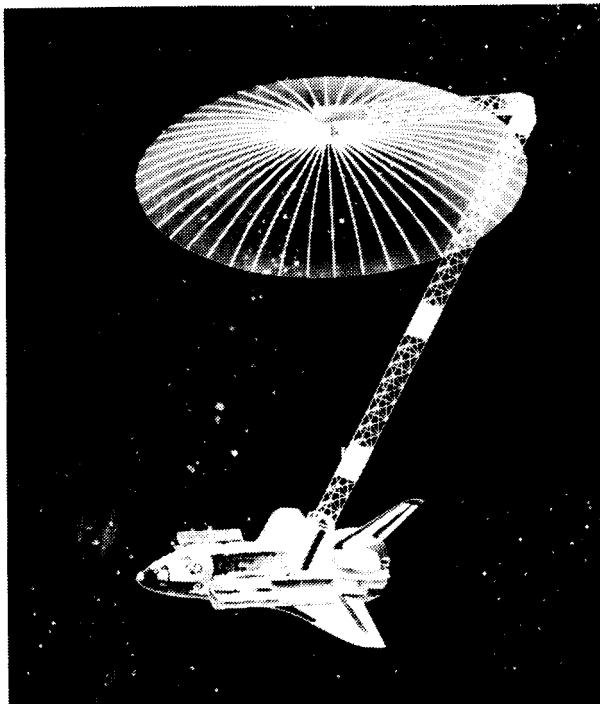
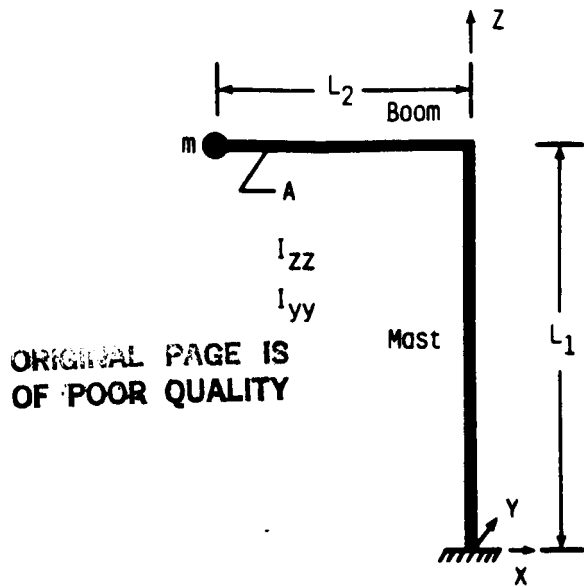


Fig. 7 COFS-II candidate configuration.



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Fig. 8 Mathematical model of COFS-II candidate configuration.

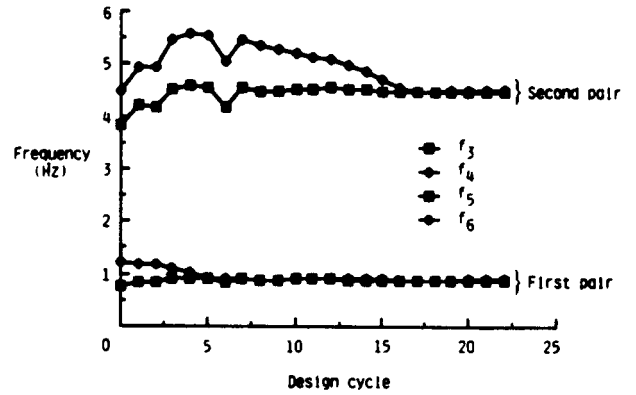


Fig. 9 Convergence of frequency pairs for COFS-II study.

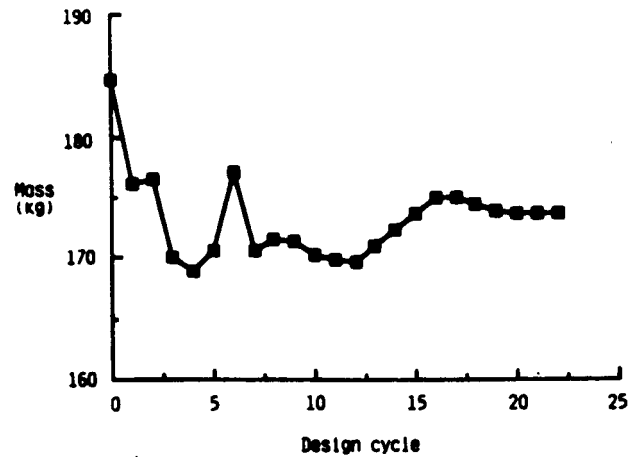


Fig. 10 Mass as a function of design cycle for COFS-II study.

Standard Bibliographic Page

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16. Abstract  As spacecraft structural concepts increase in size and flexibility, the vibration frequencies become more closely-spaced. The identification and control of such closely-spaced frequencies present a significant challenge. To validate system identification and control methods prior to actual flight, simpler space structures will be flown. To challenge the above technologies, it will be necessary to design these structures with closely-spaced or coupled vibration modes. Thus, there exists a need to develop a systematic method to design a structure which has closely-spaced vibration frequencies. This paper describes an optimization procedure which is used to design a large flexible structure to have closely-spaced vibration frequencies. The procedure uses a general-purpose finite element analysis program for the vibration and sensitivity analyses and a general-purpose optimization program. Results are presented from two studies. The first study uses a detailed model of a large flexible structure to design a structure with one pair of closely-spaced frequencies. The second study uses a simple equivalent beam model of a large flexible structure to obtain a design with two pairs of closely-spaced frequencies.			
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