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Flexibility Effects on Tooth Contact Location in Spiral Bevel Gear Transmissions

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SUMMARY

An analytical method to predict the shift of the contact ellipse between the meshing teeth in a spiral bevel gear set is presented in this report. The contact ellipse shift of interest is the motion of the nominal tooth contact location on each tooth from the ideal pitch point to the point of contact between the two teeth considering the elastic motions of the gears and their supporting shafts. This is the shift of the pitch point from the ideal, unloaded position on each tooth to the nominal contact location on the tooth when the gears are fully loaded. It is assumed that the major contributors of this motion are the elastic deflections of the gear shafts, the slopes of the shafts under load and the radial deflections of the four gear shaft bearings. The motions of the two pitch point locations on the pinion and the gear tooth surfaces are calculated in a Fortran program which also calculates the size and orientation of the Hertzian contact ellipse on the tooth faces. Based on the curvatures of the two spiral bevel gear teeth and the size of the contact ellipse, the program also predicts the basic dynamic capacity of the tooth pair. A complete numerical example is given to illustrate the use of the program.

INTRODUCTION

Spiral bevel gears are important elements for transmitting power. The design of an efficient gear box consisting of spiral bevel gears is based on gear geometry, bearing and shaft sizing, applied loading, and material properties. The gear tooth interaction is considerably more complex than that found in spur or helical gears. The loaded region of the gear mesh shifts due to the deflections and rotations of the gear and the pinion. A primary cause of these error inducing motions is the flexibility of the gear support shaft and bearings. If this shift is sufficiently large, the dynamic capacity of the gear mesh may be significantly reduced from the capacity based on the tooth curvature at the ideal pitch point.

In this report, a method to predict the shift of the loaded region on the tooth as a result of the gear tooth load and the elastic deflections of the gear support shafting and bearings is developed. The calculation of the gear mesh dynamic capacity based on the tooth curvatures, material strength and the applied load is presented also. A Fortran computer program to calculate this shift and the mesh dynamic capacity assuming constant tooth curvature is presented as well.

The gear box is modeled as a single output reduction such as those found in the OH-58 light duty helicopter. This geometry is shown in figure 1a, and is composed of a single spiral bevel gear drive with a spiral bevel pinion input and the shafts and bearings that support the gear and pinion. The analysis is based on the assumption that the transmission gear box is adequately lubricated and well designed.

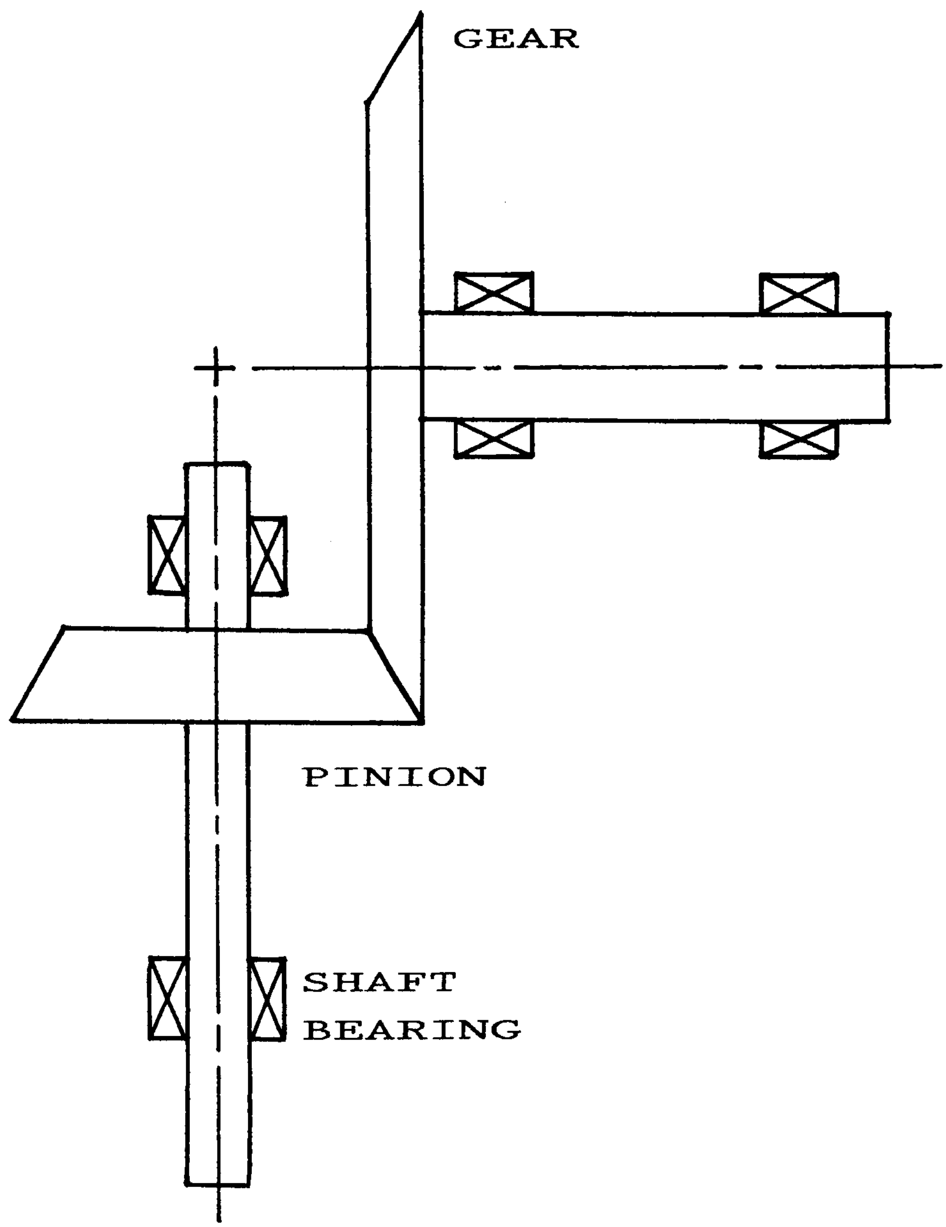


Figure 1a. Spiral Bevel Gear Unit.

Due to the interactions of the deflections of the component parts of the reduction, the analysis is performed in a modular form. In the first stage, the gear geometry and loading is defined. In the second stage, the elastic deflections of the bearings and shafts and the slopes of the shafts at the gears are determined. In the third stage, the motions of the gear teeth caused by each elastic deflection are determined. The total deflections of the gear teeth are the algebraic sum of these motions. In the fourth stage, the interaction between the tooth geometry and motion is studied to determine the prediction for the influence of these elastic motions on the gear teeth contact.

Two separate analyses are presented in this fourth stage. In the first, a relative motion vector analysis is performed to model the relative displacement of the unloaded pitch points and the change in the shaft angle at the point of contact between the two gears due to these elastic deflections. These quantities are required for the Tooth Contact Analysis [1-3] of the mesh performed by a major bevel gear manufacturer. In the second, the geometry and curvatures of the gear and pinion teeth are combined with the separate elastic motions of the two gear teeth to predict the shift of the contact ellipse under load. In this analysis, it is assumed that the curvatures are constant over the surfaces of the teeth.

Finally, the basic dynamic capacity of the mesh is calculated from the loading, tooth curvatures and material properties. The computer program can simulate a number of different support geometries with the gear and pinion separately supported either between two bearings or overhung from a bearing quill behind the gear. Several different roller and ball bearing types can also be simulated with and without preload.

When working with spiral bevel gears, it is necessary to have a thorough understanding of the basic terms that define these gears and the kinematic properties that govern the meshing of the two gears. In the classic work on spiral bevel gears, Wildhaber [1] presents much of this information. Additional papers by Baxter [2] and Coleman [3] expand on this fundamental theory and describe the basis of a Tooth Contact Analysis program which is used to analyze the kinematic action of the two spiral bevel gears in mesh, based on their manufacture.

Litvin and Coy [4] have also completed works dealing with the theory of spiral bevel gear generation, and design. They have presented two types of spiral bevel geometry, and the line of action along the bearing contact can be determined. A significant result of this work is the ability to estimate the kinematic errors induced in the transmission by errors in gear manufacturing and assembly. Litvin Rahman, and Goldrich [5] have extended this work to obtain general mathematical models for the synthesis and optimization of spiral bevel gears. In this work the tooth surface is described as a conjugate envelope of the cutter surface in its generating positions. It is possible to have two different cutters for the gear and the pinion. Their generating surfaces may both be conical or one may be conical while the second is a more general surface of revolution.

The tooth curvature analysis presented in this report is built directly upon this work of Litvin [4,5]. Direct relationships have been presented between the principal curvatures and the directions of the tool generating surfaces and the generated curvatures and directions on the bevel gear teeth. Using these relationships, the model for the contact point shift was made possible. In addition, the Hertzian contact ellipse and the mesh dynamic capacity can be found, once the tooth curvatures and directions are known. The conjugate analysis of Litvin [5] has made this possible.

Taha, Ettles, and MacPherson [6] have presented the interaction of the structural rigidity and performance for a helicopter reduction gear box. Their study used finite element models for the housing and presented the influence of deflections on bearing roller load distribution, bearing fatigue life and absolute deflections of the gear and pinion for the unloaded pitch point. This work demonstrated the importance of rigidity to minimize bearing life in a transmission.

In preliminary work to this effort, Savage, Brikmanis, Lewicki and Coy [7], and Savage, Knorr and Coy [8] have presented system reliability models for bevel gear reductions and helicopter transmissions. Both works describe analytical reliability simulations of transmission systems based on the reliabilities of the bearings and gears in the transmissions. Both works assume that the loaded geometry of the transmission is identical to the ideal unloaded geometry. This work concerns itself with predicting the change in gear mesh geometry due to loading.

The reliability model for gear teeth is based on the research of Coy, Townsend, and Zaretsky [9-12] and Lewicki [13]. In this model, the gears are assumed to be sufficiently well designed that bending failure at the tooth root is not a factor in the gear tooth life. The mode of failure which cannot be avoided is that of pitting near the pitch point on the gear teeth.

SPIRAL BEVEL GEAR GEOMETRY

Recently there has been extensive interest in determining the effects of loading and small profile changes on the kinematic efficiency, wear and life of spiral bevel gears. The interest originates from the desire to improve operating and maintenance procedures in high performance transmissions of helicopters and other aircraft.

In order to model the service life of these gears, a quantitative understanding of the geometrical characteristics is required.

These gears are called "spiral bevel" because the tooth face is constructed in a spiral on the pitch cone of the gear. This is done to obtain gradual engagement and disengagement of successive teeth in the mesh. The spiral angle, ψ, is nearly constant along the tooth and is generated by the circular cutter location. Spiral bevel gears are used in high performance transmissions because their curved teeth provide smoother and quieter operation.

A schematic representation of a single input bevel gear is shown in figure 1b. The pitch cone center distance, A_0, is a distance from the apex of the bevel cones to the back edge of the tooth face.

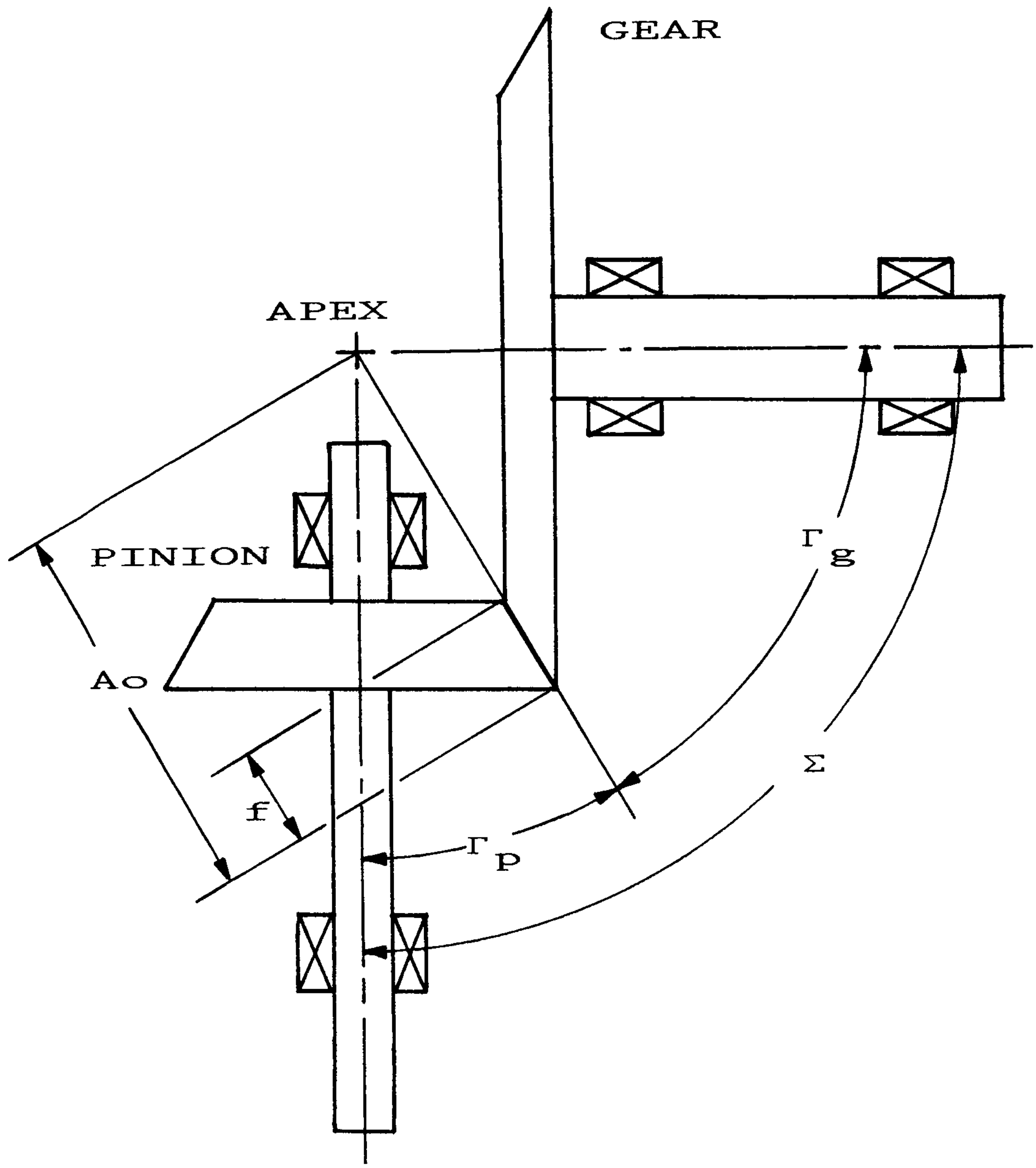


Figure 1b. Spiral Bevel Gear Geometry.

The pitch cone distance is measured along the pitch line of the two pitch cones and it is used as the major measure of the bevel size. The face width, f, is also measured along the pitch line.

The following parameters define the pitch cone geometry: The number of the gear and pinion teeth, N_g and N_p respectively, the shaft angle, Σ. The shaft angle is defined as the angle between the gear and pinion shaft. The shafts may or may not be perpendicular to each other. The pitch angles are half of the cone angle and are directly related to the above parameters in the following equations:

$$\tan \Gamma_g = \frac{\sin \Sigma}{(N_p/N_g) + \cos \Sigma} \tag{1}$$

$$\tan \Gamma_p = \frac{\sin \Sigma}{(N_g/N_p) + \cos \Sigma} \tag{2}$$

Throughout this analysis, the pitch point is considered to be the contact point of the gear mesh. The pitch point is located on the pitch ray at the midpoint of the tooth face. The distance from the cone apex to the pitch point is:

$$D_o = A_o - \frac{f}{2} \tag{3}$$

The pitch diameters of the equivalent spur gears for the spiral bevel gear and pinion are defined accordingly:

$$D_g = 2\ D_o \sin \Gamma_g \tag{4}$$

$$D_p = 2\ D_o \sin \Gamma_p \tag{5}$$

Beside the size and the shape of the pitch surface, the geometry of the meshing gear is also defined. In figure 2, the spiral bevel gear is shown at the pitch plane, which is tangent to the pitch cones at the line of contact. In the same figure, the spiral angle, ψ, is also shown. The spiral angle is defined as the angle between the pitch ray and the tangent to the circular cutter at the midpoint of the tooth, and it is measured at the mean radius of the gear.

A right-handed advance of the spiral along the axis of the gear toward the cone apex is defined as positive. The figure shows a right-hand spiral. The spiral angle is of the same magnitude in both the gear and the pinion but of opposite hands. The diametral pitch of the gear and the pinion is defined at the mean radius by the following equation:

$$P_d = \frac{N_g}{2\, D_o \sin \Gamma_g} \tag{6}$$

Equation 6 shows that the diametral pitch is a direct function of the number of the gear teeth and the pitch cone geometry.

As shown in figure 3 the normal tooth is defined at the midplane of the tooth which corresponds to the section A-A in figure 2. The geometry also includes the normal pressure angle ϕ_n, and the addendum and dedendum distances as well. The addendum and dedendum distances are defined as:

$$a_b = \frac{a}{P_d} \tag{7}$$

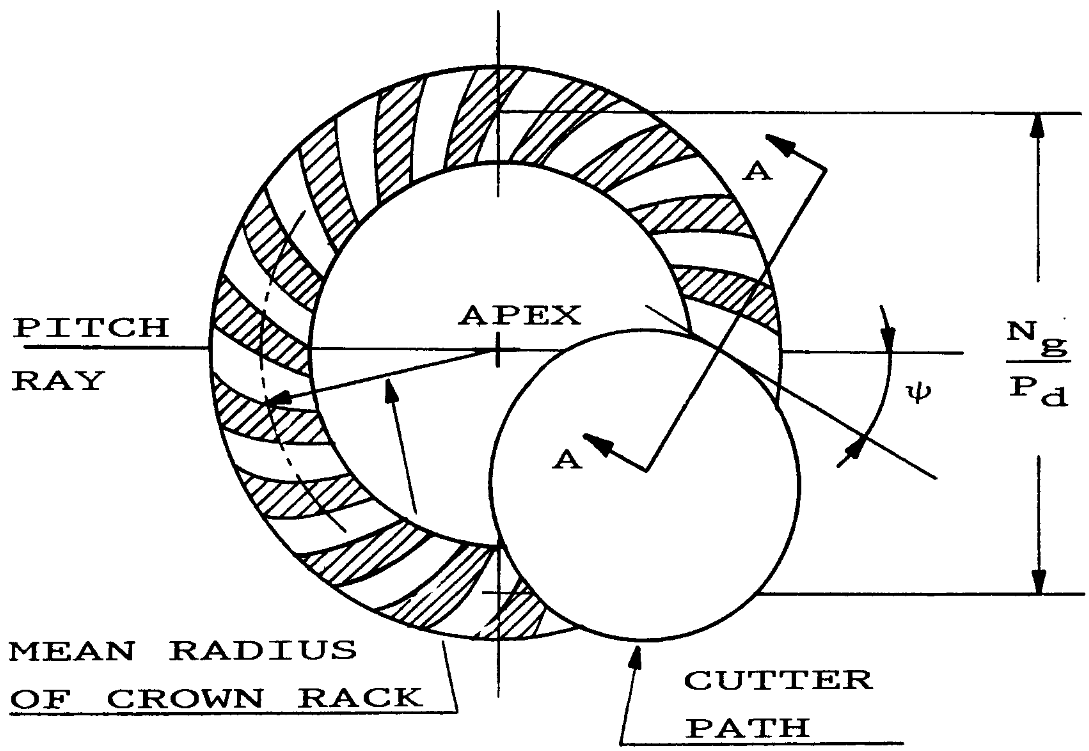
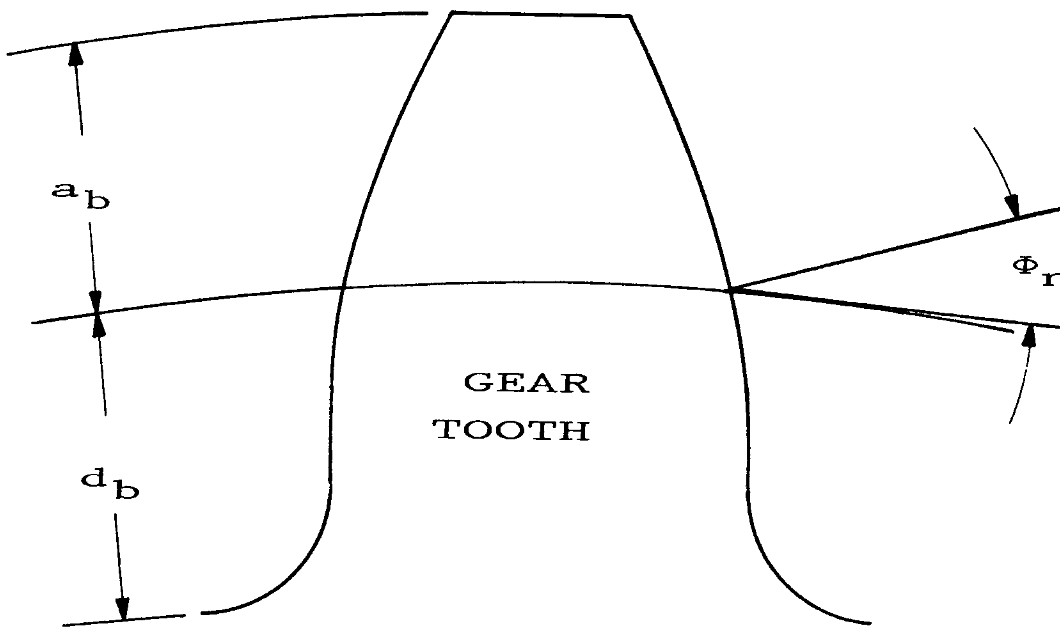


Figure 2. Spiral Angle.



SECTION A-A

Figure 3. Normal Tooth.

$$d_b = \frac{b}{P_d} \tag{8}$$

The constants a and b in equations 7 and 8, respectively, are functions of the gear type. Dudley [15] provides equations for the calculation of the gear tooth dimensions of many bevel gear types.

In this study, the teeth at the pitch point are modeled as spur gear teeth. Figure 4 shows in a drawing the correlation of the bevel and the reference spur gear. As can be seen, the radius of the reference spur gear is equal to the midcone distance. The midcone distance is defined as the perpendicular distance from the centerline of the gear shaft to the midpoint of the gear face at the pitch point. From the geometry of figure 4, the midcone distance for the gear and the pinion are defined by using the distance from the cone apex to the midpoint of the gear face, D_0, and the respective pitch angle.

$$BC_g = D_o \tan \Gamma_g \tag{9}$$

$$BC_p = D_o \tan \Gamma_p \tag{10}$$

In order to define the loads transmitted at the point of the gear contact, the direction of rotation is required. In this study, a rotation of the gear is positive for clockwise rotation when looking at the cone apex from the back of the gear at the gear shaft.

The gear assembly includes the support system of bearings and their location. The support system is defined by the position of the gear with respect to the bearing locations. Two bearing

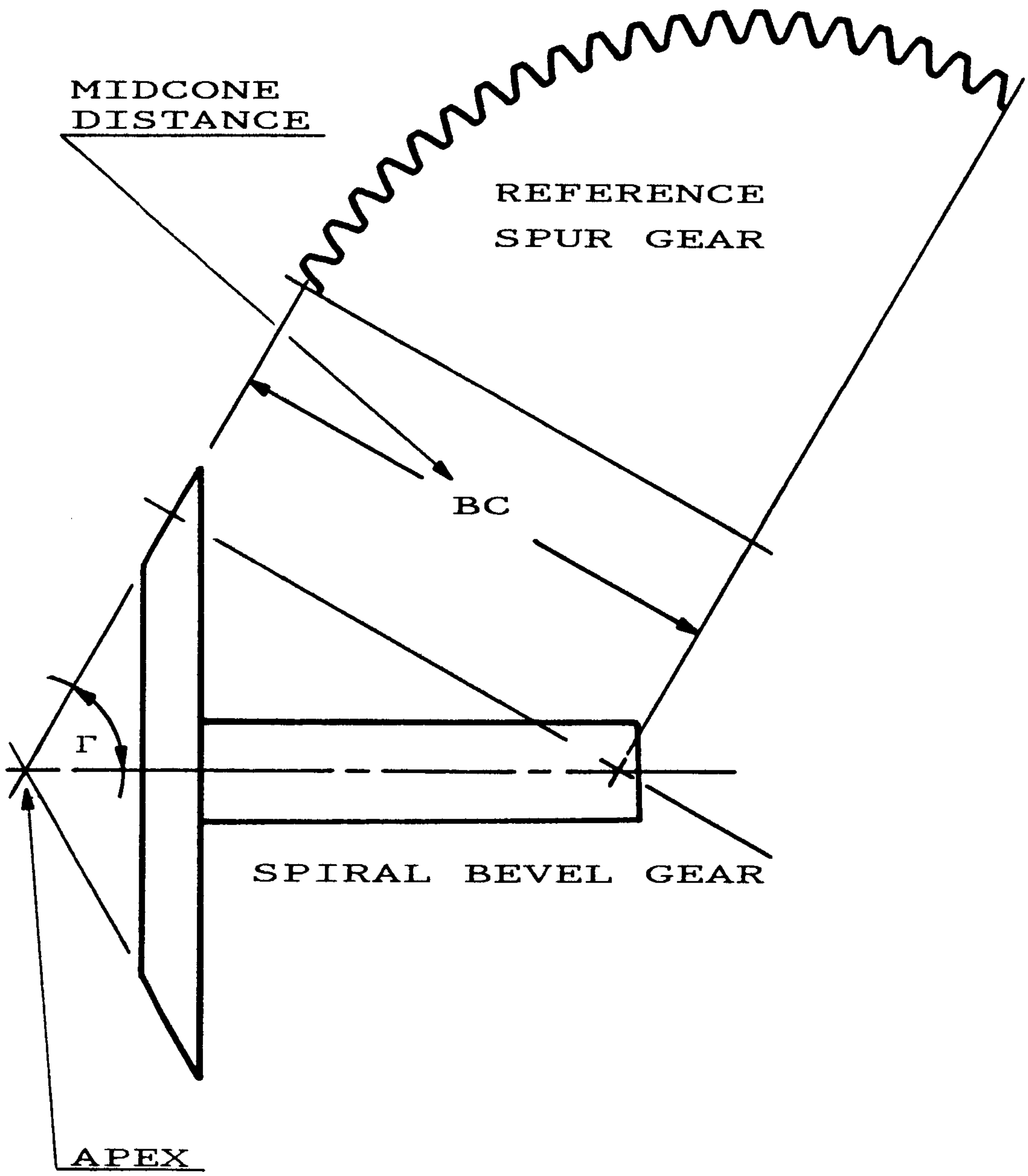
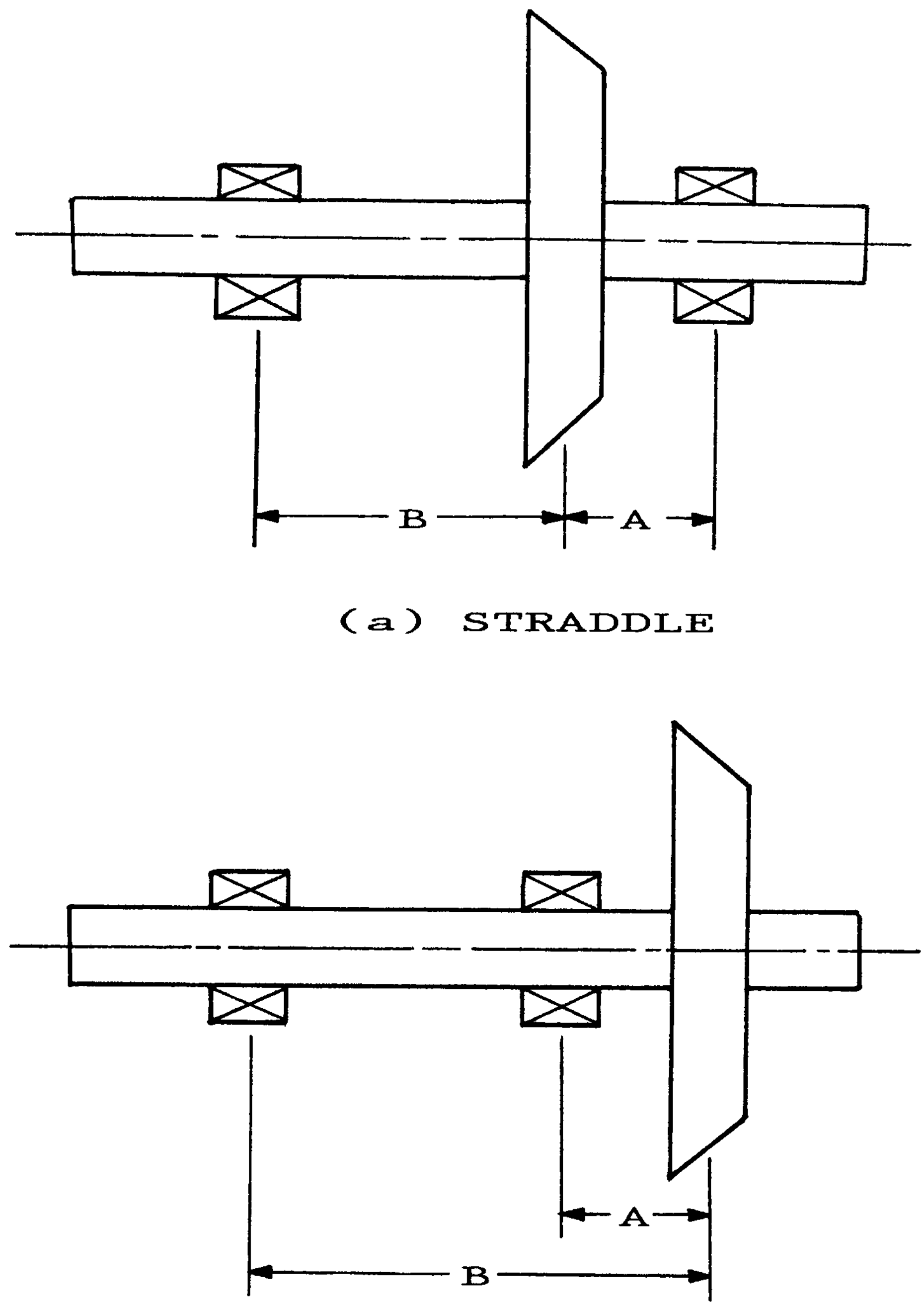


Figure 4. Composite Spiral Bevel Gear and Reference Spur Gear.

configurations are commonly used, straddle and overhung. In the first case, the bearings straddle the bevel gear, while in the second, the gear is overhung from the two bearing quill. Both configurations are shown in figure 5. In both cases, distance A is defined from the gear to the right bearing, while distance B is defined from the gear to the left bearing. The distances are measured from the midpoint of the gear to the midpoint of the bearing. It should be noted that for a straddle mounted gear, distance A is considered positive. In the overhung configuration, distance A is considered to be negative. The axial thrust loads produced by the gear mesh are carried by the bearing which is identified as the thrust bearing.



(a) STRADDLE

(b) OVERHUNG

Figure 5. Bearing Mount Configurations.

LOADING ANALYSIS

The load components may be calculated from the applied input torque and the spiral bevel gear geometry.

The total force acting normal to the gear tooth is assumed to be concentrated at the average radius of the pitch cone. This force is divided into three orthogonal components, which are aligned relative to the axis of the gear alone [16]. The three components shown in figure 6 are: The transmitted or tangential load, W_t, which produces the torque transmitted to the gear; the axial, or the thrust load, W_a; and the radial load, W_r. The axial and the radial load components are directly related to geometry of the gear teeth that transmit the tangential load. According to Shigley [16], the loads for the output gear are computed as:

$$W_t = \frac{T_g}{2(A_o - f/2)\sin\Gamma_g} \tag{11}$$

$$W_a = \frac{W_t}{\cos\psi}(\tan\phi_n \sin\Gamma_g - \sin\psi\cos\Gamma_g) \tag{12}$$

$$W_r = \frac{W_t}{\cos\psi}(\tan\phi_n \cos\Gamma_g + \sin\psi\sin\Gamma_g) \tag{13}$$

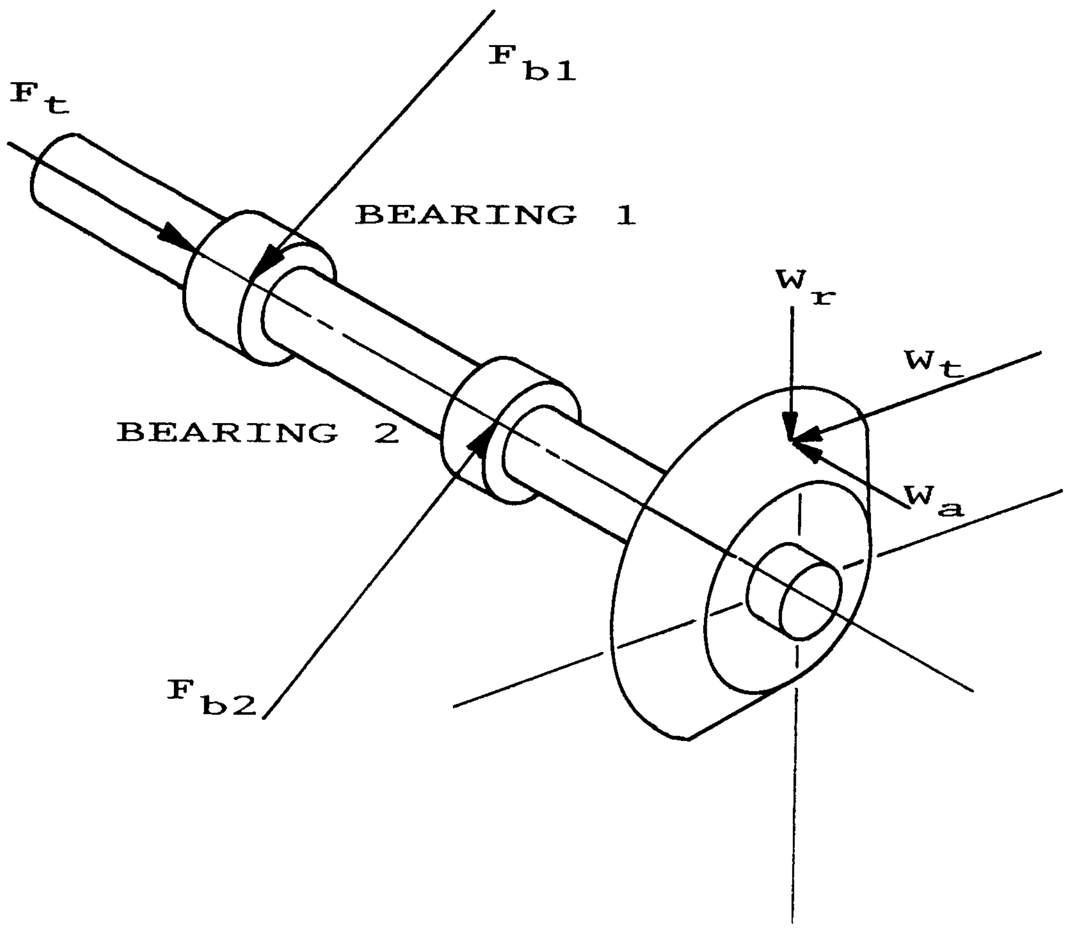


Figure 6. Spiral Bevel Gear Forces.

where T_g is the torque to the output gear. A positive sign for an axial or radial load indicates that it is directed towards the cone center.

The sign of the last term in equations 12 and 13 depends on the direction of rotation and the hand of the spiral and whether the gear is driving or being driven. There are four possible cases to be considered. These equations are valid for a right-hand spiral driving gear rotating clockwise, or a left-hand spiral rotating counterclockwise. For a driven gear, these equations are also valid for a right-hand spiral gear rotating counterclockwise or a left-hand spiral rotating clockwise. For the other four conditions with power flow in the opposite direction, the signs of the last terms in these equations are switched. The rotation is figured by looking at the gear from the side opposite the apex.

Only for right angle drives are all the force components exerted by the gear on the pinion equal and opposite. The opposite of a radial pinion load is an axial gear load, and the opposite of an axial pinion load is a radial gear load.

In all cases, however, the tangential and the total resultant tooth load of one gear must be equal and opposite to the corresponding force on the mating gear. The thrust load which acts in the axial direction of the shaft is equal to the gear axial force and is carried by the thrust bearing.

$$F_t = W_a \tag{14}$$

The normal tooth load which is the resultant of the three components is given by:

$$W_n = \sqrt{W_t^2 + W_a^2 + W_r^2} \tag{15}$$

The radial forces on the two bearings are the resultants of the radial and tangential components of the reactions acting on each bearing as shown in figure 6. By taking moments with respect to the points where the bearings are located, the bearing load components can be calculated from the gear force components of equations 11 through 13. As shown in figure 7, the bearing load in each station is resolved in two planes which are perpendicular to each other. The gear tangential component acts on the horizontal plane, while the gear radial and axial components are on the vertical plane. The development of the bearing loads is the same for both support configurations.

STRADDLE CONFIGURATION

Figure 7 shows the straddle configuration modeled as a single span, simply supported beam with a concentrated load acting on it.

Tangential Plane

Taking the sum of moments with respect to the left bearing to be equal to zero:

$$\sum M_A = 0 = F_{t2}(A + B) - W_t B \tag{16}$$

or $$F_{t2} = \frac{W_t B}{A + B} \tag{17}$$

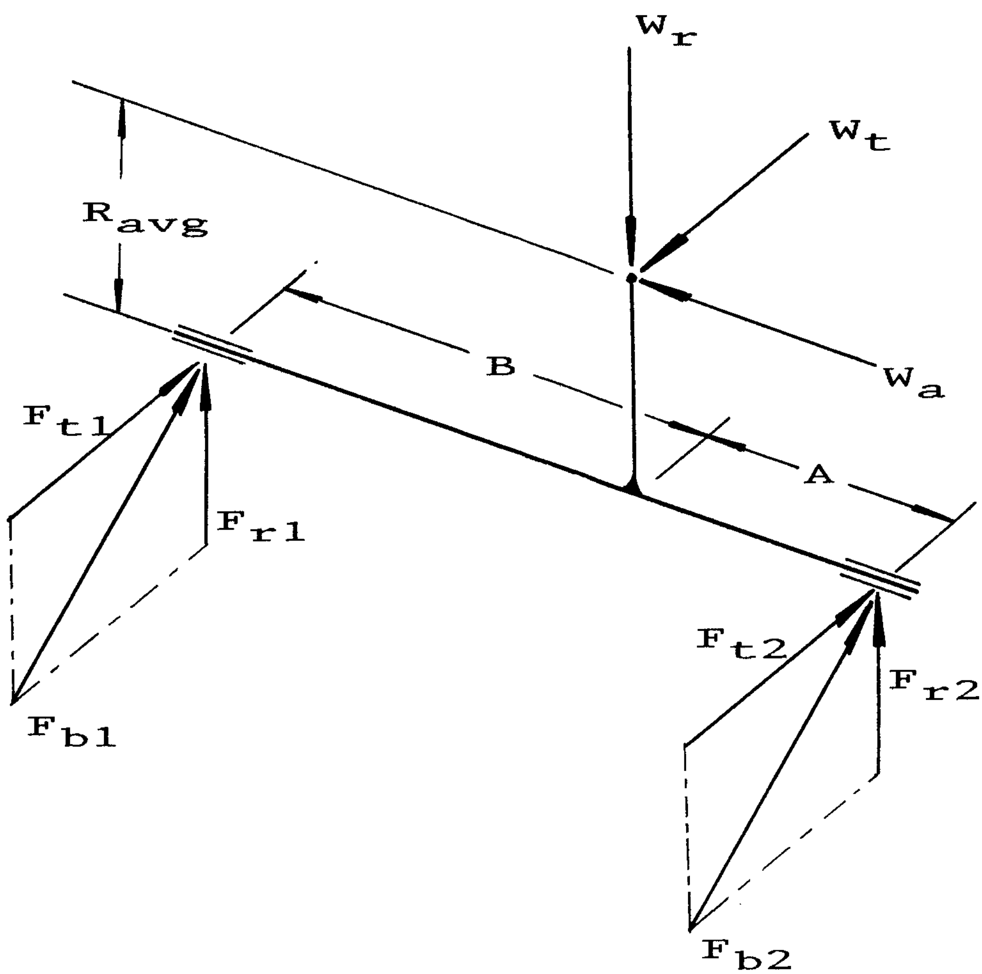


Figure 7. Force Analysis - Straddle Configuration.

Similarly, take moments with respect to the right bearing in figure 7.

$$\sum M_B = 0 = F_{t1}(A + B) - W_t\, A \tag{18}$$

or $$F_{t1} = \frac{W_t\, A}{A + B} \tag{19}$$

By rewriting the equation of diametral pitch (6) as:

$$D_o \sin \Gamma_g = \frac{N_g}{2\, P_d} = R_{avg} \tag{20}$$

Equation 20 yields the average radius of the gear. The average radius is defined at the midpoint of the gear face, and is used as the moment arm of the axial load, W_a, that causes a moment load in the radial direction.

Radial Plane

In the same way the loads in the radial plane are derived:

Sum of moments with respect to left bearing:

$$\sum M_A = 0$$

$$= F_{r2}(A + B) + \left(W_a\, N_g/(2\, P_d)\right) - W_r\, B \tag{21}$$

or $$F_{r2} = \frac{-\left(W_a\, N_g/(2\, P_d)\right) + W_r\, B}{A + B} \tag{22}$$

And with respect to the right bearing:

$$\sum M_B = 0$$

$$= F_{r1}(A + B) - (W_a N_g/(2 P_d)) - W_r A \tag{23}$$

$$\text{or} \quad F_{r1} = \frac{(W_a N_g/(2 P_d)) + W_r A}{A + B} \tag{24}$$

OVERHUNG CONFIGURATIONS

The equations for overhung configurations can be found in a similar way. Figure 8 shows the gear shaft as a single span, simply supported beam with an overhang at the right end. The beam is loaded with a concentrated load at the outer end. As mentioned in the spiral bevel gear geometry section, distance A is considered to be negative for the overhung configurations.

Tangential Plane

Moments with respect to the left bearing:

$$\sum M_A = 0 = F_{t2}(B - A) - W_t B \tag{25}$$

$$\text{or} \quad F_{t2} = \frac{W_t B}{B - A} \tag{26}$$

Moments with respect to the right bearing:

$$\sum M_B = 0 = F_{t1}(B - A) - W_t A \tag{27}$$

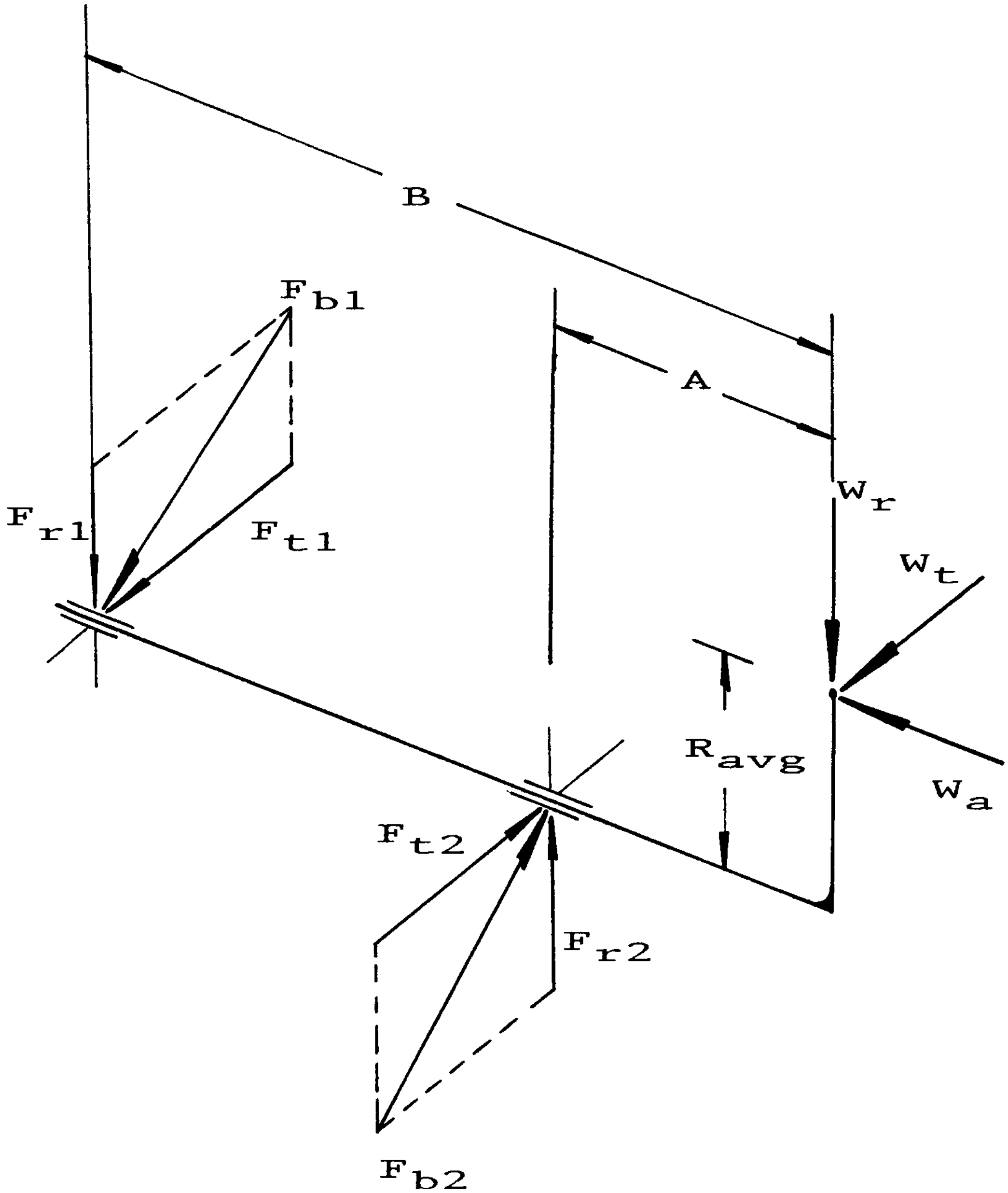


Figure 8. Force Analysis - Overhung Configuration.

$$\text{or} \quad F_{t1} = \frac{W_t\ A}{B - A} \tag{28}$$

Radial Plane

Similarly:

$$\sum M_A = 0$$

$$= F_{r2}(B - A) + (W_a\ N_g/(2\ P_d)) - W_r\ B \tag{29}$$

$$\text{or} \quad F_{r2} = \frac{-(W_a\ N_g/(2\ P\ )) + W_r\ B}{B - A} \tag{30}$$

$$\sum M_B = 0$$

$$= F_{r1}(B - A) + (W_a\ N_g/(2\ P_d)) - W_r\ A \tag{31}$$

$$\text{or} \quad F_{r1} = \frac{-(W_a\ N_g/(2\ P_d)) + W_r\ A}{B - A} \tag{32}$$

The following equations give the combined load for each bearing station. It should be noted that these equations hold true for both bearing configurations.

Combined Left Bearing Load

$$F_{b1} = \sqrt{F_{t1}^{\ 2} + F_{r1}^{\ 2}} \tag{33}$$

Combined Right Bearing Load

$$F_{b2} = \sqrt{F_{t2}^{\ 2} + F_{r2}^{\ 2}} \tag{34}$$

These equations are valid for any gear loaded by a single pinion.

DEFLECTIONS AND SLOPES

The motion of a gear assembly may be considered to be composed of three components. These components are the shaft motion, gear body motion, and bearing motion. The first two will be analyzed in this section, while the last will be treated along with the bearing analysis.

Using simple beam theory, the equations for the deflections and the slopes due to the three loading components are derived [16]. Due to the nature of the load application, the three load components may be classified as two different types of load. The tangential and the radial loads are considered as concentrated loads acting on the gear shaft. The axial load is treated as a moment load, since it acts parallel to the shaft.

STRADDLE CONFIGURATION

As shown in figure 9a the gear shaft is assumed to be a simply supported beam with a concentrated load acting on it. This load may be the tangential or the radial component. According to Shigley [16], the deflection equation for this case is given by:

$$Y_i = \frac{FAx}{6\,EIL}(x^2 + A^2 - L^2) \qquad (35)$$

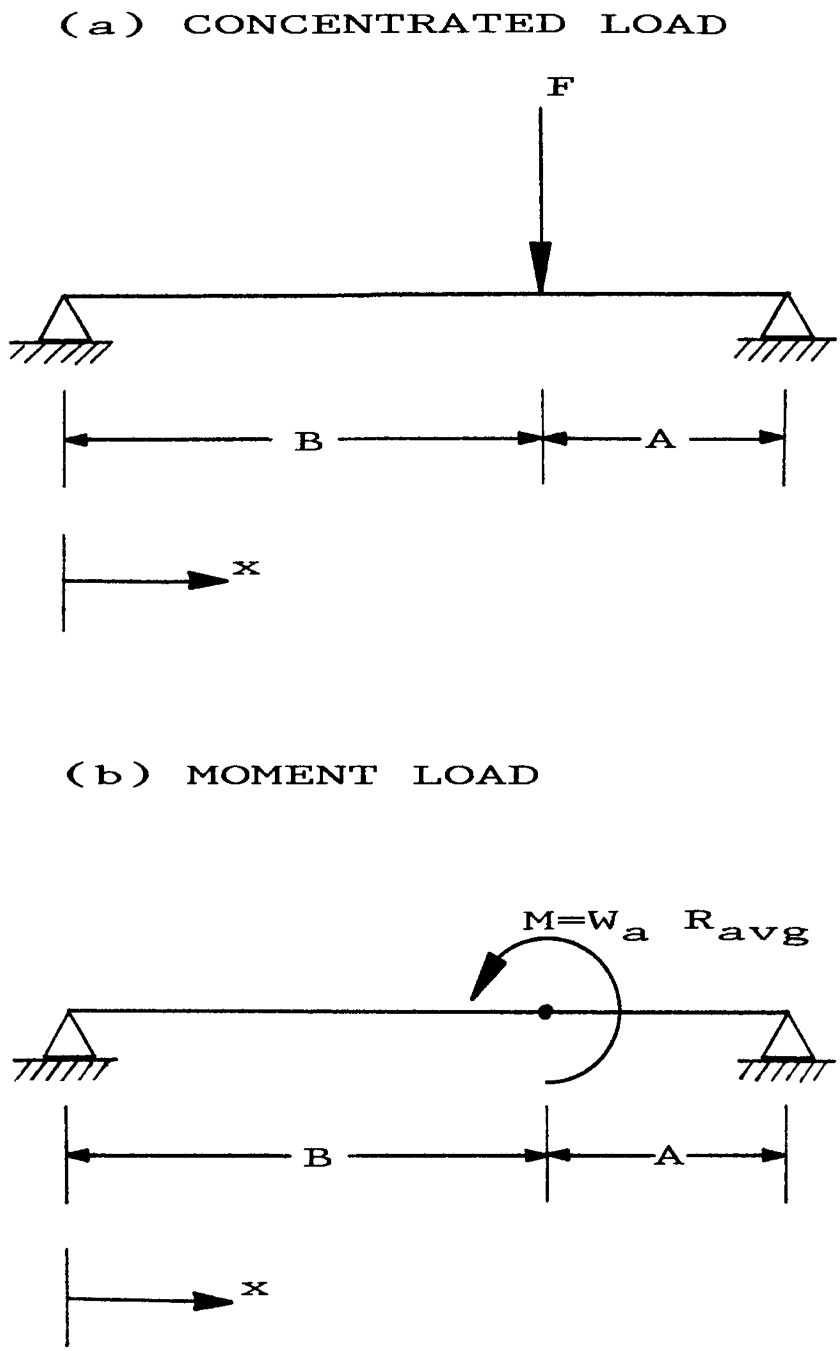


Figure 9. Straddle Configuration Loading Conditions.

Substituting x=B and L=A + B yields:

$$Y_i = \frac{F\ B^2\ A^2}{3\ EI(A + B)} \tag{36}$$

where $F = W_t$ for tangential load

$F = W_r$ for radial load

$Y_i = Y_t$ for tangential load

$Y_i = Y_r$ for radial load

The slope equation can be found by differentiating the deflection equation 35 with respect to distance x. The same slope equation is used for both tangential and radial loads.

$$\theta_i = \frac{F\ A[(x^2 + A^2 - L^2) + 2\ x^2]}{6\ EIL} \tag{37}$$

for a tangential load:

$$\theta_t = \frac{W_t\ A\ B(B - A)}{3\ EI(A + B)} \tag{38}$$

and for a radial load:

$$\theta_r = \frac{W_r\ A\ B(B - A)}{3\ EI(A + B)} \tag{39}$$

Similarly, the deflection equation due to axial load is derived using equation 40 as given by Shigley [16]. The axial load in figure 9b acts along the gear shaft direction, thus creating a moment load at the point of application with a moment arm equal to the average radius of the gear at the midpoint of the face width.

$$Y_a = \frac{M\,x}{6\,EIL}(x^2 + 3\,B^2 - 6\,BL + 2\,L^2) \tag{40}$$

Substituting $x = B$ and $L = A + B$ and $M = W_a \cdot R_{avg}$ yields

$$Y_a = \frac{W_a\,R_{avg}\,AB(B - A)}{3\,EI\,(A + B)} \tag{41}$$

The equation giving the slope due a moment load is calculated from the derivative of equation 40 with respect to x as:

$$\theta_a = \frac{M[(x^2 + 3\,B^2 - 6\,BL + 2\,L^2) + 2\,x^2]}{6\,EIL} \tag{42}$$

where $x = B$ and $L = A + B$ and $M = W_a \cdot R_{avg}$,

$$\theta_a = \frac{W_a\,R_{avg}(A^2 + B^2 - AB)}{3\,EI(A + B)} \tag{43}$$

OVERHUNG CONFIGURATION

The analysis for the overhung configuration is similar to the straddle case. However, the loading conditions are different and therefore the approach to derive the deflection and slope equations varies as well.

The gear shaft is treated as a cantilever beam with two types of loads at the free end. One concentrated load for the tangential and radial components and one moment load for the axial component. The following analysis is the same for both types of loads.

As shown in figure 10, the gear shaft is modeled as a single span, simply supported beam with overhang, and is loaded by a concentrated load at the outer end. The portion of the beam which is outside the bearing quill is considered as a cantilever beam. The total deflection at the free end is the algebraic sum of two discrete deflections.

The first deflection is given by equation 44, and it represents a pure deflection of a cantilever beam with a moment load at the free end.

$$D_1 = \frac{FA^3}{3\,EI} \tag{44}$$

The slope is found by differentiating equation 44, and is

$$S_1 = \frac{FA^2}{EI} \tag{45}$$

As shown in figure 10, the load acting at the free end also creates a rotation at the end of the cantilever. This rotation contributes to the total deflection by creating a slope to the whole beam, and is shown in equation 46.

$$S_2 = \frac{ML}{3\,EI} \tag{46}$$

where $M = F\,A$ and $L = B - A$.

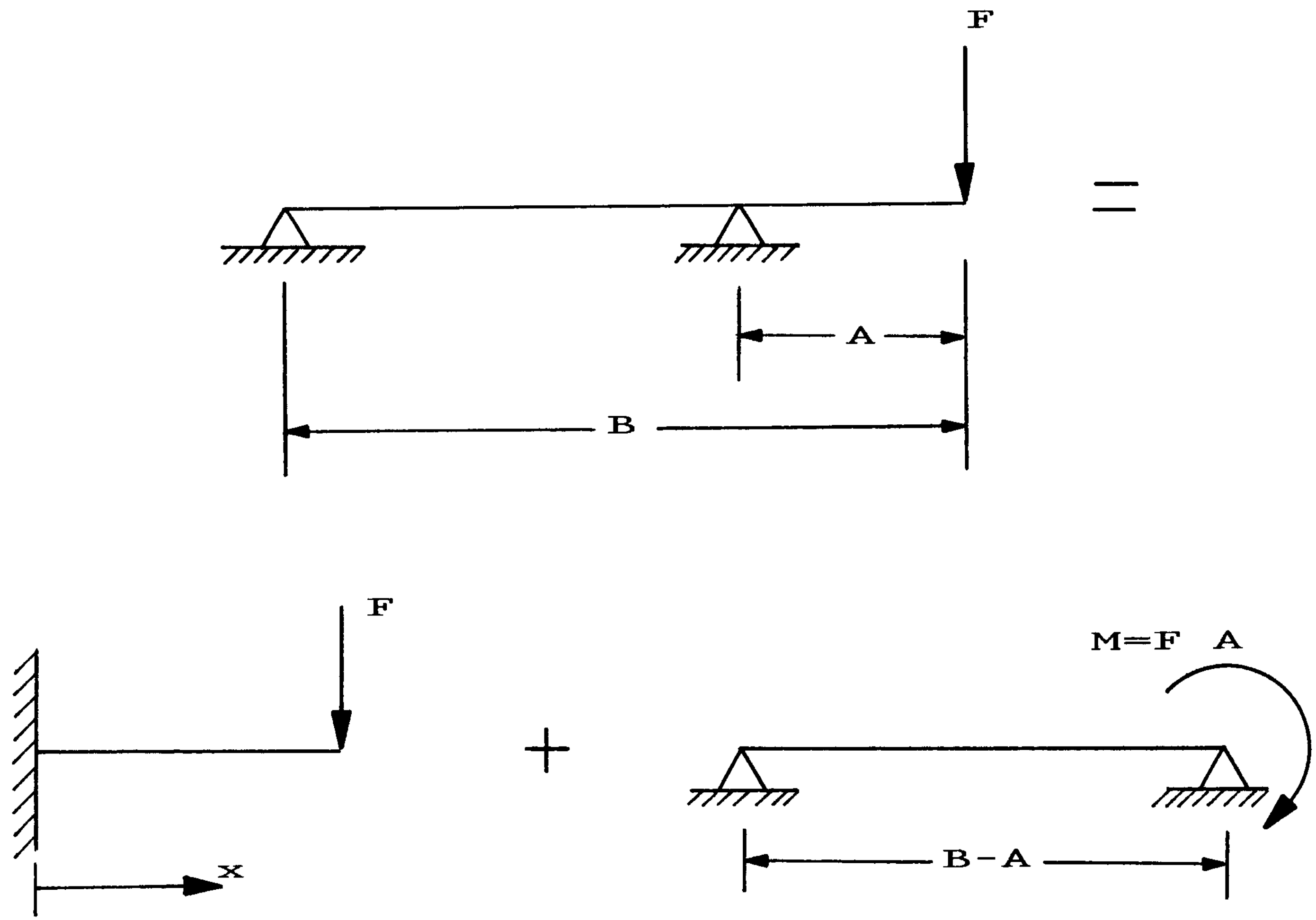


Figure 10. Overhung Configuration
Concentrated Load Analysis

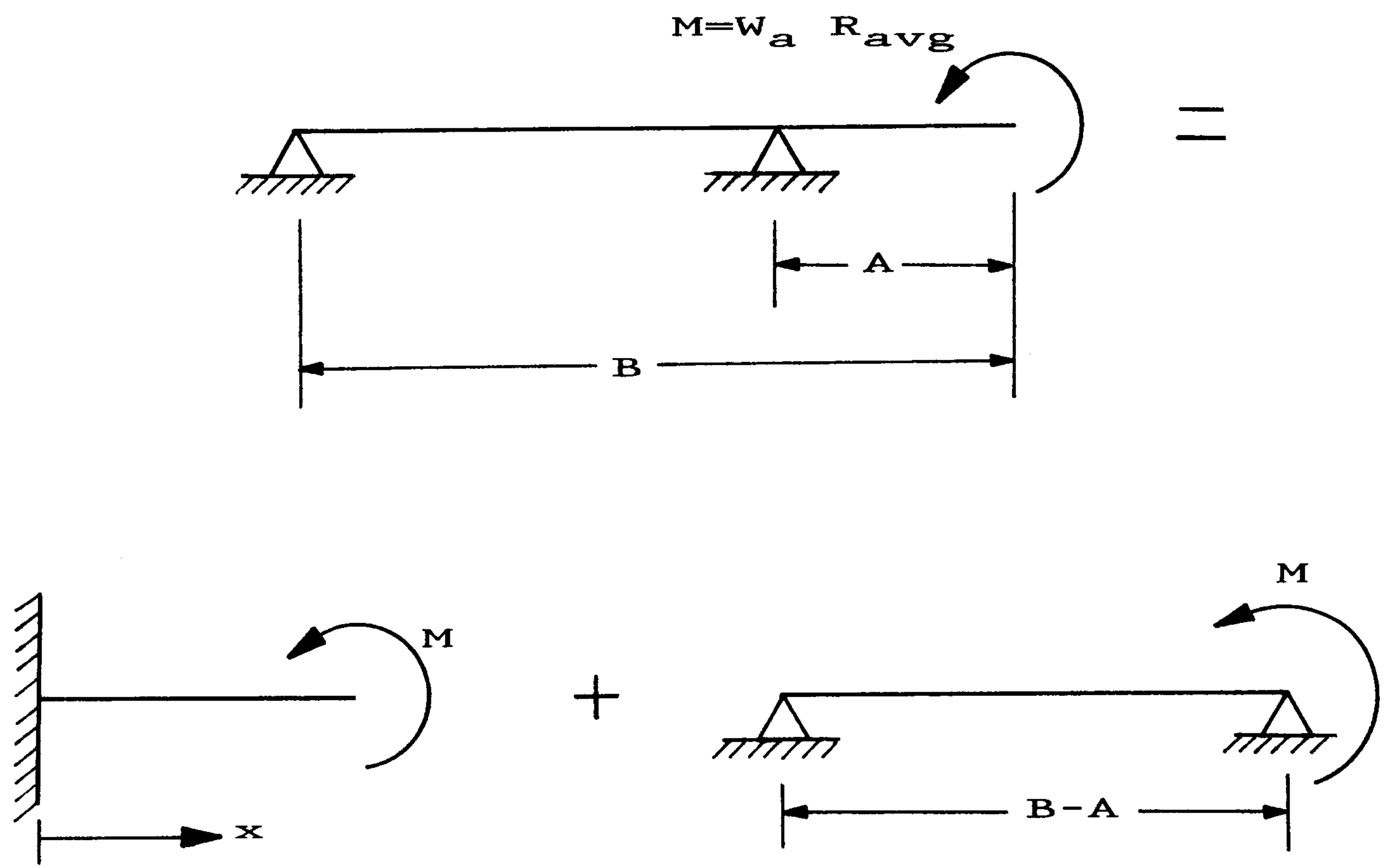


Figure 11. Overhung Configuration
Moment Load Analysis

Multiplying this slope by the cantilever length yields the second deflection:

$$D_2 = \frac{F\ A^2\ B}{3\ EI} \tag{47}$$

By adding the deflections given in equations 44 and 47, the total deflection is given by equation 48. Similarly, by adding the slopes in equations 43 and 46, the total slope is given by equation 49.

$$Y_0 = D_1 + D_2 = \frac{F\ A^2\ B}{3\ EI} \tag{48}$$

$$\theta_0 = S_1 + S_2 = \frac{F\ A(3A + 2(B - A))}{3\ EI} \tag{49}$$

Equations 48 and 49 are valid for tangential and radial loads. The load, F, can be either a tangential load, W_t, or a radial load, W_r.

Working in a similar manner, the equations for a moment load are derived.

$$D_3 = \frac{M\ A^2}{2\ EI} \tag{50}$$

By differentiating equation 50, the slope is:

$$S_3 = \frac{M\ A}{EI} \tag{51}$$

Figure 11 shows the moment load acting at the free end that creates a rotation at the end of the cantilever which contributes to the total deflection by creating a slope to the whole beam as shown in equation 52:

$$S_4 = \frac{M(B - A)}{3\ EI} \tag{52}$$

where M = F A.

This slope is multiplied by the cantilever length results in the second deflection:

$$D_4 = \frac{M\ A(B - A)}{3\ EI} \tag{53}$$

The deflections given in equations 50 and 53 are added to give the total deflection of equation 54. Similarly, by adding the slopes in equations 51 and 52, the total slope is given in equation 55.

$$Y_0 = D_3 + D_4 = \frac{M(3\ A + 2(B - A))}{6\ EI} \tag{54}$$

$$\theta_0 = S_3 + S_4 = \frac{M(3\ A + (B - A))}{6\ EI} \tag{55}$$

where $M = W_a \cdot R_{avg}$.

According to the axis system established in figure 12, the gear tooth deflection can be derived. The origin of this axis system is located at the midpoint of the gear face. Based on the loads exerted on the gear tooth, the new position may be found by considering the effect of these loads on the gear shaft. Equations 56-61

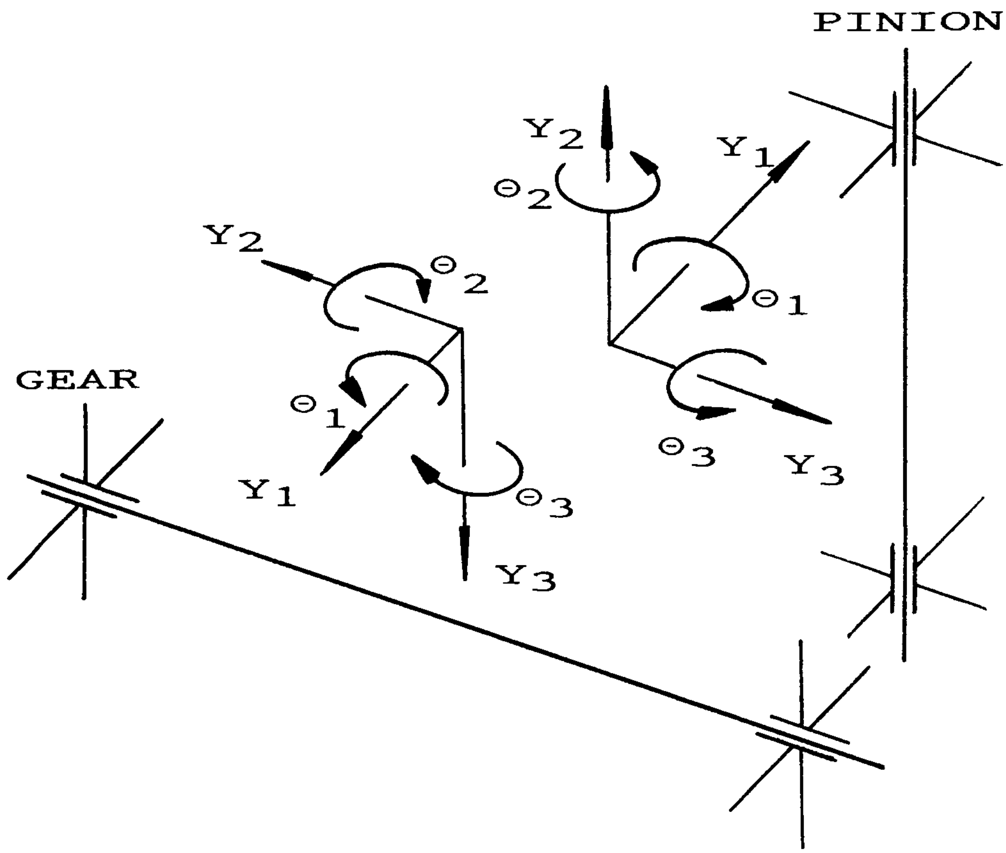


Figure 12. Reference Axis System.

describe the new position for any translation and/or rotation with respect to the axis system of figure 12. In the direction tangent to the gear tooth, Y_1, carries the deflection Y_t, which is caused by the tangential load W_t. The slope about Y_1, in equation 57, is equal to the difference of the slopes caused by the axial and radial loads.

$$Y_1 = Y_t \tag{56}$$

$$\theta_1 = \theta_a - \theta_r \tag{57}$$

As shown in equation 58, the deflection Y_2 accounts for the new position of the midpoint of the gear tooth, whose slope is equal to θ_1. The slope about Y_2 is found by considering that the gear shaft is subjected to a torsional load equal to the tangential component, W_t, and causes a torsional displacement equal to the slope, θ_2. Equation 59 is the slope about Y_2, where the negative sign is due to the fact that the tangential load is acting in the counterclockwise direction. According to the right-hand rule established in figure 13, the slope about Y_2 causes a clockwise torsional effect on the gear shaft.

$$Y_2 = R_{avg} \sin(\theta_a - \theta_r) \tag{58}$$

$$\theta_2 = \frac{-W_t R_{avg} Z}{JG} \tag{59}$$

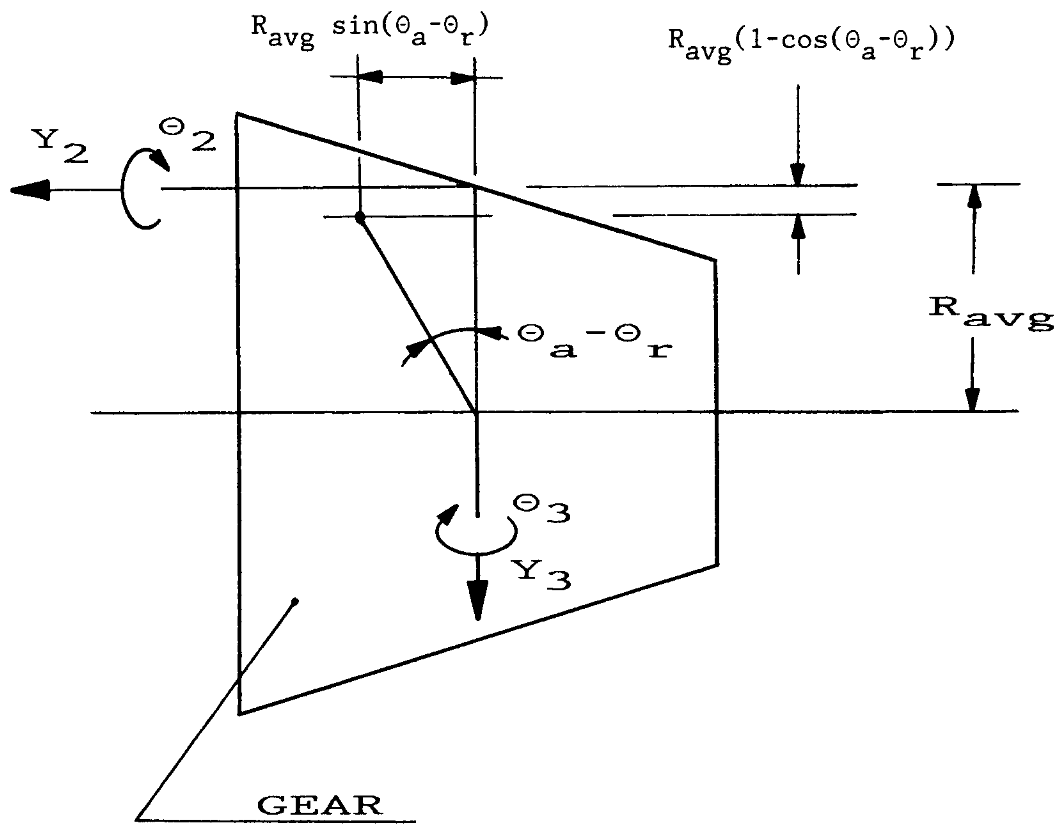


Figure 13. Pitch Point Deflection.

where $Z = A + B$ for straddle configuration

$Z = B$ for overhung configuration

J = polar moment of inertia of the shaft, in^4

$J = \pi d^4/32$

G = modulus of rigidity, lbf/in^2

$$G = \frac{E}{2(1 + \nu)}$$

where E is the modulus of elasticity, and ν is the Poisson's ratio (for steel $\nu = 0.25$).

As mentioned earlier in this chapter, the axial and the radial loads act on the same plane as the radial component; therefore, they contribute to the deflection in the radial direction. This contribution is accounted for in equation 60 along with the new position of the pitch point in the radial direction.

$$Y_3 = Y_a + Y_r + R_{avg}(1 - \cos(\theta_a - \theta_r)) \tag{60}$$

As can be seen from figure 12, the slope θ_3 is equal to the slope caused by the tangential load.

$$\theta_3 = \theta_t \tag{61}$$

The deflections at the gear center for the three directions are given by equations 62 through 64. The axis system is the one of figure 12. The gear is considered to be a disk and the deflections are the ones caused by the external loading components at the disk center. It should be noted that these deflections are describing the same gear motion but at a different position.

$$V_1 = Y_t \tag{62}$$

$$V_2 = 0 \tag{63}$$

$$V_3 = Y_r + Y_a \tag{64}$$

These deflections are the vector summations of the superimposed components of the shaft and the gear body motion.

BEARING ANALYSIS

The third component of the gear assembly deflection is due to the deflection of the bearings supporting the spiral bevel gear mesh.

The bearings used in bevel gears are usually ball or roller or tapered roller bearings with one or two rows of rolling elements. In order to determine the deflection in a bearing, the bearing geometry and the load to which the bearing is subjected have to be known.

The deflection of a bearing may be treated as the sum of two major deflections, the motion through the internal clearance of the bearing and the compression between the bearing races and the rolling elements. The deflection due to the motion through the internal clearance can be found by considering the change in the radial clearance, which is half of the diametral clearance. From the geometry of figure 14 the diametral clearance, C_d, is defined:

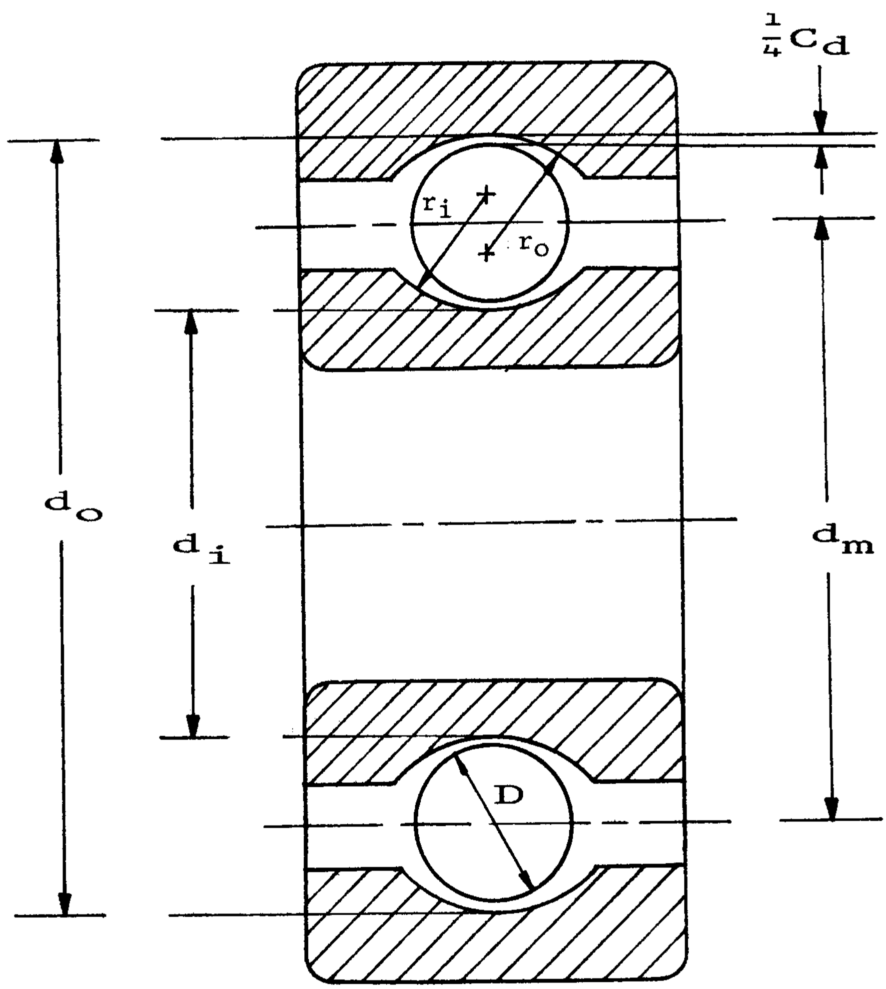


Figure 14. Ball Bearing Geometry.

$$C_d = 4(d_o - d_i - 2D\ ) \tag{65}$$

where d_o and d_i are the outer and inner race diameters, respectively, and D is the rolling element diameter.

For most practical bearings, this clearance is either zero or negative in which case the bearing is preloaded at assembly. The negative value of the diametral clearance is one way to describe the amount of preload present in the beraring. This preload affects the stiffness of the bearing because of the nonlinear Hertzian contact and the load-deflection relationship. Each rolling element in the loaded region of the bearing has Hertzian deflections with both the inner and outer bearing races. The equations presented in this section are valid for zero or negative clearance. In the case of positive clearance the bearing deflection is equal to the deflection of the bearing with zero clearance plus the amount of positive clearance.

The deflection due to the compression of the bearing components can not be found with a direct approach. This is because of the nonlinear Hertzian contact and the static indeterminancy among the rolling elements. As a starting point point though, the Hertzian analysis for a surface deflection of two bodies may be considered.

Figure 15 shows two bodies of revolution with different radii of curvatures in a pair of inter-secting principal planes. These planes pass through the contact point of the two bodies. In figure 15, the upper body is denoted as I, and the lower as II. For reference purposes, the principal planes are denoted as 1 and 2. Therefore, the radius of curvature of body I in plane 2 is designated as R_{I2}. The curvature κ is defined as the reciprocal of the corresponding radius of curvature R, and one may write:

$\kappa_{I1} = 1/R_{I1}$ = Body I curvature in the direction of the rolling velocity

$\kappa_{I2} = 1/R_{I2}$ = Body I curvature at 90° to the direction of the rolling velocity (66)

$\kappa_{II1} = 1/R_{II1}$ = Body II curvature in the direction of the rolling velocity

$\kappa_{II2} = 1/R_{II2}$ = Body I curvature at 90° to the direction of the rolling velocity

These curvatures may be positive or negative. In figure 16a is shown the case in which the center of curvature is inside the surface, known as convex; this curvature is positive. The curvature is negative when the center of curvature is outside the surface as shown for R_2 in figure 16b. This curvature is known as concave. The curvatures are used to describe the contact between surfaces of revolution in the following equations:

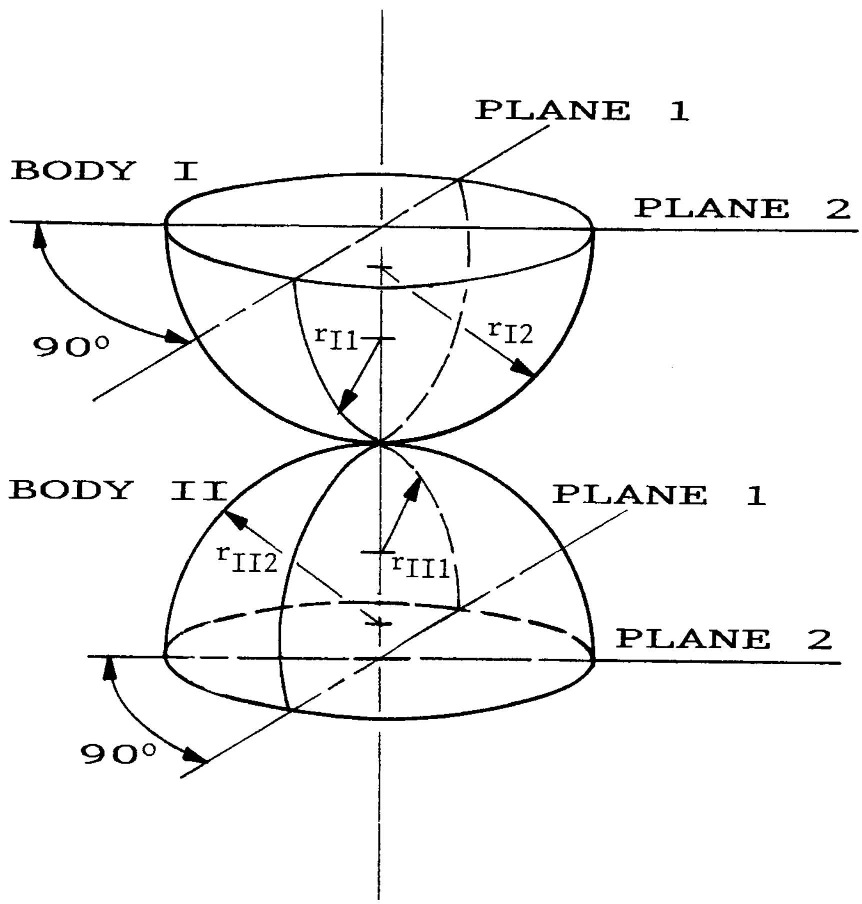
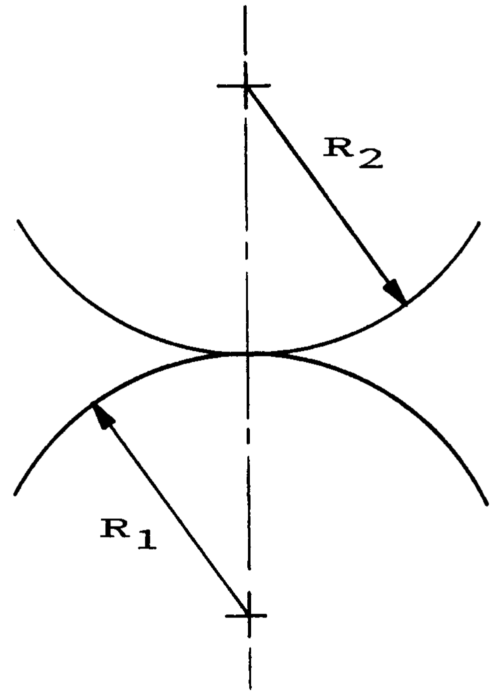
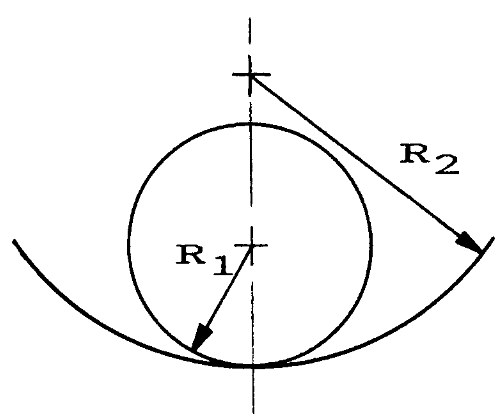


Figure 15. Contacting Surface Geometry.



(a) CONVEX



(b) CONCAVE

Figure 16. Types of Contacting Surfaces.

The curvature sum:

$$\sum\kappa = \kappa_{I1} + \kappa_{I2} + \kappa_{II1} + \kappa_{II2} \tag{67}$$

The curvature difference $F(\kappa)$:

$$F(\kappa) = \frac{(\kappa_{I1} - \kappa_{I2}) + (\kappa_{II1} - \kappa_{II2})}{\sum\kappa} \tag{68}$$

In both cases the sign convention of convex and concave curvature is applied, and $F(\kappa)$ should be positive. A complete derivation of these equations is presented by Houghton [17].

The sum and curvature difference can be calculated for the inner and outer raceways. The bearing raceways that are in contact with the rolling element have different radii of curvatures, thus the curvature sum and the curvature difference are also different. Harris [18] provides equations for ball and roller bearings which are shown in equations 69-76.

BALL BEARINGS

a. Inner Raceway

$$\sum\kappa_i = \frac{1}{D}\left(4 - \frac{1}{f_i} + \frac{2\gamma}{1-\gamma}\right) \tag{69}$$

$$F(\kappa)_i = \frac{\dfrac{1}{f_i} + \dfrac{2\gamma}{1-\gamma}}{4 - \dfrac{1}{f_i} + \dfrac{2\gamma}{1-\gamma}} \tag{70}$$

b. Outer raceway

$$\sum\kappa_0 = \frac{1}{D}\left(4 - \frac{1}{f_0} + \frac{2\gamma}{1-\gamma}\right) \tag{71}$$

$$F(\kappa)_0 = \frac{\frac{1}{f_0} + \frac{2\gamma}{1-\gamma}}{4 - \frac{1}{f_0} + \frac{2\gamma}{1-\gamma}} \tag{72}$$

ROLLER BEARINGS

a. Inner Raceway

$$\sum\kappa_i = \frac{1}{D}\left[\frac{2\gamma}{1-\gamma} + D\left(\frac{1}{R} - \frac{1}{r_i}\right)\right] \tag{73}$$

$$F(\kappa)_i = \frac{\frac{2\gamma}{1-\gamma} - D\left(\frac{1}{R} - \frac{1}{r_i}\right)}{\frac{2\gamma}{1-\gamma} + D\left(\frac{1}{R} - \frac{1}{r_i}\right)} \tag{74}$$

b. Outer raceway

$$\sum\kappa_0 = \frac{1}{D}\left[\frac{2\gamma}{1-\gamma} + D\left(\frac{1}{R} - \frac{1}{r_0}\right)\right] \tag{75}$$

$$F(\kappa)_0 = \frac{\frac{2\gamma}{1-\gamma} - D\left(\frac{1}{R} - \frac{1}{r_0}\right)}{\frac{2\gamma}{1-\gamma} + D\left(\frac{1}{R} - \frac{1}{r_0}\right)} \tag{76}$$

where

$$\gamma = \frac{D\cos a}{d_m} \tag{77}$$

the bearing pitch diameter is given by:

$$d_m = \frac{d_i + d_0}{2} \tag{78}$$

In these equations, D is the ball or roller diameter, a is the contact angle, and r_i, r_0 are the raceway groove curvature radii. In the same equations, f_i and f_0 designate the ratio r/D of the inner and outer raceway groove radii to the ball or roller element diameter, and R is the roller contour radius, for roller bearings.

The curvature difference equation is considered to be an auxiliary function of the elliptical parameters a and b, the semi-major and semi-minor axis of the projected ellipse surface, respectively [18]. This function is described by equation 79.

$$F(k) = \frac{(q^2 + 1)E - 2F}{(q^2 - 1)E} \tag{79}$$

where E and F are complete elliptic integrals of the first and second kind, respectively, and q is the elliptical eccentricity. These integrals are given by equations 80 and 81, respectively.

$$F = \int_0^{\pi/2} [1 - (1 - \frac{1}{q^2})\sin^2\phi]^{1/2} \, d\phi \tag{80}$$

$$E = \int_0^{\pi/2} [1 - (1 - \frac{1}{q^2})\sin^2\phi]^{1/2} \, d\phi \tag{81}$$

in which ϕ is the auxiliary angle.

For bearings whose surfaces have point contact, it has been determined [18] that the ellipse size is given by equations 82 and 83:

$$a = a^*\left[\frac{3Q}{2\sum\kappa}\left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_1^2}{E_2}\right)\right]^{1/3} \tag{82}$$

$$b = b^*\left[\frac{3Q}{2\sum\kappa}\left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_1^2}{E_2}\right)\right]^{1/3} \tag{83}$$

The deflection for the contact of the two bodies is found by:

$$\delta = \delta^*\left[\frac{3Q}{2\sum\kappa}\left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_1^2}{E_2}\right)\right]^{2/3} \frac{\sum\kappa}{2} \tag{84}$$

where Q is the normal load acting on the two bodies in contact. The dimensionless parameters, a*, b*, δ*, are given by equations 85-87:

$$a^* = \left(\frac{2\ q^2\ E}{\pi}\right)^{1/3} \tag{85}$$

$$b^* = \left(\frac{2\ E}{\pi q}\right)^{1/3} \tag{86}$$

$$\delta^* = \frac{2\ F}{\pi}\left(\frac{\pi}{2\ q^2\ E}\right)^{1/3} \tag{87}$$

A complete table giving values of the dimensionless parameters, a*, b*, δ*, has been developed by Harris [18], to record the evaluation of the integrals of equations 80 and 81, for all possible ball bearing geometries.

In the case of bearings with line contact, the equations for deflection can be calculated in a similar way. Palmgren [19] has developed an equation to calculate the deflection between the inner or outer race and the rolling element:

$$\delta = 0.0003\, E_e^{2.7} \frac{Q^{0.9}}{L_w^{0.9}} \tag{88}$$

where Q is the normal load acting on the two bodies, L_w is the contact length and E_e is the material constant. The material constant is given by equation 89:

$$E_e = \sqrt[3]{\frac{11500(E_{01} + E_{02})}{E_{01}\, E_{02}}} \tag{89}$$

$$E_{01} = \frac{E_1}{1 - \frac{1}{\nu_1^2}} \qquad E_{02} = \frac{E_2}{1 - \frac{1}{\nu_2^2}}$$

where E_1, E_2 are the moduli of elasticity in kg/mm^2, and ν_1, ν_2 are the Poisson's ratios of the two bodies in contact.

The method previously described could be used to find the contact deflection in bearings. However, due to the assumptions used in the Hertz analysis, it can not be used directly.

The first assumption is that there is only one point of contact between the two bodies, which is not the case in bearings since there is more than one roller element in contact with the inner and outer races at all times. The second assumption is that the load is acting in the same direction as the deflection. This assumption is only valid for a single rolling element directly under the load in a bearing with a zero contact angle. For all other rolling elements, the assumption is not valid.

Furthermore, the Hertz analysis is for the rolling element in contact with one of the bearing races. However, the rolling element is in contact with both the inner and outer race, thus both contact points contribute to the total bearing deflection. In order to get an accurate value for the total bearing deflection, the Hertzian analysis can still be used, but in a modified version, to account for the above considerations.

By observing equations 84 and 88, it can be seen that they can take the form:

$$\delta \propto Q^{1/n} \tag{90}$$

where n = 3/2 for ball and spherical bearings, and 10/9 for roller bearings.

By inverting relation 90 and expressing it in an equation format,

$$Q = K\delta^{n} \tag{91}$$

where K is a load-deflection factor.

Solving for K yields:

$$K = \frac{Q}{\delta^{n}} \tag{92}$$

By raising equation 84 to n=3/2 and simplifying:

$$\delta = 0.530\ Q[\delta^{*3/2}\ \sum\kappa^{1/2}(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_1^2}{E_2})] \tag{93}$$

Substituting this into equation 92 produces the load-deflection factor for ball and spherical bearings:

$$K = \frac{1.886}{\delta^{*3/2}\ \sum\kappa^{1/2}(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_1^2}{E_2})} \tag{94}$$

Similarly, the load-deflection factor for roller bearings with line contact may be found by using the deflection of equation 88 in equation 92, or:

$$K = \frac{Q}{[0.0003\ E_e^{2.7}\ \frac{Q^{0.9}}{L_w^{0.8}}]^{10/9}} \tag{95}$$

or

$$K = \frac{L_w^{8.9}}{0.000122\ E_e^{3}} \tag{96}$$

where L_w is in mm and E_e^3 in mm^2/kg.

Equation 96 can be easily converted in English units yielding:

$$K = \frac{658665.714\ L_w^{8.9}}{E_e^{3}} \tag{97}$$

where L_w is in inches and E_e^3 in in^2/lb.

The total normal approach-deflection between the raceways under load separated by a rolling element is the sum of approaches between the rolling element and each raceway [18]. Hence,

$$\delta_t = \delta_i + \delta_0 \tag{98}$$

Since the deflection at the inner and outer races of a bearing can be expressed as a relationship of the applied load, Q, and the corresponding load-deflection factor, K, of the contacting bodies, equation 91 can be written as:

$$Q = K_i \ \delta_i^{\,n} \tag{99}$$

$$Q = K_0 \ \delta_0^{\,n} \tag{100}$$

Since the load acting at each of the contact points is the same, it can be expressed as:

$$Q = K_j \ \delta_j^{\,n} \tag{101}$$

Due to the fact that the rolling element acts in series between the two races, the total load-deflection factor, K_j, can be evaluated by equation 102.

$$K_j = \left[\frac{1}{(\frac{1}{K_i})^{1/n} + (\frac{1}{K_0})^{1/n}}\right]^n \tag{102}$$

To account for load sharing among the rolling elements, Harris [18] has shown that, for a rigidly supported bearing subjected to radial loading, the radial deflection at any angular position of the rolling element is given by:

$$\delta_\psi = \delta_r \cos\psi - 0.5\, C_d \qquad (103)$$

where C_d is the diametral clearance and ψ is the angular position of the roller. The ring radial shift, δ_r, occurs at ψ=0. Figure 17 illustrates a radial bearing with clearance and a radial deflection at an angle ψ. Harris [18] has shown that equation 103 can be rearranged in terms of the maximum deflection.

$$\delta_\psi = \delta_r\left(1 - \frac{1}{2e}(1 - \cos\psi)\right) \qquad (104)$$

in which

$$e = \frac{1}{2}\left(1 - \frac{C_d}{2\,\delta_r}\right) \qquad (105)$$

From equation 91 it is clear that

$$\frac{Q}{Q_{max}} = \left(\frac{\delta_\psi}{\delta_r}\right)^n \qquad (106)$$

Therefore, from equations 104 and 106:

$$Q_\psi = Q_{max}\left[1 - \frac{1}{2e}(1 - \cos\psi)\right]^n \qquad (107)$$

If static equilibrium is to exist, the applied radial load must be equal to the sum of the vertical components of the rolling element loads:

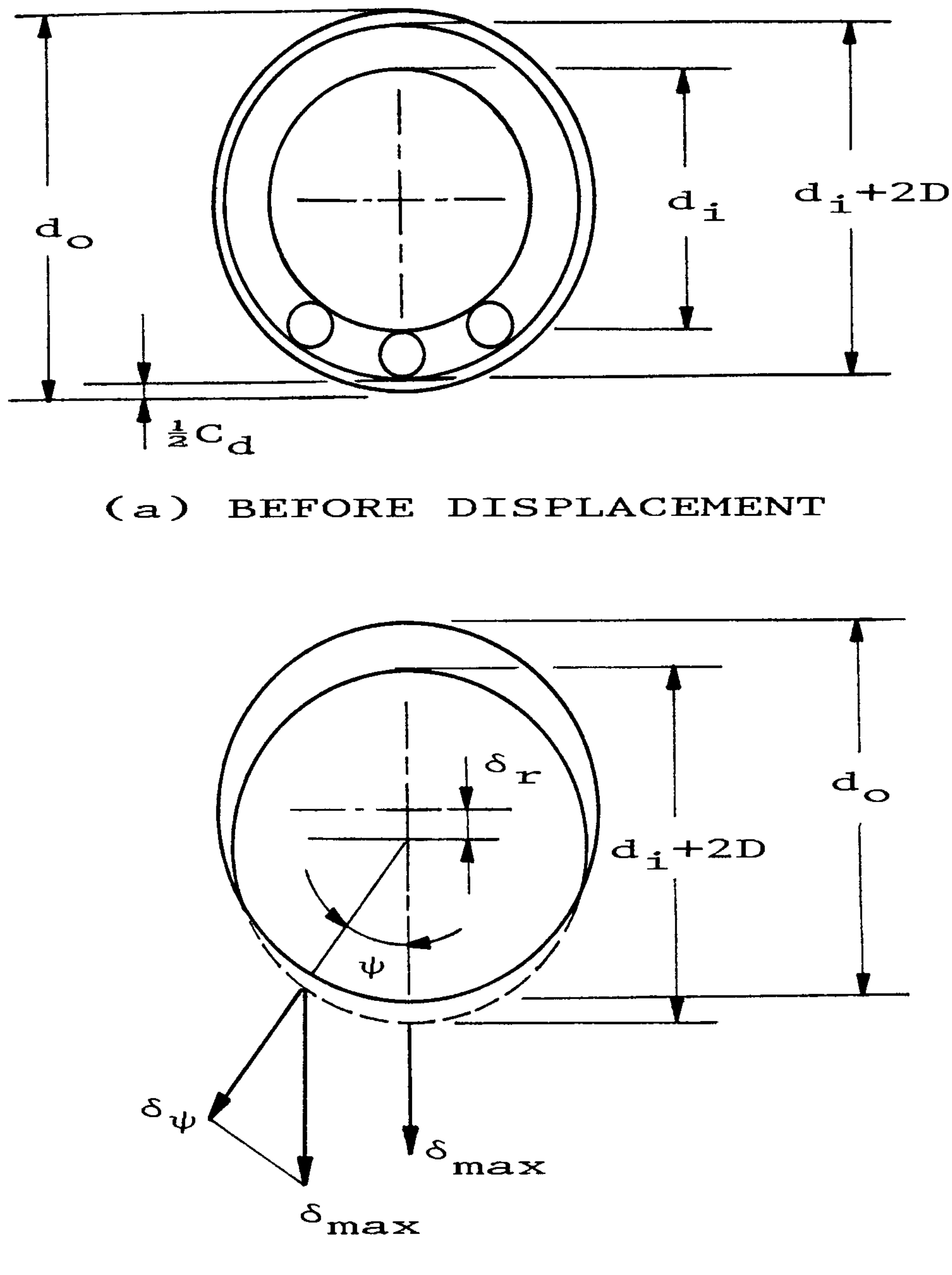


Figure 17. Bearing Ring Deflection.

$$F_r = \sum_{\psi=0}^{\psi=\pm\pi} Q_\psi \cos \psi \tag{108}$$

$$\text{or} \quad F_r = Q_{max} \sum_{\psi=0}^{\psi=\pm\pi} [1 - \frac{1}{2e}(1 - \cos \psi)]^n \cos \psi \tag{109}$$

which has the following equivalent integral form:

$$F_r = Z\, Q_{max} \frac{1}{2\pi} \int_{-\psi_\ell}^{+\psi_\ell} [1 - \frac{1}{2e}(1 - \cos \psi)]^n \cos \psi \, dy \tag{110}$$

where Z is the number of rolling elements in the bearing and ψ_ℓ is the angle over which the load is distributed on the bearing. By defining:

$$J_r(e) = \frac{1}{2\pi} \int_{-\psi_\ell}^{+\psi_\ell} [1 - \frac{1}{2e}(1 - \cos \psi)]^n \cos \psi \, dy \tag{111}$$

Harris [18] has evaluated the integral numerically and has tabulated the values as a function of e for a range of values from 0 to 5. The integral was evaluated for point and line contact. The results were plotted and are shown in figure 18. Using equation 111, equation 110 can then be written in the form:

$$F_r = Z\, Q_{max}\, J_r(e) \tag{112}$$

The value of Q_{max} is found by evaluating equation 103 at ψ=0.

$$Q_{max} = K_j\, \delta^n_{\psi=0} = K_j(\delta_r - 0.5\, C_d)^n \tag{113}$$

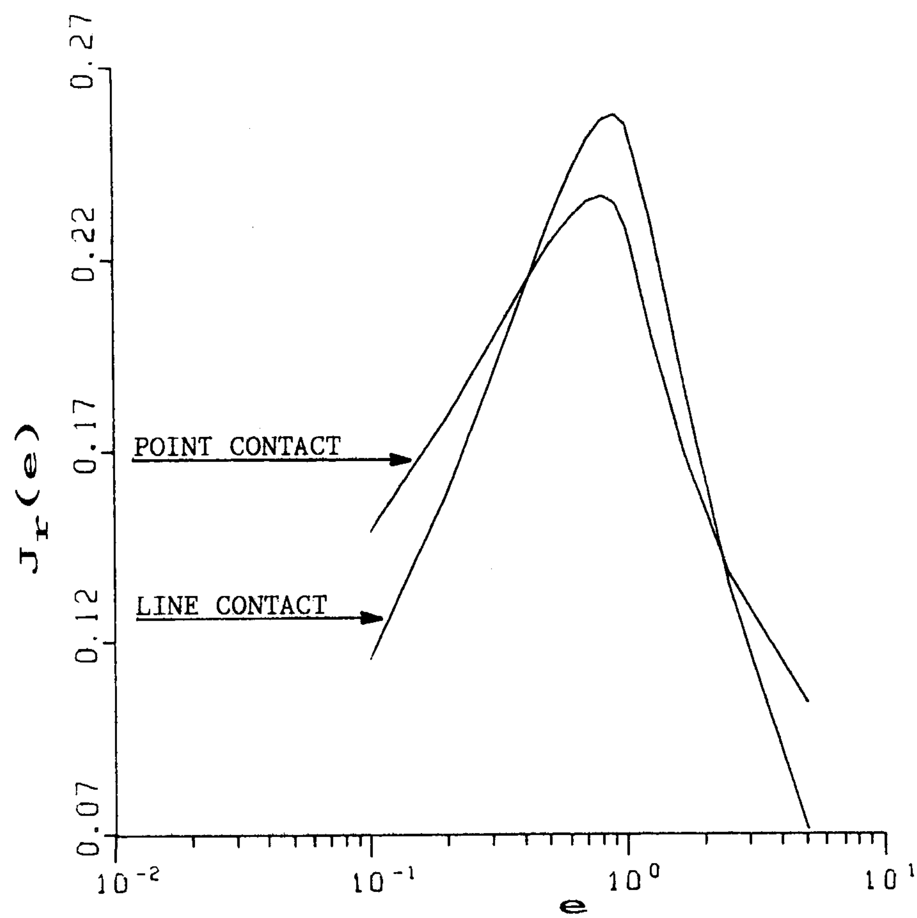


Figure 18. $J_r(e)$ vs. e for Radial Bearings.

or in terms of equation 112:

$$F_r = Z\, K_j(\delta_r - 0.5\, C_d)^n J_r(e) \qquad (114)$$

Due to the nonlinear relationship of the applied load and the bearing deflection, equation 114 cannot be solved directly. An incremental search method is used to calculate the deflection that corresponds to the externally applied load. In this method, the bearing deflection is the value to be determined that corresponds to a value $F_r(\delta_r)$, that equals the externally applied load within a small prescribed error range.

The sum and the difference equations 69 through 77 are first calculated. Depending on the bearing type the inner and outer race deflections for zero diametral clearance are calculated per equations 84 or 88, along with the resultant load-deflection factor, K_j, of equation 102. Then e is calculated in equation 105 which results in a value of $J_r(e)$. As a first guess for the deflection, equation 98 gives the value for δ_r for zero diametral clearance, which is used to calculate F_r in equation 114. The method compares the difference between the externally applied load and the value found from equation 114, against a small convergence criterion. Should these figures be within the limits of the criterion, the scheme converges to the right value of deflection δ_r. Otherwise, an increment is added to the previous approximation and the scheme is repeated. The increment is halved and reversed whenever the error changes sign. The process continues until the above mentioned difference is less or equal to the convergence criterion.

In a bearing for which the angle of contact is not equal to zero, the load that is in the direction of the deflection can be found by equation 115:

$$F_{appl} = \frac{F_r}{\cos\ \alpha} \tag{115}$$

where F_r is the applied load to which the bearing is subjected, and α is the contact angle. For the case of the bearing that has two rows of rolling elements, the total load applied to the bearing is equally shared by the two rows of elements. For ball bearings that are subjected to pure radial load and zero radial clearance, Stribeck [20] concluded that:

$$F_{appl} = \frac{4.37\ F_r}{Z\ \cos\ \alpha} \tag{116}$$

where Z is the number of rolling elements.

Based on this transformation, the deflection calculated at an angle equal to the contact angle from the radial direction can be found as:

$$D_r = \delta_r \cos\ \alpha \tag{117}$$

According to the Cartesian axis system established in figure 19a, the bearing loads in the tangential and radial directions, F_t and F_r, respectively, are combined in equation 119, in one resultant load, F_{tot} per bearing station. The bearing loads were determined in equations 33 and 34 and are shown in figure 19b.

$$\beta = \tan^{-1}(F_r/F_t) \tag{118}$$

$$F_{tot} = \sqrt{F_t^{\,2} + F_r^{\,2}} \tag{119}$$

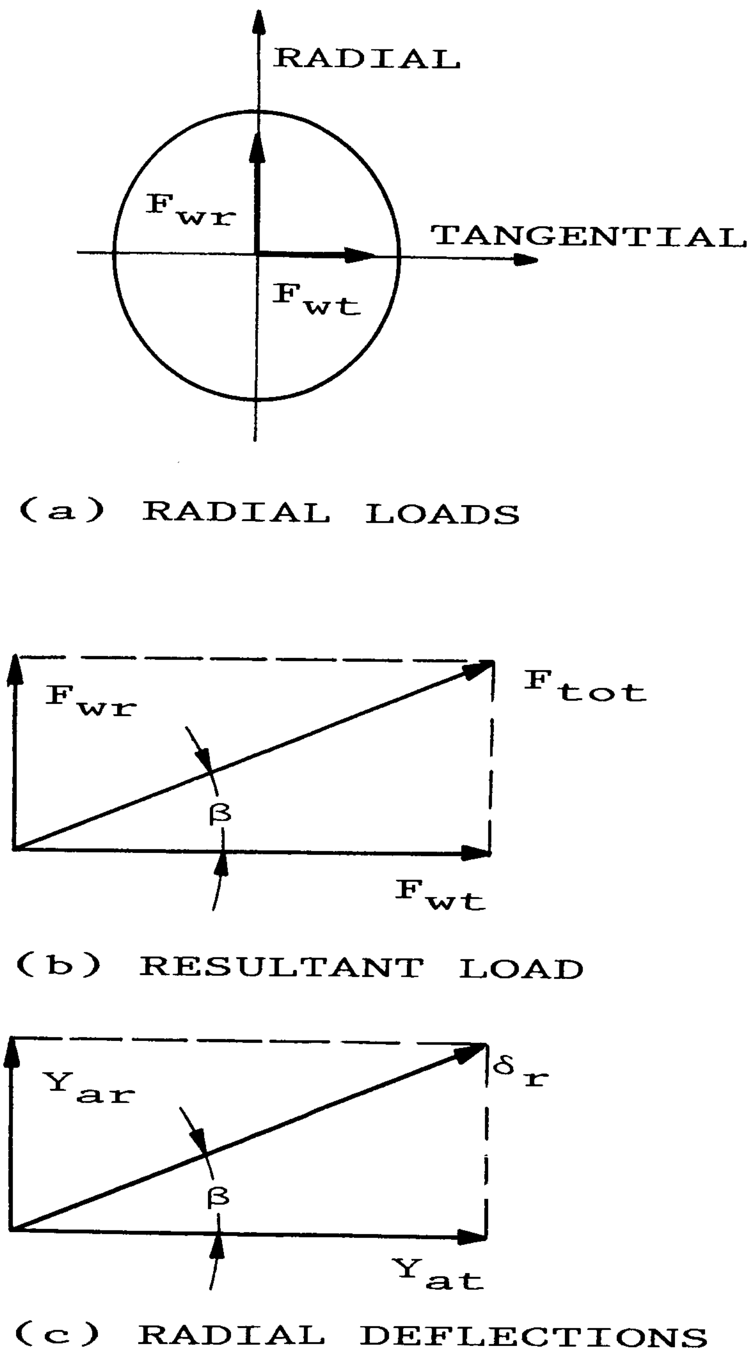


Figure 19. Bearing Force and Deflection Components.

Based on the information provided for the bearing design parameters, the computer program calculates the bearing deflection, δ_r, for each station on the same shaft. This deflection is then analyzed back into two components in the tangential direction, Y_{at}, and radial direction, Y_{ar}, of figure 19c.

$$Y_{at} = \delta_r \cos \beta \tag{120}$$

$$Y_{ar} = \delta_r \sin \beta \tag{121}$$

By designating the left bearing deflection as Y_1, and right as Y_2, the deflection Y_b due to the tangential load, W_t, or the radial load, W_r, is determined. Therefore, the left bearing deflection in the tangential direction is designated as Y_{1t}, and in the radial direction as Y_{1r}. For the straddle configuration of figure 20a, the bearing deflection in the tangential load direction is:

$$Y_{bwt} = \frac{Y_{1t} A + Y_{2t} B}{A + B} \tag{122}$$

From the same figure the slope of the deflected shaft is found as:

$$\theta_{bwt} = \frac{Y_{2t} - Y_{2t}}{A + B} \tag{123}$$

Due to the fact that the two bearing deflections on the same shaft are not equal, the new position of the shaft is calculated by

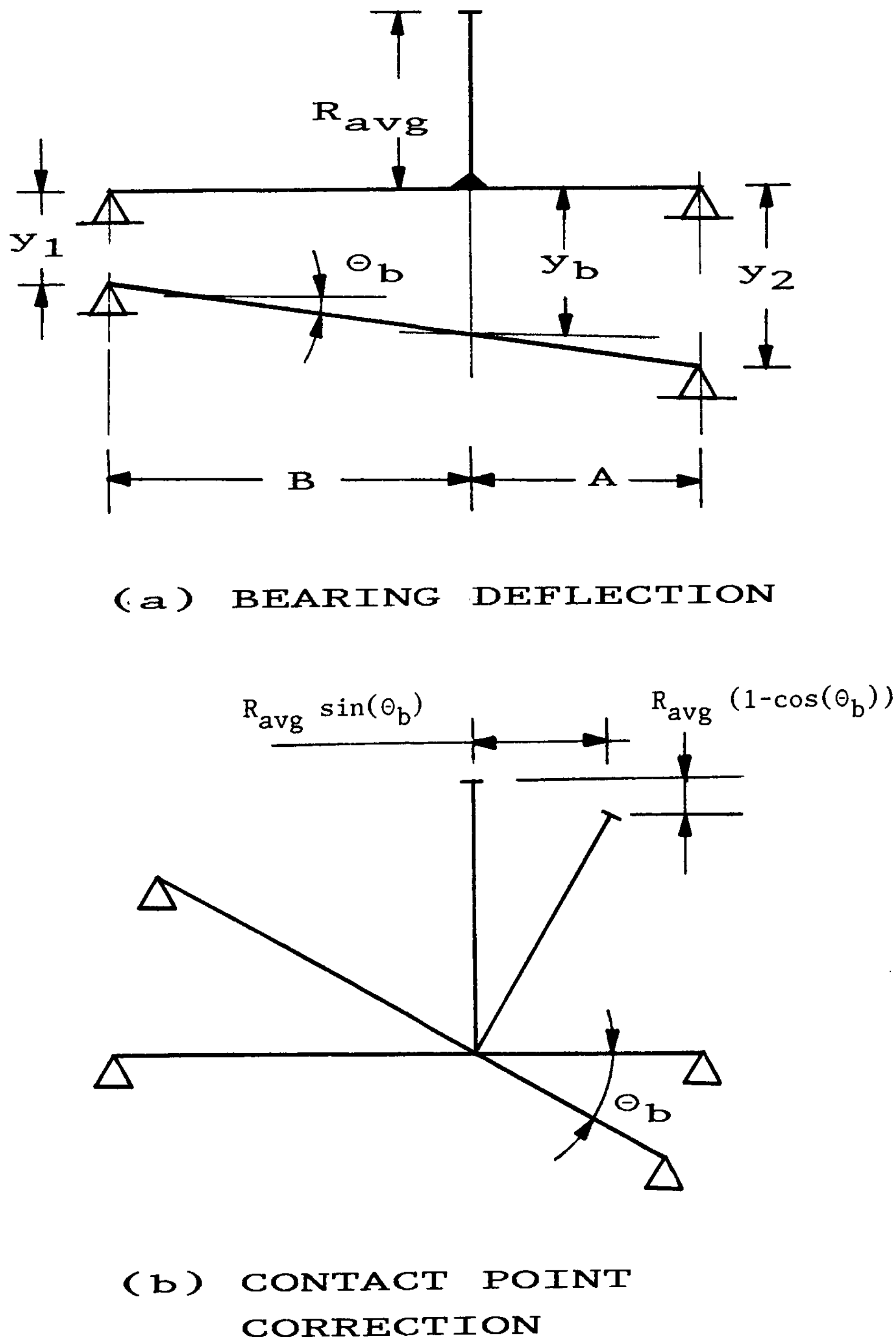


Figure 20. Straddle Configuration Bearing Support Deflection.

considering the slope of the shaft with respect to the original undeflected position.

$$\theta_{bwr} = \frac{Y_{2r} - Y_{1r}}{A + B} \tag{124}$$

In the radial load direction, the deflection is similar to the tangential, with an additional term that accounts for the new position of the pitch point in the radial direction, caused by the slope, θ_{bwr}, as shown in figure 20b.

$$Y_{bwr} = \frac{Y_{1r}\ A + Y_{2r}\ B}{A + B} + R_{avg}(1 - \cos(\theta_{bwr})) \tag{125}$$

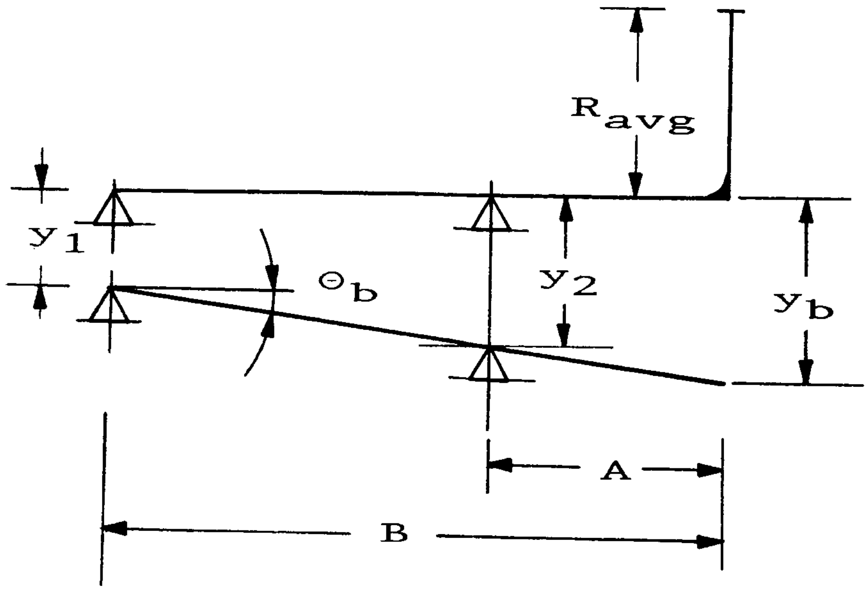
From the same figure, the deflection in the axial direction changes the position of the pitch point by:

$$Y_{bwa} = R_{avg} \sin(\theta_{bwr}) \tag{126}$$

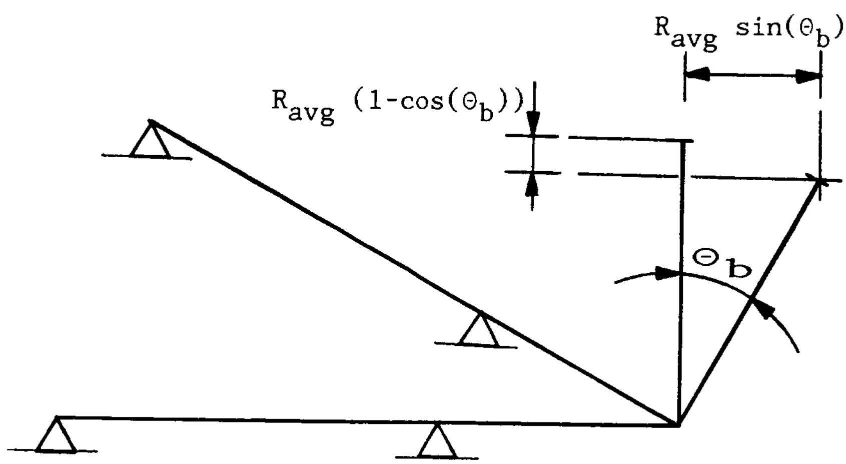
The axial load does not cause an out-of-plane slope, thus it does not affect the bearing deflection of any bearing station.

Similarly, for an overhung configuration, shown in figure 21a, the deflections in the three directions of the acting loads and the shaft slope are given by equations 127 through 131. The bearing deflection due to the tangential load is given as:

$$Y_{bwt} = \frac{Y_{2t}\ B + Y_{1t}\ A}{B - A} \tag{127}$$



(a) BEARING DEFLECTION



(b) CONTACT POINT CORRECTION

Figure 21. Overhung Configuration Bearing Support Deflection.

The unequal deflections of the two bearing stations in the tangential direction create a slope, θ_{bwt}, as:

$$\theta_{bwt} = \frac{Y_{2t} - Y_{1t}}{B - A} \tag{128}$$

In the radial direction the slope is found by:

$$\theta_{bwr} = \frac{Y_{2r} - Y_{1r}}{B - A} \tag{129}$$

The pitch point's new location, shown in figure 21b, is added to the deflection in the radial deflection

$$Y_{bwr} = \frac{Y_{2r}\ B + Y_{1r}\ A}{B - A} + R_{avg}(1 - \cos(\theta_{bwr})) \tag{130}$$

From figure 21b also, the pitch point has moved in the axial direction:

$$Y_{bwa} = R_{avg} \sin(\theta_{bwr}) \tag{131}$$

The motion of the gear assembly due to the bearing internal deflection is the third and last of the three motions. The total deflections and slopes due to these three motions is the algebraic sum of all three contributing components. These three motions analyzed in this section, comprise the total absolute motion consisting of three translations and three rotations. The term "absolute" refers to the nominal point of contact at no load, with no relative motion of one gear with respect to the other.

Equations 132 through 137 give the total no-loaded deflections and slopes, shown in figure 22, of the gear or the pinion, in each of the three directions and motions analyzed in the last two sections.

TOTAL DEFLECTIONS

Tangential component : $D_{t1} = Y_t + Y_{bwt}$ (132)

Axial component : $D_{t2} = Y_a + Y_2 + Y_{bwa}$ (133)

Radial component: : $D_{t3} = Y_r + Y_3 + Y_{bwr}$ (134)

TOTAL SLOPES

Tangential component : $\theta_{t1} = \theta_t + \theta_1 + \theta_{bwt}$ (135)

Axial component : $\theta_{t2} = \theta_a + \theta_2$ (136)

Radial component : $\theta_{t3} = \theta_r + \theta_3 + \theta_{bwr}$ (137)

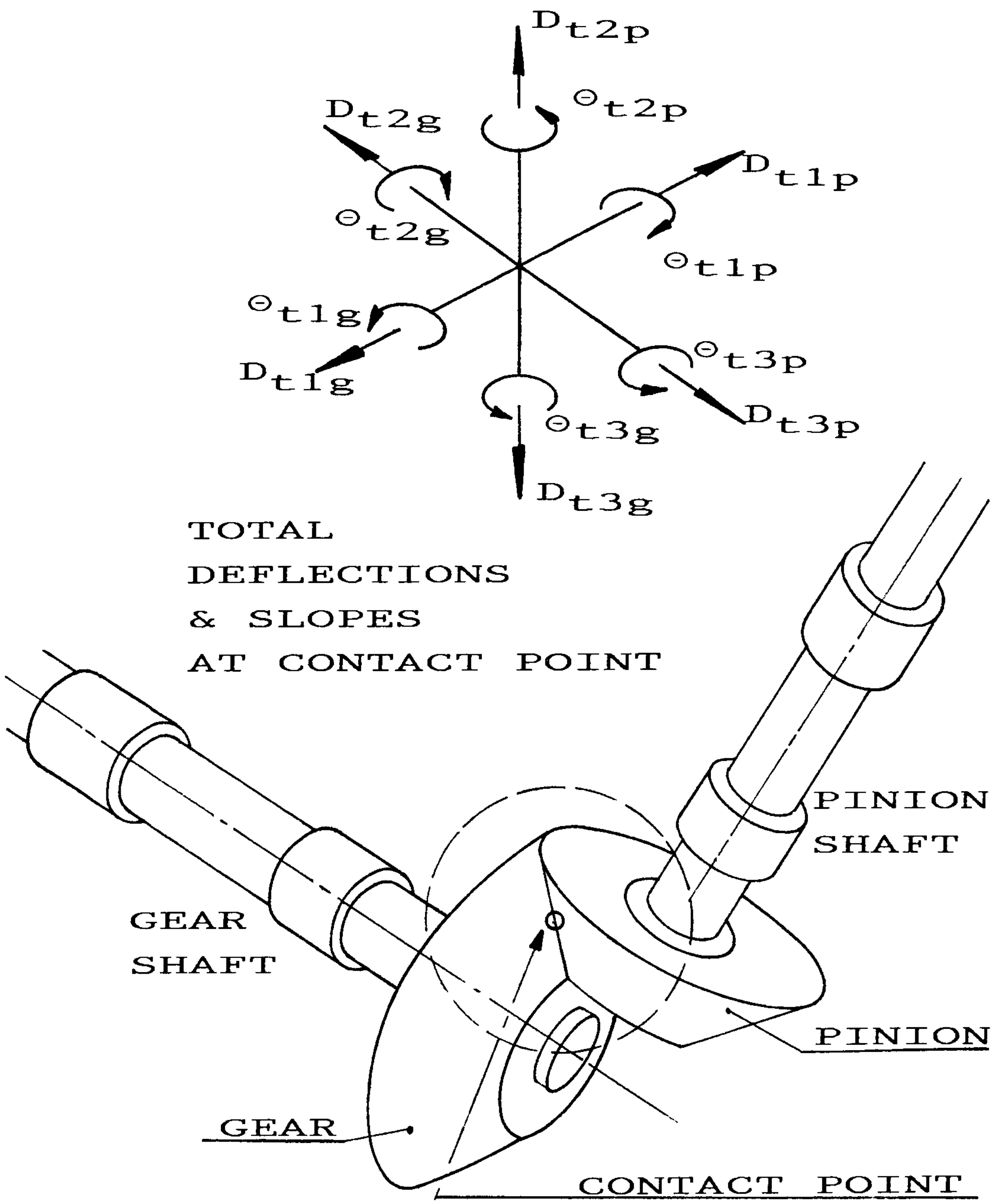


Figure 22. Total Deflections and Slopes for Spiral Gear and Pinion.

RELATIVE MOTION

In dealing with the motion of a gear assembly in the previous sections, the motion has been considered and analyzed as an absolute motion of three translations and three rotations at the nominal (no load) point of contact and at the gear center as well.

Another way to estimate the motion of the loaded point of contact and its curvatures on the two gears is by considering the relative motion of the two gears at their nominal point of contact. Based on the gear tooth and assembly geometry, the actual (loaded) point of contact is determined in this section.

Consider the axis system in figure 23, the original of which is located at the midpoint of the tooth face. This coordinate frame can be defined by three axes: Z_1, is the common tangential component of the gear and the pinion; Z_2 the pinion axial component; and Z_3 the gear axial component. Due to the fact that the shaft angle, Σ, of the gear mesh may not be a right angle, the axis system need not be orthogonal.

The axis system combines the three translational and three rotational relative motions of the gear and the pinion in one coordinate system. The relative translations are indicated by Z, while the relative rotations are indicated by A. Figure 23 shows these

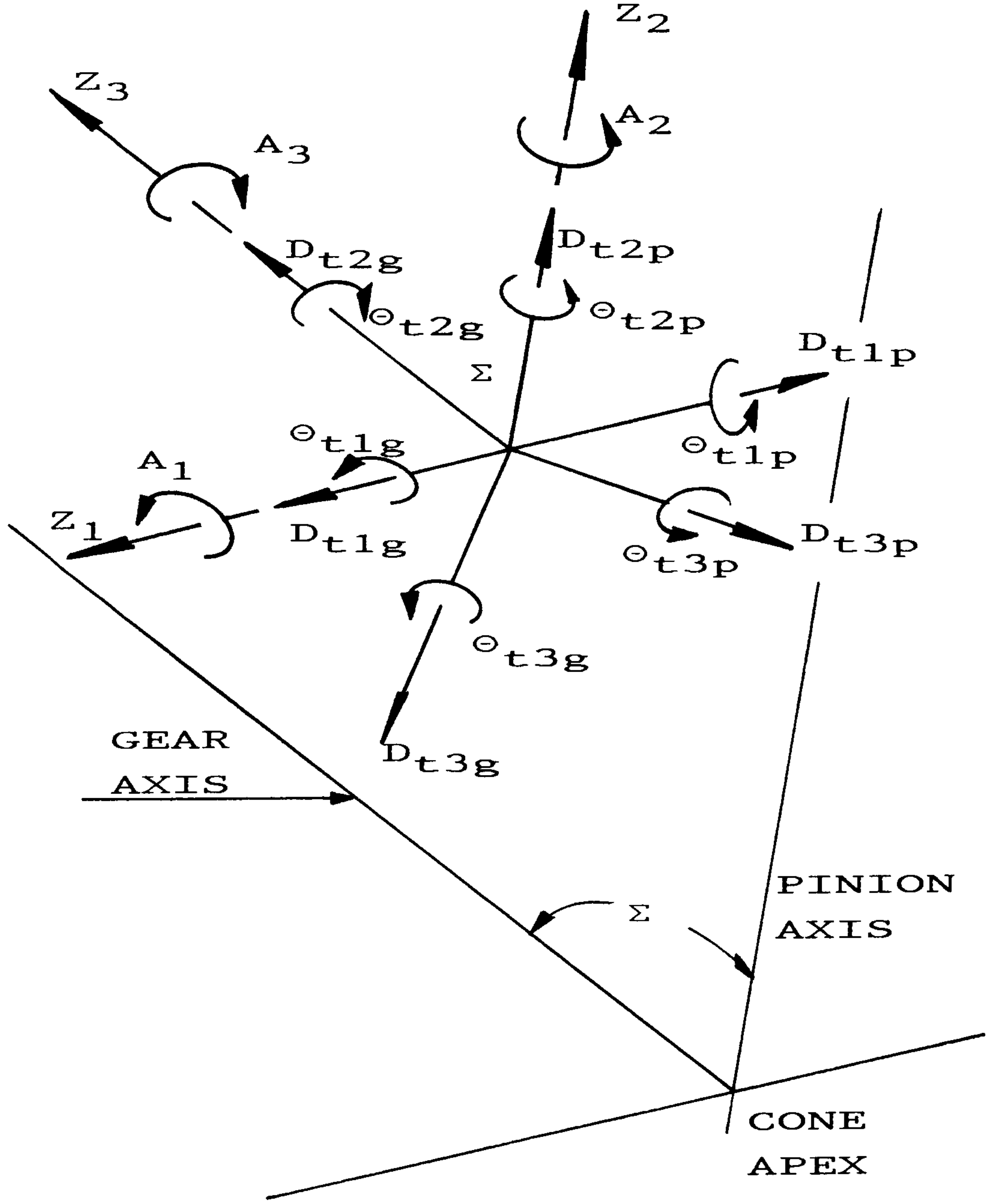


Figure 23. Relative Motion Coordinate Axis System.

translations and rotations in the positive sense. In the same figure the subscript g designates the gear parameters, while the subscript p, the pinion's. Z_1 and A_1 are in the same direction as the gear's tangential deflection and slope, $D_{t_1 g}$ and $\theta_{t_1 g}$, respectively. Z_2 and A_2 are in the direction of the pinion's total axial deflection, $D_{t_2 p}$, and slope, $\theta_{t_2 p}$ and Z_3 and A_3 are colinear with the gear's total axial deflection, $D_{t_2 g}$, and slope, $\theta_{t_2 g}$.

The relative rotation can be defined in terms of the projections of the total deflections and slopes of the two gears on the axes of the coordinate frame of figure 23. The total axial and radial deflections and slopes act in the same plane that is defined by Z_2 and Z_3. This plane is perpendicular to the plane of the common tangential component in the direction of Z_1. The projections of these deflections and slopes are the only ones to contribute to the pinion axial, Z_2 and A_2, and the gear axial, Z_3 and A_3, relative translational and rotational motion components.

Assuming that the pinion is fixed, and the two mating gears have point contact, the following equations give the rotation and translation of the contact point caused by the relative motion of the gear relative to the pinion. In the case of the reverse situation, that is, the motion of the pinion relative to the gear, the deflections and rotations have the same magnitude but the opposite signs. The analysis is also based on the assumption that the finite rotations of the one gear with respect to the other are sufficiently small that they can be treated as vector quantities.

The relative rotation is first analyzed. Along the common tangential axis, A_1, the relative rotation of the gear with respect to pinion is the algebraic sum of the corresponding total tangential slopes. The relative rotation, which is in the same plane with the total slopes, can be expressed as the algebraic difference of the slopes. By virtue of the right-hand rule, the pinion rotation is in the opposite direction of the gear's. Therefore, the relative rotation can be expressed as:

$$A_1 = \theta_{t1g} - (- \theta_{t1p}) \tag{138}$$

The pinion axial, A_2, and the gear axial, A_3, components of figure 23 can be seen independently in figure 24. The total axial and radial slopes of the gear and the pinion are projected on the axes of the system yielding the relative rotations in the axial direction. The pinion axial component, A_2, is comprised of three discrete rotations. These rotations are: the pinion radial, θ_{t_3p}, and axial, θ_{t_2p}, and the gear radial, θ_{t_3g}.

Due to the choice of this coordinate axis system, the projection of the gear axial rotation, θ_{t_2g}, in the direction of A_2, is equal to zero. Similarly, the pinion axial rotation, θ_{t_2p}, does not contribute to the relative rotation in the direction of A_3.

As shown in figure 24, the vector of the pinion axial rotation, θ_{t_3p}, is on the diagonal of a parallelogram whose sides are the projections along the A_2 and A_3 rotational directions. Similarly, the gear axial rotation, θ_{t_3g}, is along the diagonal of a parallelogram composed of the projections of its respective components along the same A_2 and A_3 rotational directions. The gear or pinion rotation in any of these directions is the algebraic difference of the participating components. Therefore, the pinion component, A_{2p}, in the pinion axial rotation direction, is written as:

$$A_{2p} = + \theta_{t2p} + \frac{\theta_{t3p}}{\tan \Sigma} \tag{139}$$

and the gear component, A_{2g}, in the same direction is:

$$A_{2g} = - \frac{\theta_{t3g}}{\sin \Sigma} \tag{140}$$

The pinion axial rotation, A_2, is the algebraic difference of A_{2p} and A_{2g}.

$$A_2 = - A_{2p} + A_{2g} \tag{141}$$

Similarly, the gear axial component, A_3, is the difference of the projections of the same two rotations, but on the gear axial component axis, A_3.

The pinion component, A_{3p}, along A_3 is:

$$A_{3p} = \frac{\theta_{t3p}}{\tan \Sigma} \tag{142}$$

and the gear component, A_{3g}, in the same direction:

$$A_{3g} = + \theta_{t2g} + \frac{\theta_{t3g}}{\sin \Sigma} \tag{143}$$

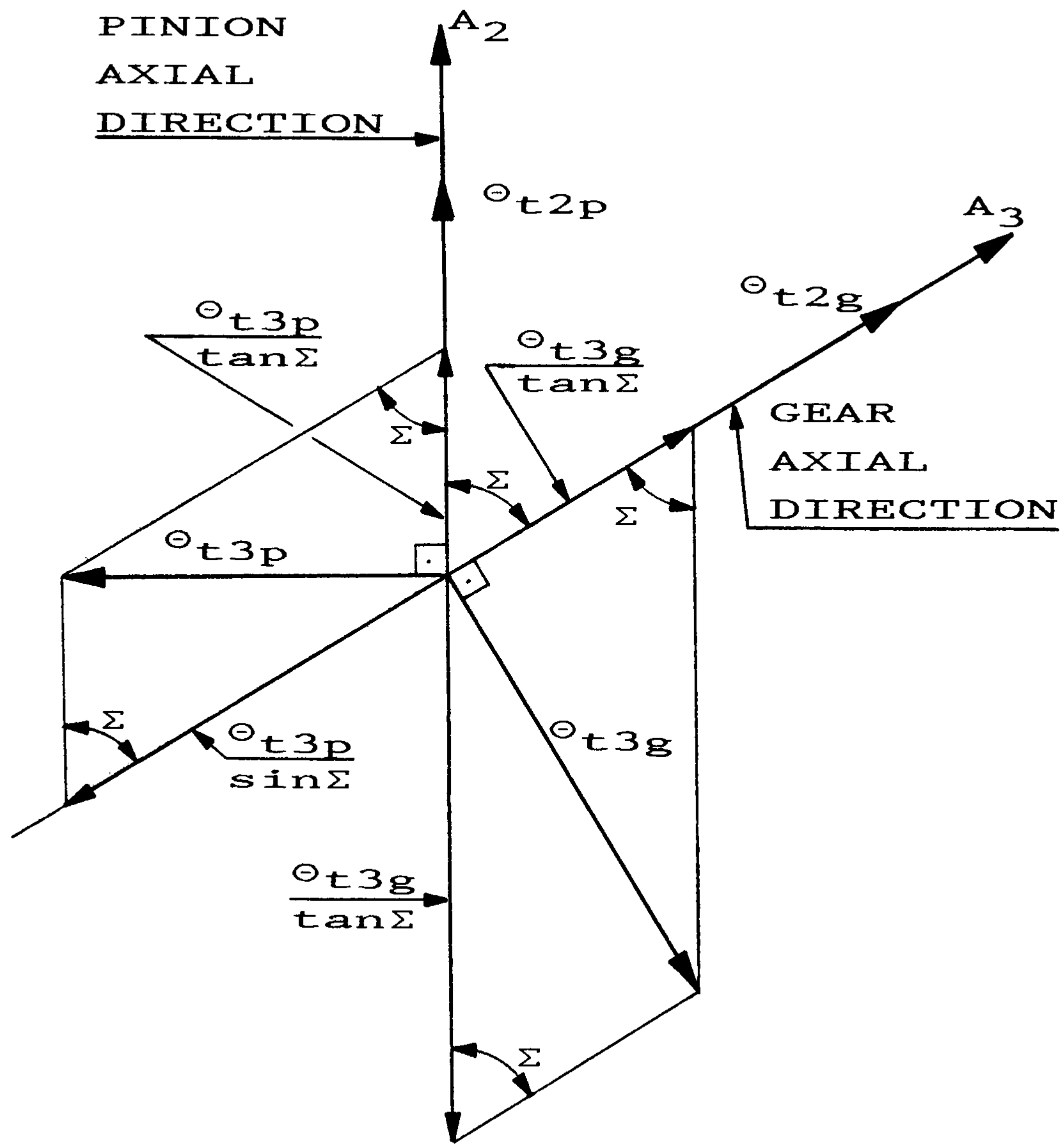


Figure 24. Relative Rotation Components.

The gear axial rotation, A_3, then is:

$$A_3 = - A_{3p} + A_{3g} \tag{144}$$

The relative translational motion of the gear with respect to the pinion can also be seen in figure 23. The common tangential translation, Z_1, is in the same plane with the relative rotation, A_1. Therefore, equation 145 can be derived in a similar manner.

$$Z_1 = D_{t1g} - (- D_{t1p}) + \Delta Z_1 \tag{145}$$

where ΔZ_1 is the relative motion of the contact points along the pitch ray of the gear and the pinion, given by equations 146 and 147, respectively.

$$\theta_g = R_{avg,g} \ A_3 \tag{146}$$

$$\theta_p = R_{avg,p} \ A_2 \tag{147}$$

Therefore,

$$\Delta Z_1 = \theta_g - \theta_p \tag{148}$$

or $$\Delta Z_1 = R_{avg,g} \ A_3 - R_{avg,p} \ A_2 \tag{149}$$

The relative translation along the Z_2 and Z_3 coordinate axes, shown in figure 25, can be determined in the same way.
The component projections in the pinion axial, Z_2 follow.

The pinion components are:

$$Z_{2p} = + D_{t2p} + \frac{D_{t3p}}{\tan \Sigma}. \tag{150}$$

and the gear components are:

$$Z_{2g} = - \frac{D_{t3g}}{\sin \Sigma} \tag{151}$$

Their difference results in the pinion axial translation, Z_2, which is also adjusted for the relative motion:

$$Z_2 = - Z_{2p} + Z_{2g} \tag{152}$$

and along the gear axial direction, the pinion component is:

$$Z_{3p} = - \frac{D_{t3p}}{\tan \Sigma} \tag{153}$$

and the gear component:

$$Z_{3g} = D_{t2g} + \frac{D_{t3g}}{\sin \Sigma} \tag{154}$$

Their difference yields the gear relative translation, Z_3:

$$Z_3 = - Z_{3p} + Z_{3g} \tag{155}$$

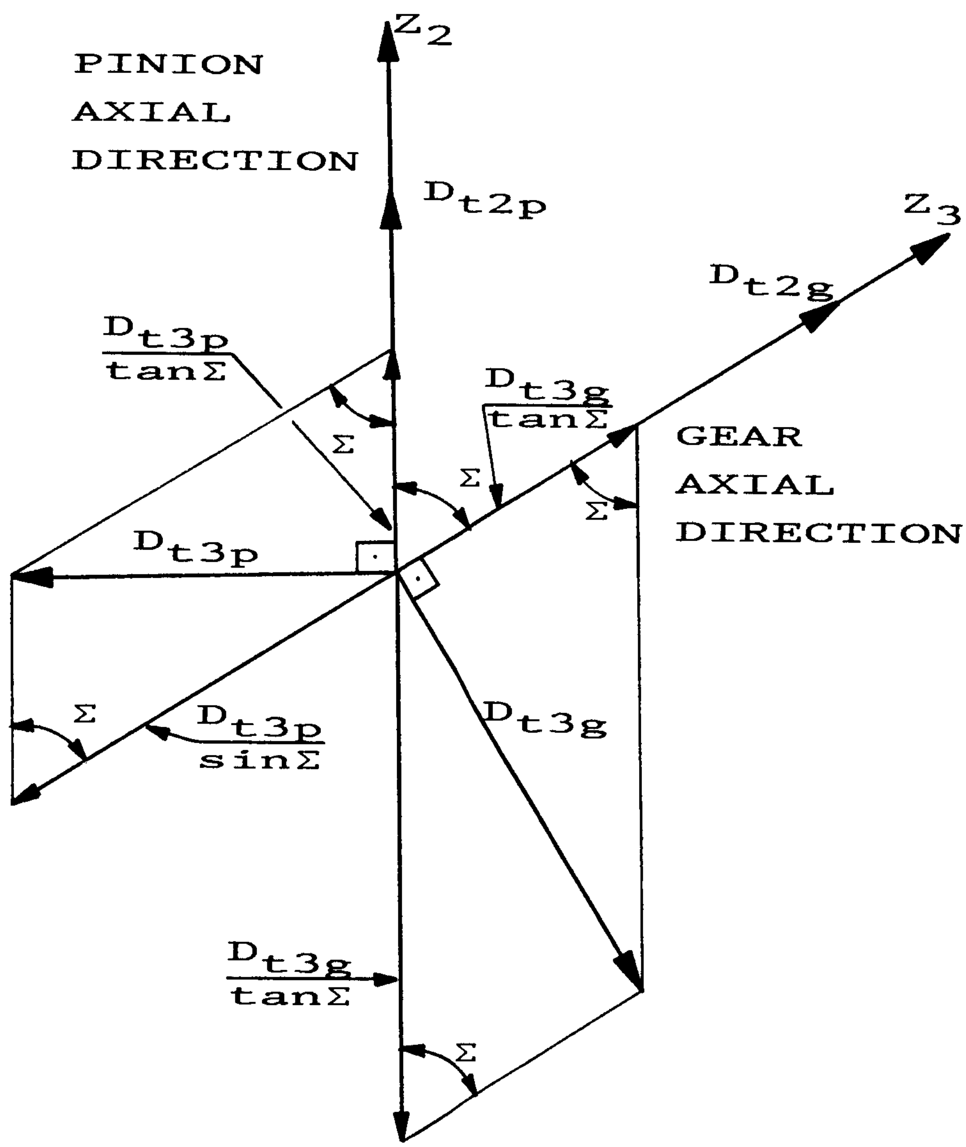


Figure 25. Relative Translation Components.

CONTACT ANALYSIS

The determination of the bearing contact for spiral bevel gears is analyzed in this section. It comprises the last part of the deflection analysis.

By considering the gear and shaft deflection found in the Deflection analysis section, along with the bearing motion of the Bearing analysis section, the total deflection and slope of the gear mesh are determined. Based on this analysis, the contact point deflection can be determined.

Due to the fact that the geometry of the spiral gear tooth surface is very complicated, the determination of the principal curvatures and principal directions along with the Hertzian contact stresses is a difficult problem. Litvin [5] has developed the equations to determine these principal curvatures and directions. The analysis is based on the assumption that there is a direct relationship between the principal curvature and its direction of the gear tooth surface and the gear cutting tool surface. Therefore, the curvature of the gear tooth surface is determined without using the complicated equations of the tooth geometry.

A novel approach to understanding and analyzing the geometry of spiral bevel gears is to consider the fact that one of the generrating surfaces of these gears is conical, and the other is a surface of revolution as shown in figure 26. These surfaces are in linear tangency along a circle of radius, r_d, passing through the pitch point of the gear tooth. They are also in point contact at the same point that moves along the circumference of the circle. In order to approach and solve the Hertzian contact problem, Litvin [5] has used methods of kinematic and analytic geometry and kinematic relations between motions of contact point.

The result of this analysis is the principal curvatures of the two gear teeth in contact. These curvatures are used to determine the size of the contact ellipse, and the maximum developed contact stresses. Finally, the principal curvatures are utilized to calculate the new location of the contact point with respect to the nominal pitch point in the middle of the tooth face. Furthermore, the dynamic capacity of the gear assembly is also determined.

Litvin's analysis is based on the gear and pinion geometry, along with the gear's and pinion's cutter machine settings. The geometry is described by the number of teeth for the gear and the pinion, N_g and N_p, spiral angle, ψ, pressure angle, ϕ_n, pitch angles, Γ_g and Γ_p, and the mean cone distance. D_o. The pitch angles and the mean cone distance are determined by equations 1-3 found in the spiral bevel gear geometry analysis. The gear cutter settings are provided by the cutting machine's manufacturer. Table 1 is a set of manufacturing parameter values which are needed to determine the principal curvatures and their directions for the gear and the pinion.

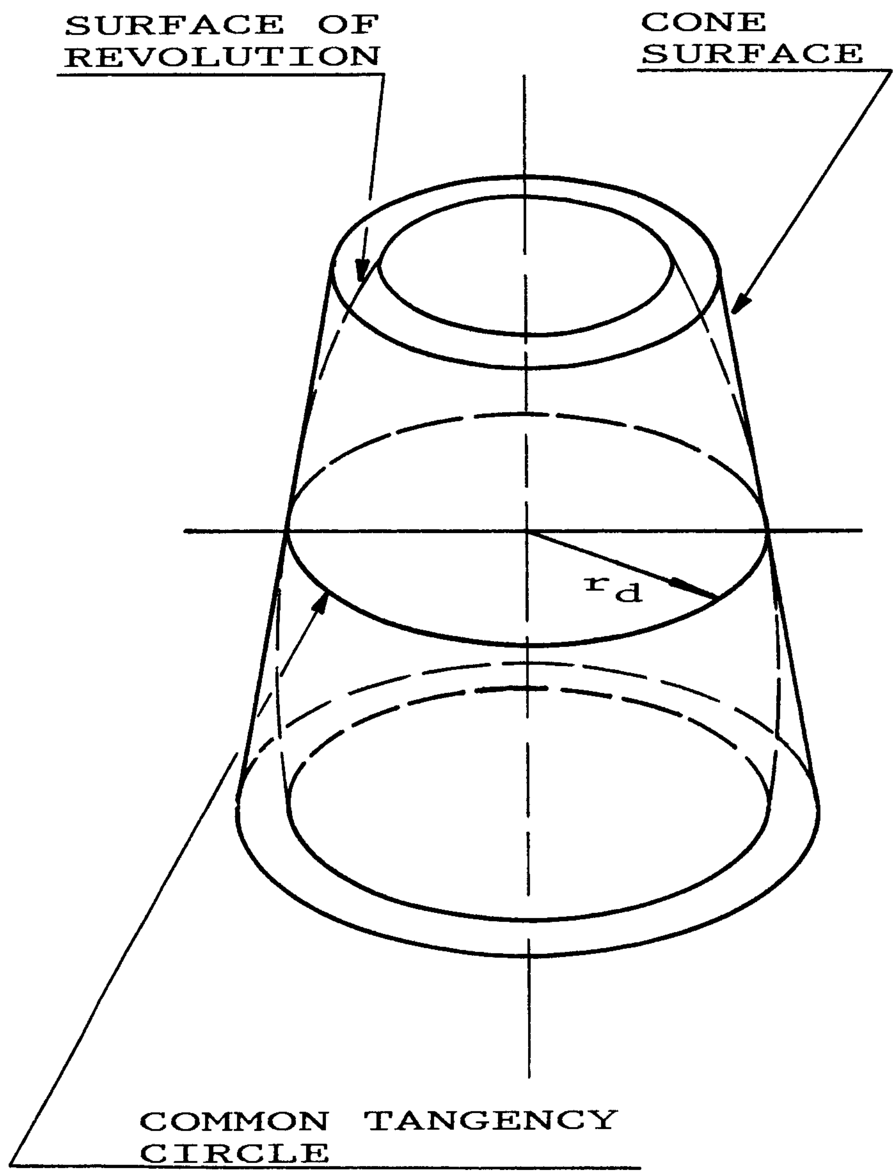


Figure 26. Generating Surfaces.

CONTACT ELLIPSE SIZE AND COMPRESSIVE STRESS

The contact ellipse size can be determined by using equations provided by Roark and Young [21]. The semi-major and semi-minor axis lengths of the contact ellipse are calculated as:

$$a = \alpha \sqrt[3]{W_n \; K_D \; C_E} \tag{156}$$

$$b = \beta \sqrt[3]{W_n \; K_D \; C_E} \tag{157}$$

The relative motion of approach, y, along the axis of the applied loading is calculated as:

$$y = \lambda \sqrt[3]{\frac{W_n^{\,2} \; C_E^{\,3}}{K_D}} \tag{158}$$

In these equations, W_n designates the normal tooth load determined by equation 15, K_D is a curvature constant given by equation 159, and C_E of equation 160 is a material constant of the two teeth in contact.

$$K_D = \frac{1.5}{\kappa_{I1} + \kappa_{I2} + \kappa_{II1} + \kappa_{II2}} \tag{159}$$

where κ_{I1}, κ_{I2}, κ_{II1}, κ_{II2} are the principal curvatures of the two bodies as described in Bearing Analysis section and whose equations are given in Litvin's analysis [5].

$$C_E = \frac{1 - \nu_1^{\,2}}{E_1} + \frac{1 - \nu_2^{\,2}}{E_2} \tag{160}$$

TABLE 1

Gear Geometry and Cutter Settings

1 - Spiral Angle (deg.)
2 - Mean Cone Distance (mm.)
3 - Head Cutter Diameter (mm.)
4 - Outside Tooth Height (mm.)
5 - Gear Face Width (mm.)

1	2	3	4	5
0-15 15-29 29-40	36- 58 40- 62 40- 55	88.9	8	10-20
0-15 15-29 29-40	40- 65 45- 70 45- 60	100	9	10-20
0-15 15-29 29-40	50- 80 55- 90 55- 75	125	10	12-25
0-15 15-29 29-40	60-100 70-110 70- 90	152.4	10	15-30
0-15 15-29 29-40	65-105 72-110 72- 95	160	12	16-32
0-15 15-29 29-40	75-120 85-135 85-115	190.5	15	20-40
0-15 15-29 29-40	80-130 90-140 90-120	200	15	20-40
0-15 15-29 29-40	90-150 100-160 100-135	288.6	15	20-40
0-15 15-29 29-40	100-160 110-175 140-190	250	18	25-50

TABLE 1 (continued)

1	2	3	4	5
0-15 15-29 29-40	120-200 140-215 140-190	304.8	20	30- 65
0-15 15-29 29-40	120-200 140-220 140-190	315	24	32- 65
0-15 15-29 29-40	160-250 180-280 180-240	400	30	40- 80
0-15 15-29 29-40	180-300 200-320 200-280	457.2	28	50-100
0-15 15-29 29-40	200-320 225-350 225-300	500	36	50-100
0-15 15-29 29-40	250-400 290-440 290-380	630	45	70-125
0-15 15-29 29-40	320-520 360-560 360-480	800	60	80-160
15-29	400-650	1000	70	100-200

where ν_1 and ν_2 are the Poisson's ratios and E_1 and E_2 are the Young's moduli of elasticity of the contacting teeth. The constants α, β, and λ are a function of an auxiliary angle, θ, given by equation 161, and have been evaluated [21] for a range of 0 to 90 degrees.

$$\cos\theta = \frac{K_D}{1.5}\sqrt{(K_{I1} - K_{I2})^2 + (K_{II1} - K_{II2})^2 + 2[(K_{I1} - K_{I2}) + (K_{II1} - K_{II2})]\cos 2\phi} \qquad (161)$$

where ϕ is the angle between the principal curvatures K_{I1} and K_{II2} shown in Figure 27.

The maximum contact stress can be determined as a function of the normal load acting on the gear tooth and the contact ellipse size:

$$\max \sigma_c = \frac{1.5\ W_n}{a\ b} \qquad (162)$$

CONTACT POINT DEFLECTION

The total axial and radial deflections determined in Bearing analysis section are shown in the β_D axis system shown in figure 28. To differentiate between the pitch point deflection components and the directions in which they act, let β_{D1} be the unit vector in the direction of the D_{t1} displacement. Then β_{E1} is a unit vector in the direction of the E_{t1} displacement component. The coordinate system has its origin at the pitch point on the gear tooth, with axes extending along the pitch ray and the midcone plane. This coordinate

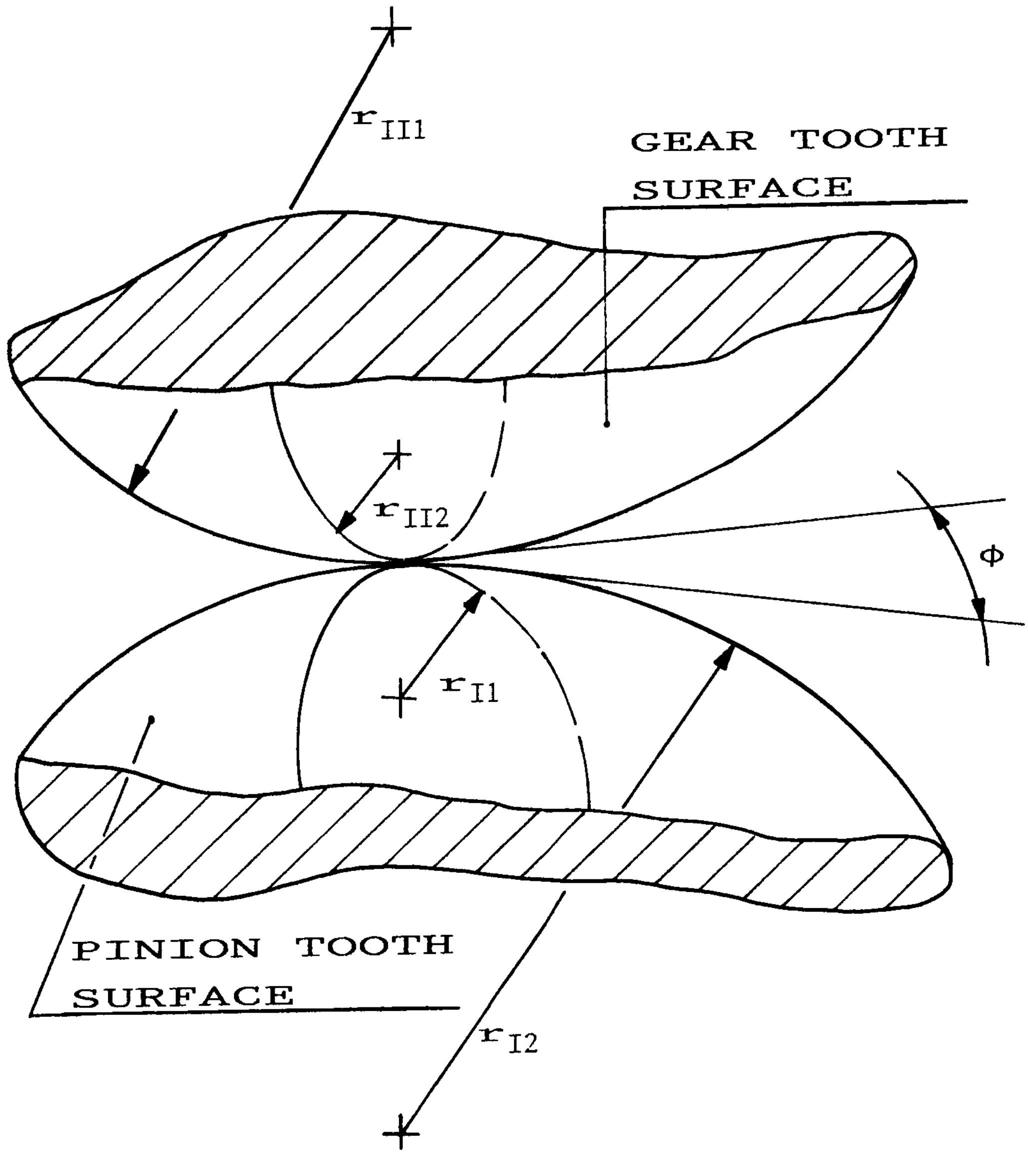


Figure 27. Contact Angle.

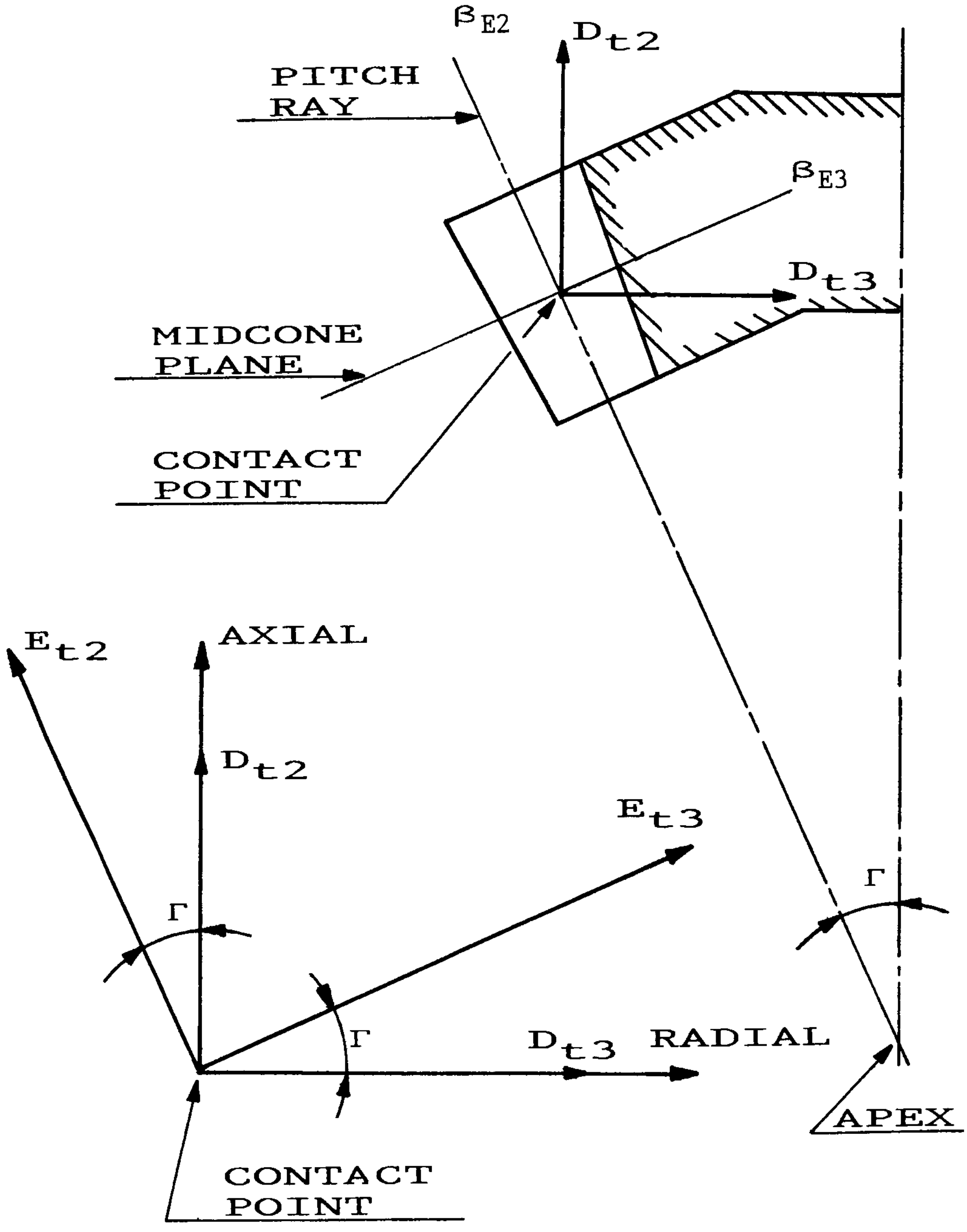


Figure 28. Contact Point Displacement in β_{E_2}, β_{E_3} Direction.

system is the β_{D1} coordinate system rotated through the half cone angle about the β_{D1} axis.

The deflections shown are those of the pitch point. The deflections in the midcone plane direction, E_{t_3}, and along the pitch ray, E_{t_2}, are described by the following equations.

$$E_{t2} = D_{t2} \cos \Gamma - D_{t3} \sin \Gamma \tag{163}$$

$$E_{t3} = D_{t2} \sin \Gamma + D_{t3} \cos \Gamma \tag{164}$$

These equations are valid for the gear or the pinion.

Since the pitch ray is not tangent to the tooth surface, shown in figure 29, the motion along the pitch ray has to be transformed to a motion tangent to the tooth in the plane tangent to the pitch cone. In order to obtain this tangent direction, one has to rotate the β_{E_2} direction through the spiral angle, ψ, about the β_{E_3} direction.

In figure 29, a complete representation of the β_{E_1} coordinate frame and the total deflections in these three directions are shown. From the same figure, the motion in the tangent direction, H_{t_2}, can be determined by considering the motion along the pitch ray, E_{t_2}, and in the bevel gear tangential deflection, E_{t_1}, as a function of the spiral angle where:

$$E_{t1} = D_{t1} \tag{165}$$

The tooth tangent displacement, H_{t_2}, acts in a direction β_{H_2} which is one of the axes of a β_H coordinate frame rotated from the β_E

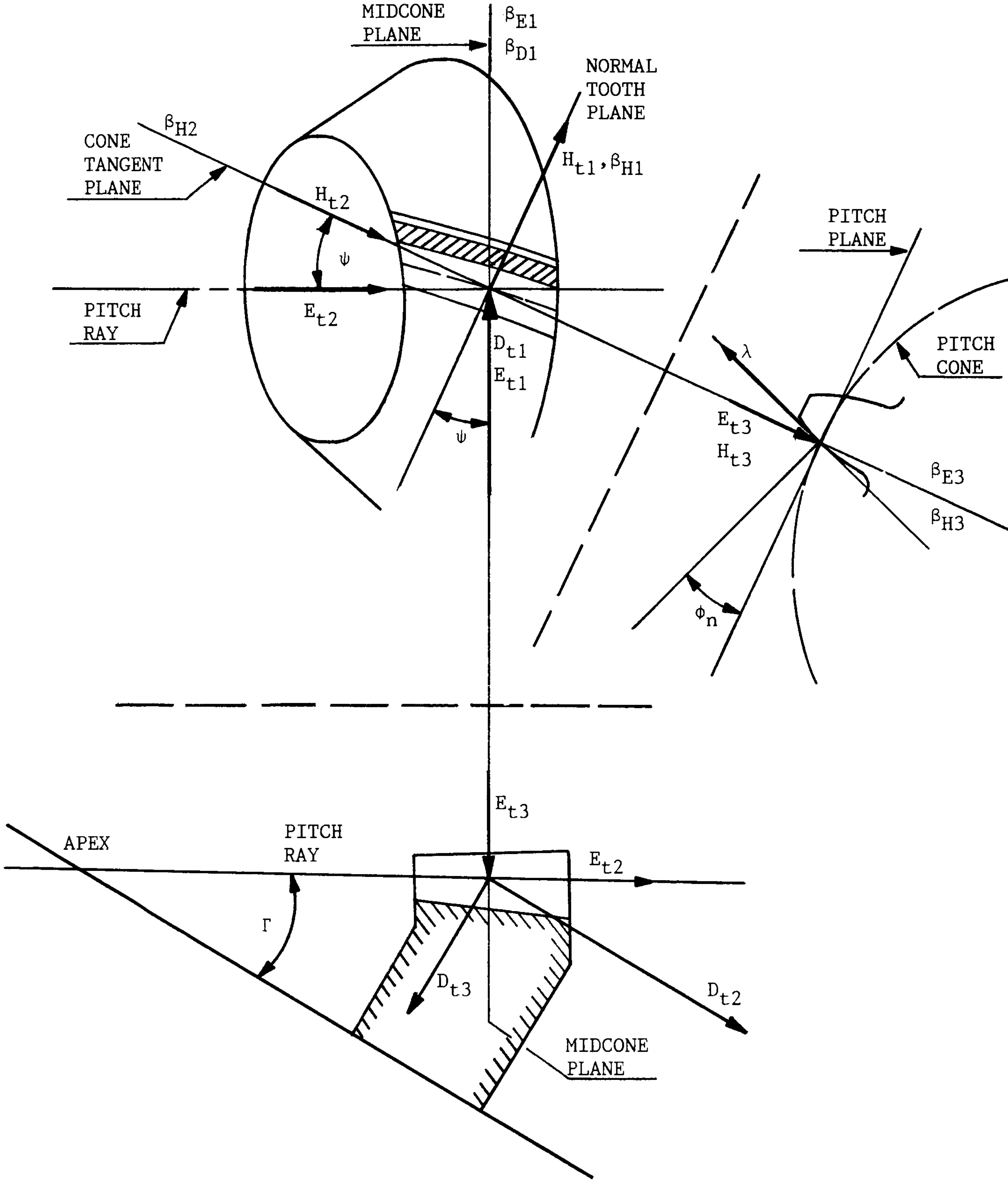


Figure 29. Displacements on Spiral Bevel Tooth

coordinate frame about the β_{E3} axis through the spiral angle, ψ. The displacements of the pitch point in this tangent coordinate frame, β_H, are:

$$H_{t1} = E_{t2} \sin \psi + E_{t1} \cos \psi \tag{166}$$

$$H_{t2} = E_{t2} \cos \psi - E_{t1} \sin \psi \tag{167}$$

$$H_{t3} = E_{t3} \tag{168}$$

The slope vector components in the β_D coordinate frame are the θ_{ti} vector components. In the β_E coordinate frame these slope vector components become:

$$A_{t1} = \theta_{t1} \tag{169}$$

$$A_{t2} = \theta_{t2} \cos \Gamma - \theta_{t3} \sin \Gamma \tag{170}$$

$$A_{t3} = \theta_{t2} \sin \Gamma + \theta_{t3} \cos \Gamma \tag{171}$$

In order to treat the tangential motion of the contact point, the motion of the unloaded pitch point and the motion of the primary center of curvature will be combined. By this combination, the tangential location of the loaded pitch point on both the gear and the pinion will be estimated. These two points lie in a plane normal to the tooth surface, which makes an angle ϕ_n with the pitch plane of the bevel gear which is tangent to the pitch cone. A unit vector, λ, is shown in figure 29. The components of this unit vector in the β_E coordinate frame, as shown in figure 30, are:

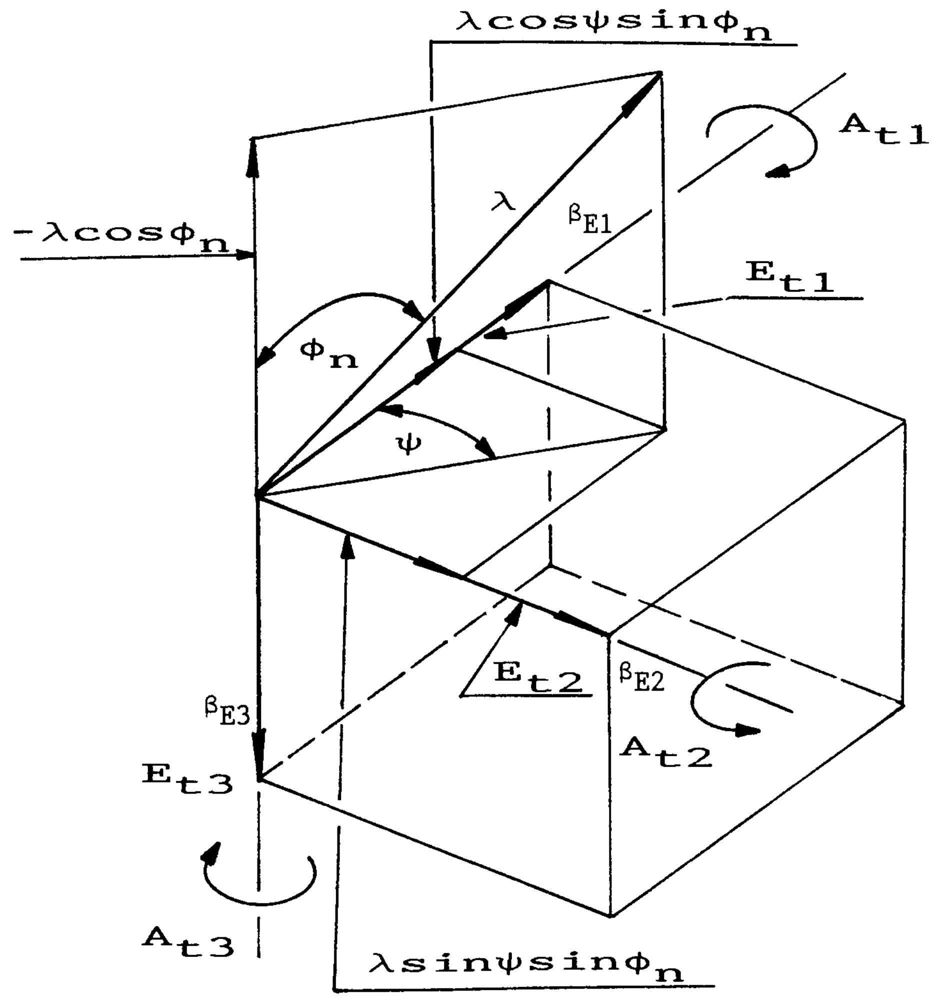


Figure 30. Unit Vector λ and Its Components.

$$\lambda_1 = \cos\psi \sin\phi_n \tag{172}$$

$$\lambda_2 = \sin\psi \sin\phi_n \tag{173}$$

$$\lambda_3 = -\cos\phi_n \tag{174}$$

The rotation vector components in the direction which acts in the normal plane containing the centers of primary curvature can be found as the dot product of λ with A.

$$\theta_r = \overline{\lambda} \cdot \overline{A} = A_{t1} \cos\psi \sin\phi_n + $$

$$+ A_{t2} \sin\psi \sin\phi_n - A_{t3} \cos\phi_n \tag{175}$$

Figure 31 shows the normal plane defined by λ, which contains the tooth tangent direction, β_{H2}, and the primary centers of curvature for both gear and pinion.

The new contact point location can be determined in this plane by using similar triangles. The new position is determined by considering the motion components along the tooth in the plane tangent to the pitch cone, H_{t2}, plus the new positions of the center of curvatures in the same plane due to the angles, θ_r, of the gear and the pinion.

$$D_f = H_{t2g} + R_g \sin\theta_{rg} +$$

$$+ \frac{R_g}{R_p + R_g}[(H_{t2p} + R_p \sin\theta_{rp}) -$$

$$- (H_{t2g} + R_g \sin\theta_{rg})] \tag{176}$$

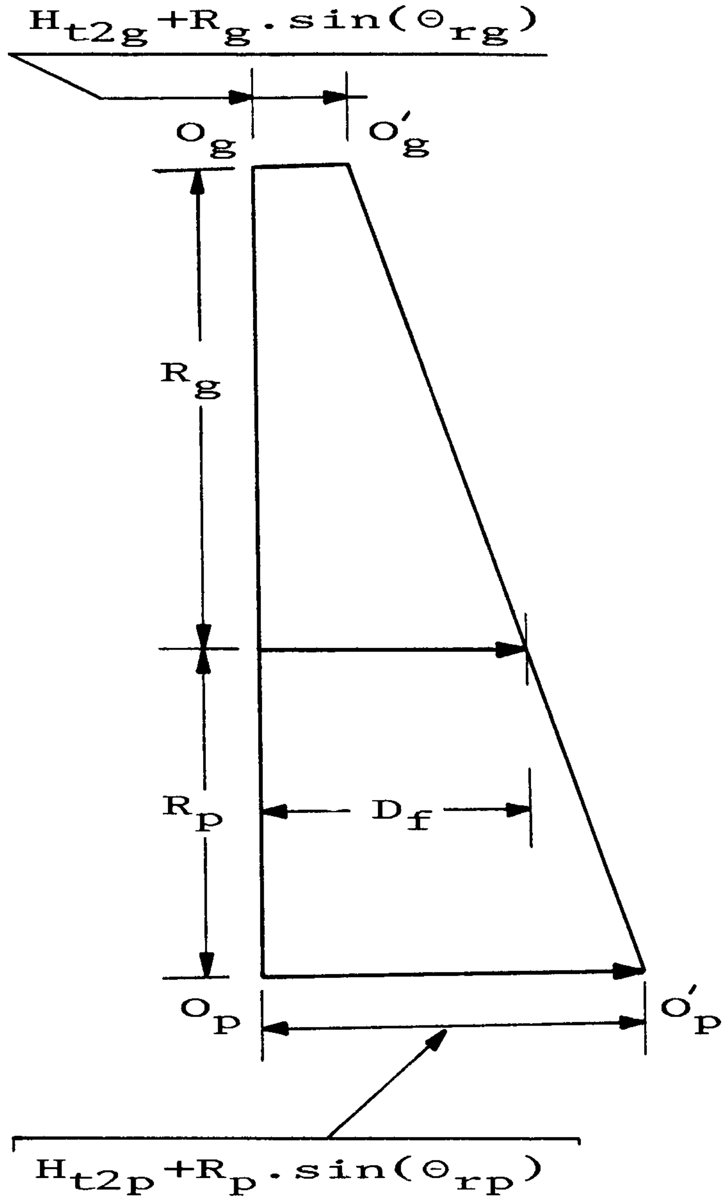


Figure 31. Contact Point Motion in β_{H_2} Direction.

This equation describes the absolute motion from the unloaded pitch point at the tooth center to the loaded pitch point in the tangent direction, D_f. The shift of the contact point on the gear in this direction is the difference between this motion and the absolute motion of the unloaded pitch point:

$$D_{pg} = D_f - H_{t2g} \tag{177}$$

and for the pinion:

$$D_{pp} = D_f - H_{t2p} \tag{178}$$

This is shown in figure 32.

The radial motion of the contact point along the tooth is determined by considering the base circles of a gear system shown in figure 33a. The gear motion in the plane affects the tooth contact by moving the base circles apart, thus increasing the pressure angle, ϕ_n, and the midcone radii, R_g, R_p, for the gear and the pinion, respectively.

The midcone plane intersects the pitch circle in an ellipse. Due to the presence of the spiral angle, the shape of the tooth of a spiral bevel gear at the pitch point is nearly the same as the shape of a bevel gear having a midcone distance, R_e. The midcone radii for the gear and the pinion are found from analytic geometry [22] as:

$$R_{eg} = \frac{R_g}{\cos^2\psi} \tag{179}$$

$$R_{ep} = \frac{R_p}{\cos^2\psi} \tag{180}$$

C-2

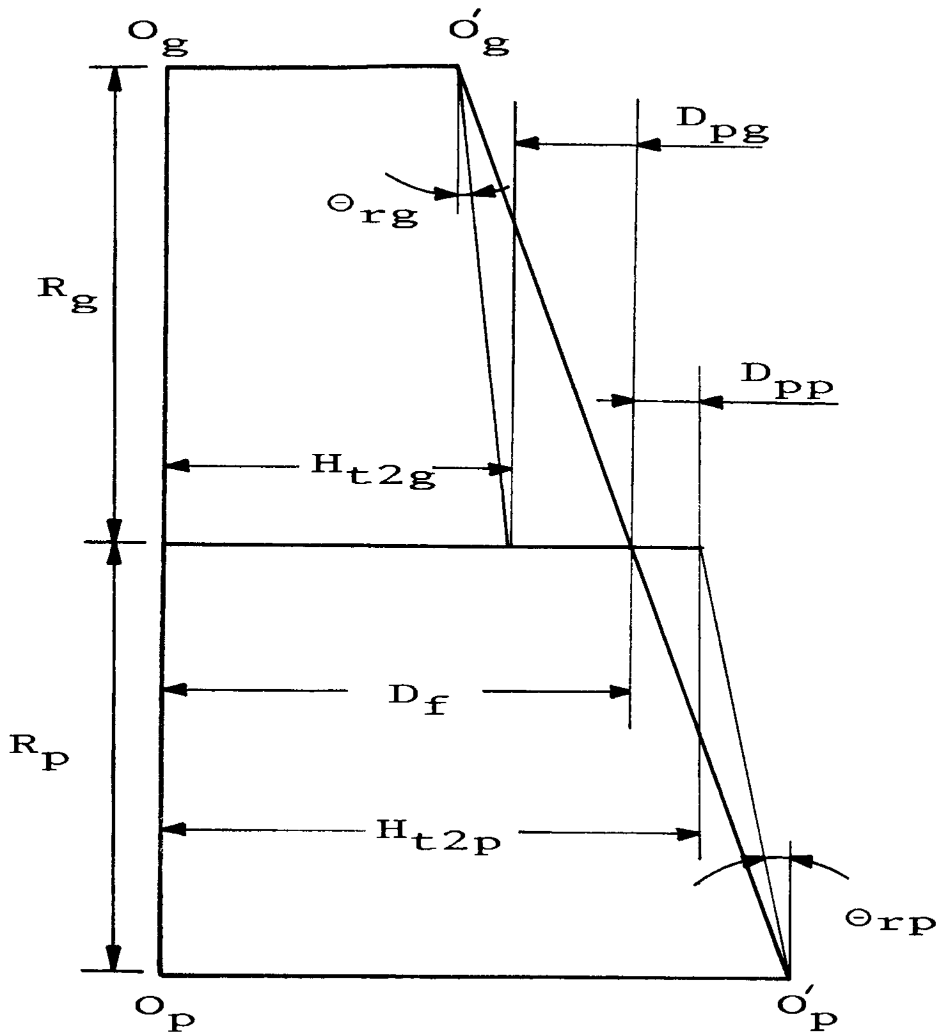


Figure 32. Contact Point Motion in Tangential Plane.

Since the sum of the two base circle radii, $R_{bp} + R_{bg}$, is fixed, the distance between the center of the two base circles is:

$$C_m = R_{eg} + R_{ep} = \frac{R_{bg} + R_{bp}}{\cos \phi_n} \tag{181}$$

Due to the radial displacement of the base circle centers in the midcone plane, the distance between these centers increases to:

$$\begin{aligned} C_m' = C_m &+ H_{t3g} + H_{t3p} \\ &- R_{eg}(1 - \cos \theta_{rmg}) - R_{ep}(1 - \cos \theta_{rmp}) \end{aligned} \tag{182}$$

where θ_{rm} is the rotation vector component normal to the midcone plane for the gear and the pinion, and is found from the rotations about β_{E1} and β_{E2}, A_{t1} and A_{t2} found in equations 169 and 170.

$$\theta_{rm} = A_{t1} \cos \psi + A_{t2} \sin \psi \tag{183}$$

The new pressure angle, ϕ_n', is determined by considering that the sum of the base circle radii remains constant.

$$\phi_n' = \cos^{-1}\left(\frac{C_m \cos \phi_n}{C_m'}\right) \tag{184}$$

The new midcone radii which are shown in figure 33b for the gear and the pinion can be determined by equations 185 and 186, respectively.

$$R_g' = \frac{R_{eg} \cos \phi_n}{\cos \phi_n'} \tag{185}$$

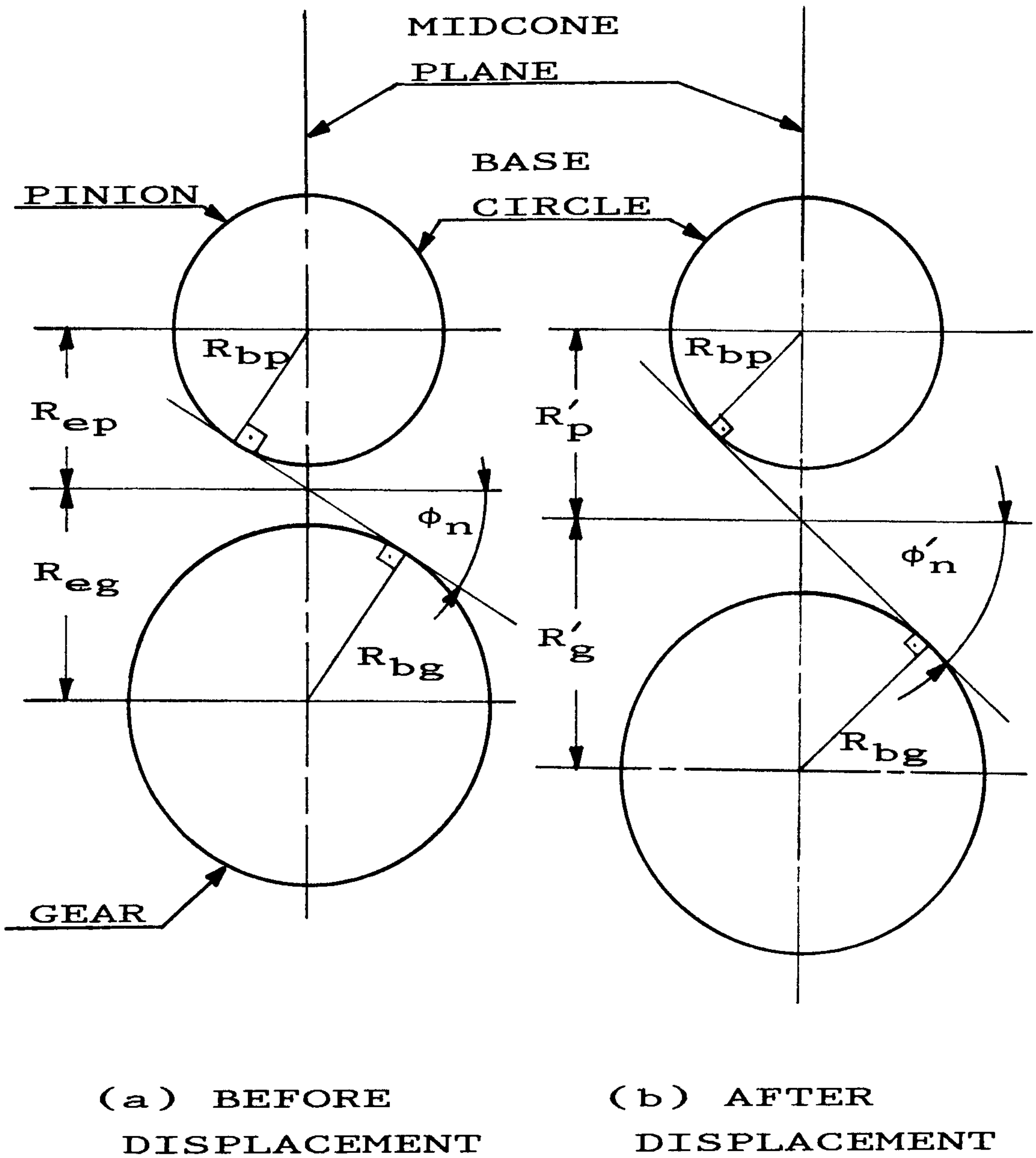


Figure 33. Contact Point Motion in Midcone Plane.

$$R_p' = \frac{R_{ep} \cos \phi_n}{\cos \phi_n'} \tag{186}$$

The gear pitch point moves radially along the tooth by an amount, D_n, which is shown in figure 34. This figure shows the unloaded and loaded pitch radii, R_{eg} and R_g', and their two pressure angles, ϕ_n and ϕ_n'. A chordal approximation of the motion is:

$$D_{ng} = \sqrt{R_{eg}^2 + R_g'^2 - 2 R_{eg} R_g' \cos(\phi_n' - \phi_n)} \tag{187}$$

Similarly, for the pinion:

$$D_{np} = \sqrt{R_{ep}^2 + R_p'^2 - 2 R_{ep} R_p' \cos(\phi_n' - \phi_n)} \tag{188}$$

The new position of the contact point on the gear or the pinion, as shown in figure 35, is found by adding the motions along the pitch ray and the midcone plane. The new contact point on the gear is separated from the old by:

$$D_{totg} = \sqrt{D_{pg}^2 + D_{ng}^2} \tag{189}$$

and on the pinion:

$$D_{totp} = \sqrt{D_{pp}^2 + D_{np}^2} \tag{190}$$

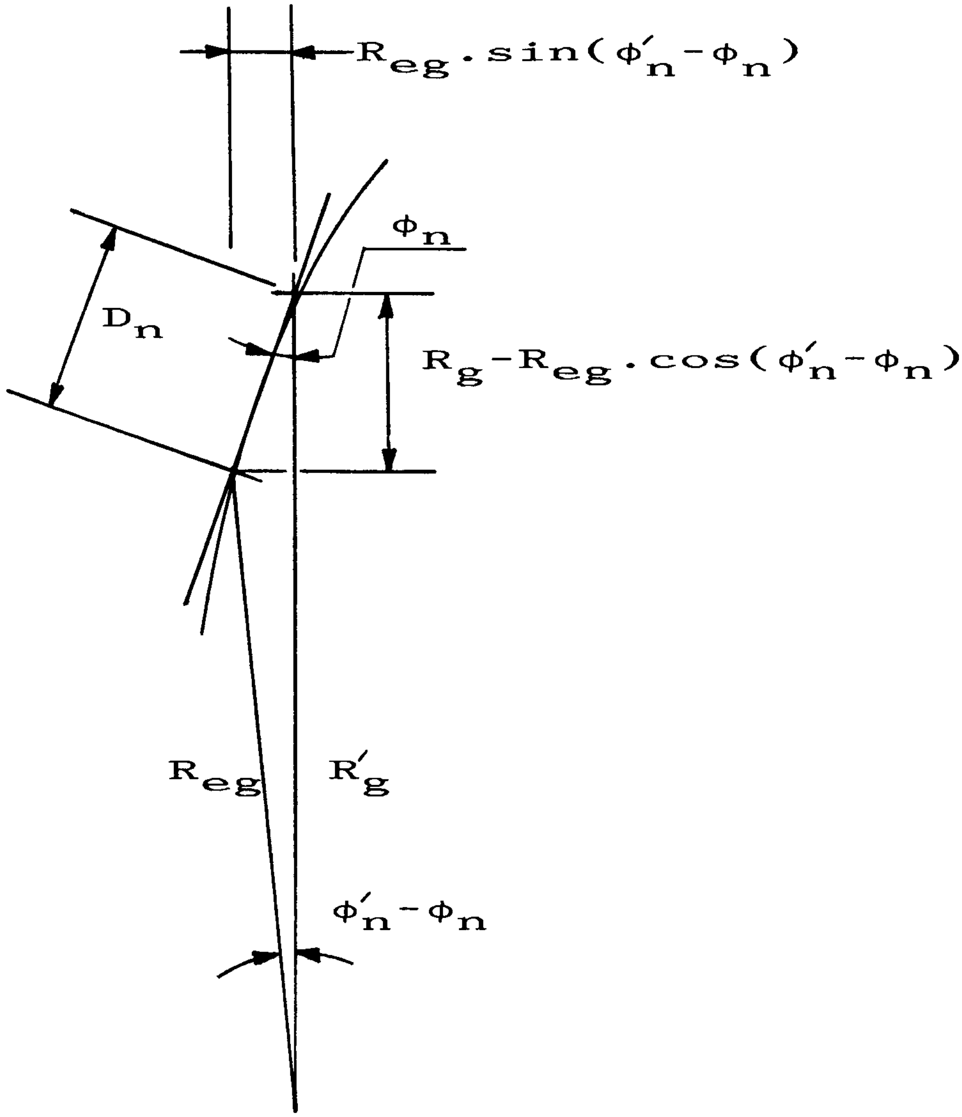


Figure 34. Radial Motion of Gear Pitch Point.

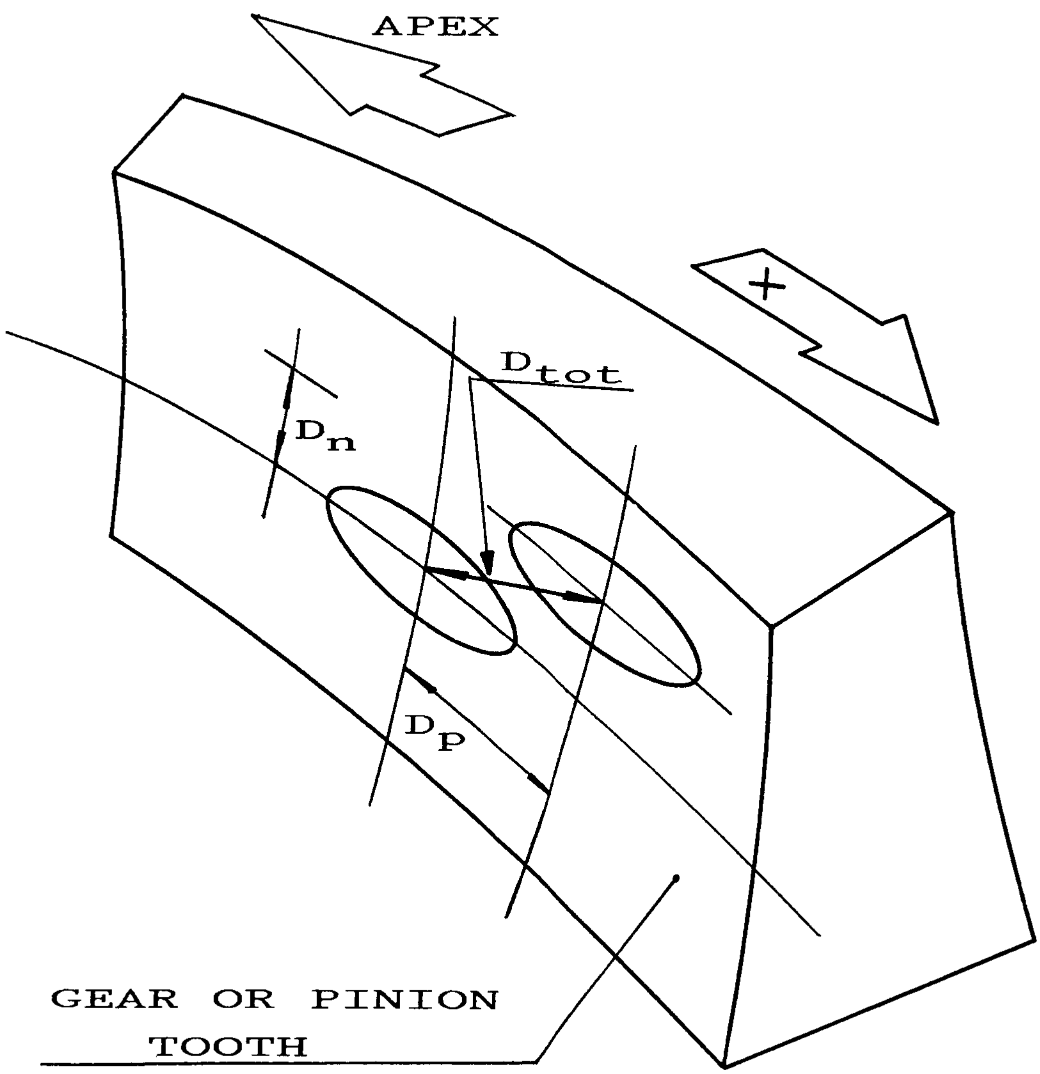


Figure 35. Total Contact Point Motion in Gear and Pinion.

DYNAMIC CAPACITY

The dynamic capacity of a gear mesh is defined as the normal load that may be carried with a 90 percent probability of survival for a life of one million cycles. The dynamic capacity of a gear tooth is proportional to the Hertzian contact pressure squared for applications in which the major axis of the contact ellipse is significantly larger than the minor axis. This elliptical contact shape is similar to that found in the case of two cylinders in contact. In this case the ratio of the major to minor axis of the contact ellipse is infinity and the area of contact is a rectangular strip. The contact between two cylinders can then be visualized as a 2-dimensional case.

In dealing with spiral bevel gears, the teeth contact can be seen as a contact of two spheres. Since the contact area is of circular shape, the ratio of the major to minor axis is equal to 1. The spherical contact implies a 3-dimensional relationship, thus the dynamic capacity is proportional to the Hertzian pressure cubed. However, the size and shape of the contact ellipse show that the contact area can be approximated more accurately as a 2-dimensional case, as if it were a contact area between two cylindrical surfaces.

Due to the significant difference in size of the major axis as compared to the minor, the curvature in the direction of the major axis is small. Therefore, the curvature in the direction of the tooth rotation dominates, and provides a more accurate estimate of the dynamic capacity. Based on these assumptions and definitions the dynamic capacity of a gear tooth can be calculated as:

$$C = \frac{2\ B_1\ a}{\kappa_{I1} + \kappa_{II2}} \tag{191}$$

where B_1 is a material constant, a is the semi-major axis of the contact ellipse, and κ_{I1}, κ_{II2} are the principal curvatures of the two gears in contact in the direction of the tooth rotation.

The material constant, B_1, is an experimental factor of load-stress relationship and is based upon test values. The value for case-hardened AISI 9310 Vacuum Arc Remelt steel gears can be determined as B_1 = 35,000 psi [7]. The values for this constant may be used to calculate the limiting surface loads between two curved surfaces of the same combination of materials [14].

The gears involved in the reduction have a certain load - the dynamic capacity - that will cause 10 percent of a large sample of gears to fail by pitting before or at one million applications of that load. Once the dynamic capacity is known, it can be used to determine the 90 percent reliability life in one million cycles for the applied load on the component [7]. The load-life relation for the gear can then be used to find the 90 percent reliability life for the actual gear load.

PROGRAM STRUCTURE AND USE

The computer-aided analysis of a spiral bevel gear assembly is performed using a program written in FORTRAN 77. The program can run in an interactive or in a batch file mode. At any prompt of input data entry, the user has the choice of the mode.

In the interactive mode, the user has to answer a series of prompts to enter the proper information on gear geometry, gear assembly, and bearing specifications. After a large block of data has been entered, an input summary of this block is printed to verify the correctness of the data. A prompt will ask whether any changes on the last set have to be made. In the case of any wrong input(s), the program will return to the beginning of the last section of input and the information is entered again.

In a batch file mode, the information will be read from a data file. At the end of the data entry, a verification is needed to resume the execution. For any incorrect number(s) found in the file, the program prompts the user to enter the data file's entries in the interactive mode described above.

The program was written in a modular form. There is a main program where the introductory information for the geometry of the gear mesh is entered, and the preliminary calculations are performed.

Depending on the bearing support configuration of the gear and/or the pinion, the program branches to two subroutines. The subroutines cover the straddle and the overhung configurations independently. The calculations performed are similar in both subroutines. For proper execution of the subroutines, two additional sets of information are required: The mounting geometry of the gears and bearings and the characteristics of the bearings. The sets of data inputs in the main program and the subroutines are shown in tables 2, 3 and 4.

Inside the subroutines, the deflections discussed and analyzed in the Deflection analysis section are calculated first. The bearing deflection is also calculated within the same subroutine by branching to other subroutines that perform the calculations of the Bearing analysis section .

The output of the two major subroutines is returned to the main program, and the relative motion analysis is performed. The program branches to a subroutine to figure the contact point geometry, principal curvatures and contact ellipse size. These results are used by another subroutine, and the new contact point location and dynamic capacity are determined. A complete flowchart of the program logic is shown in figure 36, and the source code is listed in Appendix A.

In the output of the program, the complete dimensions and the loading components of the gear mesh are displayed. Furthermore, the various deflections and slopes, the contact analysis and the dynamic capacity results are also printed by the program. Table 5 lists the sequence of the transmission analysis results.

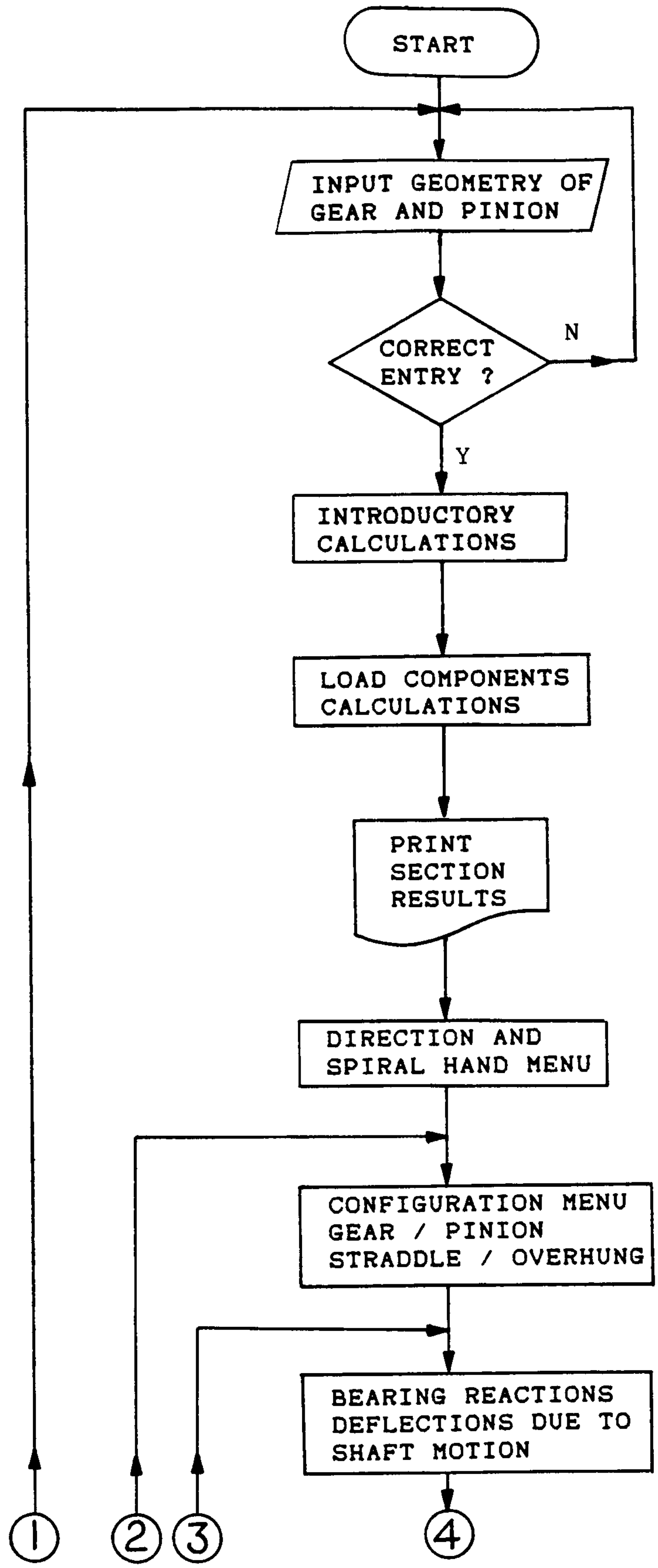


Figure 36. Computer Program Flow Chart.

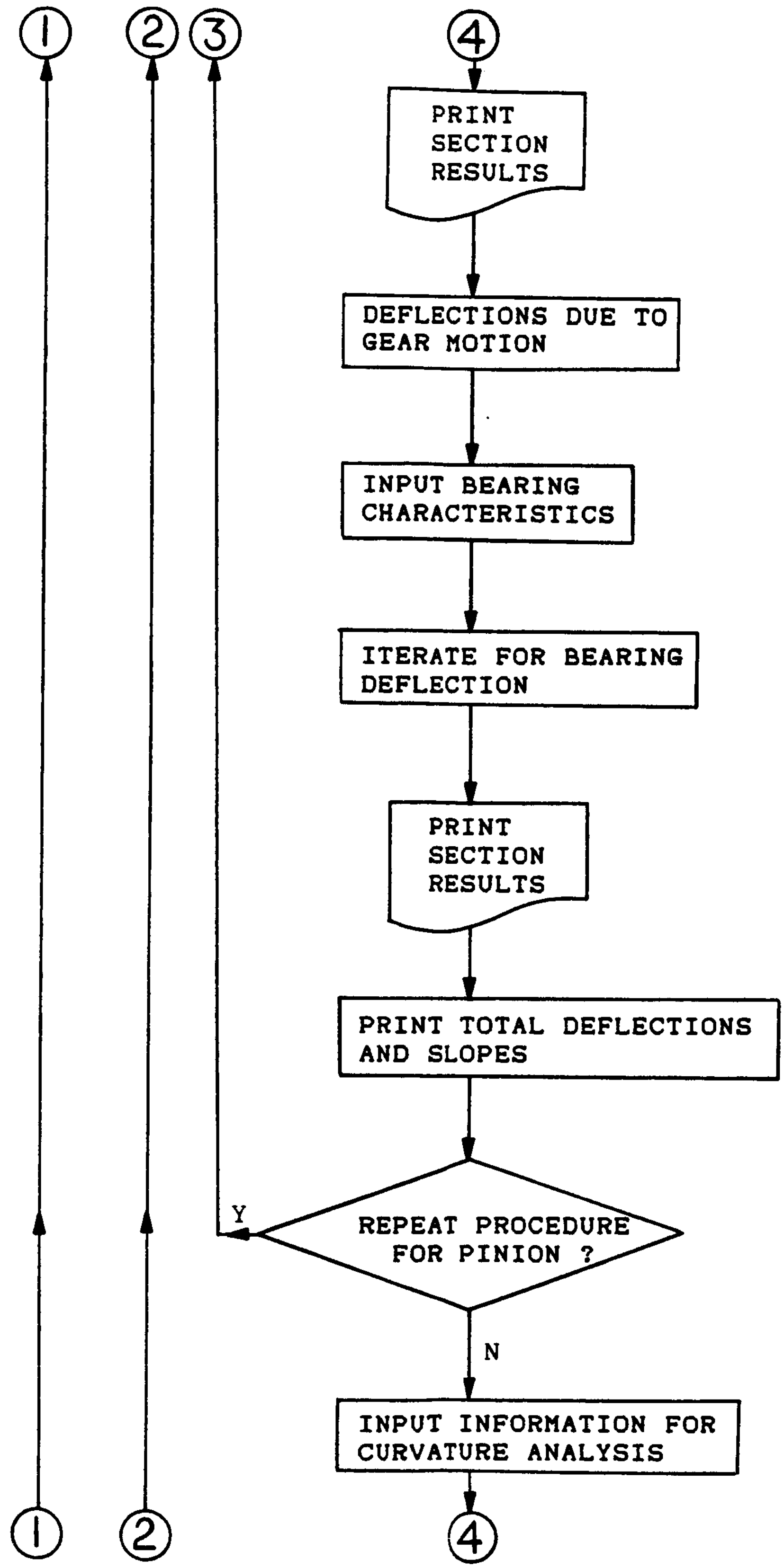


Figure 36 (continued)

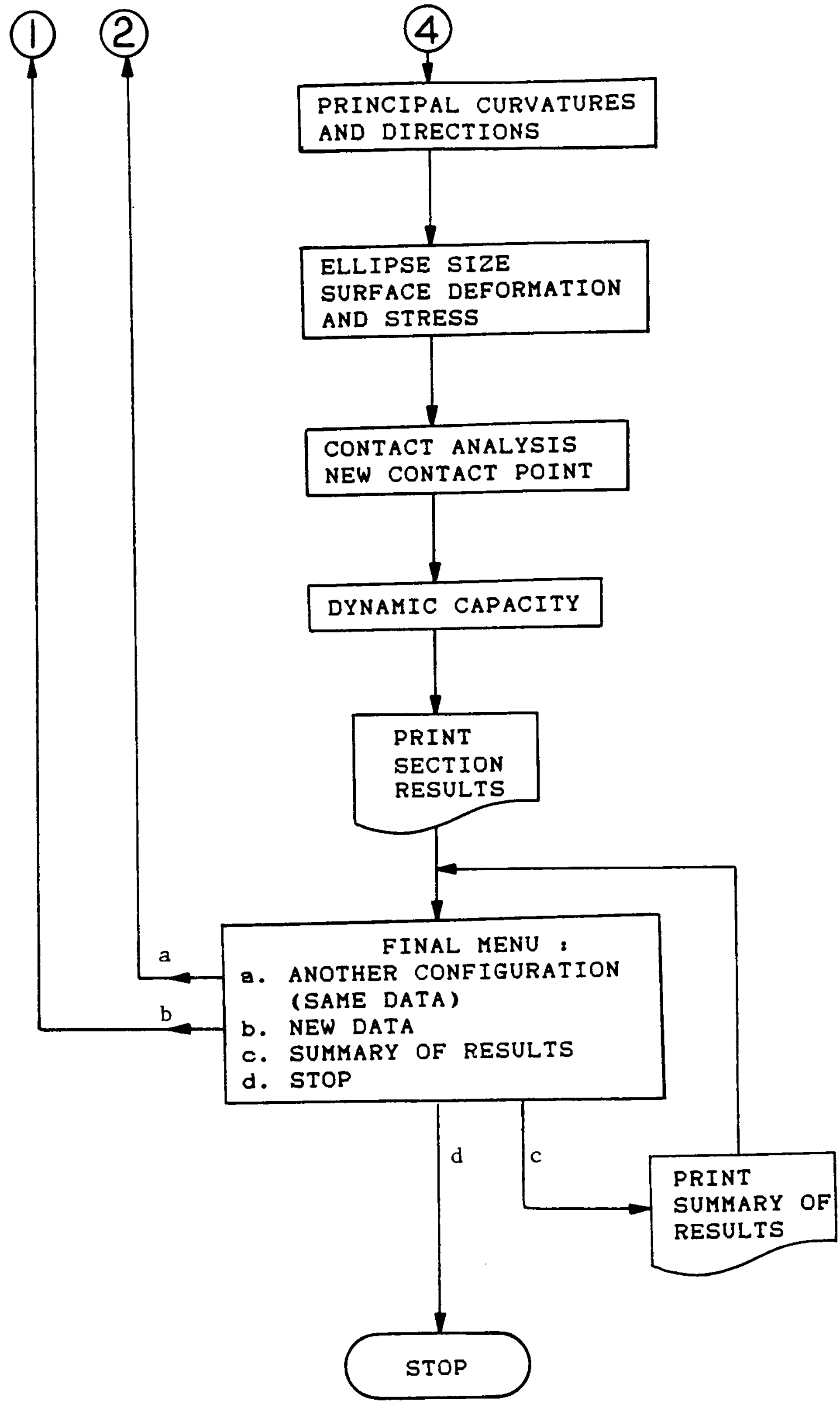


Figure 36 (continued)

TABLE 2

Inputs to Define the Geometry of
the Spiral Bevel Gear Mesh

1. Number of the gear teeth
2. Number of the pinion teeth
3. Cone distance of the gear mesh (in)
4. Face width of the gear mesh (in)
5. Normal pressure angle (deg)
6. Spiral angle (deg)
7. Shaft angle between the gear shaft centerline and the pinion shaft centerline (deg)
8. Input torque applied to the pinion (lbs-inch)
9. Gear addendum distance at the heel of the tooth (in)
10. Gear dedendum distance at the heel of the tooth (in)
11. Pinion addendum distance at the heel of the tooth (in)
12. Pinion dedendum distance at the heel of the tooth (in)
13. Spiral hand and direction of the driving gear

TABLE 3

Inputs to Define the Mounting of the Spiral
Bevel Gear Unit Gears and Bearings

1. The case of mounting - Straddle or Overhung mounting
2. Distance A - Gear to the right bearing
3. Distance B - Gear to the left bearing

TABLE 4

Inputs to Define the Characteristics of the Spiral Bevel Gear Mesh Bearings

The bearings can be of the following types:

1. Single Row Ball Bearings
2. Double Row Ball Bearings
3. Single Row Roller Bearings
4. Double Row Roller Bearings

For all types the inputs required are:

1. Inner raceway diameter (inches)
2. Inner raceway groove radius (inches)
3. Inner raceway elastic modulus ($lbs/inch^2$)
4. Inner raceway Poisson ratio
5. Outer raceway diameter (inches)
6. Outer raceway groove radius (inches)
7. Outer raceway elastic modulus ($lbs/inch^2$)
8. Outer raceway Poisson ratio
9. Rolling element diameter (inches)
10. Rolling element groove radius (inches)
11. Rolling element elastic modulus ($lbs/inch^2$)
12. Rolling element Poisson ratio
13. Number of rolling elements in the bearing
14. Angle of contact between the rolling element and the inner/outer raceway

TABLE 5

Output of the Program

For each gear the output will be as follows:

1. The gear mesh geometry
2. The loading force components on the gear
3. The forces on the bearings
4. The shaft motion deflections and slopes
5. The gear motion deflections and slopes
6. The bearing characteristics
7. The bearing deflection and slope components
8. The total deflection and slopes
9. The deflections at the gear center

For the curvature analysis the following will be the output:

1. The principal curvatures
2. The unit vectors along the principal curvatures
3. The contact ellipse size
4. The contact deformation and maximum stress

For the contact analysis the following will be the output:

1. The deflection along the pitch ray
2. The deflection along the midcone plane
3. The total deflection on the gear and the pinion

A summary of the results is also printed in a tabulated form (optional).

NUMERICAL EXAMPLE

The analysis presented in the previous chapters along with the computer program can be used to analyze the spiral bevel mesh shown in figure 37. In this example, the mesh being analyzed is part of a 320 horsepower single input helicopter transmission. The input speed is 6180 rpm and the output speed is 1654 rpm, producing a gear reduction of 3.736:1. The input torque of 3232 lbs-in will be transferred through 95 degrees from an approximately hori-zontal input pinion to a vertical output gear shaft. The geometry of the spiral bevel gear unit and the bearing characteristics are listed in table 6. Appendix B shows how the input information is entered into the program. The program was run with the geometry given in table 6 and the output can be found in Appendix C.

The loading on each component in the transmission is first calculated. If the geometry of the transmission changes, the loads on the components will change as well. This change of loading will subsequently affect the deflections of the assembly, the principal curvatures and the contact geometry. The loading analysis results are shown in table 7.

After the intermediate step of loading calculations, the various deflections due to the three motions discussed in the sections

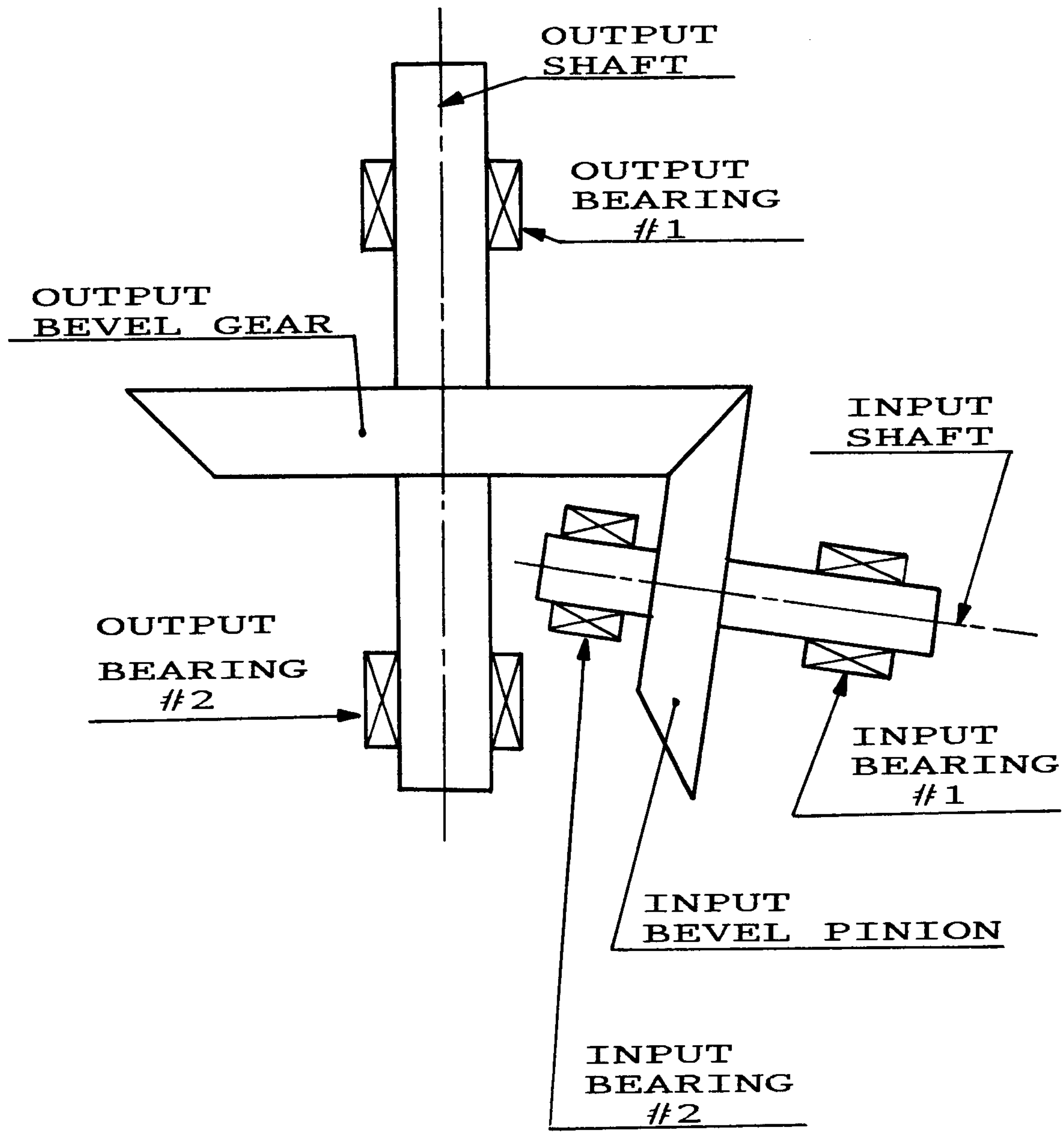


Figure 37. Example Spiral Bevel Mesh.

of Deflection and Bearing Analysis are calculated. These calculations are valuable to the designer since any change in shaft, gear or bearing geometry will affect the final contact analysis. The results are shown in table 8. Finally, the principal curvatures of the gear and pinion teeth are computed and used in calculating the contact geometry, contact point deflection, and the dynamic capacity, shown in table 9. The tooth profile and contact ellipses at the original and new position for the gear and the pinion are shown in figure 38.

The contact point deflection informs the designer whether the new point is on the tooth face or at the edge of it. This will affect the strength and the life of the tooth itself.

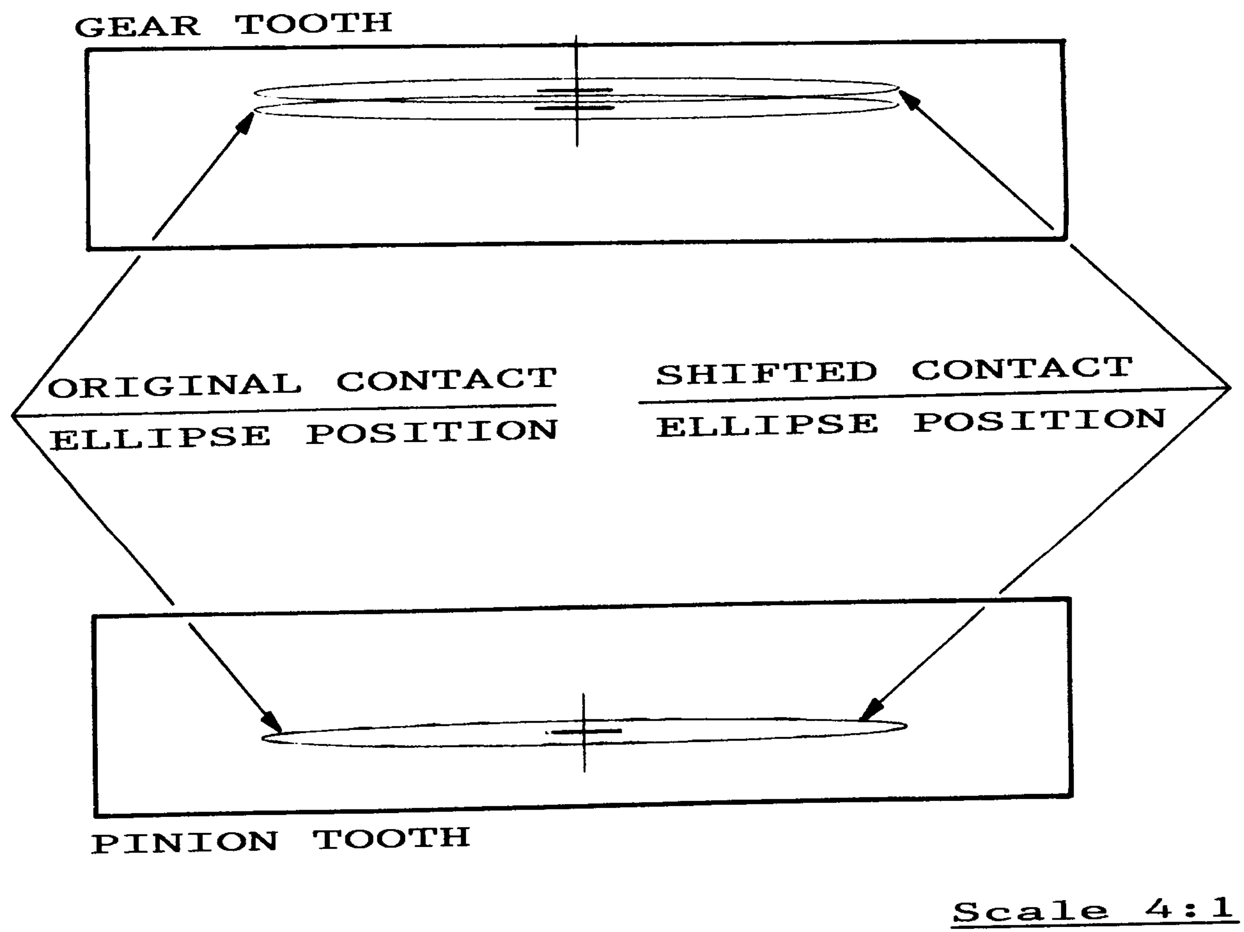


Figure 38. Contact Ellipse Shift in Gear and Pinion.

TABLE 6

Example of Spiral Bevel Gear Unit Input

Gear mesh geometry		
Number of the gear teeth		71
Number of the pinion teeth		19
Cone distance	132.08 mm	(5.2 in.)
Face width	32.512 mm	(1.28 in.)
Normal pressure angle		20 deg.
Spiral angle		30 deg.
Shaft angle		95 deg.
Input torque (to pinion)	365.00 N-m	(3232 lbs-in)
Gear addendum distance	2.0574 mm	(0.081 in.)
Gear dedendum distance	4.1402 mm	(0.163 in.)
Pinion addendum distance	4.445 mm	(0.175 in.)
Pinion dedendum distance	2.3622 mm	(0.093 in.)
Spiral hand of the driving gear		Clockwise
Direction of the driving gear		Left hand
Gear mounting		
Straddle mount		
Distance A	38.10 mm	1.50 in.
Distance B	5.08 mm	0.20 in.
Gear Bearing #1		
Single Row Roller Bearing		
Inner raceway		
Diameter	90.424 mm	(3.56 in.)
Groove radius	90.424 mm	(3.56 in.)
Elastic modulus	2.0685E08 KPa	(30000000 psi)
Poisson ratio		0.25
Outer raceway		
Diameter	112.5855 mm	(4.4325 in.)
Groove radius	112.5855 mm	(4.4325 in.)
Elastic modulus	2.0685E08 KPa	(30000000 psi)
Poisson ratio		0.25

TABLE 6 (continued)

Rolling element		
Diameter	11 mm	(0.433 in.)
Groove radius	11 mm	(0.433 in.)
Elastic modulus	2.0685E08 KPa	(30000000 psi)
Poisson ratio		0.25
Number of rolling elements		24
Contact length	21.6 mm	(0.85 in.)
Gear bearing #2		
Double Row Ball Bearing		
Inner raceway		
Diameter	84.836 mm	(3.34 in.)
Groove radius	5.842 mm	(0.23 in.)
Elastic modulus	2.0685E08 KPa	(30000000 psi)
Poisson ratio		0.25
Outer raceway		
Diameter	100.330 mm	(3.95 in.)
Groove radius	5.842 mm	(0.23 in.)
Elastic modulus	2.0685E08 KPa	(30000000 psi)
Poisson ratio		0.25
Rolling element		
Diameter	7.874 mm	(0.310 in.)
Groove radius	4.572 mm	(0.180 in.)
Elastic modulus	2.0685E08 KPa	(30000000 psi)
Poisson ratio		0.25
Number of rolling elements		25
Angle of contact		26.0 deg.
Pinion mounting		
Straddle mount		
Distance A	33.528 mm	(1.32 in.)
Distance B	48.26 mm	(1.90 in.)

TABLE 6 (continued)

Pinion Bearing #1

Double Row Ball Bearing

Inner raceway		
Diameter	63.246 mm	(2.49 in.)
Groove radius	8.382 mm	(0.33 in.)
Elastic modulus	2.0685Eo8 KPa	(30000000 psi)
Poisson ratio		0.25
Outer raceway		
Diameter	84.620 mm	(3.3315 in.)
Groove radius	8.382 mm	(0.33 in.)
Elastic modulus	2.0685E08 KPa	(30000000 psi)
Poisson ratio		0.25
Rolling element		
Diameter	10.744 mm	(0.423 in.)
Groove radius	5.537 mm	(0.218 in.)
Elastic modulus	2.0685E08 KPa	(30000000 psi)
Poisson ratio		0.25
Number of rolling elements		14
Angle of contact		35.0 deg.

Pinion Bearing #2

Single Row Roller Bearing

Inner raceway		
Diameter	52.578 mm	(2.07 in.)
Groove radius	52.578 mm	(2.07 in.)
Elastic modulus	2.0685E08 KPa	(30000000 psi)
Poisson ratio		0.25
Outer raceway		
Diameter	68.58 mm	(2.70 in.)
Groove radius	68.58 mm	(2.70 in.)
Elastic modulus	2.0685E08 KPa	(30000000 psi)
Poisson ratio		0.25

TABLE 6 (continued)

Rolling element		
Diameter	8.077 mm	(0.318 in.)
Groove radius	8.001 mm	(0.315 in.)
Elastic modulus	2.0685E08 KPa	(30000000 psi)
Poisson ratio		0.25
Number of rolling elements		18
Contact length	9.804 mm	0.386 in.

TABLE 7

Intermediate and Loading Analysis Results

	Gear	Pinion	
Pitch Angle	79.733	15.267	deg.
Diametral Pitch	7.912	7.912	$in.^{-1}$
Average Radius	4.487	1.201	in.
Load			
Tangential	2691.670	2691.670	lbs.
Axial	836.142	1797.072	lbs.
Radial	1730.784	682.112	lbs.
Normal	3307.541	3307.541	lbs.

TABLE 8

Summary of Deflection and Slope Results

	Gear	Pinion	
Deflections (Shaft and Gear Motion)			
Y_1 - Tangential	0.9E-060	0.000055	in.
Y_2 - Axial	0.000135	0.000020	in.
Y_3 - Radial	-0.48E-05	0.000024	in.
Slopes (Shaft and Gear Motion)			
θ_1 - About Y_1	0.001724	0.000952	deg.
θ_2 - About Y_2	-0.028794	-0.024157	deg.
θ_3 - About Y_3	-0.000222	0.000724	deg.
Total Deflections (Shaft + Gear + Bearing Motion)			
D_{t1} - Tangential	0.89E-04	0.000310	in.
D_{t2} - Axial	0.001356	0.000215	in.
D_{t3} - Radial	0.87E-04	0.000235	in.
Total Slopes (Shaft + Gear + Bearing Motion)			
θ_{t1} - About D_{t1}	0.003176	0.011420	deg.
θ_{t2} - About D_{t2}	-0.028794	-0.024157	deg.
θ_{t3} - About D_{t3}	0.015368	0.010006	deg.

TABLE 8 (continued)

Deflections at gear center

V_1	0.9E-06	0.000055	in.
V_2	0.0	0.0	in.
V_3	0.000130	0.000044	in.

Relative motion (gear relative to pinion)

Relative translation

Z_1	-0.001719	in.
Z_2	-0.13E-04	in.
Z_3	0.000159	in.

Relative translation

A_1	0.014596	deg.
A_2	0.009606	deg.
A_3	-0.020094	deg.

TABLE 9

Contact Analysis Results

Ellipse size

Semi-major axis	0.4233	in.
Semi-minor axis	0.0156	in.
Ratio (major/minor)	27.087	
Tooth surface deformation	0.00109	in.
Max. contact stress	0.75E06	psi

Contact Deflections

	Gear	Pinion	
D_p - Tangent Plane	0.000422	0.000231	in.
D_t - Normal Plane	0.000000	0.000000	in.
D_{tot} - Total	0.000422	0.000231	in.

Basic dynamic capacity	14900.0	lbs.

DISCUSSION OF RESULTS

The analysis presented in this study is a model of the effect of shaft and bearing design on spiral bevel gear performance. Helicopter transmissions are designed with the objective of transmitting high power in a minimum weight package. In order to achieve this objective, some elastic flexibility must be accepted in the transmission. The purpose of this study is to develop a design tool which can be used by the designer to evaluate the necessary trade off between transmission weight and rigidity. Although all deflections do not adversely affect transmission performance, the deflections inside the bevel gear mesh dramatically reduce transmission performance. These deflections change the kinematic interaction of the gear teeth, increase the kinematic transmission error, increase the transmission noise and reduce the life and dynamic capacity of the bevel gear mesh.

Use of this model should assist the designer in detemining transmission configurations which are light in weight but which also support the bevel mesh in such a way as to minimize the gear mesh deflections and the shift of the Hertzian contact ellipse location on the tooth surface.

The model is incorporated in a Fortran computer program which has been written with interactive input modules for ease of use. The program requests the input information in a series of prompts. After a screen of data has been entered, the program then echoes the given information and requests the designer to verify its accuracy. If any information has been incorrectly read by the computer, the user may correct it and repeat the review. Once the screen of data has been accepted, the user need not re-enter it if later data is corrected. If multiple designs are to be checked at the same session, an input data file may be used to bypass the interactive data entry process and speed the program execution.

Since the model is not specific to a single support geometry configuration or a fixed set of spiral bevel gear parameters, the program may be used to evaluate the contact shift magnitude for many significantly different designs. It is hoped that systematic use of this program can assist the helicopter transmission designer in improving the effective power to weight ratio of his transmission.

SUMMARY OF RESULTS

A method to predict the shift of the loaded region on the teeth of the gears in a spiral bevel gear reduction is presented in this report. This shift is a result of the gear tooth load and the elastic deflections of the gear support shafting and bearings. The deflections considered are those caused by the radial bearing deflections, the shaft deflections and the shaft slopes.

The modeled gear box is a single output, single input, bevel gear reduction such as found in the OH-58 light duty helicopter. This geometry is composed of a single spiral bevel gear drive with a spiral bevel pinion input and the shafts and bearings that support the gear and pinion. The calculation of the gear mesh dynamic capacity based on the tooth curvatures, material strength and the applied load is presented also. A Fortran computer program to calculate this shift and the mesh dynamic capacity assuming constant tooth curvature is included.

The analysis is performed in a modular fashion. In the first stage, the gear geometry and loading are defined. In the second stage, the elastic defections of the bearings and shafts and the slopes of the shafts at the gears are determined. In the third stage, the motions of the gear teeth caused by each elastic deflection are determined. The total deflections of the gear teeth are the algebraic sum of these motions. In the fourth stage, the interaction between the tooth geometry and motion is analyzed to obtain a prediction for the effects of these elastic motions on the gear tooth contact. Two separate analyses are presented in this fourth stage. In the first, a relative motion vector analysis is performed to model the relative displacement of the unloaded pitch points and the change in the shaft angle at the point of contact between the two gears due to these elastic deflections. These quantities are required for the Tooth Contact Analysis of the mesh performed by a major bevel gear manufacturer. In the second analysis, the geometry and curvatures of the gear and the pinion teeth are combined with the separate elastic motions of the two gear teeth to predict the shift of the contact ellipse under load. In this analysis, it is assumed that the curvatures are constant over the surfaces of the teeth.

Finally the basic dynamic capacity of the mesh is calculated from the loading, tooth curvatures and material properties. The computer program can simulate a number of different support geometries with the gear and the pinion separately supported, either between two bearings or overhung from a bearing quill behind the gear. Several different roller and ball bearing types can also be simulated with and without preload.

This report includes the development of theory that supports the computer aided simulation. A listing of the program in Fortran source code is appended. The application of this model and the use of the program are illustrated with an example. A listing of the computer input and output for this example are appended to the report as well.

APPENDICES

APPENDIX A

PROGRAM LISTING

```
C
C>
C>> *------------------------------------------------------------*
C>> *                 SPIRAL BEVEL GEAR PERFORMANCE              *
C>> *------------------------------------------------------------*
C
C>> *----------- DEFLECTION AND CONTACT ANALYSIS ----------------*
C
C>> *------------------------------------------------------------*
C>> * BEARING ANALYSIS IS PERFORMED WITH LINEAR INTERPOLATION *
C>> *------------------------------------------------------------*
C>
C
C>    INPUT INFORMATION
C
      CHARACTER*6,DFILE
      CHARACTER*3,ANS
      REAL NP,NG
      COMMON/INFO/NP,NG,AO,F,E
      COMMON/SPEC/ADDG,ADDP,DEDG,DEDP
      COMMON/DEFLE/YPRG,YPRP,YPAG,YPAP,YPTG,YPTP,
     1        THE1P,THE1G,THE2P,THE2G,THE3P,THE3G
      COMMON/OUT/DPG,DPP,DNG,DNP,DTOTG,DTOTP,RATG,RATP
      RAD(A)=PI*A/180.
      DEG(A)=180.*A/PI
      PI=3.1415927
      WRITE(1,110)
110   FORMAT(//,4X,'HOW DO YOU WANT TO ENTER THE ',/,
     1          4X,'INTRODUCTORY INFORMATION ?',//,
     2          4X,' 1.  USING THE TERMINAL  ',/,
     3          4X,' 2.  USING A DATA FILE',/)
      READ(1,*)NCHOI
      GOTO (4444,4455)NCHOI
4455  WRITE(1,4456)
4456  FORMAT(//,4X,'ENTER DATA FILE NAME',/)
      READ(1,4457)DFILE
4457  FORMAT(1A6)
      OPEN(66,FILE=DFILE)
      READ(66,*)NG,NP,AO,F,PHIN,PSI,GPAN,TP,DG,DP,
     1          ADDG,ADDP,DEDG,DEDP
      CLOSE (66)
      GOTO 1232
4444  WRITE(1,10)
10    FORMAT(//,
```

```
      1 4X, 'ENTER NUMBER OF GEAR TEETH, NG',/)
       READ(1,*)NG
       WRITE(1,11)
 11    FORMAT(//,
      1 4X, 'ENTER NUMBER OF PINION TEETH, NP',/)
       READ(1,*)NP
       WRITE(1,14)
 14    FORMAT(//,
      1 4X, 'ENTER PITCH CONE CENTER DIST., AO, IN',/)
       READ(1,*)AO
       WRITE(1,15)
 15    FORMAT(//,
      1 4X, 'ENTER GEAR FACE WIDTH, F, IN',/)
       READ(1,*)F
       WRITE(1,18)
 18    FORMAT(//,
      1 4X, 'ENTER NORMAL PRESSURE ANGLE, PHIN, DEG',/)
       READ(1,*)PHIN
       WRITE(1,19)
 19    FORMAT(//,
      1 4X, 'ENTER SPIRAL ANGLE, PSI, DEG',/)
       READ(1,*)PSI
       WRITE(1,20)
 20    FORMAT(//,4X,
      1 'ENTER MESH ANGLE OF GEAR AND PINION, SIGMA, DEG',/)
       READ(1,*)GPAN
       WRITE(1,21)
 21    FORMAT(//,
      1 4X, 'ENTER TORQUE APPLIED TO THE PINION, LBS-IN',/)
       READ(1,*)TP
       WRITE(1,22)
 22    FORMAT(//,
      1 4X, 'ENTER THE GEAR SHAFT DIAMETER, DG, IN',/)
       READ(1,*)DG
       WRITE(1,23)
 23    FORMAT(//,
      1 4X, 'ENTER THE PINION SHAFT DIAMETER, DP, IN',/)
       READ(1,*)DP
       WRITE(1,24)
 24    FORMAT(//,
      1 4X, 'ENTER THE GEAR ADDENDUM DISTANCE, DEDG, IN',/)
       READ(1,*)ADDG
       WRITE(1,25)
 25    FORMAT(//,4X,
      1 'ENTER THE PINION ADDENDUM DISTANCE, DEDP, IN',/)
```

```
      READ(1,*)ADDP
      WRITE(1,26)
26    FORMAT(//,4X,
     1 'ENTER THE GEAR DEDENDUM DISTANCE, DEDG, IN',/)
      READ(1,*)DEDG
      WRITE(1,27)
27    FORMAT(//,4X,
     1 'ENTER THE PINION DEDENDUM DISTANCE, DEDP, IN',/)
      READ(1,*)DEDP
1232  WRITE(1,1111)NG,NP,AO,F,PHIN,PSI,GPAN,TP,DG,DP
1111  FORMAT(//,16X,'I N P U T   S U M M A R Y',//,
     14X,' NUMBER OF GEAR TEETH, NG                          =',
     @9X,I2,/,
     24X,' NUMBER OF PINION TEETH, NP                        =',
     @9X,I2,/,
     34X,' PITCH CONE DISTANCE CENTER, AO                    =',
     $F15.3,' INCHES',/,
     44X,' GEAR FACE WIDTH, F                                =',
     $F15.3,' INCHES',/,
     74X,' NORMAL PRESSURE ANGLE, PHIN                       =',
     $F15.3,' DEGREES',/,
     84X,' SPIRAL ANGLE, PSI                                 =',
     $F15.3,' DEGREES',/,
     94X,' MESH ANGLE OF GEAR AND PINION, SIGMA              =',
     $F15.3,' DEGREES',/,
     14X,' TORQUE APPLIED TO THE PINION                      =',
     $F15.3,' LBS/IN',/,
     24X,' GEAR SHAFT DIAMETER, DG                           =',
     $F15.3,' INCHES',/,
     34X,' PINION SHAFT DIAMETER, DP                         =',
     $F15.3,' INCHES')
      WRITE(1,1112)ADDG,ADDP,DEDG,DEDP
1112  FORMAT(
     14X,' GEAR ADDENDUM DISTANCE, ADDG,                     =',
     $F15.3,' INCHES',/,
     24X,' GEAR DEDENDUM DISTANCE, ADDP                      =',
     $F15.3,' INCHES',/,
     34X,' PINION ADDENDUM DISTANCE, DEDG                    =',
     $F15.3,' INCHES',/,
     44X,' PINION DEDENDUM DISTANCE, DEDP                    =',
     $F15.3,' INCHES',//,
     67X,'  IS THIS DATA CORRECT ? (Y/N)',/)
151   READ(1,33)ANS
33    FORMAT(1A3)
      IF(ANS.EQ.'Y'.OR.ANS.EQ.'N')GOTO 152
```

```
      WRITE(1,153)
 153  FORMAT(//,4X,
     1 '**** PLEASE ANSWER Y-YES OR N-NO ****',/)
      GOTO 151
 152  IF (ANS.EQ.'N')GOTO 4444
C
C>    INTRODUCTORY CALCULATIONS
C
      SIG=RAD(GPAN)
      PGRAT=NP/NG
      GAMAG=SIN(SIG)/(PGRAT+COS(SIG))
      GAMAP=SIN(SIG)/((1./PGRAT)+COS(SIG))
      GAG=ATAN(GAMAG)
      GAP=ATAN(GAMAP)
      GAMG=DEG(GAG)
      GAMP=DEG(GAP)
      PD=(2.*(AO-F/2)*SIN(GAG))
      PDG=NG/PD
      PDP=PDG
      RAVGG=(AO-F/2.)*SIN(GAG)
      RAVGP=(AO-F/2.)*SIN(GAP)
      SMOIG=PI*DG**4/64
      SMOIP=PI*DP**4/64
      E=30E06
C
C>    LOAD CALCULATIONS
C
      RPSI=RAD(PSI)
      TG=TP/PGRAT
      WTG=TG/RAVGG
      WTP=TP/RAVGP
      WTXG=(WTG/COS(RPSI))
      WTXP=(WTP/COS(RPSI))
      RPHI=RAD(PHIN)
      WA1G=TAN(RPHI)*SIN(GAG)
      WR1G=TAN(RPHI)*COS(GAG)
      WA1P=TAN(RPHI)*SIN(GAP)
      WR1P=TAN(RPHI)*COS(GAP)
      WA2G=SIN(RPSI)*COS(GAG)
      WR2G=SIN(RPSI)*SIN(GAG)
      WA2P=SIN(RPSI)*COS(GAP)
      WR2P=SIN(RPSI)*SIN(GAP)
      WRITE(1,1000)
 1000 FORMAT(//,4X,'NOTE: PINION IS THE DRIVING GEAR',///,
     $      4X,'  DRIVING AND DIRECTION MENU ',//,
```

```
     1      4X,' ENTER YOUR CHOICE (NUMBER 1 - 8) :',/,
     2      4X,'  1.        DRIVING   RH CW',/,
     3      4X,'  2.                  LH CCW',/,
     4      4X,'  3.        DRIVEN    RH CCW',/,
     5      4X,'  4.                  LH CW',/,
     6      4X,'  ----------------------',/,
     7      4X,'  5.        DRIVING   RH CCW',/,
     8      4X,'  6.                  LH CW',/,
     9      4X,'  7.        DRIVEN    RH CW',/,
     1      4X,'  8.                  LH CCW',//)
      READ(1,*)NCHOI
      GOTO (100,100,100,100,200,200,200,200),NCHOI
  100 WAP=WTXP*(WA1P-WA2P)
      WRP=WTXP*(WR1P+WR2P)
      WAG=WTXG*(WA1G+WA2G)
      WRG=WTXG*(WR1G-WR2G)
      GOTO 300
  200 WAP=WTXP*(WA1P+WA2P)
      WRP=WTXP*(WR1P-WR2P)
      WAG=WTXG*(WA1G-WA2G)
      WRG=WTXG*(WR1G+WR2G)
  300 WNG=SQRT(WTG**2+WAG**2+WRG**2)
      WNP=SQRT(WTP**2+WAP**2+WRP**2)
C
C>    PRINT RESULTS
C
      WRITE(1,310)WTG,WAG,WRG,WNG,GAMG,PDG,RAVGG,
     1              WTP,WAP,WRP,WNP,GAMP,PDP,RAVGP
  310 FORMAT(//,5X,'     PARTIAL RESULTS',//,
     1           5X,'      GEAR SECTION ',//,
     2    5X,'TANGENTIAL LOAD, LBS                         = ',
     @    F15.3,/
     3    5X,'AXIAL      LOAD, LBS                         = ',
     @    F15.3,/
     4    5X,'RADIAL     LOAD, LBS                         = ',
     @    F15.3,/
     5    5X,'NORMAL     LOAD, LBS                         = ',
     @    F15.3,/
     6    5X,'GEAR CONE PITCH ANGLE, DEG                   = ',
     @    F15.3,/
     7    5X,'DIAMETRAL PITCH  (1/IN)                      = ',
     @    F15.3,/
     8    5X,'AVERAGE RADIUS, IN                           = ',
     1    F15.3,//,5X,'      PINION SECTION ',//,
     2    5X,'TANGENTIAL LOAD, LBS                         = ',
```

```
     @    F15.3,/
     3    5X,'AXIAL      LOAD, LBS                              = ',
     @    F15.3,/
     4    5X,'RADIAL     LOAD, LBS                              = ',
     @    F15.3,/
     5    5X,'NORMAL     LOAD, LBS                              = ',
     @    F15.3,/
     6    5X,'PINION CONE PITCH ANGLE, DEG                      = ',
     @    F15.3,/
     7    5X,'DIAMETRAL PITCH  (1/IN)                           = ',
     @    F15.3,/
     8    5X,'AVERAGE RADIUS, IN                                = ',
     9    F15.3,//,5X,
     @    'DOES EVERYTHING LOOK O.K.  SO FAR ? (Y/N)',//)
161  READ(1,33)ANS
     IF(ANS.EQ.'Y'.OR.ANS.EQ.'N')GOTO 162
     WRITE(1,153)
     GOTO 161
162  IF (ANS.EQ.'N')GOTO 4444
750  WRITE (1,754)
754  FORMAT(//,4X,'   BEARING SUPPORT CONFIGURATION :  ',//,
    1  4X,'   STRADDLE    (GR --------- BRNG --------- GR)',/,
    2  4X,'   OVERHUNG   (BRNG --------------BRNG------ GR)',//,
    3  4X,'   THERE ARE 4 POSSIBLE SYSTEM CONFIGURATIONS :',//,
    4  4X,'      1.  STRADDLE GEAR   AND   STRADDLE PINION',/,
    5  4X,'      2.  STRADDLE GEAR   AND   OVERHUNG PINION',/,
    6  4X,'      3.  OVERHUNG GEAR   AND   STRADDLE PINION',/,
    7  4X,'      4.  OVERHUNG GEAR   AND   OVERHUNG PINION',//,
    8  4X,' ENTER YOUR CHOICE (1 - 4) :')
     READ(1,*)ICASE
     GOTO (755,756,757,758)ICASE
755  CALL STRAD (SMOIG,PDG,WTG,WAG,WRG,'GEAR  ',RAVGG,
    1 Y1G,Y2G,Y3G,V1G,V2G,V3G,THE1G,THE2G,THE3G,YPTG,YPRG,YPAG)
     CALL STRAD (SMOIP,PDP,WTP,WAP,WRP,'PINION',RAVGP,
    1 Y1P,Y2P,Y3P,V1P,V2P,V3P,THE1P,THE2P,THE3P,YPTP,YPRP,YPAP)
     GOTO 9997
756  CALL STRAD (SMOIG,PDG,WTG,WAG,WRG,'GEAR  ',RAVGG,
    1 Y1G,Y2G,Y3G,V1G,V2G,V3G,THE1G,THE2G,THE3G,YPTG,YPRG,YPAG)
     CALL OVERH (SMOIP,PDP,WTP,WAP,WRP,'PINION',RAVGP,
    1 Y1P,Y2P,Y3P,V1P,V2P,V3P,THE1P,THE2P,THE3P,YPTP,YPRP,YPAP)
     GOTO 9997
757  CALL OVERH (SMOIG,PDG,WTG,WAG,WRG,'GEAR  ',RAVGG,
    1 Y1G,Y2G,Y3G,V1G,V2G,V3G,THE1G,THE2G,THE3G,YPTG,YPRG,YPAG)
     CALL STRAD (SMOIP,PDP,WTP,WAP,WRP,'PINION',RAVGP,
    1 Y1P,Y2P,Y3P,V1P,V2P,V3P,THE1P,THE2P,THE3P,YPTP,YPRP,YPAP)
```

```
      GOTO 9997
 758  CALL OVERH (SMOIG,PDG,WTG,WAG,WRG,'GEAR   ',RAVGG,
     1 Y1G,Y2G,Y3G,V1G,V2G,V3G,THE1G,THE2G,THE3G,YPTG,YPRG,YPAG)
      CALL OVERH (SMOIP,PDP,WTP,WAP,WRP,'PINION',RAVGP,
     1 Y1P,Y2P,Y3P,V1P,V2P,V3P,THE1P,THE2P,THE3P,YPTP,YPRP,YPAP)
C
C>    BEVEL GEAR DEFORMATION (BASED ON RELATIVE MOTION)
C
C>    THE FOLLOWING EQUATIONS DEFINE THE
C>    MOTION OF THE GEAR *RELATIVE* TO PINION.
C
C>    NOTE : FOR THE OPPOSITE CASE SIGNS ARE REVERSED
C
C>        Ai = RELATIVE ROTATION
C>        Zi = RELATIVE TRANSLATION
C
C>    A1, Z1 = COMMON TANGENTIAL COMPONENT
C>    A2, Z2 = PINION AXIAL COMPONENT
C>    A3, Z3 = GEAR AXIAL COMPONENT
C>       DZ1 = ADDITIONAL GEAR CONTACT ROTATION
C
 9997 A1=RAD(THE1G)+RAD(THE1P)
      A2=-RAD(THE2P)-RAD(THE3P)/TAN(SIG)-RAD(THE3G)/SIN(SIG)
      A3=RAD(THE2G)+RAD(THE3G)/TAN(SIG)+RAD(THE3P)/SIN(SIG)
      AA1=DEG(A1)
      AA2=DEG(A2)
      AA3=DEG(A3)
      DTHG=RAVGG*A3
      DTHP=RAVGP*A2
      DZ1=DTHG-DTHP
      Z1=Y1G+Y1P+DZ1
      Z2=-Y2P-Y3P/TAN(SIG)-Y3G/SIN(SIG)
      Z3=Y2G+Y3G/TAN(SIG)+Y3P/SIN(SIG)
C
C>    PRINT RESULTS
C
      WRITE(1,8876)Z1,Z2,Z3,DZ1,AA1,AA2,AA3
 8876 FORMAT(/,4X,'     BEVEL DEFORMATION RESULTS',//,
     1   4X,' Z1 - COMMON TANGENTIAL COMPONENT                  =',
     $     F15.8,' INCHES',/,
     2   4X,' Z2 - PINION AXIAL COMPONENT                       =',
     $     F15.8,' INCHES',/,
     3   4X,' Z3 - GEAR AXIAL COMPONENT                         =',
     $     F15.8,' INCHES',//,
     3   4X,'DZ1 - ADDITIONAL MOTION (GEAR WRT PINION)          =',
```

```
     $     F15.8,' INCHES',//,
     3    4X,' A1 - COMMON TANGENTIAL COMPONENT (ROTATION)      =',
     $     F15.8,' DEGREES',/
     3    4X,' A2 - PINION AXIAL ROTATION  (ROTATION)           =',
     $     F15.8,' DEGREES',/
     3    4X,' A3 - GEAR AXIAL COMPONENT   (ROTATION)           =',
     $     F15.8,' DEGREES',/)
C
C>
C>>   CURVATURE ANALYSIS AND ELLIPSE SIZE
C>
C
      WRITE(1,8976)
 8976 FORMAT(/,4X,
     1 'DO YOU WANT CURVATURE AND CONTACT ANALYSIS ? (Y/N)',/)
      READ(1,33)ANS
      IF(ANS.EQ.'N')GOTO 8877
      CALL CURVA(NG,NP,PSI,WNG,GAG,GAP,RK12,RK22,RK11,RK21)
C
C>    CALCULATION OF THE NEW CONTACT POINT
C>    CONSIDERING THE CURVATURES RK12-GEAR, RK11-PINION
C
      ROG=(1.0/RK12)
      ROP=(1.0/RK11)
      IF((NCHOI.EQ.2).OR.(NCHOI.EQ.4).OR.(NCHOI.EQ.6).OR.
     1    (NCHOI.EQ.8)) GOTO 7766
      RPSIP=RPSI
      RPSIG=-RPSI
      GOTO 7777
 7766 RPSIP=-RPSI
      RPSIG=RPSI
 7777 CALL CONTA(PDP,AO,F,RPHI,ROG,ROP,GAG,GAP,RPSIP,RPSIG)
C
C>    LAST MENU - SUMMARY OF RESULTS
C
 8877 WRITE(1,9987)
 9987 FORMAT(//,4X,' WHAT DO YOU WANT TO DO NOW ? ',//,
     1  4X,' 1.  RUN ANOTHER CONFIGURATION WITH SAME DATA ',/,
     2  4X,' 2.  ENTER NEW DATA',/,
     3  4X,' 3.  SUMMARY OF RESULTS (DEFLECTIONS & SLOPES)',/,
     4  4X,' 4.  EXIT PROGRAM',//)
      READ(1,*)KCHO
      GOTO (750,4444,5555,9999)KCHO
 5555 WRITE(1,5556)
     1  Y1G,Y1P,Y2G,Y2P,Y3G,Y3P,THE1G,THE1P,THE2G,THE2P,
```

```
   2  THE3G,THE3P,YPTG,YPTP,YPAG,YPAP,YPRG,YPRP
5556 FORMAT(//,32X,
   @   '******  S U M M A R Y   O F   R E S U L T S   ******',//,
   @ 13X,' SECTION                                              GEAR',
   @   '                        PINION ',///,
   1    4X,' Y1 - TANGENTIAL DEFLECTION .................... =',
   $    F15.8,8X,F15.8,'      INCHES',/,
   2    4X,' Y2 - AXIAL DEFLECTION ......................... =',
   $    F15.8,8X,F15.8,'      INCHES',/,
   3    4X,' Y3 - RADIAL DEFLECTION ........................ =',
   $    F15.8,8X,F15.8,'      INCHES',//,
   4    4X,' THETA1 -(IN RADIAL-AXIAL PLANE) ABOUT Y1 ..... =',
   $    F15.8,8X,F15.8,'      DEGREES',/,
   5    4X,' THETA2 -(IN TANGENTIAL-RADIAL PLANE) ABOUT Y2  =',
   $    F15.8,8X,F15.8,'      DEGREES',/,
   6    4X,' THETA3 -(IN AXIAL-TANGENTIAL PLANE) ABOUT Y3 . =',
   $    F15.8,8X,F15.8,'      DEGREES',//,
   @    20X,' TOTAL DEFLECTIONS (INCLUDING BEARING STIFFNESS)',
   1//,4X,' Y1 - TANGENTIAL DEFLECTION .................... =',
   $    F15.8,8X,F15.8,'      INCHES',/,
   2    4X,' Y2 - AXIAL DEFLECTION ......................... =',
   $    F15.8,8X,F15.8,'      INCHES',/,
   3    4X,' Y3 - RADIAL DEFLECTION ........................ =',
   $    F15.8,8X,F15.8,'      INCHES',//)
      WRITE(1,5558)V1G,V1P,V2G,V2P,V3G,V3P
5558 FORMAT(20X,' DEFLECTIONS AT GEAR CENTER',//,
   1    4X,' V1 ............................................ =',
   $    8X,F15.8,'      INCHES',/,
   2    4X,' V2 ............................................ =',
   $    8X,F15.8,'      INCHES',/,
   3    4X,' V3 ............................................ =',
   $    8X,F15.8,'      INCHES',//)
      WRITE(1,5557)Z1,Z2,Z3,AA1,AA2,AA3
5557 FORMAT(20X,' BEVEL DEFORMATION (GEAR RELATIVE TO PINION)',
   1//,4X,' Z1 - COMMON TANGENTIAL COMPONENT ............. =',
   $     11X,F15.8,12X,'      INCHES',/,
   2    4X,' Z2 - PINION AXIAL COMPONENT .................. =',
   $     11X,F15.8,12X,'      INCHES',/,
   3    4X,' Z3 - GEAR AXIAL COMPONENT .................... =',
   $     11X,F15.8,12X,'      INCHES',//,
   4    4X,' A1 - COMMON TANGENTIAL COMPONENT (ROTATION) .. =',
   $     11X,F15.8,12X,'      DEGREES',/,
   5    4X,' A2 - PINION AXIAL ROTATION  (ROTATION) ....... =',
   $     11X,F15.8,12X,'      DEGREES',/,
   6    4X,' A3 - GEAR AXIAL COMPONENT   (ROTATION) ....... =',
```

```
     $    11X,F15.8,12X,'      DEGREES',/)
      WRITE(1,5559)DPG,DPP,DNG,DNP,DTOTG,DTOTP,RATG,RATP
 5559 FORMAT(/,4X,'     CONTACT POINT DEFLECTION ',//,
     1    4X,' DEFLECTION ON THE TANGENT PLANE............... =',
     $    F15.8,8X,F15.8,'      INCHES',/,
     2    4X,' DEFLECTION ON THE NORMAL PLANE ............... =',
     $    F15.8,8X,F15.8,'      INCHES',/,
     3    4X,' TOTAL CONTACT POINT DEFLECTION ............... =',
     $    F15.8,8X,F15.8,'      INCHES',/,
     8    4X,' DEFLECTION RATIO (NORMAL PLANE) .............. =',
     $    F15.8,8X,F15.8,'      INCHES',//)
      GOTO 8877
 9999 STOP
      END
C
C>
C>>    SUBROUTINE STRADDLE
C>
C
C>     INPUT ARGUMENTS
C             SMOI : SECOND MOMENT OF INERTIA
C               PD : DIAMETRAL PITCH
C              WTG : TANGENTIAL LOAD
C              WAG : AXIAL LOAD
C              WRG : RADIAL LOAD
C            TITLE : GEAR or PINION
C             RAVG : AVERAGE RADIUS
C
C>     OUTPUT ARGUMENTS
C               Yi : TOTAL DEFLECTIONS
C               Vi : DEFLECTION AT GEAR CENTER
C             THEi : TOTAL SLOPES
C                    (i=1-TANGENTIAL, 2-AXIAL, 3-RADIAL)
C              YPi : TOTAL DEFLECTION (INCLUDING BEARING STIFFNESS)
C                    (i=T-TANGENTIAL, A-AXIAL, R-RADIAL)
C>
      SUBROUTINE STRAD(SMOI,PD,WTG,WAG,WRG,TITLE,RAVG,
     1     Y1,Y2,Y3,V1,V2,V3,THE1,THE2,THE3,YPT,YPR,YPA)
      REAL NP,NG
      COMMON/INFO/NP,NG,AO,F,E
      COMMON/ELEM/NRE
      CHARACTER*6,TITLE,NCHE
      RAD(A)=PI*A/180.
      DEG(A)=180.*A/PI
      PI=3.1415927
```

```
      WRITE(1,1617)TITLE
 1617 FORMAT(//,7X,5('*'),2X,1A6,1X,'SECTION',2X,5('*'),//,
     @           4X,'    STRADDLE CONFIGURATION',//,
     $           4X,' THE GEAR SETUP LOOKS LIKE THIS :',//,
     $           4X,'BRNG                    GR         BRNG',/,
     1           4X,' *------------------*----------* ',/,
     2           4X,' *------- B --------*--- A ---* ',//,
     3           4X,
     @ ' THE NEXT TWO QUESTIONS REFER TO THIS SETUP',/)
      WRITE(1,16)
 16   FORMAT(//,
     1 4X,'ENTER DISTANCE FROM GEAR TO RIGHT BEARG, A, IN',/)
      READ (1,*)A
      WRITE(1,17)
 17   FORMAT(//,
     1 4X,'ENTER DISTANCE FROM GEAR TO LEFT BEARG, B, IN',/)
      READ(1,*)B
C
C>    REACTIONS
C
      SPAN=B+A
C
C>    FAZG = LEFT Z-FORCE
C>    FBZG = RIGHT Z-FORCE
C
      FAZG=WTG*A/SPAN
      FBZG=WTG*B/SPAN
C
C>    FAXG = LEFT X-FORCE
C>    FBXG = RIGHT X-FORCE
C
      FAXG=(WAG*RAVG+WRG*A)/SPAN
      FBXG=(-WAG*RAVG+WRG*B)/SPAN
      FAY=-WAG
      FR22=SQRT(FBZG**2+FBXG**2)
      FR11=SQRT(FAZG**2+FAXG**2)
      IF(TITLE.EQ.'GEAR  ') G1=NG
      IF(TITLE.EQ.'PINION') G1=NP
 454  R1=WAG*G1/(2.*PD)
      FR1=((WTG**2)*(B**2)+(R1-WRG*B)**2)**0.5/SPAN
      FR2=((WTG**2)*(A**2)+(R1+WRG*A)**2)**0.5/SPAN
      WRITE(1,4545)FR11,FR22
 4545 FORMAT(//,4X,' SIMPLIFIED RADIAL FORCES ',//,
     $   4X,' RADIAL FORCE AT LEFT BEARING, FR2            =',
     3   F15.3, ' LBS',/,
```

```
     $   4X,' RADIAL FORCE AT RIGHT BEARING, FR1             =',
     3   F15.3, ' LBS',/)
C
C>    DEFLECTIONS AND SLOPES
C
C
C>    DEFLECTION DUE TO TANGENTIAL LOAD, WT
C
      YWTG=WTG*A*A*B*B/(3.*E*SMOI*(A+B))
C
C>    DEFLECTION DUE TO RADIAL LOAD, WR
C
      YWRG=WRG*A*A*B*B/(3.*E*SMOI*(A+B))
C
C>    DEFLECTION DUE TO AXIAL LOAD, WA
C
      YWAG=WAG*RAVG*A*B*(2.*B-2*A)/(6.*E*SMOI*(A+B))
C
C
C>    THA = SLOPE DUE TO AXIAL LOAD
C>    THR = SLOPE DUE TO RADIAL LOAD
C>    THT = SLOPE DUE TO TORSION OF TANGENTIAL LOAD
C
C>    Y1  = DEFLECTION DUE TO TANGENTIAL LOAD
C>    Y2  = DEFLECTION DUE TO RADIAL+AXIAL+ROTATION
C>    Y3  = DEFLECTION DUE TO AXIAL+RADIAL SLOPE
C
      THA=WAG*RAVG*(A*A+B*B-A*B)/(3.0*E*SMOI*(A+B))
      THEA=ATAN(THA)
      THR=WRG*A*B*(B-A)/(3.0*E*SMOI*(A+B))
      THER=ATAN(THR)
      THT=WTG*A*B*(B-A)/(3.0*E*SMOI*(A+B))
      THET=ATAN(THT)
      Y1=YWTG
      Y2=RAVG*SIN(THEA-THER)
      Y3=YWAG+YWRG+RAVG*(1-COS(THEA-THER))
      THE1=DEG(THEA)-DEG(THER)
      THE2R=-WTG*RAVG*(A+B)/(2.*SMOI*0.385*E)
      THE2=DEG(THE2R)
      THE3=DEG(THET)
      WRITE(1,400)FAZG,FAXG,FR2,FBZG,FBXG,FR1,FAY,YWTG,YWRG,YWAG
  400 FORMAT(//,
     @ '*****************************************************',/,
     @ '*              STRADDLE CONFIGURATION RESULTS       *',/,
     @ '*****************************************************',//
```

```
     1 4X,'      REACTIONS AND DEFLECTIONS',//,
     1 4X,'NOTE : THE FOLLOWING AXIS CONVENTION IS USED  ',//,
     2 4X,'                          !X',/,
     2 4X,'                          !',/,
     3 4X,'                          !Z_______Y',//,
     @ 4X,' Z-AXIS REACTION AT LEFT BEARING                  =',
     @ F15.3,' LBS',/,
     @ 4X,' X-AXIS REACTION AT LEFT BEARING                  =',
     @ F15.3,' LBS',/,
     $ 4X,' RADIAL FORCE AT LEFT BEARING, FR2                =',
     3 F15.3,' LBS',/,
     @ 4X,' Z-AXIS REACTION AT RIGHT BEARING                 =',
     @ F15.3,' LBS',/,
     @ 4X,' X-AXIS REACTION AT RIGHT BEARING                 =',
     @ F15.3,' LBS',/,
     $ 4X,' RADIAL FORCE AT RIGHT BEARING, FR1               =',
     3 F15.3,' LBS',/,
     $ 4X,' THRUST REACTION AT LEFT BEARING                  =',
     3 F15.3,' LBS',//,
     4 4X,' DEFLECTION DUE TO TANGENTIAL LOAD, WT            =',
     $ F15.8,' INCHES',/,
     5 4X,' DEFLECTION DUE TO RADIAL LOAD, WR                =',
     $ F15.8,' INCHES',/,
     6 4X,' DEFLECTION DUE TO AXIAL LOAD, WA                 =',
     $ F15.8,' INCHES',/)
C
C>     BEARING STIFFNESS PRECALCULATIONS
C
C>     LEFT BEARING - INPUT = LOAD (X,Z -DIRECTION)
C>                    OUTPUT = BEARING DEFLECTION
C
       WRITE(1,417)
 417   FORMAT(/,4X,60('-'),/)
       WRITE(1,410)
 410   FORMAT(/,6X,'******* LEFT BEARING DEFLECTION *******',//,
     1          6X,'** DEFLECTIONS IN X AND Z DIRECTION  **',/)
       WRITE(1,417)
       CALL BINFO(ITYPE,AC,DI,RI,EI,PRI,DO,RO,EO,PRO,D,
     1 RR,ER,PRR,NCHE)
       IF (NCHE.EQ.'N')GOTO 9999
       AC1=AC
       RESA=SQRT(FAXG**2+FAZG**2)
       BETA=ATAN(FAXG/FAZG)
       CALL BEARI(RESA,DELA,ITYPE,AC,DI,RI,EI,PRI,DO,RO,EO,PRO,D,
     1 RR,ER,PRR,NCHE)
```

```
      YAXG=DELA*SIN(BETA)
      YAZG=DELA*COS(BETA)
C
C>    RIGHT BEARING - INPUT = LOAD (X,Z -DIRECTION)
C>                   OUTPUT = BEARING DEFLECTION
C
      WRITE(1,417)
      WRITE(1,411)
 411  FORMAT(/,
     @ 6X,'******* RIGHT BEARING DEFLECTION *******',//,
     1 6X,'**  DEFLECTIONS IN X AND Z DIRECTION   **',/)
      WRITE(1,417)
      CALL BINFO(ITYPE,AC,DI,RI,EI,PRI,DO,RO,EO,PRO,D,
     1 RR,ER,PRR,NCHE)
      IF (NCHE.EQ.'N')GOTO 9999
      AC1=AC
      RESB=SQRT(FBXG**2+FBZG**2)
      BETB=ATAN(FBXG/FBZG)
      CALL BEARI(RESB,DELB,ITYPE,AC,DI,RI,EI,PRI,DO,RO,EO,PRO,D,
     1 RR,ER,PRR,NCHE)
      YBXG=DELB*SIN(BETB)
      YBZG=DELB*COS(BETB)
C
C>    SLOPES + DEFLECTIONS DUE TO BEARING STIFFNESS
C
      SLOR=ATAN((YAXG-YBXG)/SPAN)
      SLORD=DEG(SLOR)
C
C>    YPR = DEFLECTION OF GEAR TOOTH DUE TO RADIAL LOAD
CV
      YPR=((YAXG*A+YBXG*B)/SPAN)+RAVG*(1-COS(SLOR))
      SLOT=ATAN((YAZG-YBZG)/SPAN)
      SLOTD=DEG(SLOT)
C
C>    YPT = DEFLECTION OF GEAR TOOTH DUE TO TANGENTIAL LOAD
C
      YPT=((YAZG*A+YBZG*B)/SPAN)
C
C>    AXIAL LOAD Y-DIRECTION
C
C>    YPA = DEFLECTION OF GEAR TOOTH DUE TO AXIAL LOAD
C
      YPA=RAVG*SIN(SLOR)
      WRITE(1,4195)YPT,YPA,YPR
 4195 FORMAT(//,4X,' PURE BEARING DEFLECTIONS DUE TO 3 LOADS',/,
```

```
     1    4X,' YT - TANGENTIAL DEFLECTION                              =',
     $    F15.8,  ' INCHES',/,
     2    4X,' YA - AXIAL DEFLECTION                                   =',
     $    F15.8,  ' INCHES',/,
     3    4X,' YR - RADIAL DEFLECTION                                  =',
     $    F15.8,  ' INCHES',/)
      WRITE(1,419)YAXG,YBXG,SLORD,YAZG,YBZG,SLOTD
419   FORMAT(//,4X,' BEARING DEFLECTION & SLOPE COMPONENTS',/,
     1    4X,' LEFT BEARING - RADIAL           (Yax)                   =',
     $    F15.8,' INCHES',/,
     2    4X,' RIGHT BEARING - RADIAL          (Ybx)                   =',
     $    F15.8,' INCHES',/,
     3    4X,' SLOPE ON RADIAL PLANE OF THE BEARING                    =',
     $    F15.8,' DEGREES',/,
     4    4X,' LEFT BEARING - TANGENTIAL       (Yaz)                   =',
     $    F15.8,' INCHES',/,
     5    4X,' RIGHT BEARING - TANGENTIAL      (Ybz)                   =',
     $    F15.8,' INCHES',/,
     6    4X,' SLOPE ON TANGENTIAL PLANE OF THE BEARING                =',
     $    F15.8,' DEGREES',//)
      WRITE(1,420)Y1,Y2,Y3,THE1,THE2,THE3
420   FORMAT(//,4X,' TOTAL DEFLECTIONS AND SLOPES',//,
     1    4X,' Y1 - TANGENTIAL DEFLECTION                              =',
     $    F15.8,' INCHES',/,
     2    4X,' Y2 - AXIAL DEFLECTION                                   =',
     $    F15.8,' INCHES',/,
     3    4X,' Y3 - RADIAL DEFLECTION                                  =',
     $    F15.8,' INCHES',//,
     4    4X,' THETA1 -(IN RADIAL-AXIAL PLANE) ABOUT Y1                =',
     $    F15.8,' DEGREES',/,
     5    4X,' THETA2 -(IN TANGENTIAL-RADIAL PLANE) ABOUT Y2           =',
     $    F15.8,' DEGREES',/,
     6    4X,' THETA3 -(IN AXIAL-TANGENTIAL PLANE) ABOUT Y3            =',
     $    F15.8,' DEGREES',//)
C
C>     TOTAL DEFLECTIONS (INCLUDING BEARING STIFFNESS)
C
      YPT=YPT+Y1
      YPA=YPA+Y2
      YPR=YPR+Y3
      THE1=THE1+SLOTD
      THE3=THE3+SLORD
      V1=Y1
      V2=0.0
      V3=Y2+Y3
```

```
      WRITE(1,430)YPT,YPA,YPR,THE1,THE2,THE3,V1,V2,V3
 430  FORMAT(/,4X,' TOTAL DEFLECTIONS AND SLOPES  ',
     @           ' (INCLUDING BEARING STIFFNESS)',/,
     1   4X,' Y1 - TANGENTIAL DEFLECTION                       =',
     $   F15.8,' INCHES',/,
     2   4X,' Y2 - AXIAL DEFLECTION                            =',
     $   F15.8,' INCHES',/,
     3   4X,' Y3 - RADIAL DEFLECTION                           =',
     $   F15.8,' INCHES',//,
     4   4X,' THETA1 - ABOUT Y1                                =',
     $   F15.8,' DEGREES',/,
     5   4X,' THETA2 - ABOUT Y2                                =',
     $   F15.8,' DEGREES',/,
     6   4X,' THETA3 - ABOUT Y3                                =',
     $   F15.8,' DEGREES',//,
     7   4X,'     DEFLECTIONS AT GEAR CENTER',//,
     8   4X,' V1                                               =',
     $   F15.8,' INCHES',/,
     9   4X,' V2                                               =',
     $   F15.8,' INCHES',/,
     1   4X,' V3                                               =',
     $   F15.8,' INCHES',/)
 9999 RETURN
      END
C
C>
C>>   SUBROUTINE OVERHUNG
C>
C
C>    INPUT ARGUMENTS
C           SMOI : SECOND MOMENT OF INERTIA
C             PD : DIAMETRAL PITCH
C            WTG : TANGENTIAL LOAD
C            WAG : AXIAL LOAD
C            WRG : RADIAL LOAD
C          TITLE : GEAR or PINION
C           RAVG : AVERAGE RADIUS
C
C>    OUTPUT ARGUMENTS
C             Yi : TOTAL DEFLECTIONS
C             Vi : DEFLECTION AT GEAR CENTER
C           THEi : TOTAL SLOPES
C                  (i=1-TANGENTIAL, 2-AXIAL, 3-RADIAL
C            YPi : TOTAL DEFLECTION (INCLUDING BEARING STIFFNESS)
C                  (i=T-TANGENTIAL, A-AXIAL, R-RADIAL
```

```
C>
      SUBROUTINE OVERH(SMOI,PD,WTG,WAG,WRG,TITLE,RAVG,
     1      Y1,Y2,Y3,V1,V2,V3,THE1,THE2,THE3,YPT,YPR,YPA)
      REAL NP,NG
      COMMON/INFO/NP,NG,AO,F,E
      COMMON/ELEM/NRE
      CHARACTER*6,TITLE,NCHE
      RAD(A)=PI*A/180.
      DEG(A)=A*180/PI
      PI=3.1415927
      WRITE(1,1617)TITLE
 1617 FORMAT(//,7X,5('*'),2X,1A6,1X,'SECTION',2X,5('*'),//,
     @            4X,'     OVERHUNG CONFIGURATION   ',//,
     $            4X,' THE GEAR SETUP LOOKS LIKE THIS :',//,
     $            4X,'BRNG                        BRNG      GR',/,
     1            4X,' *-------------------*-----------* ',/,
     2            4X,' *------------ B -------------* ',/,
     3            4X,'                         *--- A ---* ',//,
     4            4X,
     5   ' THE NEXT TWO QUESTIONS REFER TO THIS SETUP',/)
      WRITE(1,16)
 16   FORMAT(//,
     1 4X,'ENTER DISTANCE FROM GEAR TO RIGHT BEARG, A',/)
      READ (1,*)A
      WRITE(1,17)
 17   FORMAT(//,
     1 4X,'ENTER DISTANCE FROM GEAR TO LEFT BEARG, B',/)
      READ(1,*)B
C
C>    REACTIONS
C
      SPAN=B-A
C
C>    FBZG = RIGHT Z-FORCE
C>    FAZG = LEFT Z-FORCE
C
      FBZG=WTG*B/SPAN
      FAZG=WTG*A/SPAN
C
C>    FBXG = RIGHT X-FORCE
C>    FAXG = LEFT X-FORCE
C
      FBXG=(-WAG*RAVG+WRG*B)/SPAN
      FAXG=(-WAG*RAVG+WRG*A)/SPAN
      FBY=-WAG
```

```
      FR11=SQRT(FBZG**2+FBXG**2)
      FR22=SQRT(FAZG**2+FAXG**2)
      IF(TITLE.EQ.'GEAR  ') G1=NG
      IF(TITLE.EQ.'PINION') G1=NP
 454  R1=WAG*G1/(2.*PD)
      FR1=((WTG**2)*(B**2)+(R1-WRG*B)**2)**0.5/SPAN
      FR2=((WTG**2)*(A**2)+(R1-WRG*A)**2)**0.5/SPAN
       WRITE(1,4545)FR22,FR11
 4545 FORMAT(//,4X,' SIMPLIFIED RADIAL FORCES ',//,
     $   4X,' RADIAL FORCE AT LEFT  BEARING, FR2                   =',
     3   F15.3,' LBS',/,
     $   4X,' RADIAL FORCE AT RIGHT BEARING, FR1                   =',
     3   F15.3,' LBS',//)
C
C>     DEFLECTIONS AND SLOPES
C
C>     DEFLECTION DUE TO TANGENTIAL LOAD, WT
C
      YWTG=WTG*A*A*B/(3.*E*SMOI)
C
C>     DEFLECTION DUE TO RADIAL LOAD, WR
C
      YWRG=WRG*A*A*B/(3.*E*SMOI)
C
C>     DEFLECTION DUE TO AXIAL LOAD, WA
C
      YWAG=WAG*RAVG*A*(3.*A+2.*SPAN)/(6.*E*SMOI)
C
C>     SLOPES AND DEFLECTIONS FOR THE THREE LOADINGS
C
C>     THA = SLOPE DUE TO AXIAL LOAD
C>     THR = SLOPE DUE TO RADIAL LOAD
C>     THT = SLOPE DUE TO TORSION OF TANGENTIAL LOAD
C
C>     Y1  = DEFLECTION DUE TO TANGENTIAL LOAD
C>     Y2  = DEFLECTION DUE TO RADIAL+AXIAL SLOPE
C>     Y3  = DEFLECTION DUE TO AXIAL+RADIAL+ROTATION
C
      THA=WAG*RAVG*(3.*A+SPAN)/(3.0*E*SMOI)
      THEA=ATAN(THA)
      THR=WRG*A*(3.*A+2.*SPAN)/(3.0*E*SMOI)
      THER=ATAN(THR)
      THT=WTG*A*(3.*A+2.*SPAN)/(3.0*E*SMOI)
      THET=ATAN(THT)
      Y1=YWTG
```

```
      Y2=RAVG*SIN(THEA-THER)
      Y3=YWAG+YWRG+RAVG*(1-COS(THEA-THER))
      THE1=DEG(THEA)-DEG(THER)
      THE2R=-WTG*RAVG*B/(2.*SMOI*0.385*E)
      THE2=DEG(THE2R)
      THE3=DEG(THET)
      WRITE(1,400)FAZG,FAXG,FR2,FBZG,FBXG,FR1,FBY,YWTG,YWRG,YWAG
 400  FORMAT(//,
     @'**************************************************',/,
     @'*            OVERHUNG CONFIGURATION RESULTS          *',/,
     @'**************************************************',//
     1 4X,'      REACTIONS AND DEFLECTIONS',//,
     1 4X,'NOTE :  THE FOLLOWING AXIS CONVENTION IS USED   ',//,
     2 4X,'                            !X',/,
     2 4X,'                            !',/,
     3 4X,'                            !Z______Y',//,
     @ 4X,' Z-AXIS REACTION AT LEFT BEARING                 =',
     @ F15.3,' LBS',/,
     @ 4X,' X-AXIS REACTION AT LEFT BEARING                 =',
     @ F15.3,' LBS',/,
     $ 4X,' RADIAL FORCE AT LEFT BEARING, FR2               =',
     3 F15.3,' LBS',/,
     @ 4X,' Z-AXIS REACTION AT RIGHT BEARING                =',
     @ F15.3,' LBS',/,
     @ 4X,' X-AXIS REACTION AT RIGHT BEARING                =',
     @ F15.3,' LBS',/,
     $ 4X,' RADIAL FORCE AT RIGHT BEARING, FR1              =',
     3 F15.3,' LBS',/,
     $ 4X,' THRUST FORCE AT RIGHT BEARING                   =',
     3 F15.3,' LBS',//,
     4 4X,' DEFLECTION DUE TO TANGENTIAL LOAD, WT           =',
     $ F15.8,' INCHES',/,
     5 4X,' DEFLECTION DUE TO RADIAL LOAD, WR               =',
     $ F15.8,' INCHES',/,
     6 4X,' DEFLECTION DUE TO AXIAL LOAD, WA                =',
     $ F15.8,' INCHES',//)
C
C>     BEARING STIFFNESS PRECALCULATIONS
C
C>     RADIAL LOADS X-DIRECTION
C
C
C>     RIGHT BEARING - INPUT = LOAD (X,Z -DIRECTION)
C>                    OUTPUT = BEARING DEFLECTION
C
```

```
      WRITE(1,417)
 417  FORMAT(/,4X,60('-'),/)
      WRITE(1,410)
 410  FORMAT(/,6X,'******* RIGHT BEARING DEFLECTION *******',//,
     1          6X,'**  DEFLECTIONS IN X AND Z DIRECTION  **',/)
      WRITE(1,417)
      CALL BINFO(ITYPE,AC,DI,RI,EI,PRI,DO,RO,EO,PRO,D,
     1 RR,ER,PRR,NCHE)
      IF(NCHE.EQ.'N')GOTO 9999
      AC1=AC
      RESB=SQRT(FBXG**2+FBZG**2)
      BETB=ATAN(FBXG/FBZG)
      CALL BEARI(RESB,DELB,ITYPE,AC,DI,RI,EI,PRI,DO,RO,EO,PRO,D,
     1 RR,ER,PRR,NCHE)
      YBXG=DELB*SIN(BETB)
      YBZG=DELB*COS(BETB)
C
C>    LEFT BEARING - INPUT = LOAD (X,Z -DIRECTION)
C>                  OUTPUT = BEARING DEFLECTION
C
      WRITE(1,417)
      WRITE(1,411)
 411  FORMAT(/,6X,'******* LEFT BEARING DEFLECTION *******',//,
     1          6X,'** DEFLECTIONS IN X AND Z DIRECTION  **',/)
      WRITE(1,417)
      CALL BINFO(ITYPE,AC,DI,RI,EI,PRI,DO,RO,EO,PRO,D,
     1 RR,ER,PRR,NCHE)
      IF(NCHE.EQ.'N')GOTO 9999
      AC1=AC
      RESA=SQRT(FAXG**2+FAZG**2)
      BETA=ATAN(FAXG/FAZG)
      CALL BEARI(RESA,DELA,ITYPE,AC,DI,RI,EI,PRI,DO,RO,EO,PRO,D,
     1 RR,ER,PRR,NCHE)
      YAXG=DELA*SIN(BETA)
      YAZG=DELA*COS(BETA)
C
C>    SLOPES + DEFLECTIONS DUE TO BEARING STIFFNESS
C
      SLOR=ATAN((YBXG - YAXG)/SPAN)
      SLORD=DEG(SLOR)
C
C>    YPR = DEFLECTION OF GEAR TOOTH DUE TO RADIAL LOAD
C
      YPR=((YBXG*B-YAXG*A)/SPAN)+RAVG*(1-COS(SLOR))
C
```

```
C>    TANGENTIAL LOAD CALCULATIONS Z-DIRECTION
C
      SLOT=ATAN((YBZG-YAZG)/SPAN)
      SLOTD=DEG(SLOT)
C
C>    YPT = DEFLECTION OF GEAR TOOTH DUE TO TANGENTIAL LOAD
C
      YPT=(YBZG*B-YAZG*A)/SPAN
C
C>    AXIAL LOAD Y-DIRECTION
C
C>    YPA = DEFLECTION OF GEAR TOOTH DUE TO AXIAL LOAD
C
      YPA=RAVG*SIN(SLOR)
      WRITE(1,4195)YPT,YPA,YPR
 4195 FORMAT(//,4X,' PURE BEARING DEFLECTIONS DUE TO 3 LOADS',/,
     1    4X,' YT - TANGENTIAL DEFLECTION                     =',
     $     ' INCHES',/,
     2    4X,' YA - AXIAL DEFLECTION                          =',
     $     ' INCHES',/,
     3    4X,' YR - RADIAL DEFLECTION                         =',
     $     ' INCHES',/)
      WRITE(1,419)YAXG,YBXG,SLORD,YAZG,YBZG,SLOTD
 419  FORMAT(//,4X,' BEARING DEFLECTION & SLOPE COMPONENTS',/,
     1    4X,' LEFT BEARING - RADIAL           (Yax)          =',
     $    F15.8,' INCHES',/,
     2    4X,' RIGHT BEARING - RADIAL          (Ybx)          =',
     $    F15.8,' INCHES',/,
     3    4X,' SLOPE ON RADIAL PLANE OF THE BEARING           =',
     $    F15.8,' DEGREES',/,
     4    4X,' LEFT BEARING - TANGENTIAL       (Yaz)          =',
     $    F15.8,' INCHES',/,
     5    4X,' RIGHT BEARING - TANGENTIAL      (Ybz)          =',
     $    F15.8,' INCHES',/,
     6    4X,' SLOPE ON TANGENTIAL PLANE OF THE BEARING       =',
     $    F15.8,' DEGREES',//)
      WRITE(1,420)Y1,Y2,Y3,THE1,THE2,THE3
 420  FORMAT(//,4X,' TOTAL DEFLECTIONS AND SLOPES',//,
     1    4X,' Y1 - TANGENTIAL DEFLECTION                     =',
     $    F15.8,' INCHES',/,
     2    4X,' Y2 - AXIAL DEFLECTION                          =',
     $    F15.8,' INCHES',/,
     3    4X,' Y3 - RADIAL DEFLECTION                         =',
     $    F15.8,' INCHES',//,
     4    4X,' THETA1 -(IN RADIAL-AXIAL PLANE) ABOUT Y1       =',
```

```
     $    F15.8,' DEGREES',/,
     5    4X,' THETA2 -(IN TANGENTIAL-RADIAL PLANE) ABOUT Y2  =',
     $    F15.8,' DEGREES',/,
     6    4X,' THETA3 -(IN AXIAL-TANGENTIAL PLANE) ABOUT Y3   =',
     $    F15.8,' DEGREES',//)
C
C>     TOTAL DEFLECTIONS AND SLOPES (INCLUDING BEARING STIFFNESS)
C
       YPT=YPT+Y1
       YPA=YPA+Y2
       YPR=YPR+Y3
       THE1=THE1+SLOTD
       THE3=THE3+SLORD
       V1=Y1
       V2=0.0
       V3=Y2+Y3
       WRITE(1,430)YPT,YPA,YPR,THE1,THE2,THE3,V1,V2,V3
  430  FORMAT(/,4X,' TOTAL DEFLECTIONS AND SLOPES  ',
     @              ' (INCLUDING BEARING STIFFNESS)',//,
     1    4X,' Y1 - TANGENTIAL DEFLECTION                     =',
     $    F15.8,' INCHES',/,
     2    4X,' Y2 - AXIAL DEFLECTION                          =',
     $    F15.8,' INCHES',/,
     3    4X,' Y3 - RADIAL DEFLECTION                         =',
     $    F15.8,' INCHES',//,
     4    4X,' THETA1 - ABOUT Y1                              =',
     $    F15.8,' DEGREES',/,
     5    4X,' THETA2 - ABOUT Y2                              =',
     $    F15.8,' DEGREES',/,
     6    4X,' THETA3 - ABOUT Y3                              =',
     $    F15.8,' DEGREES',//,
     7    4X,'     DEFLECTIONS AT GEAR CENTER',//,
     8    4X,' V1                                             =',
     $    F15.8,' INCHES',/,
     9    4X,' V2                                             =',
     $    F15.8,' INCHES',/,
     1    4X,' V3                                             =',
     $    F15.8,' INCHES',/)
 9999  RETURN
       END
C
C>
C>>     SUBROUTINE TO CALCULATE THE NEW POSITION OF
C>>     THE CONTACT POINT
C>
```

```
C
C>     INPUT ARGUMENTS
C              PD : DIAMETRAL PITCH
C              AO : PITCH CONE DISTANCE
C               F : FACE WIDTH
C            RPHI : PRESSURE ANGLE
C             ROG : RADIUS OF CURVATURE OF GEAR
C             ROP : RADIUS OF CURVATURE OF PINION
C             GAG : PITCH ANGLE OF GEAR
C             GAP : PITCH ANGLE OF PINION
C
C>     OUTPUT ARGUMENTS
C
C            NONE : (RESULTS PRINTED WITHIN THE SUBROUTINE)
C
C>
      SUBROUTINE CONTA(PDP,AO,F,RPHI,ROG,ROP,GAG,GAP,RPSIP,RPSIG)
      COMMON/OUT/DPG,DPP,DNG,DNP,DTOTG,DTOTP,RATG,RATP
      COMMON/DEFLE/YPRG,YPRP,YPAG,YPAP,YPTG,YPTP,
     1        THE1P,THE1G,THE2P,THE2G,THE3P,THE3G
      COMMON/SPEC/ADDG,ADDP,DEDG,DEDP
      RAD=180.0/3.1415927
      PHID=RPHI*RAD
      GAGD=GAG*RAD
      GAPD=GAP*RAD
      TH1PR=THE1P/RAD
      TH1GR=THE1G/RAD
      TH2PR=THE2P/RAD
      TH2GR=THE2G/RAD
      TH3PR=THE3P/RAD
      TH3GR=THE3G/RAD
C
C>
C>>    CONTACT POINT MOTION ALONG THE NORMAL TOOTH PLANE
C>
C
      E1P=YPTP
      E1G=YPTG
      E2P=YPAP*COS(GAP)-YPRP*SIN(GAP)
      E2G=YPAG*COS(GAG)-YPRG*SIN(GAG)
      E3P=YPAP*SIN(GAP)+YPRP*COS(GAP)
      E3G=YPAG*SIN(GAG)+YPRG*COS(GAG)
      A1P=TH1PR
      A1G=TH1GR
      A2P=TH2PR*COS(GAP)-TH3PR*SIN(GAP)
```

```
      A2G=TH2GR*COS(GAG)-TH3GR*SIN(GAG)
      A3P=TH2PR*SIN(GAP)+TH3PR*COS(GAP)
      A3G=TH2GR*SIN(GAG)+TH3GR*COS(GAG)
C
C>    DEFLECTION OF GEAR AND PINION CENTER OF CURVATURE
C
      H1P=E2P*SIN(RPSIP)+E1P*COS(RPSIP)
      H1G=E2G*SIN(RPSIG)+E1G*COS(RPSIG)
      H2P=E2P*COS(RPSIP)-E1P*SIN(RPSIP)
      H2G=E2G*COS(RPSIG)-E1G*SIN(RPSIG)
      H3P=E3P
      H3G=E3G
C
C>    DEFLECTION OF GEAR AND PINION CENTER OF CURVATURE
C
      THRG=A1G*SIN(RPHI)*COS(RPSIG)+A2G*SIN(RPHI)*SIN(RPSIG)-
     1      A3G*COS(RPHI)
      THRP=A1P*SIN(RPHI)*COS(RPSIP)+A2P*SIN(RPHI)*SIN(RPSIP)-
     1      A3P*COS(RPHI)
      OG=H2G+ROG*SIN(THRG)
      OP=H2P+ROP*SIN(THRP)
      DF=OG+(ROG/(ROP+ROG))*(OP-OG)
      DPG=DF-H2G
      DPP=DF-H2P
C
C>
C>>   CONTACT POINT MOTION ALONG THE TANGENT TOOTH PLANE
C>
C
      RBCG=(AO-F/2.0)*TAN(GAG)/((COS(RPSIG))**2)
      RBCP=(AO-F/2.0)*TAN(GAP)/((COS(RPSIP))**2)
      C=RBCG+RBCP
      RNUM=C*COS(RPHI)
      THRMG=A1G*COS(RPSIG)+A2G*SIN(RPSIG)
      THRMP=A1P*COS(RPSIP)+A2P*SIN(RPSIP)
      RDEN=C+H3G+H3P-
     1 (RBCG*(1.0-COS(THRMG))+RBCP*(1.0-COS(THRMP)))
      RPPR=ACOS(RNUM/RDEN)
      RBCPG=RBCG*COS(RPHI)/COS(RPPR)
      RBCPP=RBCP*COS(RPHI)/COS(RPPR)
      DNG=SQRT(RBCG**2+RBCPG**2-
     1 (2.0*RBCG*RBCPG*COS(RPPR-RPHI)))
      DNP=SQRT(RBCP**2+RBCPP**2-
     1 (2.0*RBCP*RBCPP*COS(RPPR-RPHI)))
      DTOTG=SQRT(DPG**2+DNG**2)
```

```
      DTOTP=SQRT(DPP**2+DNP**2)
      RATG=DNG*(ROG/(ROG+ROP))
      RATP=DNP*(ROP/(ROG+ROP))
      WRITE(1,8777)DPG,DPP,DNG,DNP,DTOTG,DTOTP,
     1            E2G,E2P,E3G,E3P,RATG,RATP
 8777 FORMAT(/,4X,'     CONTACT POINT DEFLECTION ',//,
     1   4X,' DEFLECTION ALONG THE TANGENT PLANE, (GEAR)      =',
     $   F15.8,' INCHES',/,
     2   4X,' DEFLECTION ALONG THE TANGENT PLANE, (PINION)    =',
     $   F15.8,' INCHES',//,
     3   4X,' DEFLECTION ALONG THE NORMAL PLANE, (GEAR)       =',
     $   F15.8,' INCHES',/,
     4   4X,' DEFLECTION ALONG THE NORMAL PLANE, (PINION)     =',
     $   F15.8,' INCHES',//,
     5   4X,' TOTAL CONTACT POINT DEFLECTION, (GEAR)          =',
     $   F15.8,' INCHES',/,
     6   4X,' TOTAL CONTACT POINT DEFLECTION, (PINION)        =',
     $   F15.8,' INCHES',/,
     7   4X,' DEFLECTION ON THE PITCH RAY, E2 (GEAR)          =',
     $   F15.8,' INCHES',/,
     8   4X,' DEFLECTION ON THE PITCH RAY, E2 (PINION)        =',
     $   F15.8,' INCHES',/,
     9   4X,' DEFLECTION ON THE BACKCONE PLANE, E3 (GEAR)     =',
     $   F15.8,' INCHES',/,
     1   4X,' DEFLECTION ON THE BACKCONE PLANE, E3 (PINION)   =',
     $   F15.8,' INCHES',/,
     2   4X,' DEFLECTION RATIO (BACKCONE PLANE), (GEAR)       =',
     $   F15.8,' INCHES',/,
     3   4X,' DEFLECTION RATIO (BACKCONE PLANE), (PINION)     =',
     $   F15.8,' INCHES',//)
       IF (DNG.GE.ADDG.OR.DNG.GE.DEDG)WRITE(1,8820)
       IF (DNP.GE.ADDP.OR.DNP.GE.DEDP)WRITE(1,8821)
 8820 FORMAT(/,4X,'             **** WARNING **** ',/,
     @         4X,'   --------  GEAR DEFLECTION  --------',/,
     1         4X,'    DEFLECTION ON NORMAL PLANE EXCEEDS',/,
     2         4X,'     ADDENDUM OR DEDENDUM DISTANCES',//)
 8821 FORMAT(/,4X,'             **** WARNING **** ',/,
     @         4X,'   --------  PINION DEFLECTION  --------',/,
     1         4X,'   DEFLECTION ON NORMAL PLANE EXCEEDS',/,
     2         4X,'     ADDENDUM OR DEDENDUM DISTANCES',//)
      RETURN
      END
C
C>
C>>  SUBROUTINE FOR A GIVEN GEOMETRY FINDS BEARING DEFLECTION
```

```
C>
C
C>     INPUT ARGUMENTS
C           ITYPE : TYPE OF BEARING
C                     1=SINGLE ROW BALL BEARING
C                     2=DOUBLE ROW BALL BEARING
C                     3=SINGLE ROW ROLLER BEARING
C                     4=DOUBLE ROW ROLLER BEARING
C              FR : RADIAL LOAD SEEN BY BEARING
C              AC : BEARING CONTACT ANGLE
C              DI : INNER RACEWAY'S DIAMETER
C              RI : INNER RACEWAY'S GROOVE RADIUS
C              EI : INNER RACEWAY'S YOUNGS MODULUS
C             PRI : INNER RACEWAY'S POISIONS RATIO
C              DO : OUTER RACEWAY'S DIAMETER
C              RO : OUTER RACEWAY'S GROOVE RADIUS
C              EO : OUTER RACEWAY'S YOUNGS MODULUS
C             PRO : OUTER RACEWAY'S POISIONS RATIO
C               D : ROLLING ELEMENT DIAMETER
C              RR : ROLLING ELEMENT GROOVE RADIUS
C                    (EQUAL TO .5 * D FOR BALL BEARINGS)
C              ER : ROLLING ELEMENT YOUNGS MODULUS
C             PRR : ROLLING ELEMENT POISIONS RATIO
C             NRE : NUMBER OF ROLLING ELEMENTS
C              CL : LINE CONTACT LENGTH
C            NCHE : FLAG CONTROLLING THE EXISTENCE OF A DATA FILE
C
C>     OUTPUT ARGUMENTS
C
C            DMAX : BEARING DEFLECTION
C
C>
      SUBROUTINE BEARI
     1 (FR,DMAX,ITYPE,AC,DI,RI,EI,PRI,DO,RO,EO,PRO,D,
     2 RR,ER,PRR,NCHE)
      EXTERNAL FORCE
      COMMON/BINF/KT,CD,AN,JRE,ITC
      COMMON/BOUT/EPSI
      COMMON/ELEM/NRE
      REAL KI, KO, KT, JRE
      CHARACTER*6,NCHE
      FR=ABS(FR)
      ITC = 0
      IF(RR .EQ. 99. .OR. RR .EQ. RI) ITC = 1
      IF(ITC .NE. 1) GO TO 10
```

```
      WRITE(1,9)
 9    FORMAT(//,
     1 4X,' SINCE THE ROLLING ELEMENT'S RADIUS IS EQUAL TO',/,
     2 4X,' INNER GRACEWAY'S GROOVE RADIUS OR A BIG NUMBER, ',/,
     3 4X,' THE BEARING HAS A LINE CONTACT AND THE CONTACT',/,
     4 4X,' LENGTH (CL) IS REQUIRED, ENTER  CL (INCHES)',/)
      READ(1,*)CL
 10   GO TO(20,30,40,50), ITYPE
 20   WRITE(1,1101)
      GO TO 60
 30   WRITE(1,1102)
      GO TO 60
 40   WRITE(1,1103)
      GO TO 60
 50   WRITE(1,1104)
 60   WRITE(1,1105)
     1 DI,RI,EI,PRI,DO,RO,EO,PRO,D,RR,ER,PRR,NRE,AC
      IF(ITC .EQ. 1) GO TO 70
      WRITE(1,1106)
      GO TO 80
 70   WRITE(1,1107) CL
 80   CONTINUE
C
C>    END OF INPUT SECTION NOW START CALCULATIONS
C
      DM = .5 * (DI + DO)
      CD = DO - DI - (2. * D)
      AC = AC * 3.14159 / 180.
      PHI = D * COS(AC) / DM
      WRITE(1,2220)CD
 2220 FORMAT(//,4X,
     1'** DIAMETRAL CLEARANCE (Cd)... = ',F15.6,
     2' INCHES **',/)
C
C>    FIND THE LOAD ACTING ON ONE ROLLER ELEMENT
C
      TFR = 4.37 * FR / (NRE * COS(AC))
      IF(ITYPE .EQ. 1 .OR. ITYPE .EQ. 3) GO TO 90
      TFR = TFR /  2.0
 90   CONTINUE
C
C>    FIND CURVATURE DIFFERENCE FOR BALL BEARINGS
C
      IF(ITYPE .EQ. 3 .OR. ITYPE .EQ. 4) GO TO 100
      FII = D / RI
```

```
      BI = 2. * PHI / (1. - PHI)
      SPI = (4. - FII + BI) / D
      FPI = (FII + BI) / (4. - FII + BI)
      FOI = D / RO
      BO = 2. * PHI / (1. + PHI)
      SPO = (4. - FOI - BO) / D
      FPO = (FOI - BO) / (4. - FOI - BO)
      CALL BDEFP(FPI,SPI,ER,EI,PRR,PRI,BDI,KI,TFR)
      CALL BDEFP(FPO,SPO,ER,EO,PRR,PRO,BDO,KO,TFR)
      GO TO 120
C
C>    FIND CURVATURE DIFFERENCE FOR ROLLER BEARINGS
C
  100 BI = 2. / (1. - PHI)
      C = D * ((1.0/RR) - (1.0/RI))
      SPI = (BI + C) / D
      FPI = (BI - C) / (BI + C)
      BO = 2. / (1. + PHI)
      C = D * ((1.0/RR) - (1.0/RO))
      SPO = (BO + C) / D
      FPO = (BO - C) / (BO + C)
      IF(ITC .EQ. 1) GO TO 110
      CALL BDEFP(FPI,SPI,ER,EI,PRR,PRI,BDI,KI,TFR)
      CALL BDEFP(FPO,SPO,ER,EO,PRR,PRO,BDO,KO,TFR)
      GO TO 120
  110 CALL BDEFL(ER,PRR,EI,PRI,CL,TFR,BDI,KI)
      CALL BDEFL(ER,PRR,EO,PRO,CL,TFR,BDO,KO)
  120 CONTINUE
C
C>    NOW FIND THE BEARING DEFLECTION BY USING
C>    THE INTERVAL HALVING ROUTINE 'HALVE'
C
      IF(ITC.EQ.1) GO TO 130
      AN = 1.5
      GO TO 140
  130 AN=10./9.
  140 DMAXI=BDI+BDO
      CPD=0.0
      IF(CD.GT.0.0) CPD=CD
      IF(CD.GT.0.0) CD=0.0
      ANI=1.0/AN
      KT=(1.0/((1.0/KI)**ANI + (1.0/KO)**ANI))**AN
      ERR=0.005*FR
      DXI=DMAXI/5.0
      CALL HALVE(FORCE,DMAXI,DXI,FR,ERR,DMAX,DX,JJ,IERR)
```

```
      IF(IERR.EQ.0)GOTO 160
      WRITE(1,2211)JJ
2211  FORMAT(//,4X,'NO SUCCESSFUL ITERATION, MAX OF ',
     1 I3,' EXCEEDED',/)
160   CONTINUE
      IF(ITYPE.EQ.1.OR.ITYPE .EQ. 3) GO TO 170
      DMAX=DMAX*COS(AC)
170   IF(EPSI.GT.100.0)DMAX=-CD/400.0
      DMAX=DMAX+CPD
      WRITE(1,1108) DMAX
      RETURN
1000  FORMAT(I1,3X,F9.3,3X,F7.4)
1001  FORMAT(F7.4,3X,F7.4,3X,F10.1,3X,F5.3)
1002  FORMAT(F7.4,3X,F7.4,3X,F10.1,3X,F5.3,3X,I2)
1003  FORMAT(F6.4)
1101  FORMAT(/,
     1 6X,'THE BEARING IS A SINGLE ROW BALL BEARING')
1102  FORMAT(/,
     1 6X,'THE BEARING IS A DOUBLE ROW BALL BEARING')
1103  FORMAT(/,
     1 6X,'THE BEARING IS A SINGLE ROW ROLLER BEARING')
1104  FORMAT(/,
     1 6X,'THE BEARING IS A DOUBLE ROW ROLLER BEARING')
1105  FORMAT(/,
     16X,'WITH THE FOLLOWING DIMENSIONS :'
     @,/,6X,'INNER RACEWAY DIAMETER . . . . = ',F15.4,
     @'    INCHES'
     @,/,6X,'INNER RACEWAY GROOVE RADIUS. . = ',F15.4,
     @'    INCHES'
     @,/,6X,'INNER RACEWAY ELASTIC MODULUS. = ',F15.4,
     @' LBF/IN**2'
     @,/,6X,'INNER RACEWAY POISONS RATIO  . = ',F15.4
     @,/,6X,'OUTER RACEWAY DIAMETER . . . . = ',F15.4,
     @'    INCHES'
     @,/,6X,'OUTER RACEWAY GROOVE RADIUS. . = ',F15.4,
     @'    INCHES'
     @,/,6X,'OUTER RACEWAY ELASTIC MODULUS. = ',F15.4,
     @' LBF/IN**2'
     @,/,6X,'OUTER RACEWAY POISONS RATIO  . = ',F15.4
     @,/,6X,'ROLLING ELEMENT DIAMETER . . . = ',F15.4,
     @'    INCHES'
     @,/,6X,'ROLLING ELEMENT GROOVE RADIUS. = ',F15.4,
     @'    INCHES'
     @,/,6X,'ROLLING ELEMENT ELASTIC MODULUS= ',F15.4,
     @' LBF/IN**2'
```

```
     @,/,6X,'ROLLING ELEMENT POISONS RATIO    = ',F15.4
     @,/,6X,'NUMBER OF ROLLING ELEMENTS . .   = ',8X,I2
     @,/,6X,'ANGLE OF CONTACT . . . . . . .   = ',F15.4,
     @'     DEGREES')
 1106 FORMAT(/,6X,
     1 '******* THE BEARING HAS POINT CONTACT *******',/)
 1107 FORMAT(/,6X,
     1'******* THE BEARING HAS LINE CONTACT WITH A',/,
     @6X,'CONTACT LENGTH OF ',F6.4,' INCHES *******')
 1108 FORMAT(6X,'******* THE BEARING DEFLECTION = ',F10.8,
     1   ' INCHES *******')
      END
C
C>
C>>    SUBROUTINE 'HALVE'
C>
C
C>     INPUT ARGUMENTS
C               XI :  INITIAL GUESS FOR X
C              DXI :  INITIAL INCREMENT IN X
C                V :  VALUE OF Y TO BE OBTAINED
C                E :  ACCEPTIBLE ERROR IN Y
C
C>     OUTPUT ARGUMENTS
C                X :  THE FOUND VALUE OF X
C               DX :  THE LAST INCREMENT VALUE
C                J :  THE NUMBER OF ITERATIONS USED
C             IERR :  AN ERROR CODE
C                       IERR = 0   -   ITERATION IS SUCCESSFUL
C                       IERR = 1   -   ITERATION IS NOT SUCCESSFU
      SUBROUTINE HALVE(SUBF,XI,DXI,V,E,X,DX,J,IERR)
      X=XI
      DX=DXI
      IERR=0
      J=0
      DO 10 I = 1,500
      CALL SUBF(X,Y)
      ESA=Y-V
      ECA=ABS(ESA)
      IF(ECA.LT.E) RETURN
      IF(J.EQ.0) GO TO 20
      SIGN=ESA*ESB
      ECB=ABS(ESB)
      IF(ECA.GT.ECB.OR.SIGN.LT.0.0)  DX=-DX/2.0
   20 ESB=ESA
```

```
      X=X+DX
      J=J+1
   10 CONTINUE
      IERR=1
      RETURN
      END
C
C>
C>>   SUBROUTINE 'FORCE'
C>
C
C>      INPUT ARGUMENTS
C                D : BEARING DEFLECTION
C
C>      OUTPUT ARGUMENTS
C                F : FORCE BASED ON D
C>
      SUBROUTINE FORCE(D,F)
      COMMON/BINF/KT,CD,AN,JRE,ITC
      COMMON/BOUT/EPSI
      COMMON/ELEM/NRE
      REAL JRE,KT
      EPSI=0.5*(1.0-(CD/(2.0*D)))
      CALL BJRES(EPSI,JRE,ITC,NRE)
      CHECK=D-0.5*CD
      IF(CHECK.GT.0.0)GOTO 2215
      WRITE(1,2220)
 2220 FORMAT(//,8X,
     1'***** IMPOSSIBLE SITUATION : D < Cd  *****',/)
      GOTO 9999
 2215 F=NRE*KT*CHECK**AN*JRE
 9999 RETURN
      END
C
C>
C>>   THIS SUBROUTINE CALCULATES THE DEFLECTION BETWEEN THE ROLLER
C>>   ELEMENT AND THE RACE DUE TO THE HERTZIAN CONTACT STRESSES,
C>>   WITH POINT CONTACT.
C>
C
C>      INPUT ARGUMENTS
C              FP : CURVATURE DIFFERENCE
C              SP : CURVATURE SUM
C              ER : ROLLING ELEMENT MODULUS OF ELASTICITY
C              E2 : INNER/OUTER RACE MODULUS OF ELASTICITY
```

```
C            PRR : ROLLING ELEMENT POISSON'S RATIO
C            PR2 : INNER/OUTER RACE POISSON'S RATIO
C            TFR : APPLIED LOAD AT THE ANGLE OF CONTACT
C
C>     OUTPUT ARGUMENTS
C           BDEF : BEARING DEFLECTION AT THE ANGLE OF CONTACT
C              K : LOAD-DEFLECTION FACTOR
C
C>
      SUBROUTINE BDEFP(FP,SP,ER,E2,PRR,PR2,BDEF,K,TFR)
      REAL K
      CALL BDS(FP,DS)
      Q = ((1.0-PRR**2)/ER + (1.0-PR2**2)/E2) * 1.5*TFR/SP
      BDEF = (DS * SP / 2.0) * (Q**.666)
      K = 1.8856181 / (((1.0 - PRR**2)/ER + (1.0 - PR2**2)/E2)
     @ * DS**1.5 * SP**.5)
      RETURN
      END
C
C>
C>>   THIS SUBROUTINE CALCULATES THE DEFLECTION BETWEEN THE ROLLER
C>>   ELEMENT AND THE RACE DUE TO THE HERTZIAN CONTACT STRESSES,
C>>   WITH LINE CONTACT.
C>
C
C>    INPUT ARGUMENTS
C             ER : ROLLING ELEMENT MODULUS OF ELASTICITY
C            PRR : ROLLING ELEMENT POISSON'S RATIO
C             E1 : INNER/OUTER RACE MODULUS OF ELASTICITY
C            PR1 : INNER/OUTER RACE POISSON'S RATIO
C             CL : CONTACT LENGTH
C            TFR : APPLIED LOAD AT THE ANGLE OF CONTACT
C
C>    OUTPUT ARGUMENTS
C             BD : BEARING DEFLECTION AT THE ANGLE OF CONTACT
C             SK : LOAD-DEFLECTION FACTOR
C
C>
      SUBROUTINE BDEFL(ER,PRR,E1,PR1,CL,TFR,BD,SK)
      ERT = ER * 4.448222 / 9.806 / 25.4**2.
      E1 = E1 * 4.448222 / 9.806 / 25.4**2.
      FRM = TFR * 4.448222 / 9.806
      CLT = CL * 25.4
      ETR = ERT / (1. - PRR**2.)
      ET1 = E1 / (1. - PR1**2.)
```

```
      ET = (11550. * (ETR + ET1) / (ETR * ET1))**(1./3.)
      BD = .0003 * ET**2.7 * FRM**.9 / CLT**.8
      BD = BD / 25.4
      SK = 658478.28 * CLT**(8./9.) / (ET**3)
      RETURN
      END
C
C>
C>>    THIS SUBROUTINE USES A PIECEWISE LINEAR SCHEME
C>>    TO INTERPOLATE THE VALUES FOR THE RADIAL INTEGRAL Jr(e).
C>
C
C>     INPUT ARGUMENTS
C               X : X-COMPONENT OF INTERPOLATION
C             ITC : BEARING CONTACT FLAG ; 0=POINT, 1=LINE
C             NRE : NUMBER OF BEARING ROLLING ELEMENTS
C
C>     OUTPUT ARGUMENTS
C               P : Y-VALUE (INTERPOLATED)
C>
      SUBROUTINE BJRES(X,P,ITC,NRE)
      DIMENSION E(16), DP(16), DL(16)
C
C>
C>>      ***** LOAD DISTRIBUTION INTEGRAL Jr(E) *****
C>>    DATA TAKEN FROM BOOK BY T. HARRIS, 2nd Edition, 1984
C>>    FOR INFORMATION SEE TABLE 6.1, PAGE 164
C>
C
      DATA DP/0.1156,0.1590,0.1892,0.2117,0.2288,0.2416,0.2505,
     @         0.2559,0.2576,0.2546,0.2289,0.1871,0.1339,0.0711,
     @         0.0500,0.0/
      DATA DL/0.1268,0.1737,0.2055,0.2286,0.2453,0.2568,0.2636,
     @         0.2658,0.2628,0.2523,0.2078,0.1589,0.1075,0.0544,
     @         0.0400,0.0/
      DATA E/0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,
     @        1.0,1.25,1.67,2.50,5.0,10.0,1000.0/
C
C>     ENTER INPUT INFORMATION (POINT or LINE CONTACT)
C
      ND=16
      IF (X.EQ.0.0) GO TO 100
      IF (X.GT.E(ND)) GOTO 888
      IF(ITC.NE.1) ITC=0
      IF (X.GE.E(1).AND.ITC.EQ.0)GOTO 551
```

```
      IF (X.GE.E(1).AND.ITC.EQ.1)GOTO 800
      XO=0.0
      X1=E(1)
      YO=1.0/NRE
      IF(ITC.EQ.1)Y1=DL(1)
      IF(ITC.EQ.0)Y1=DP(1)
      GOTO 115
C
C>    POINT CONTACT : ITC = 0
C
 551  CONTINUE
      DO 10 I=1,ND
          IF (I.EQ.ND)GOTO 330
          IF (X.EQ.E(ND)) GOTO 666
          IF (X.GE.E(I).AND.X.LT.E(I+1))GOTO 330
 10   CONTINUE
 330  XO=E(I)
      X1=E(I+1)
      YO=DP(I)
      Y1=DP(I+1)
      GOTO 115
 800  CONTINUE
C
C>    LINE CONTACT : ITC = 1
C
      DO 20 I=1,ND
          IF (I.EQ.ND)GOTO 440
          IF (X.EQ.E(ND)) GOTO 777
          IF (X.GE.E(I).AND.X.LT.E(I+1))GOTO 440
 20   CONTINUE
 440  XO=E(I)
      X1=E(I+1)
      YO=DL(I)
      Y1=DL(I+1)
C
C>    INTERPOLATION SCHEME
C
 115  AA=Y1-YO
      BB=X1-XO
      CC=X-XO
      P=YO+(AA*CC/BB)
      GOTO 110
 666  P=DP(ND)
      GOTO 110
 777  P=DL(ND)
```

```
      GOTO 110
 888  P=0.0
      GOTO 110
 100  P=1.0/NRE
 110  CONTINUE
      RETURN
      END
C
C>
C>>    THIS SUBROUTINE USES A PIECEWISE LINEAR SCHEME
C>>    TO INTERPOLATE THE VALUES FOR THE STRESS ELLIPSE DATA
C>
C
C>     INPUT ARGUMENTS
C               X : X-COMPONENT OF INTERPOLATION
C
C>     OUTPUT ARGUMENTS
C               P : Y-VALUE (INTERPOLATED)
C>
      SUBROUTINE BDS(X,P)
      DIMENSION FP(23),DS(23)
C
C>
C>>     ***** DIMENSIONLESS CONTACT DEFORMATION DATA *****
C>>    DATA TAKEN FROM BOOK BY T. HARRIS, 2nd Edition, 1984
C>>    FOR INFORMATION SEE TABLE 5.1, PAGE 130 (Columns 1 & 4)
C>
C
      DATA FP/0.0,0.1075,0.3204,0.4795,0.5916,0.6716,0.7332,
     1        0.7948,0.83495,0.87366,0.90999,0.93657,0.95738,
     2        0.97290,0.983797,0.990902,0.995112,0.997300,
     3        0.9981847,0.9989156,0.9994785,0.9998527,1.0/
      DATA DS/1.0,0.9974,0.9761,0.9429,0.9077,0.8733,
     1        0.8394,0.7961,0.7602,0.7169,0.6636,0.6112,
     2        0.5551,0.4960,0.4352,0.3745,0.3176,0.2705,0.2427,
     3        0.2106,0.17167,0.11995,0.0/
C
C>     ENTER INPUT INFORMATION
C
      ND=23
      IF (X.EQ.0.0) GO TO 100
      IF (X.GT.FP(ND)) GOTO 888
      IF (X.GT.FP(1).AND.X.LE.FP(ND))GOTO 551
      X0=FP(1)
      X1=FP(2)
```

```
      Y0=DS(1)
      Y1=DS(2)
      GOTO 115
 551  CONTINUE
      DO 10 I=1,ND
          IF (I.EQ.ND)GOTO 330
          IF (X.EQ.FP(ND)) GOTO 666
          IF (X.GE.FP(I).AND.X.LT.FP(I+1))GOTO 330
 10   CONTINUE
 330  X0=FP(I)
      X1=FP(I+1)
      Y0=DS(I)
      Y1=DS(I+1)
      GOTO 115
 800  CONTINUE
C
C>    INTERPOLATION SCHEME
C
 115  AA=Y1-Y0
      BB=X1-X0
      CC=X-X0
      P=Y0+(AA*CC/BB)
      GOTO 110
 666  P=DS(ND)
      GOTO 110
 888  P=0.0
      GOTO 110
 100  P=1.0
 110  CONTINUE
      RETURN
      END
C
C>
C>>   SUBROUTINE TO PROVIDE BASIC BEARING INFORMATION
C>
C
C>    INPUT ARGUMENTS
C           NCHE : FLAG CONTROLLING THE
C                  EXISTENCE OF A DATA FILE
C
C>    OUTPUT ARGUMENTS
C
C          ITYPE : TYPE OF BEARING
C                      1=SINGLE ROW BALL BEARING
C                      2=DOUBLE ROW BALL BEARING
```

```
C                    3=SINGLE ROW ROLLER BEARING
C                    4=DOUBLE ROW ROLLER BEARING
C             FR : RADIAL LOAD SEEN BY BEARING
C             AC : BEARING CONTACT ANGLE
C             DI : INNER RACEWAY'S DIAMETER
C             RI : INNER RACEWAY'S GROOVE RADIUS
C             EI : INNER RACEWAY'S YOUNGS MODULUS
C            PRI : INNER RACEWAY'S POISIONS RATIO
C             DO : OUTER RACEWAY'S DIAMETER
C             RO : OUTER RACEWAY'S GROOVE RADIUS
C             EO : OUTER RACEWAY'S YOUNGS MODULUS
C            PRO : OUTER RACEWAY'S POISIONS RATIO
C              D : ROLLING ELEMENT DIAMETER
C             RR : ROLLING ELEMENT GROOVE RADIUS
C                   (EQUAL TO .5 * D FOR BALL BEARINGS)
C             ER : ROLLING ELEMENT YOUNGS MODULUS
C            PRR : ROLLING ELEMENT POISIONS RATIO
C            NRE : NUMBER OF ROLLING ELEMENTS
C             CL : LINE CONTACT LENGTH
C
C>
      SUBROUTINE BINFO(ITYPE,AC,DI,RI,EI,PRI,DO,RO,EO,PRO,D,
     1 RR,ER,PRR,NCHE)
      COMMON/ELEM/NRE
      CHARACTER*6,DFILE,ANS,NCHE
      WRITE(1,10)
 10   FORMAT(//,
     1 4X,'HOW DO YOU WANT TO ENTER THE BEARING',/,
     2 4X,'INTRODUCTORY INFORMATION ?',//,
     3 4X,' 1. USING THE TERMINAL ',/,
     4 4X,' 2. USING A DATA FILE',/)
      READ(1,*)NCHOI
      GOTO (500,1000)NCHOI
 500  WRITE(1,11)
 11   FORMAT(//,4X,'ENTER THE FOLLOWING PARAMETERS : ',/,
     $ 4X,' (USE ONE BLANK SPACE OR COMMA BETWEEN YOUR ENTRIES',
     1 //,4X,' 1.   ITYPE--TYPE OF BEARING ',/,
     2 4X,'               1=SINGLE ROW BALL BEARING',/,
     3 4X,'               2=DOUBLE ROW BALL BEARING',/,
     4 4X,'               3=SINGLE ROW ROLLER BEARING',/,
     5 4X,'               4=DOUBLE ROW ROLLER BEARING',/,
     6 4X,' 2.   AC    --BEARING CONTACT ANGLE',/)
      READ(1,*)ITYPE,AC
      WRITE(1,12)
 12   FORMAT (/,
```

```
     1 4X,' 3.   DI    --INNER RACEWAY`S DIAMETER',/,
     2 4X,' 4.   RI    --INNER RACEWAY`S GROOVE RADIUS',/,
     3 4X,' 5.   EI    --INNER RACEWAY`S YOUNGS MODULUS',/,
     4 4X,' 6.   PRI   --INNER RACEWAY`S POISONS RATIO',/)
      READ(1,*)DI,RI,EI,PRI
      WRITE(1,13)
13    FORMAT (/,
     1 4X,' 7.   DO    --OUTER RACEWAY`S DIAMETER',/,
     2 4X,' 8.   RO    --OUTER RACEWAY`S GROOVE RADIUS',/,
     3 4X,' 9.   EO    --OUTER RACEWAY`S YOUNGS MODULUS',/,
     4 4X,'10.   PRO   --OUTER RACEWAY`S POISONS RATIO',/)
      READ(1,*)DO,RO,EO,PRO
      WRITE(1,14)
14    FORMAT (/,
     1 4X,'11.   D     --ROLLING ELEMENT DIAMETER',/,
     2 4X,'12.   RR    --ROLLING ELEMENT GROOVE RADIUS',/,
     3 4X,'              EQUAL TO .5 * D FOR BALL BEARINGS',/,
     4 4X,'13.   ER    --ROLLING ELEMENT YOUNGS MODULUS',/,
     5 4X,'14.   PRR   --ROLLING ELEMENT POISONS RATIO',/,
     6 4X,'15.   NRE   --NUMBER OF ROLLING ELEMENTS',//)
      READ(1,*)D,RR,ER,PRR,NRE
1106  WRITE(1,1105)DI,RI,EI,PRI,DO,RO,EO,PRO,D,RR,ER,PRR,NRE,AC
1105  FORMAT(/,6X,'WITH THE FOLLOWING DIMENSIONS :'
     @,/,6X,'INNER RACEWAY DIAMETER . . . . = ',F15.3,
     @'    INCHES'
     @,/,6X,'INNER RACEWAY GROOVE RADIUS. . = ',F15.3,
     @'    INCHES'
     @,/,6X,'INNER RACEWAY ELASTIC MODULUS. = ',F15.3,
     @' LBF/IN**2'
     @,/,6X,'INNER RACEWAY POISONS RATIO  . = ',F15.3
     @,/,6X,'OUTER RACEWAY DIAMETER . . . . = ',F15.3,
     @'    INCHES'
     @,/,6X,'OUTER RACEWAY GROOVE RADIUS. . = ',F15.3,
     @'    INCHES'
     @,/,6X,'OUTER RACEWAY ELASTIC MODULUS. = ',F15.3,
     @' LBF/IN**2'
     @,/,6X,'OUTER RACEWAY POISONS RATIO  . = ',F15.3
     @,/,6X,'ROLLING ELEMENT DIAMETER . . . = ',F15.3,
     @ '    INCHES'
     @,/,6X,'ROLLING ELEMENT GROOVE RADIUS. = ',F15.3,
     @ '    INCHES'
     @,/,6X,'ROLLING ELEMENT ELASTIC MODULUS= ',F15.3,
     @ ' LBF/IN**2'
     @,/,6X,'ROLLING ELEMENT POISONS RATIO  = ',F15.3
     @,/,6X,'NUMBER OF ROLLING ELEMENTS . . = ',9X,I2
```

```
     @,/,6X,'ANGLE OF CONTACT . . . . . . . . = ',F15.3,
     @'    DEGREES')
      WRITE(1,15)
15    FORMAT (//,
     1 4X,' IS EVERYTHING O.K.  SO FAR ? (Y/N)',/)
151   READ(1,1515)NCHE
1515  FORMAT(1A6)
      IF(NCHE.EQ.'Y'.OR.NCHE.EQ.'N')GOTO 152
      WRITE(1,153)
153   FORMAT(//,
     1 4X,'**** PLEASE ANSWER Y-YES OR N-NO ****',/)
      GOTO 151
152   IF (NCHE.EQ.'N')GOTO 500
      GOTO 999
1000  WRITE(1,197)
197   FORMAT(/,
     $ 4X,' HAVE YOU CREATED THE DATA FILES ? (Y/N)',//,
     1 4X,'*** IN ORDER TO  PREPARE THE DATA FILES ***',/,
     2 4X,'*** ENTER N-NO  TO EXIT THE PROGRAM .    ***',/)
161   READ(1,1515)NCHE
      IF(NCHE.EQ.'Y'.OR.NCHE.EQ.'N')GOTO 162
      WRITE(1,153)
      GOTO 161
162   IF (NCHE.EQ.'Y')GOTO 444
      WRITE(1,198)
198   FORMAT (//,
     1 4X,
     @' THE USER HAS TO ENTER THE FOLLOWING PARAMETERS VIA',/,
     2 4X,
     @' A DATA FILE, THEREFORE ONE DATA FILE NAME IS ENTERED' ,/,
     3 4X,
     @' FIRST (FOR EACH STATION).  THE DATA FILE STRUCTURE ',/,
     4 4X,
     @' IS AS FOLLOWING (USE BLANK SPACE OR COMMA BETWEEN THE #): ',/
     5 4X,' 1ST LINE    ITYPE,AC',/,
     6 4X,' 2ND LINE    DI,RI,EI,PRI',/,
     7 4X,' 3RD LINE    DO,RO,EO,PRO',/,
     8 4X,' 4TH LINE    D,RR,ER,PRR,NRE ',//,
     9 4X,
     @' SEE EXPLANATION OF THESE VARIABLES IN THE CODE PROGRAM',/)
      STOP
444   WRITE(1,450)
450   FORMAT (//,
     1 4X,' ENTER NOW THE DATA FILE NAME :(6 CHARS OR LESS)',//)
      READ(1,451)DFILE
```

```
 451   FORMAT (1A6)
C
C>     INPUT BEARING DATA FROM A DATA FILE
C
       OPEN(64,FILE=DFILE)
       READ(64,*) ITYPE,AC
       READ(64,*) DI,RI,EI,PRI
       READ(64,*) DO,RO,EO,PRO
       READ(64,*) D,RR,ER,PRR,NRE
       CLOSE(64)
C
C>     CHECK WHETHER THE INNER - OUTER DIAMETER DIFFERENCE
C>      IS <=> ROLLING ELEMENT DIAMETER
C
 999   DIFFE=1.10*(DO-DI)/2.0
       IF (DIFFE.GT.D)GOTO 9999
       WRITE(1,9998)DIFFE,D
 9998 FORMAT(//,
      1 4X,'              ***** WARNING *****',//,
      2 4X,
      @'ROLLING ELEMENT DIAMETER IS GREATER/EQUAL TO INNER-OUTER',/,
      3 4X,
      @'RACE DIAMETER DIFFERENCE (DIAMETRAL CLEARANCE) AND EXCEEDS',/,
      4 4X,
      @'THE ALLOWED 10% OF INNER-OUTER RACE/ELEMENT INTERFERENCE.',//,
      5 4X,
      @'THE ROLLING ELEMENT SHOULD BE AT LEAST :',F8.3,' INCHES',/,
      6 4X,
      @' BUT IT HAS BEEN FOUND TO BE EQUAL TO  :',F8.3,' INCHES',//,
      7 4X,'          *** THE PROGRAM RESUMES OR STOPS ***',/,
      8 4X,'          *** DEPENDING ON THE WAY YOU CHOSE ***',/,
      9 4X,'          *** TO ENTER THE BEARING DATA  ***',//)
       IF (NCHOI.EQ.1)GOTO 500
       STOP
 9999 RETURN
       END
C
C>
C>>    SUBROUTINE TO CALCULATE THE PRINCIPAL CURVATURES & DIRECTIONS
C>>    OF A GEAR AND PINION TOOTH SURFACES FOR SPIRAL BEVEL GEAR
C>
C
C>     INPUT ARGUMENTS
C            TN2 :  GEAR TEETH NUMBER
C            TN1 :  PINION TEETH NUMBER
```

```
C             BPR : GEAR MEAN SPIRAL ANGLE
C           PLOAD : APPLIED NORMAL LOAD ON THE TOOTH SURFACE
C            GMA2 : GEAR PITCH ANGLE
C            GMA1 : PINION PITCH ANGLE
C
C      FROM DATA FILE
C
C            PHIC : GEAR BLADE PRESSURE ANGLE
C             RCP : RADIUS OF GEAR CUTTER
C             RCF : RADIUS OF PINION CUTTER
C              RL : MEAN CONE DISTANCE
C
C>     OUTPUT ARGUMENTS
C            RK12 : FIRST CURVATURE OF GEAR
C            RK22 : SECOND CURVATURE OF GEAR
C            RK11 : FIRST CURVATURE OF PINION
C            RK21 : SECOND CURVATURE OF PINION
C>
      SUBROUTINE CURVA(TN2,TN1,BPR,PLOAD,GMA2,GMA1,
     1                     RK12,RK22,RK11,RK21)
      DIMENSION CO(17),AL(17),BE(17)
      REAL LAMD,LA(17)
      CHARACTER*6,DFILE
      CHARACTER*3,ANS
C
C>
C>>    DATA FOUND IN BOOK "FORMULAS FOR STRESS AND STAIN"
C>>    BY ROARK & YOUNG'S BOOK, (See Table 33, page 518)
C>
C
      DATA CO/0.00,0.10,0.20,0.30,0.40,0.50,0.60,0.70,0.75,
     1           0.80,0.85,0.90,0.92,0.94,0.96,0.98,0.99/
      DATA AL/1.000,1.070,1.150,1.242,1.351,1.486,1.661,1.905,
     1  2.072,2.292,2.600,3.093,3.396,3.824,4.508,5.937,7.774/
      DATA BE/1.000,0.936,0.878,0.822,0.769,0.717,0.664,0.608,
     1  0.578,0.544,0.507,0.461,0.438,0.412,0.378,0.328,0.287/
      DATA LA/0.750,0.748,0.743,0.734,0.721,0.703,0.678,0.644,
     1  0.622,0.594,0.559,0.510,0.484,0.452,0.410,0.345,0.288/
      PI=3.1415927
      RAD=PI/180.0
      GMA1DG=GMA1/RAD
      GMA2DG=GMA2/RAD
      WRITE(1,109)
  109 FORMAT(//,4X,'***** CURVATURE ANALYSIS *****',//)
      WRITE(1,110)
```

```
110  FORMAT(//,4X,'HOW DO YOU WANT TO ENTER THE ',/,
   1          4X,'INTRODUCTORY INFORMATION ?',//,
   2          4X,' 1. USING THE TERMINAL ',/,
   3          4X,' 2. USING A DATA FILE',/)
     READ(1,*)NCHOI
     GOTO (4444,4455)NCHOI
4455 WRITE(1,4456)
4456 FORMAT(//,4X,'ENTER DATA FILE NAME',/)
     READ(1,4457)DFILE
4457 FORMAT(1A6)
     OPEN(77,FILE=DFILE)
     READ(77,*)PHIC,RCP,RCF,RL
     CLOSE (77)
     GOTO 1232
4444 WRITE(1,11)
11   FORMAT(//,
    1 4X,'ENTER GEAR BLADE PRESSURE ANGLE, Phic, DEG. ',/)
     READ(1,*)PHIC
     WRITE(1,12)
12   FORMAT(//,
    1 4X,'ENTER RADIUS OF GEAR CUTTER, Rcp, IN',/)
     READ(1,*)RCP
     WRITE(1,14)
14   FORMAT(//,
    1 4X,'ENTER RADIUS OF PINION CUTTER, Rcf, IN',/)
     READ(1,*)RCF
     WRITE(1,15)
15   FORMAT(//,
    1 4X,'ENTER MEAN CONE DISTANCE, RL, IN',/)
     READ(1,*)RL
1232 CONTINUE
     E1=30.0E06
     E2=30.0E06
     PR1=0.25
     PR2=0.25
     B1=35000.0
     WRITE(1,151)E1,E2,PR1,PR2,B1
151  FORMAT(//,
    1 4X,' THE YOUNG MODULUS AND THE POISSON RATIO',/,
    2 4X,' FOR THE GEAR AND THE PINION ARE SET AS : ',//,
    3 4X,' GEAR ELASTIC MODULUS,   E1 = ',F15.4,' LBF/IN**2',/,
    4 4X,' PINION ELASTIC MODULUS, E2 = ',F15.4,' LBF/IN**2',/,
    5 4X,' GEAR POISSON RATIO,     PR1 = ',F15.4,/,
    6 4X,' PINION POISSON RATIO,   PR2 = ',F15.4,/,
    7 4X,' MATERIAL CONSTANT,      B1 = ',F15.4,' PSI',//,
```

```
     8 4X,' DO YOU WANT TO CHANGE THEM ? (Y/N)  ',/)
 1515 READ(1,152)ANS
 152  FORMAT(1A3)
      IF(ANS.EQ.'Y'.OR.ANS.EQ.'N')GOTO 165
      WRITE(1,153)
 153  FORMAT(//,4X,'**** PLEASE ANSWER Y-YES OR N-NO ****',/)
      GOTO 1515
 165  IF (ANS.EQ.'N')GOTO 1233
      WRITE(1,700)
 700  FORMAT(//,
     1 4X,'ENTER MODULUS OF ELASTICITY FOR GEAR AND PINION',/)
      READ(1,*)E1,E2
      WRITE(1,710)
 710  FORMAT(//,
     1 4X,'ENTER POISSON RATIO FOR GEAR AND PINION',/)
      READ(1,*)PR1,PR2
      WRITE(1,720)
 720  FORMAT(//,
     1 4X,'ENTER MATERIAL CONSTANT FOR THE MESH',/)
      READ(1,*)B1
C
C>    CALCULATIONS
C
 1233 PHP=PHIC*RAD
      BP=BPR*RAD
      BPDG=BP/RAD
      TPDG=90.0-BPDG
      TP=TPDG*RAD
      SNPHP=SIN(PHP)
      CSPHP=COS(PHP)
      SNBP=SIN(BP)
      CSBP=COS(BP)
      SNTP=SIN(TP)
      CSTP=COS(TP)
      SNGM1=SIN(GMA1)
      CSGM1=COS(GMA1)
      SNGM2=SIN(GMA2)
      CSGM2=COS(GMA2)
      WRITE(1,55)TN1,TN2,PHIC,BPR,GMA1DG,
     1 GMA2DG,RCP,RCF,RL,PLOAD
 55   FORMAT(//,4X,'     INPUT SUMMARY',//,
     1 4X,'PINION TEETH ................... = ',F10.4,/,
     2 4X,'GEAR TEETH ..................... = ',F10.4,/,
     3 4X,'GEAR BLADE PRESSURE ANGLE ...... = ',F10.4,
     @ ' DEGREES',/,
```

```
     4 4X,'GEAR MEAN SPIRAL ANGLE .........  = ',F10.4,
     @ ' DEGREES',/,
     5 4X,'PINION PITCH ANGLE .............  = ',F10.4,
     @ ' DEGREES',/,
     6 4X,'GEAR PITCH ANGLE ...............  = ',F10.4,
     @ ' DEGREES',/,
     7 4X,'RADIUS OF GEAR CUTTER ..........  = ',F10.4,
     @ ' INCHES',/,
     8 4X,'RADIUS OF PINION CUTTER ........  = ',F10.4,
     @ ' INCHES',/,
     9 4X,'MEAN CONE DISTANCE .............  = ',F10.4,
     @ ' INCHES',/,
     1 4X,'NORMAL TOOTH LOAD ..............  = ',F10.4,
     @ ' LBS',//)
C
C>
C>>   DETERMINATION OF GEAR PRINCIPAL CURVATURES AND DIRECTIONS
C>
C
      RK1P=-CSPHP/RCP
      RK2P=0.0
      F=-2.0*SNBP*CSBP/(2.0*RL*(SNGM2/CSGM2))
      G=-(SNPHP**2*SNBP**2-CSBP**2)/(RL*SNPHP*(SNGM2/CSGM2))
      S=-(SNPHP**2*SNBP**2+CSBP**2)/(RL*SNPHP*(SNGM2/CSGM2))
      U=2.0*F
      D=(RK1P-RK2P+G)
      SP22=ATAN2(U,D)
      SP2=SP22/2.0
      SP2DG=SP2/RAD
      WRITE(1,111)SP2DG
  111 FORMAT(
     1 4X,'GEAR - ANGLE BETWEEN THE TWO PRINCIPAL DIRECTIONS',/,
     2 4X,'SIGMA = ',F10.4,' DEGREES',/)
      SNSP2=SIN(SP2)
      CSSP2=COS(SP2)
      CSP22=COS(SP22)
      RI12X=-CSPHP*SNSP2
      RI12Y=CSTP*CSSP2+SNPHP*SNTP*SNSP2
      RI12Z=-(SNTP*CSSP2-SNPHP*CSTP*SNSP2)
      RI22X=-CSPHP*CSSP2
      RI22Y=-CSTP*SNSP2+SNPHP*SNTP*CSSP2
      RI22Z=SNTP*SNSP2+SNPHP*CSTP*CSSP2
      A=RK1P+RK2P+S
      B=(RK1P-RK2P+G)/CSP22
C
```

```
C>    RK12, RK22 ARE THE PRINCIPAL CURVATURES OF THE GEAR
C
      RK12=ABS((A+B)/2.0)
      RK22=ABS((A-B)/2.0)
C
C>
C>>   DETERMINATION OF PINION PRINCIPAL
C>>   CURVATURES  AND DIRECTIONS
C>
C
      RK1F=-CSPHP/RCF
      RK2F=0.0
      F=CSGM1*CSTP*SNTP/(RL*SNGM1)
      G=(SNPHP**2*CSTP**2*CSGM1**2-
     1  CSGM1**2*SNTP**2)/(RL*SNPHP*CSGM1*SNGM1)
      S=(SNPHP**2*CSTP**2*CSGM1**2+
     1  CSGM1**2*SNTP**2)/(RL*SNPHP*CSGM1*SNGM1)
      U=2.0*F
      D=RK1F-RK2F+G
      SF12=ATAN2(U,D)
      SF1=SF12/2.0
      SF1DG=SF1/RAD
      WRITE(1,22)SF1DG
 22   FORMAT(/,
     1 4X,
     @ 'PINION - ANGLE BETWEEN THE TWO PRINCIPAL DIRECTIONS',/,
     2 4X,'SIGMA = ',F10.4,' DEGREES',/)
      SNSF1=SIN(SF1)
      CSSF1=COS(SF1)
      CSF12=COS(SF12)
      RI11X=-CSPHP*SNSF1
      RI11Y=CSTP*CSSF1+SNPHP*SNTP*SNSF1
      RI11Z=-(SNTP*CSSF1-SNPHP*CSTP*SNSF1)
      RI21X=-CSPHP*CSSF1
      RI21Y=-CSTP*SNSF1+SNPHP*SNTP*CSSF1
      RI21Z=SNTP*SNSF1+SNPHP*CSTP*CSSF1
      A=RK1F+RK2F+S
      B=(RK1F-RK2F+G)/CSF12
C
C>    RK11, RK21 ARE THE PRINCIPAL CURVATURES OF THE PINION
C
      RK11=(A+B)/2.0
      RK21=(A-B)/2.0
C
C>    NORMAL
```

```
C
      RNX2=SNPHP
      RNY2=CSPHP*CSBP
      RNZ2=CSPHP*SNBP
C
C>    DETERMINATION OF ELLIPSE SIZE
C
      CS21=(RI11X*RI12X+RI11Y*RI12Y+RI11Z*RI12Z)
      SN21=RI11X*RNY2*RI12Z+RI12X*RNZ2*RI11Y+RNX2*RI12Y*RI11Z-
     1    (RI12X*RNY2*RI11Z+RNX2*RI11Y*RI12Z+RI11X*RNZ2*RI12Y)
      S21=ATAN2(SN21,CS21)
      S21DG=S21/RAD
C
C>    G1 & G2 : AUXILIARY FUNCTIONS TO DETERMINE
C>              SIZE OF CONTACT ELLIPSE
C
      G1=RK11-RK21
      G2=RK12-RK22
      RK1=RK11+RK21
      RK2=RK12+RK22
      S212=S21*2.0
      CSS21=COS(S21)
      SNS21=SIN(S21)
      CS212=COS(S212)
      SN212=SIN(S212)
      P=G1*SN212
      Q=G2-G1*CS212
      ALPH2=ATAN2(P,Q)
      ALPH=ALPH2/2.0
      ALPDG=ALPH/RAD
C
C>    AA & BB : AUXILIARY FUNCTIONS USED
C>              IN BEARING CONTACT
C
      AA=(RK1-RK2-(G1**2-2.0*G1*G2*CS212+G2**2)**0.5)/4.0
      BB=(RK1-RK2+(G1**2-2.0*G1*G2*CS212+G2**2)**0.5)/4.0
C
C>
C>>   CALCULATE THE ELLIPSE SIZE BY USING THE EQUATIONS
C>>   FOUND IN BOOK "FORMULAS FOR STRESS AND STRAIN"
C>>   BY ROARK & YOUNG (5th Edition)
C>>       (See Table 33, page 518)
C>
C
      RKD=1.5/(RK12+RK22+RK11+RK21)
```

```
      CE=(1.0-PR1**2)/E1 + (1.0-PR2**2)/E2
C
C>
C>>   LINEAR INTERPOLATION SCHEME FOR ALPHA, BETA, LAMBDA
C>
C
      RR1=(RK11-RK21)**2+(RK12-RK22)**2
      RR2=2.0*(RK11-RK21)*(RK12-RK22)*COS(2.0*S21)
      QUA=(PLOAD*RKD*CE)**0.33333
      COSTH=(RKD/1.5)*SQRT(RR1+RR2)
C
C>      A = SEMI MAJOR AXIS
C>      B = SEMI MINOR AXIS
C>    S21 = ANGLE BETWEEN PRINCIPAL CURVATURES
C
      CALL INTER(COSTH,CO,AL,ALFA)
      SA=ALFA*QUA
      CALL INTER(COSTH,CO,BE,BETA)
      SB=BETA*QUA
      RATIO=SA/SB
      CALL INTER(COSTH,CO,LA,LAMD)
      DLTA=LAMD*(((PLOAD*CE)**2)/RKD)**0.33333
      SIGMA=1.5*PLOAD/(SA*SB)
C
C>    CALCULATE THE DYNAMIC CAPACITY, LBS
C
      BASCA=B1*2.0*SA/(RK11+RK22)
      WRITE(1,333)RK12,RK22,RK11,RK21
 333  FORMAT(/,
     @ 4X,'RKxy : PRINCIPAL CURVATURES x OF SURFACE y',//,
     1 4X,'RK12  .......................... = ',F10.4,/,
     2 4X,'RK22  .......................... = ',F10.4,/,
     3 4X,'RK11  .......................... = ',F10.4,/,
     4 4X,'RK21  .......................... = ',F10.4,/)
      WRITE(1,444)RI11X,RI11Y,RI11Z,RI21X,RI21Y,RI21Z
 444  FORMAT(/,4X,
     @ '3-D UNIT VECTORS ALONG FIRST PRINCIPAL DIRECTIONS',
     1 //,4X,'RI11X ....................... = ',F10.4,/,
     2 4X,'RI11Y ......................... = ',F10.4,/,
     3 4X,'RI11Z ......................... = ',F10.4,/,
     4 4X,'RI21X ......................... = ',F10.4,/,
     5 4X,'RI21Y ......................... = ',F10.4,/,
     6 4X,'RI21Z ......................... = ',F10.4,/)
      WRITE(1,555)RI12X,RI12Y,RI12Z,RI22X,RI22Y,RI22Z
 555  FORMAT(/,4X,
```

```
     @ '3-D UNIT VECTORS ALONG SECOND PRINCIPAL DIRECTIONS',
     1 //,4X,'RI12X ........................... = ',F10.4,/,
     2 4X,'RI12Y .......................... = ',F10.4,/,
     3 4X,'RI12Z .......................... = ',F10.4,/,
     4 4X,'RI22X .......................... = ',F10.4,/,
     5 4X,'RI22Y .......................... = ',F10.4,/,
     6 4X,'RI22Z .......................... = ',F10.4,/)
      WRITE(1,666)S21DG,ALPDG,SA,SB,RATIO,DLTA,SIGMA,BASCA
 666  FORMAT(/,
     1 4X,'ANGLE BTWN PRNCPL DIRECTIONS ... = ',F10.4,
     @ ' DEGREES',/,
     2 4X,'CCW ANGLE OF I12 FROM MINOR AXIS = ',F10.4,
     @ ' DEGREES',/,
     3 4X,'SEMI MAJOR AXIS ................ = ',F10.4,
     @ ' INCHES ',/,
     4 4X,'SEMI MINOR AXIS ................ = ',F10.4,
     @ ' INCHES ',/,
     5 4X,'RATIO (MAJOR/MINOR) ............ = ',F10.4,/,
     6 4X,'SURFACE DEFORMATION ............ = ',E10.4,
     @ ' INCHES ',/,
     7 4X,'MAXIMUM CONTACT STRESS ......... = ',E10.4,
     8    ' LBS/IN**2',//,
     9 4X,'BASIC DYNAMIC CAPACITY ......... = ',E10.4,
     1    ' LBS/IN**2',//)
      RETURN
      END
C
C>
C>>   SUBROUTINE TO RERFORM A LINEAR INTERPOLATION USED
C>>   FOR CURVATURE ANALYSIS FOR DATA BY ROARK & YOUNG
C>
C
C>    INPUT ARGUMENTS
C              X : X-COMPONENT OF INTERPOLATION
C             XA : X-ARRAY CONTAINING THE INTERPOLATING VALUES
C             YA : Y-ARRAY CONTAINING THE Y-VALUE (INTERPOLATED)
C
C>    OUTPUT ARGUMENTS
C              P : Y-VALUE (INTERPOLATED)
C>
      SUBROUTINE INTER(X,XA,YA,P)
      DIMENSION XA(17),YA(17)
C
C>    ENTER INPUT INFORMATION
C
```

```
      ND=17
      IF (X.EQ.0.0) GO TO 100
      IF (X.GT.XA(ND)) GOTO 666
      IF (X.GT.XA(1).AND.X.LE.XA(ND))GOTO 551
      XO=XA(1)
      X1=XA(2)
      YO=YA(1)
      Y1=YA(2)
      GOTO 115
 551  CONTINUE
      DO 10 I=1,ND
          IF (I.EQ.ND)GOTO 330
          IF (X.EQ.XA(ND)) GOTO 666
          IF (X.GE.XA(I).AND.X.LT.XA(I+1))GOTO 330
 10   CONTINUE
 330  XO=XA(I)
      X1=XA(I+1)
      YO=YA(I)
      Y1=YA(I+1)
      GOTO 115
 800  CONTINUE
C
C>    INTERPOLATION SCHEME
C
 115  AA=Y1-YO
      BB=X1-XO
      CC=X-XO
      P=YO+(AA*CC/BB)
      GOTO 110
 666  P=YA(ND)
      GOTO 110
 100  P=YA(1)
 110  CONTINUE
      RETURN
      END
```

APPENDIX B

PROGRAM INPUT

```
HOW DO YOU WANT TO ENTER THE
INTRODUCTORY INFORMATION ?

 1. USING THE TERMINAL
 2. USING A DATA FILE

2

ENTER DATA FILE NAME

SPINFO

               I N P U T   S U M M A R Y

NUMBER OF GEAR TEETH, NG                       =      71
NUMBER OF PINION TEETH, NP                     =      19
PITCH CONE DISTANCE CENTER, AO                 =       5.200 INCHES
GEAR FACE WIDTH, F                             =       1.280 INCHES
NORMAL PRESSURE ANGLE, PHIN                    =      20.000 DEGREES
SPIRAL ANGLE, PSI                              =      30.000 DEGREES
MESH ANGLE OF GEAR AND PINION, SIGMA           =      95.000 DEGREES
TORQUE APPLIED TO THE PINION                   =    3232.000 LBS/IN
GEAR SHAFT DIAMETER, DG                        =       2.450 INCHES
PINION SHAFT DIAMETER, DP                      =       2.160 INCHES
GEAR ADDENDUM DISTANCE, ADDG,                  =       0.081 INCHES
GEAR DEDENDUM DISTANCE, ADDP                   =       0.163 INCHES
PINION ADDENDUM DISTANCE, DEDG                 =       0.175 INCHES
PINION DEDENDUM DISTANCE, DEDP                 =       0.093 INCHES

     IS THIS DATA CORRECT ? (Y/N)

Y

NOTE: PINION IS THE DRIVING GEAR

  DRIVING AND DIRECTION MENU

ENTER YOUR CHOICE (NUMBER 1 - 8) :
  1.        DRIVING   RH CW
  2.                  LH CCW
  3.        DRIVEN    RH CCW
  4.                  LH CW
  ------------------------
  5.        DRIVING   RH CCW
```

```
     6.                 LH CW
     7.        DRIVEN   RH CW
     8.                 LH CCW

6

        PARTIAL RESULTS

         GEAR SECTION

   TANGENTIAL LOAD, LBS                        =        2691.670
   AXIAL      LOAD, LBS                        =         836.142
   RADIAL     LOAD, LBS                        =        1730.784
   NORMAL     LOAD, LBS                        =        3307.541
   GEAR CONE PITCH ANGLE, DEG                  =          79.733
   DIAMETRAL PITCH  (1/IN)                     =           7.912
   AVERAGE RADIUS, IN                          =           4.487

         PINION SECTION

   TANGENTIAL LOAD, LBS                        =        2691.669
   AXIAL      LOAD, LBS                        =        1797.072
   RADIAL     LOAD, LBS                        =         682.112
   NORMAL     LOAD, LBS                        =        3307.541
   PINION CONE PITCH ANGLE, DEG                =          15.267
   DIAMETRAL PITCH  (1/IN)                     =           7.912
   AVERAGE RADIUS, IN                          =           1.201

   DOES EVERYTHING LOOK O.K. SO FAR ? (Y/N)

Y

      BEARING SUPPORT CONFIGURATION :

   STRADDLE    (GR --------- BRNG --------- GR)
   OVERHUNG  (BRNG --------------BRNG------ GR)

   THERE ARE 4 POSSIBLE SYSTEM CONFIGURATIONS :

     1. STRADDLE GEAR  AND  STRADDLE PINION
     2. STRADDLE GEAR  AND  OVERHUNG PINION
     3. OVERHUNG GEAR  AND  STRADDLE PINION
     4. OVERHUNG GEAR  AND  OVERHUNG PINION

   ENTER YOUR CHOICE (1 - 4) :
```

```
1

        *****  GEAR   SECTION  *****

         STRADDLE CONFIGURATION

      THE GEAR SETUP LOOKS LIKE THIS :

     BRNG                  GR        BRNG
      *------------------*----------*
      *------- B --------*--- A ---*

     THE NEXT TWO QUESTIONS REFER TO THIS SETUP

     ENTER DISTANCE FROM GEAR TO RIGHT BEARG, A, IN

1.5

     ENTER DISTANCE FROM GEAR TO LEFT BEARG, B, IN

0.2

     --------- BEARING ANALYSIS INPUT ----------

     -------------  GEAR SECTION ---------------

     ------------------------------------------
       ******* LEFT BEARING DEFLECTION *******
       ** DEFLECTIONS IN X AND Z DIRECTION  **
     ------------------------------------------

     HOW DO YOU WANT TO ENTER THE BEARING
     INTRODUCTORY INFORMATION ?

      1. USING THE TERMINAL
      2. USING A DATA FILE

2

     HAVE YOU CREATED THE DATA FILES ? (Y/N)

     *** IN ORDER TO  PREPARE THE DATA FILES ***
     *** ENTER N-NO  TO EXIT THE PROGRAM .    ***

Y
```

```
ENTER NOW THE DATA FILE NAME :(6 CHARS OR LESS)

BN1

SINCE THE ROLLING ELEMENT'S RADIUS IS EQUAL TO
INNER GRACEWAY'S GROOVE RADIUS OR A BIG NUMBER,
THE BEARING HAS A LINE CONTACT AND THE CONTACT
LENGTH (CL) IS REQUIRED, ENTER  CL (INCHES)

0.80

THE BEARING IS A SINGLE ROW ROLLER BEARING
WITH THE FOLLOWING DIMENSIONS :

INNER RACEWAY DIAMETER . . . . =           3.5665     INCHES
INNER RACEWAY GROOVE RADIUS. . =          99.0000     INCHES
INNER RACEWAY ELASTIC MODULUS. =    30000000.0000 LBF/IN**2
INNER RACEWAY POISONS RATIO  . =           0.2500
OUTER RACEWAY DIAMETER . . . . =           4.4325     INCHES
OUTER RACEWAY GROOVE RADIUS. . =          99.0000     INCHES
OUTER RACEWAY ELASTIC MODULUS. =    30000000.0000 LBF/IN**2
OUTER RACEWAY POISONS RATIO  . =           0.2500
ROLLING ELEMENT DIAMETER . . . =           0.4335     INCHES
ROLLING ELEMENT GROOVE RADIUS. =          99.0000     INCHES
ROLLING ELEMENT ELASTIC MODULUS=    30000000.0000 LBF/IN**2
ROLLING ELEMENT POISONS RATIO  =           0.2500
NUMBER OF ROLLING ELEMENTS . . =          24
ANGLE OF CONTACT . . . . . . . =           0.0000     DEGREES

******* THE BEARING HAS LINE CONTACT WITH A
******* CONTACT LENGTH OF 0.8000 INCHES
** DIAMETRAL CLEARANCE (Cd)... = -0.001000 INCHES **

** THE BEARING DEFLECTION =  0.00017343 INCHES **

-------------------------------------------
  ******* RIGHT BEARING DEFLECTION *******
  **  DEFLECTIONS IN X AND Z DIRECTION  **
-------------------------------------------

HOW DO YOU WANT TO ENTER THE BEARING
INTRODUCTORY INFORMATION ?

 1. USING THE TERMINAL
```

```
   2. USING A DATA FILE

2

   HAVE YOU CREATED THE DATA FILES ? (Y/N)

   *** IN ORDER TO  PREPARE THE DATA FILES ***
   *** ENTER N-NO  TO EXIT THE PROGRAM     ***

Y

   ENTER NOW THE DATA FILE NAME :(6 CHARS OR LESS)

BN2

   THE BEARING IS A DOUBLE ROW BALL BEARING
   WITH THE FOLLOWING DIMENSIONS :

   INNER RACEWAY DIAMETER . . . . =          3.3425     INCHES
   INNER RACEWAY GROOVE RADIUS. . =          0.2340     INCHES
   INNER RACEWAY ELASTIC MODULUS. =   30000000.0000 LBF/IN**2
   INNER RACEWAY POISONS RATIO  . =          0.2500
   OUTER RACEWAY DIAMETER . . . . =          3.9540     INCHES
   OUTER RACEWAY GROOVE RADIUS. . =          0.2340     INCHES
   OUTER RACEWAY ELASTIC MODULUS. =   30000000.0000 LBF/IN**2
   OUTER RACEWAY POISONS RATIO  . =          0.2500
   ROLLING ELEMENT DIAMETER . . . =          0.3100     INCHES
   ROLLING ELEMENT GROOVE RADIUS. =          0.1800     INCHES
   ROLLING ELEMENT ELASTIC MODULUS=   30000000.0000 LBF/IN**2
   ROLLING ELEMENT POISONS RATIO  =          0.2500
   NUMBER OF ROLLING ELEMENTS . . =        25
   ANGLE OF CONTACT . . . . . . . =        26.0000     DEGREES

   ******* THE BEARING HAS POINT CONTACT *******
   ** DIAMETRAL CLEARANCE (Cd)... = -0.008500 INCHES **

   ** THE BEARING DEFLECTION = 0.00032017 INCHES **


      *****  PINION SECTION  *****

      STRADDLE CONFIGURATION

    THE GEAR SETUP LOOKS LIKE THIS :
```

```
BRNG                       GR           BRNG
 *-------------------------*-----------*
 *-------- B ---------*--- A ---*

THE NEXT TWO QUESTIONS REFER TO THIS SETUP

ENTER DISTANCE FROM GEAR TO RIGHT BEARG, A, IN
1.32

ENTER DISTANCE FROM GEAR TO LEFT BEARG, B, IN
1.90

--------- BEARING ANALYSIS INPUT ----------

------------ PINION SECTION -------------

----------------------------------------------
  ******* LEFT BEARING DEFLECTION *******
  ** DEFLECTIONS IN X AND Z DIRECTION  **
----------------------------------------------

HOW DO YOU WANT TO ENTER THE BEARING
INTRODUCTORY INFORMATION ?

 1.  USING THE TERMINAL
 2.  USING A DATA FILE
2

HAVE YOU CREATED THE DATA FILES ? (Y/N)

*** IN ORDER TO  PREPARE THE DATA FILES ***
*** ENTER N-NO  TO EXIT THE PROGRAM .    ***
Y

ENTER NOW THE DATA FILE NAME :(6 CHARS OR LESS)
BN3

THE BEARING IS A DOUBLE ROW BALL BEARING
WITH THE FOLLOWING DIMENSIONS :
```

```
INNER RACEWAY DIAMETER . . . . =              2.4880     INCHES
INNER RACEWAY GROOVE RADIUS. . =              0.3300     INCHES
INNER RACEWAY ELASTIC MODULUS. =      30000000.0000 LBF/IN**2
INNER RACEWAY POISONS RATIO  . =              0.2500
OUTER RACEWAY DIAMETER . . . . =              3.3315     INCHES
OUTER RACEWAY GROOVE RADIUS. . =              0.3300     INCHES
OUTER RACEWAY ELASTIC MODULUS. =      30000000.0000 LBF/IN**2
OUTER RACEWAY POISONS RATIO  . =              0.2500
ROLLING ELEMENT DIAMETER . . . =              0.4228     INCHES
ROLLING ELEMENT GROOVE RADIUS. =              0.2813     INCHES
ROLLING ELEMENT ELASTIC MODULUS=      30000000.0000 LBF/IN**2
ROLLING ELEMENT POISONS RATIO  =              0.2500
NUMBER OF ROLLING ELEMENTS . . =           14
ANGLE OF CONTACT . . . . . . . =             35.0000     DEGREES

******* THE BEARING HAS POINT CONTACT *******
** DIAMETRAL CLEARANCE (Cd)... = -0.002100 INCHES **

** THE BEARING DEFLECTION = 0.00079574 INCHES **

--------------------------------------------
  ******* RIGHT BEARING DEFLECTION *******
  **  DEFLECTIONS IN X AND Z DIRECTION  **
--------------------------------------------

HOW DO YOU WANT TO ENTER THE BEARING
INTRODUCTORY INFORMATION ?

 1. USING THE TERMINAL
 2. USING A DATA FILE

2

HAVE YOU CREATED THE DATA FILES ? (Y/N)

*** IN ORDER TO  PREPARE THE DATA FILES ***
*** ENTER N-NO  TO EXIT THE PROGRAM     ***

Y

ENTER NOW THE DATA FILE NAME :(6 CHARS OR LESS)

BN4

SINCE THE ROLLING ELEMENT'S RADIUS IS EQUAL TO
```

```
INNER GRACEWAY'S GROOVE RADIUS OR A BIG NUMBER,
THE BEARING HAS A LINE CONTACT AND THE CONTACT
LENGTH (CL) IS REQUIRED, ENTER  CL (INCHES)

.386

THE BEARING IS A SINGLE ROW ROLLER BEARING
WITH THE FOLLOWING DIMENSIONS :

INNER RACEWAY DIAMETER . . . . =            2.0680     INCHES
INNER RACEWAY GROOVE RADIUS. . =           99.0000     INCHES
INNER RACEWAY ELASTIC MODULUS. =     30000000.0000 LBF/IN**2
INNER RACEWAY POISONS RATIO  . =            0.2500
OUTER RACEWAY DIAMETER . . . . =            2.6980     INCHES
OUTER RACEWAY GROOVE RADIUS. . =           99.0000     INCHES
OUTER RACEWAY ELASTIC MODULUS. =     30000000.0000 LBF/IN**2
OUTER RACEWAY POISONS RATIO  . =            0.2500
ROLLING ELEMENT DIAMETER . . . =            0.3180     INCHES
ROLLING ELEMENT GROOVE RADIUS. =           99.0000     INCHES
ROLLING ELEMENT ELASTIC MODULUS=     30000000.0000 LBF/IN**2
ROLLING ELEMENT POISONS RATIO  =            0.2500
NUMBER OF ROLLING ELEMENTS . . =           18
ANGLE OF CONTACT . . . . . . . =            0.0000     DEGREES

******* THE BEARING HAS LINE CONTACT WITH A
******* CONTACT LENGTH OF 0.3860 INCHES
** DIAMETRAL CLEARANCE (Cd)... = -0.006000 INCHES **

** THE BEARING DEFLECTION = 0.00001500 INCHES **

DO YOU WANT CURVATURE AND CONTACT ANALYSIS ? (Y/N)

Y

------- CURVATURE ANALYSIS INPUT --------

HOW DO YOU WANT TO ENTER THE
INTRODUCTORY INFORMATION ?

 1. USING THE TERMINAL
 2. USING A DATA FILE

2

ENTER DATA FILE NAME
```

```
CFILE

    THE YOUNG MODULUS AND THE POISSON RATIO
    FOR THE GEAR AND THE PINION ARE SET AS :

    GEAR ELASTIC MODULUS,    E1 =   30000000.0000 LBF/IN**2
    PINION ELASTIC MODULUS, E2 =   30000000.0000 LBF/IN**2
    GEAR POISSON RATIO,     PR1  =          0.2500
    PINION POISSON RATIO,  PR2  =          0.2500
    MATERIAL CONSTANT,       B1 =      35000.0000 PSI

    DO YOU WANT TO CHANGE THEM ? (Y/N)

N

        INPUT SUMMARY

    PINION TEETH ...................  =     19.0000
    GEAR TEETH .....................  =     71.0000
    GEAR BLADE PRESSURE ANGLE ......  =     20.0000 DEGREES
    GEAR MEAN SPIRAL ANGLE .........  =     30.0000 DEGREES
    PINION PITCH ANGLE .............  =     15.2672 DEGREES
    GEAR PITCH ANGLE ...............  =     79.7328 DEGREES
    RADIUS OF GEAR CUTTER ..........  =      6.0000 INCHES
    RADIUS OF PINION CUTTER ........  =      6.0000 INCHES
    MEAN CONE DISTANCE .............  =      4.5600 INCHES
    NORMAL TOOTH LOAD ..............  =   3307.5415 LBS
```

APPENDIX C

PROGRAM OUTPUT

```
---------------------------------------------------
------------------ GEAR SECTION -------------------
---------------------------------------------------

***************************************************
*          STRADDLE CONFIGURATION RESULTS         *
***************************************************

         REACTIONS AND DEFLECTIONS

    NOTE : THE FOLLOWING AXIS CONVENTION IS USED

                             !X
                             !
                             !Z______Y

Z-AXIS REACTION AT LEFT BEARING             =  2375.003 LBS
X-AXIS REACTION AT LEFT BEARING             =  3734.077 LBS
RADIAL FORCE AT LEFT BEARING, FR2           =  4425.372 LBS
Z-AXIS REACTION AT RIGHT BEARING            =   316.667 LBS
X-AXIS REACTION AT RIGHT BEARING            = -2003.293 LBS
RADIAL FORCE AT RIGHT BEARING, FR1          =  2028.163 LBS
THRUST REACTION AT LEFT BEARING             =  -836.142 LBS

DEFLECTION DUE TO TANGENTIAL LOAD, WT       =  0.00000090 IN.
DEFLECTION DUE TO RADIAL LOAD, WR           =  0.00000058 IN.
DEFLECTION DUE TO AXIAL LOAD, WA            = -0.00000541 IN.

-------------------------------------------------------------
---------------- BEARING ANALYSIS OUTPUT --------------------
-------------------------------------------------------------

PURE BEARING DEFLECTIONS DUE TO 3 LOADS
YT - TANGENTIAL DEFLECTION                  =  0.00008801 IN.
YA - AXIAL DEFLECTION                       =  0.00122094 IN.
YR - RADIAL DEFLECTION                      =  0.00009208 IN.

BEARING DEFLECTION & SLOPE COMPONENTS

LEFT BEARING - RADIAL          (Yax)        =  0.00014633 IN.
RIGHT BEARING - RADIAL         (Ybx)        = -0.00031625 IN.
SLOPE ON RADIAL PLANE OF THE BEARING        =  0.01559060 DEG.
LEFT BEARING - TANGENTIAL      (Yaz)        =  0.00009307 IN.
RIGHT BEARING - TANGENTIAL     (Ybz)        =  0.00004999 IN.
```

```
SLOPE ON TANGEN'L PLANE OF THE BEARING =  0.00145206 DEG.

TOTAL DEFLECTIONS AND SLOPES

Y1 - TANGENTIAL DEFLECTION                =  0.00000090 IN.
Y2 - AXIAL DEFLECTION                     =  0.00013499 IN.
Y3 - RADIAL DEFLECTION                    = -0.00000483 IN.

THETA1 -   ABOUT Y1                       =  0.00172375 DEG.
THETA2 -   ABOUT Y2                       = -0.02879398 DEG.
THETA3 -   ABOUT Y3                       = -0.00022227 DEG.

TOTAL DEFLECTIONS AND SLOPES   (INCLUDING BEARING STIFFNESS)

Y1 - TANGENTIAL DEFLECTION                =  0.00008890 IN.
Y2 - AXIAL DEFLECTION                     =  0.00135593 IN.
Y3 - RADIAL DEFLECTION                    =  0.00008725 IN.

THETA1 - ABOUT Y1                         =  0.00317581 DEG.
THETA2 - ABOUT Y2                         = -0.02879398 DEG.
THETA3 - ABOUT Y3                         =  0.01536832 DEG.

DEFLECTIONS AT GEAR CENTER

V1                                        =  0.00000090 IN.
V2                                        =  0.00000000 IN.
V3                                        =  0.00013016 IN.

----------------------------------------------------
---------------- PINION SECTION --------------------
----------------------------------------------------

****************************************************
*         STRADDLE CONFIGURATION RESULTS           *
****************************************************

         REACTIONS AND DEFLECTIONS

    NOTE : THE FOLLOWING AXIS CONVENTION IS USED

                      |X
                      |
                      |Z______Y

Z-AXIS REACTION AT LEFT BEARING          =  1103.417 LBS
```

```
X-AXIS REACTION AT LEFT BEARING             =   949.754 LBS
RADIAL FORCE AT LEFT BEARING, FR2           =  1455.871 LBS
Z-AXIS REACTION AT RIGHT BEARING            =  1588.252 LBS
X-AXIS REACTION AT RIGHT BEARING            =  -267.641 LBS
RADIAL FORCE AT RIGHT BEARING, FR1          =  1610.644 LBS
THRUST REACTION AT LEFT BEARING             = -1797.072 LBS

DEFLECTION DUE TO TANGENTIAL LOAD, WT       =  0.00005468 IN.
DEFLECTION DUE TO RADIAL LOAD, WR           =  0.00001386 IN.
DEFLECTION DUE TO AXIAL LOAD, WA            =  0.00001014 IN.


----------------------------------------------------------
--------------- BEARING ANALYSIS OUTPUT -----------------
----------------------------------------------------------

PURE BEARING DEFLECTIONS DUE TO 3 LOADS

YT - TANGENTIAL DEFLECTION                  =  0.00025596 IN.
YA - AXIAL DEFLECTION                       =  0.00019451 IN.
YR - RADIAL DEFLECTION                      =  0.00021135 IN.

BEARING DEFLECTION & SLOPE COMPONENTS

LEFT BEARING - RADIAL          (Yax)        =  0.00051911 IN.
RIGHT BEARING - RADIAL         (Ybx)        = -0.00000249 IN.
SLOPE ON RADIAL PLANE OF THE BEARING        =  0.00928128 DEG.
LEFT BEARING - TANGENTIAL      (Yaz)        =  0.00060310 IN.
RIGHT BEARING - TANGENTIAL     (Ybz)        =  0.00001479 IN.
SLOPE ON TANGEN'L PLANE OF THE BEARING      =  0.01046821 DEG.

TOTAL DEFLECTIONS AND SLOPES

Y1 - TANGENTIAL DEFLECTION                  =  0.00005468 IN.
Y2 - AXIAL DEFLECTION                       =  0.00001995 IN.
Y3 - RADIAL DEFLECTION                      =  0.00002399 IN.

THETA1 - ABOUT Y1                           =  0.00095206 DEG.
THETA2 - ABOUT Y2                           = -0.02415757 DEG.
THETA3 - ABOUT Y3                           =  0.00072446 DEG.

TOTAL DEFLECTIONS AND SLOPES   (INCLUDING BEARING STIFFNESS)

Y1 - TANGENTIAL DEFLECTION                  =  0.00031064 IN.
Y2 - AXIAL DEFLECTION                       =  0.00021446 IN.
```

```
Y3 - RADIAL DEFLECTION                                  =  0.00023534 IN.

THETA1 - ABOUT Y1                                       =  0.01142027 DEG.
THETA2 - ABOUT Y2                                       = -0.02415757 DEG.
THETA3 - ABOUT Y3                                       =  0.01000574 DEG.

DEFLECTIONS AT GEAR CENTER

V1                                                      =  0.00005468 IN.
V2                                                      =  0.00000000 IN.
V3                                                      =  0.00004394 IN.

BEVEL DEFORMATION RESULTS

Z1 - COMMON TANGENTIAL COMPONENT                        = -0.00171940 IN.
Z2 - PINION AXIAL COMPONENT                             = -0.00001301 IN.
Z3 - GEAR AXIAL COMPONENT                               =  0.00015950 IN.

DZ1 - ADD'L MOTION (GEAR WRT PINION)                    = -0.00177497 IN.

A1 - COMMON TANG'L COMPON.  (ROTATION)                  =  0.01459607 DEG.
A2 - PINION AXIAL ROTATION  (ROTATION)                  =  0.00960593 DEG.
A3 - GEAR AXIAL COMPON.     (ROTATION)                  = -0.02009457 DEG.

---------------------------------------------------------------
--------------- CURVATURE ANALYSIS ----------------
---------------------------------------------------------------

GEAR - ANGLE BETWEEN THE TWO PRINCIPAL DIRECTIONS
SIGMA =    -77.3693 DEG.

PINION - ANGLE BETWEEN THE TWO PRINCIPAL DIRECTIONS
SIGMA =     79.6929 DEG.

RKxy : PRINCIPAL CURVATURES x OF SURFACE y

RK12                                                    =  0.0833
RK22                                                    =  0.1639
RK11                                                    =  1.8251
RK21                                                    = -0.1512

3-D UNIT VECTORS ALONG FIRST PRINCIPAL DIRECTIONS

RI11X                                                   = -0.9245
RI11Y                                                   =  0.3809
```

```
RI11Z                                             =  0.0133
RI21X                                             = -0.1681
RI21Y                                             = -0.4389
RI21Z                                             =  0.8826

3-D UNIT VECTORS ALONG SECOND PRINCIPAL DIRECTIONS

RI12X                                             =  0.9170
RI12Y                                             = -0.1797
RI12Z                                             = -0.3562
RI22X                                             = -0.2055
RI22Y                                             =  0.5527
RI22Z                                             = -0.8077

ANGLE BTWN PRNCPL DIRECTIONS                      = 157.0622 DEG.
CCW ANGLE OF I12 FROM MINOR AXIS                  = -67.8776 DEG.
SEMI MAJOR AXIS                                   =   0.4233 IN.
SEMI MINOR AXIS                                   =   0.0156 IN.
RATIO (MAJOR/MINOR)                               =  27.0871
SURFACE DEFORMATION                               =   0.1093E-02 IN.
MAXIMUM CONTACT STRESS                            =   0.7500E+06 LBS/IN**2

BASIC DYNAMIC CAPACITY                            =   0.1490E+05 LBS/IN**2

CONTACT POINT DEFLECTION

DEFLECTION IN TANGENT PLANE, (GEAR)      =   0.00042213 IN.
DEFLECTION IN TANGENT PLANE, (PINION)    =   0.00023180 IN.

DEFLECTION IN NORMAL PLANE, (GEAR)       =   0.00000000 IN.
DEFLECTION IN NORMAL PLANE, (PINION)     =   0.00000000 IN.

TOTAL CONT. POINT DEFLECTION, (GEAR)     =   0.00042213 IN.
TOTAL CONT. POINT DEFLECTION, (PINION)   =   0.00023180 IN.
DEFLECTION ON PITCH RAY, (GEAR)          =   0.00015583 IN.
DEFLECTION ON PITCH RAY, (PINION)        =   0.00014492 IN.
DEFLECTION ON MIDCONE PLANE, (GEAR)      =   0.00134977 IN.
DEFLECTION ON MIDCONE PLANE, (PINION)    =   0.00028351 IN.
DEFL'N RATIO (MIDCONE PLANE), (GEAR)     =   0.00000000
DEFL'N RATIO (MIDCONE PLANE), (PINION)   =   0.00000000

WHAT DO YOU WANT TO DO NOW ?

1.  RUN ANOTHER CONFIGURATION WITH SAME DATA
2.  ENTER NEW DATA
3.  SUMMARY OF RESULTS (DEFLECTIONS & SLOPES)
4.  EXIT PROGRAM

4
**** STOP
```

APPENDIX D

NOMENCLATURE

Variables

A	Distance from gear to right bearing in inches
A_1	relative rotation in degrees (i = 1-3)
A_0	cone distance in inches
A_{ti}	slope vector components in b coordinate frame (i = 1-3)
a	semi-major axis length in inches
a_b	addendum distance in inches
B	distance from gear to left bearing in inches
BC	midcone distance in inches
B_1	material constant in psi
b	semi-minor axis length in inches
C	dynamic capacity in pounds
C_d	diametral clearance in inches
C_e	elasticity constant in inch^{-2}
C_m	old base circle radii sum in inches
C_m'	new base circle radii sum in inches
D	pitch diameter in inches
D_f	motion of contact point in tangent plane in inches
D_0	mean cone distance in inches
D_p	contact point motion in tangent plane in inches

C-3

Variables (continued)

D_r total bearing deflection in inches

D_n contact point motion in normal plane in inches

D_{ti} total deflection in inches (i = 1-3)

D_{tot} total contact point motion in inches

d_b dedendum distances in inches

d_i inner race diameter in inches

d_m average race diameter in inches

d_o outer race diameter in inches

E modulus of elasticity in psi

E_{t2} motion in pitch ray plane in inches

E_{t3} motion in midcone plane in inches

F bearing load in pounds

$F(\kappa)$ curvature difference in inch^{-1}

F_r radial loading on bearing in pounds

f face width in inches

G shear modulus of elasticity in psi

H_{ti} displacement in tangent coordinate frame (i = 1-3)

I area moment of inertia in inch^4

J polar moment of inertia in inch^4

K load-deflection factor

K_D curvature constant in inches

K_I principal curvature of pinion tooth in inch^{-1}

K_{II} principal curvature of pinion tooth in inch^{-1}

Variables (continued)

L_w	contact length for roller bearings in inches
P_d	diametral pitch in in^{-1}
Q	load acting on bearing in pounds
q	elliptical eccentricity in inches
R, R'	midcone radii in inches
R_I	radius of curvature of body I in inches
R_{avg}	average radius in inches
T	input torque to output gear in pound inches
V_i	deflection at the gear center in inches (i = 1-3)
W	loading components in pounds
W_n	normal loads in pounds
Y_{bw}	deflection due to bearing stiffness in inches
Y_i	deflection due to gear motion in inches (i = 1=3)
y	elastic deformation in inches
Z	number of rolling elements
Z_i	relative translation in inches (i = 1=3)
Γ	pitch angle in degrees
δ_r	internal bearing deflection in inches
θ_{bw}	slope due to bearing stiffness in degrees
θ_i	slope due to gear motion in degrees (i = 1-3)
θ_r	rotation vector component in the normal plane
θ_{rm}	rotation vector component normal to midcone plane
θ_{ti}	total slope in degrees (i = 1-3)
k	curvature in bearing contact in $inch^{-1}$

Variables (continued)

λ_i unit vector in the β_E coordinate frame (i = 1-3)

ν poisson ratio

Σ shaft angle in degrees

$\sum\kappa$ curvature sum in inch^{-1}

σ_C maximum compressive stress in psi

ϕ_n pressure angle in degrees

ψ spiral angle in degrees

Subscripts

a axial direction

b_1 left bearing

b_2 right bearing

g gear

p pinion

r radial direction

t tangential direction

1 motion in tangential direction

2 motion in axial direction

3 motion in radial direction

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16. Abstract

An analytical method to predict the shift of the contact ellipse between the meshing teeth in a spiral bevel gear set is presented in this report. The contact ellipse shift of interest is the motion of the nominal tooth contact location on each tooth from the ideal pitch point to the point of contact between the two teeth considering the elastic motions of the gears and their supporting shafts. This is the shift of the pitch point from the ideal, unloaded position on each tooth to the nominal contact location on the tooth when the gears are fully loaded. It is assumed that the major contributors of this motion are the elastic deflections of the gear shafts, the slopes of the shafts under load and the radial deflections of the four gear shaft bearings. The motions of the two pitch point locations on the pinion and the gear tooth surfaces are calculated in a Fortran program which also calculates the size and orientation of the Hertzian contact ellipse on the tooth faces. Based on the curvatures of the two spiral bevel gear teeth and the size of the contact ellipse, the program also predicts the basic dynamic capacity of the tooth pair. A complete numerical example is given to illustrate the use of the program.

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