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BIOT THEORY AND ACOUSTICAL PROPERTIES OF HIGH POROSITY FIBROUS MATERIALS AND PLASTIC FOAMS

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BIOT THEORY AND ACOUSTICAL PROPERTIES OF HIGH POROSITY FIBROUS MATERIALS AND PLASTIC FOAMS

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1. Introduction

On an experimental level, the absorption and propagation of sound waves in fibrous materials are well described by the empirical laws of Delany and Bazley [1, 2]. However, these laws do not account for the effect of vibration of the fibrous structure when it is significant, and are only valid in a limited frequency range.

The acoustical properties of plastic foams are much /222 less well known; the only systematic study comparable to that of Delany and Bazley, done by Cummings [3], has bearing on only one type of material. On a theoretical level, there are numerous studies. Attenborough did a very complete review of work done up to 1981 [4].

We refer in particular to studies done by Sides, Attenborough, and Mullholland [5] and by Moore and Lyon [6], using the theories of Zwikker and Kosten [7] and of Biot [8, 9] with some modifications.

Sides [5] uses Biot's theory [8] without the mass coupling parameter and comes up with a good approximation of Delany's and Bazley's laws [1]. Moore [6] brings out analogies between Biot's theory [8] and that of Zwikker and Kosten [7] and uses a description of the porous materials which is quite close to these two theories; however, no reference is made to experimental results.

*Numbers in the margin indicate pagination in the foreign text.

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We have developed a general formalism based on Biot's theory. This formalism allows for transmission of vibrations by a material used for thermal insulation, consisting of a glass wool layer between two layers of plaster. It also allows for sonic absorption by a panel installed on a rigid surface.

We have analyzed experimental results on fibrous materials and two plastic foams, and it was possible to reveal the effect of the mass coupling parameter and quarter-wave resonances of the structure.

2. Description of the propagation of sound in high-porosity materials according to Biot's theory

2.1. Stress-deformation equations

The material is assumed to be of elastic structure. It is also assumed to be homogeneous and isotropic on the wavelength scale. The quantities below are meaningful only in this macroscopic context.

Let σ_{ij} and e_{ij} be the components of the stress and deformation tensors for the structure; e and ϵ the expansions of the structure and of the air in the material.

Stresses are the forces per unit of surface of material acting on the structure.

Making p_f the pressure in the fluid, it is possible also to define a normal force per unit of surface of material acting on the gas:

$$s = -hp_f \tag{1}$$

h is the porosity of the material.

From Biot's energy considerations [8], the following equations between normal stresses and deformations were obtained:

$$\sigma_{xx} = 2N e_{xx} + A e + Q \varepsilon$$

$$\sigma_{yy} = 2N e_{yy} + A e + Q \varepsilon$$

$$\sigma_{zz} = 2N e_{zz} + A e + Q \varepsilon$$

$$s = Q e + \varepsilon R$$
(2)

where N is the shearing module of the structure. Biot and Willis [10] proposed two experiments to measure two intermediate quantities for evaluation of A, Q, and R.

The first quantity is the compressibility \varkappa of a sample of fibrous or porous material enveloped by a supple membrane pierced by a hole so that the pressure in the sample remains constant and equal to the exterior pressure during action on the envelope.

The second quantity is the compressibility δ of an unenveloped sample during variation of exterior pressure, i.e. the pressure of the material making up the structure, e.g. that of plain glass for glass wool. The experiments proposed by Biot are very symbolic in a dynamic regime, but the representation of \varkappa and δ remains as stated above.

For high-porosity materials like most glass or stone wools, or the foams studied for which bubble walls are in large part destroyed, δ is negligeable before \varkappa and compressibility $1/\chi$ of the air, and it can be stated that:

 $\delta = 0. \tag{3}$

On the other hand, it can be assumed, as was shown for fibrous materials [4], that the Poisson coefficient v of the structure is zero for all the high-porosity materials studied. It is therefore possible to write:

$$1/\chi = 2N/3.$$
(4)

The dynamic shearing module N can be measured at atmospheric pressure, because the effect of the air in the porous material is only a slight increase in the weight of the structure.

The general equations expressing A, Q, and R as a function of h, N, \mathcal{H} , and δ [10] are simplified if h is made equal to 1 and if equations (3) and (4) are verified. The following is obtained:

$$A = 0, Q = 0, R = 1/\gamma.$$
 (5)

According to [7], γ can be written:

$$\chi = 1.4 \cdot 10^5 \left(1 + \frac{0.8}{0.86 \,\mu \, \sqrt{j}} \frac{J_1(0.86 \,\mu \, \sqrt{j})}{J_0(0.86 \,\mu \, \sqrt{j})} \right).$$

In this expression, μ is the quantity:

$$\mu = \sqrt{8\omega k \varrho_0 / h \sigma} \,. \tag{6}$$

- σ is the specific resistance of the material to the passage of air.
- ω is the pulsation of the sound wave.
- ϱ_0 is the volumic mass of the air.

k is the shape factor from Zwikker and Kosten [7].

This factor, for a material with parallel cylindrical pores of the same diameter, is the inverse of the square of the cosine of the pore generator angle with normal on the surface. It is associated with the average inclination of the airflow with respect to normal on the surface.

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2.2. Equations for movement and quantities characterizing the propagation of sound waves

Let:

 ϱ_1 be the structure mass per unit of volume of material.

 ℓ_2 be the air mass per unit of volume of material

$$\varrho_2 = h \varrho_0$$
.

*Q*_a be the mass coupling parameter introduced by Biot [8] based on energy considerations.

$$\varrho_{11} = \varrho_1 + \varrho_a$$
$$\varrho_{22} = \varrho_2 + \varrho_a$$
$$\varrho_{12} = - \varrho_a .$$

The paired equations for the expansion waves in Biot's theory [8] are expressed:

$$\nabla^{2}(P e + Q \varepsilon) = \frac{\partial^{2}}{\partial t^{2}}(\varrho_{11} + \varrho_{12} \varepsilon) + b \frac{\partial}{\partial t}(e - \varepsilon)$$

$$\nabla^{2}(Q e + R \varepsilon) = \frac{\partial^{2}}{\partial t^{2}}(\varrho_{12} e + \varrho_{22} \varepsilon) - b \frac{\partial}{\partial t}(e - \varepsilon).$$
(8)

The quantity $b\frac{\partial}{\partial t}(e-\epsilon)$ is a damping parameter related to the viscosity of the air.

By analogy with the porous materials with cylindrical pores, it is possible to write [5, 7]:

$$b = -\frac{\sigma}{4} \frac{\mu \sqrt{j} \frac{J_1(\mu \sqrt{j})}{J_0(\mu \sqrt{j})}}{1 - \frac{2J_1(\mu \sqrt{j})}{\mu \sqrt{j} J_0(\mu \sqrt{j})}}.$$

Moore [6] remarked that ϱ_a can be identified with the quantity $(k - 1)\varrho_0$ which appears in Zwikker and Kosten [7] (page 53):

$$\varrho_a = (k - 1)\varrho_0. \tag{9}$$

From equation 8, the two propagation constants γ_a and γ_b are obtained from the expansion waves by writing:

$$e = a_1 \exp j (\gamma x - \omega t)$$

$$\varepsilon = a_2 \exp j (\gamma x - \omega t).$$

For each of these values of γ , it is possible to obtain with one or the other of equations (3) the ratios φ_a and φ_b between the expansion amplitudes in the fluid and the solid:

$$\varphi = a_2/a_1$$
.

Finally, for each of the progressive waves associated with γ_a and γ_b , two characteristic impedances Z_1 and Z_2 are defined, which are the ratio of the forces per unit of surface for the structure and the gas to the velocities of the structure and of the gas.

In the case of a planar longitudinal wave propagating in direction 0x, the following is obtained from equations (2) and (5):

$$Z_{1}^{a} = 2N \gamma_{a}/\omega$$

$$Z_{1}^{b} = 2N \gamma_{b}/\omega$$

$$Z_{2}^{a} = R \gamma_{a}/\omega$$

$$Z_{2}^{b} = R \gamma_{b}/\omega$$
(10)

 Empirical laws of Delany and Bazley and Biot's theory evaluation of the mass coupling parameter for fibrous materials

3.1. Qualitative aspect

The propagation of sound in fibrous materials for frequencies f varying between 0.01 σ and σ can be associated with a wave in the gas of the material whose characteristic impedance and propagation constant depend only on f/ σ , according to the empirical laws of Delany and Bazley [1]. We have verified the compatibility of these empirical laws with Biot's theory [8] for a common type of glass wool with porosity h near 1, Poisson coefficient v near 0, volumic mass e_1 , and specific resistance to the passage of air σ with values:

$$Q_1 = 30 \text{ kg/m}^3$$

 $\sigma = 20,000 \text{ kg m}^{-3}\text{s}^{-1}.$

The empirical laws of Delany and Bazley [1] for this material are thus verified for f varying between 200 Hz and 20,000 Hz. For N varying between 10^2 and 10^5 N/m² and for frequencies above 200 Hz, one of the two propagation constants from /224 Biot's theiry, written as γ_g , is such that $|\varphi_g|$ is between 4 and 20. The associated wave thus propagates essentially in the gas. The characteristic impedance associated with the propagation of this wave in the gas is written as Z_2^g . The other wave has a propagation constant of γ_s and a characteristic impedance of Z_1^s , which are approximately those of the structure in a vacuum.

$$\operatorname{Re}(\gamma_{s}) = w \sqrt{\varrho_{1}/2N}$$
(11)

 $|\varphi_s|$ varies between 1 and 10⁻³. The structure thus pushes against air as it moves, but since it is much heavier than air, this does not noticeably change its movement.

This phenomenon of partial decoupling of the structure was brought to light by Zwikker and Kosten [7] and exists only for f verifying:

$$\sigma/2\pi f \varrho_1 < 1, \tag{12}$$

or, for the material considered, f > 100 Hz.

For greater values of σ/f , the preceding description is no longer valid.

The contribution of this wave is slight in experiments to measure impedances, because it is difficult for the air surrounding the structure to move it, except in the cases of quarter-wave resonances of the structure, which will be examined later. This means that Delany and Bazley [1] did not reveal two waves of different velocities or phenomena linked to vibrations of the structure.

3.2. Comparison of the values of γ_g and Z_2^g obtained using Biot's theory with those obtained using empirical laws and determination of the mass coupling parameter

The quantities γ_g and Z_2^g vary very little with the rigidity of the structure, as shown in Table 1.

We have thus arbitrarily written $N = 10^3 \text{ N/m}^2$ without this noticeably influencing later results, and, for the fibrous material considered, we have compared the predictions from Biot's theory [8] with the empirical laws of Delany and Bazley [1]. The following quantities are defined with Delany's and Bazley's notations:

 $\begin{aligned} &\mathcal{A} = \operatorname{Im}(\gamma_g), \ &\mathbb{R} = \operatorname{Re}(\mathbb{Z}_2^g), \\ &\mathcal{B} = \operatorname{Re}(\gamma_g), \ &\mathbb{X} = \operatorname{Im}(\mathbb{Z}_2^g). \end{aligned}$

These quantities depend on form factor k.

In the validity zone of the empirical laws, values of f/σ were sought for which these quantities vary significantly as a function of k. Only the quantities β and R vary noticeably, and this for high values of f/σ . Putting aside $f/\sigma = 1$, which is at the extreme end of the validity zone, in Table II we have shown the variations of β and R as a function of k for $f/\sigma = 0.1$ and $f/\sigma = 0.316$. The last column represents the values obtained using empirical laws. The speed of sound in free air is written as C_0 .

Table I. Variations of γ_g and Z_g^g as a function of N for f = 800 Hz and k = 1.3.

Shearing Module N M/m ²	Propagation Constant ^{Yg} 1	Characteristic Impedance Z9 Nm ⁻³ s
10 ⁵	0,30 · 10 ² + j0,17 · 10 ²	0,64 · 10 ³ + j 0,29 · 10 ³
10 ²	0,29 · 10 ² + j0,16 · 10 ²	0,63 · 10 ³ + j 0,29 · 10 ³

Table II. Variation of the quantities β and R as a function of k for $f/\sigma = 0.1$ and 0.316.

k	$\frac{C_0\beta}{\omega} - 1$ for flo	$\frac{R}{\rho_0 C_0} - 1$ = 0,316 kg^{-1} m^3	$\frac{C_0\beta}{\omega} - 1$ for flo	$\frac{R}{\rho_0 C_0} - 1$ = 0,1 kg ⁻¹ m ³
1	0,24	0,08	0,45	0,16
1,1	0,29	0,15	0,49	0,22
1,2	0,34	0,21	0.53	0,27
1,3	0,39	0,26	0,58	0,33
1.4	0,43	0,30	0,62	0,39
1,5	0.48	0,35	0,66	0,44
Delany e		,		
Bazley	0.20	0.12	0.43	0,29

Table III. Values of α , β , R, and X obtained with the laws of Delany and Bazley (D.B.) and Biot's theory for a 0 mass coupling parameter.

$\frac{f}{\sigma} kg^{-1} m^3$	$\frac{C_0 \alpha}{\omega}$	$\frac{C_0\beta}{\omega}-1$	$\frac{R}{\varrho_0 C_0}$	$-1 \frac{X}{\varrho_0 C_0}$	Ref.
0,0316	1,34	0,96	0,68	0,96	D.B.
	1,4	1,1	0,56	0,92	Biot
0,1	0,68	0,43	0,29	0,41	D.B.
	0,67	0,45	0,16	0,40	Biot
0,316	0,35	0,20	0,12	0,18	D.B.
	0,30	0,24	0,08	0,14	Biot

For $f/\sigma = 0.316$ and 0.1, the best agreement with experimental results is obtained for values of k between 1 and 1.2, i.e., according to equation (9), for values of e_a between 0 and 0.25 kg/m³. In fact, a mass coupling parameter other than 0 does not noticeably improve agreement with experi- /225 mental results, and for mass coupling parameters greater than 0.25 kg/m³, agreement worsens. The mass coupling parameter is thus negligeable for fibrous materials.

For $\rho_a = 0$, Table III shows the values of α , β , R, and X obtained using Biot's theory [8] and Delany's and Bazley's laws [1] within their validity range. Agreement is generally excellent.

4. Modelling of longitudinally vibrating porous materials

4.1. Chain matrix and equivalent four-pole

A thickness L of material will be represented by the fourpole in Figure 1.

The chain matrix of the four-pole makes it possible to determine forces p_1 and p_2 per unit of surface on the structure and the gas, as well as velocities v_1 and v_2 of the structure and the gas on the front face using the same quantities defined on the back face.

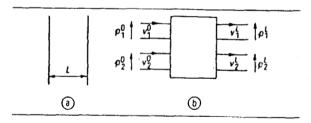
$$\begin{pmatrix} p_1^0 \\ p_2^0 \\ v_1^0 \\ v_2^0 \end{pmatrix} = \begin{pmatrix} \delta_{11} & \delta_{12} & \delta_{13} & \delta_{14} \\ \delta_{21} & \delta_{22} & \delta_{23} & \delta_{24} \\ \delta_{31} & \delta_{32} & \delta_{33} & \delta_{34} \\ \delta_{41} & \delta_{42} & \delta_{43} & \delta_{44} \end{pmatrix} \begin{pmatrix} p_1^L \\ p_2^L \\ v_1^L \\ v_2^L \end{pmatrix}$$
(13)

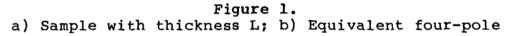
We write:

$$\Delta_1 = Z_1^a Z_2^b \varphi_b - Z_1^b Z_2^a \varphi_a$$
$$\Delta_2 = Z_1^b Z_2^a \varphi_a - Z_2^b \varphi_b Z_1^a$$

The elements of the chain matrix are expressed as follows:

$$\begin{split} \delta_{11} &= (Z_1^a Z_2^b \varphi_b \cos \gamma_a L - Z_1^b Z_2^a \varphi_a \cos \gamma_b L)/\Delta_1 \\ \delta_{21} &= (Z_2^a \varphi_a Z_2^b \varphi_b) (\cos \gamma_a L - \cos \gamma_b L)/\Delta_1 \\ \delta_{31} &= j (-Z_2^b \varphi_b \sin \gamma_a L + Z_2^a \varphi_a \sin \gamma_b L)/\Delta_1 \\ \delta_{41} &= j (-\varphi_a Z_2^b \varphi_b \sin \gamma_a L + \varphi_b Z_2^a \varphi_a \sin \gamma_b L)/\Delta_1 \\ \delta_{12} &= (Z_1^a Z_1^b) (\cos \gamma_a L - \cos \gamma_b L)/\Delta_2 \\ \delta_{22} &= (Z_2^a \varphi_a Z_1^b \cos \gamma_a L - Z_2^b \varphi_b Z_1^a \cos \gamma_b L)/\Delta_2 \\ \delta_{32} &= j (-Z_1^b \sin \gamma_a L + Z_1^a \sin \gamma_b L)/\Delta_2 \\ \delta_{32} &= j (-Z_1^a \varphi_b \sin \gamma_a L + Z_1^b \varphi_a \sin \gamma_b L)/\Delta_2 \\ \delta_{33} &= j (-Z_1^a \varphi_b \sin \gamma_a L + Z_2^b \varphi_a \varphi_b \sin \gamma_b L)/(\varphi_b - \varphi_a) \\ \delta_{33} &= j (-Z_2^a \varphi_a \varphi_b \sin \gamma_a L + Z_2^b \varphi_a \varphi_b \sin \gamma_b L)/(\varphi_b - \varphi_a) \\ \delta_{33} &= (\varphi_b \cos \gamma_a L - \varphi_a \cos \gamma_b L)/(\varphi_b - \varphi_a) \\ \delta_{34} &= (\varphi_b \cos \gamma_a L - (\cos \gamma_b L))/(\varphi_b - \varphi_a) \\ \delta_{35} &= j (Z_2^b \varphi_b \sin \gamma_b L - Z_1^a \sin \gamma_a L) \\ \delta_{44} &= \frac{j}{\varphi_a - \varphi_b} (Z_1^b \sin \gamma_b L - Z_2^a \varphi_a \sin \gamma_a L)/(\varphi_a - \varphi_b) \\ &= (\varphi_a \cos \gamma_a L - \varphi_b \cos \gamma_b L)/(\varphi_a - \varphi_b) . \end{split}$$





4.2. Chain matrix associated with a group of several superimposed layers of porous materials

The chain matrix allows easy description of the superimposed layers of porous materials.

Assume two layers A and B. At the interface of these two materials, continuity equations (14) must be verified:

$$v_{1A} = v_{1B}$$

 $p_{2A}/h_A = p_{2B}/h_B$
 $p_{1A} + p_{2A} = p_{1B} + p_{2B}$
 $h_A(v_{2a} - v_{1A}) = h_B(v_{2B} - v_{1B})$
Continuity of the structure's
velocity
Pressure continuity (14)
Continuity of the total
force per unit of surface
Continuity of air flux

We write $[\delta_A]$ and $[\delta_B]$ for chain matrices A and B and $[\delta_1]$ for the interface matrix which, according to equations (14), is expressed:

$[\delta_l] =$	[]	$1 - h_A/h_B$	0	ך 0	
	0	$h_{\rm A}/h_{\rm B}$	0	0	
	0	·0	1	0	
	0	0	$1 - h_b/h_A$	$h_{\rm B}/h_{\rm A}$	

It is the unit matrix if environments A and B have the same porosity.

The product $[\delta_A]$ $[\delta_1]$ $[\delta_B]$ is the chain matrix of group AB.

5. Effect of the structure's quarter-wave resonance on the absorption of a plastic foam panel placed on a rigid support

The presence of the structure's quarter-wave resonance can considerably modify absorption with respect to the predictions of Delany and Bazley [1]; the wave associated with the structure's vibrations can thus be of high amplitude.

For many plastic foam panels, this resonance, calculated from equation (11), is between 500 Hz and 1500 Hz.

Absorption will be obtained from impedance Z on the $\frac{226}{226}$ surface of the panel calculated by writing, in equation (13):

$$v_1^L = v_2^L = 0.$$
 (15)

For materials with porosity near 1, it is also possible to write:

$$v_2^0 = v, p_2^0 = p, p_1^0 = 0.$$
 (16)

p and v are the pressure and the acoustical velocity at the surface of the sample.

Using equations (13), (15), and (16), Z is deduced from the elements of the chain matrix and normal incidence absorption.

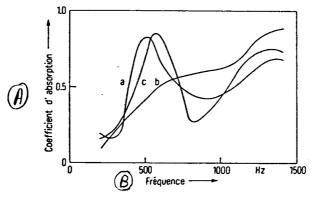


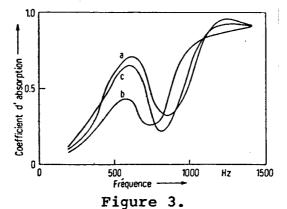
Figure 2.

Variations of the absorption coefficient of foam 1 as a function of frequency f.

Key: A - Absorption coefficient; B - Frequency.

Curve a: Experimentation,

Curve b: k = 1, zero mass coupling coefficient (Biot theory), Curve c: k = 3, mass coupling coefficient equal to twice the volumic mass of the air (Biot theory).



Variations of the absorption coefficient of foam 2 as a function of frequency f.

Key: A - Absorption coefficient; B - Frequency.

Curve a: Experimentation,

Curve b: k = 1, zero mass coupling coefficient (Biot theory), Curve c: k = 3, mass coupling coefficient equal to twice the

volumic mass of the air (Biot theory).

Absorption was calculated as a function of frequency for two plastic foams manufactured by Roth Frères (Roth Frères S.A., BP 13, 67023 Strasbourg Cedex) with porosity near 1 and Poisson coefficient assumed to be near 0.

The other characteristics of these two materials are given in Table IV.

Foa	m Thickness	Volumic mass of the structure	Specific resistance to the passage of air	Dynamic shearing module
	L m	kg/m ² 1	or arr or kg m ⁻³ s ⁻¹	N N/m ²
1 2	4.8.10 ⁻² 3.7.10 ⁻²	9.7 18	7.7.10 ³ 16.10 ³	1.4.10 ⁵ 1.4.10 ⁵

Table IV.Characteristics of foams 1 and 2.

The normal absorption calculated according to the preceding method for k = 1 and k = 3 and measured experimentally is shown as a function of frequency in Figure 2 and Figure 3. There is a certain dispersion in the experimental curves, because the measurements were made in the Kundt tube, and this method is not well adapted for supple materials [11] if the vibrations of the structure participate in absorption.

The quarter wave resonances for the two samples are localized respectively at 890 Hz and 850 Hz.

In the area of these resonances, the absorption coefficients measured are very different from those obtained with Delany's and Bazley's empirical laws [1].

With the calculation method used, the agreement between the experimental and calculated absorption coefficients is not good for k = 1, but it is much better for k = 3, i.e. when a

significant mass coupling parameter Q_a is used, on the order of twice the volumic mass of the air.

6. Conclusion

The Biot theory makes it possible to correctly predict the acoustical behavior of fibrous materials with a negligeable mass coupling parameter. Regarding plastic foams, no experimental study has been done on a large number of materials and under good conditions, i.e. in a free field on large-sized samples such that the structure can vibrate freely under conditions similar to those under which the material is used.

However, for the two samples that were studied, the $\frac{277}{277}$ Biot theory made it possible to predict absorption behavior as a function of frequency when a mass coupling parameter on the order of twice the volumic mass of the air was used.

The trajectories of air molecules around the walls of the bubbles in foams can be much more perturbed than in fibrous materials, which explains the importance of form and thus of mass coupling in foams.

Finally, note the remarkable variety of areas where the Biot theory, initially conceived to describe the behavior of rocks saturated with liquid, can be used. In addition to its applications in the area of sound waves, this theory was found to be useful in describing the propagation of ultrasonic waves, for example in glass spheres saturated with water [12].

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