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ANNUAL TECHNICAL (PROGRESS) REPORT

"COLLISIONAL DYNAMICS OF PERTURBED PARTICLE DISKS
IN THE SOLAR SYSTEM"

NASA AWARD NO. NAGW-929
7/1/86 - 6/30/87

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I. OBJECTIVES

Investigations of the collisional evolution of particulate disks subject to the gravitational perturbation of a more massive particle orbiting within the disk are underway. Both numerical N-body simulations using a novel collision algorithm and analytical kinetic theory are being employed to extend our understanding of perturbed disks in planetary rings and during the formation of the solar system. Particular problems proposed for investigation are: (1) The development and testing of general criteria for a small moonlet to clear a gap and produce observable morphological features in planetary rings; (2) The development of detailed models of collisional damping of the wavy edges observed on the Encke division of Saturn's A ring; and (3) The determination of the extent of runaway growth of the few largest planetesimals during the early stages of planetary accretion.

II. PROGRESS TO DATE

A. NUMERICAL SIMULATION STUDIES

A computational code is under development to simulate the collisional evolution of a particulate disk subject to the gravitational perturbation of a massive body orbiting within the disk. To obtain the highest possible resolution, the simulation is confined to a narrow ring of particles which is still significantly wider than the radial excursions of the particle orbits. The orbits of the perturbing mass and the disk particles are closely spaced so that the synodic period between successive encounters with the perturber is many times greater than the orbit period. As a result, encounters between disk particles and the perturber are relatively short, lasting about one orbital period. For some of our investigations, we will adopt Showalter and Burns' (1982) approach for estimating "instantaneous" changes in the particle orbits due to encounters with the perturber. An important new feature in our work, relative to that of previous investigators on this problem, is the

careful inclusion of particle-particle collisions. We plan to study their important effects. The computational code is being developed so that sizable portions of particle orbits are advanced in sufficiently short time steps so as to allow the simulation of particle collisions.

The N-body computer program under development consists in part of a modified version of a computer code developed by Hausman and Roberts (1984a) and Roberts and Hausman (1985, 1986) for the study of supersonic rarefied gas flows in gas centrifuges. In order to compute particle collisions efficiently, the code adopts Monte Carlo techniques (Bird 1969, 1976). Trajectories are followed exactly, but not the separation of pairs. Instead, at the end of each time step, pairs of particles within each (small) volume element (or "cell") are randomly chosen to collide, without regard to their separation. The rationale for this is the near uniformity of the density and velocity field within a sufficiently small volume element, so that each particle represents N real particles spread throughout the cell. Variable computational cell volumes are being incorporated into the computational code in order to achieve higher resolution in regions where density or velocity gradients are large. A numerical scheme for choosing the optimal cellular mesh is being developed. The increased efficiency resulting from this effort should permit us to simulate 10,000 to 30,000 particles, which is an order of magnitude more particles than simulated by previous investigators working on this problem, e.g., Schwarz (1981) and Showalter and Burns (1982).

In addition to the Monte Carlo code described above, we are also adapting portions of an N-body program developed by Roberts and Hausman (1984) and Hausman and Roberts (1984b) and Roberts and Stewart (1987) for the simulation of gas clouds in the interstellar medium of spiral galaxies. This second code has a collision algorithm which is slightly more accurate (and time consuming) than the Monte Carlo code since particle pairs are required to actually touch in order for a collision to occur. The graphics plotting capabilities of this code are being refined in order to be particularly useful for displaying the results of our simulations of perturbed particle disks. These capabilities are expected to provide high resolution and fine detail that can be displayed in our results. In particular, well defined shock structures should be able to be resolved in these particle simulations. Similarly sharp density contrasts are ubiquitous of perturbed regions of Saturn's rings (e.g. Cuzzi et al. 1984). To fully exploit the advantages of these two codes we are currently carrying out work to merge the capabilities of the galaxy simulation code with the efficient collision algorithm of the Monte Carlo code.

For planetesimal disks consisting of kilometer-size particles, gravitational encounters among the particles are usually as important as physical collisions (Wetherill 1980b). However, previous numerical simulations of planetesimal disks by Wetherill (1980a) indicate that the small disk particles will generally maintain relative velocities comparable to their surface escape velocities so that the two-body approximation for mutual gravitational scattering and collision cross sections should be sufficiently accurate (Wetherill and Cox 1984). Given our Monte Carlo treatment of particle collisions, we believe that sufficient accuracy will be achieved by modeling gravitational scattering as perfectly elastic collisions (This is consistent with recent analytical treatments, e.g. Shu and Stewart 1985). This, of course, does not apply to the gravitational perturbations due to the massive perturber which must be treated by numerical integration of the particle trajectories.

In order to address the problem of runaway growth of planetary embryos, we shall also simulate cases where the disk particles are allowed to be accreted by the massive perturber. Due to the nature of low-eccentricity three-body orbits, an accurate integration of trajectories which collide with the massive body would require a variable integration step size (e.g. Weidenschilling and Davis 1983) and possibly excessive computer time. In order to avoid this difficulty, we shall explore the possibility of using a statistically accurate collision cross section for slow three-body orbits, exploiting as much as possible the recent results of Wetherill and Cox (1984, 1985).

B. ANALYTICAL STUDIES

Re-examination of Radial Diffusion in Planetary Rings

According to the kinetic theory of transport processes in planetary rings reviewed by Goldreich and Tremaine (1982) and by Stewart et al. (1984) the rate of radial mass transport is governed by an effective shear viscosity that is deduced from the off-diagonal component of the velocity dispersion tensor. For rings of low optical depth the dispersion tensor may be calculated from either the Boltzmann or Krook kinetic equations, suitably modified to model the inelasticity of collisions among ring particles (Goldreich and Tremaine 1978, Shu and Stewart 1985). The theory predicts that planetary rings of low optical depth spread radially so as to decrease the average surface density of ring particles. The accuracy of this prediction has been called into question by recent numerical solutions of the Boltzmann equation that seem to indicate a tendency toward radial clumping rather than spreading at low optical depths (Brophy and Esposito 1986). The possibility of a diffusional instability at low optical depth is exciting because it would provide a much needed mechanism for forming the unexplained structure of Saturn's C ring and the surprisingly sharp density gradients at the inner edges of the A and B rings.

We have begun a re-examination of the kinetic theory of planetary rings in order to identify a possible flaw in the published theory and to formulate a new theoretical derivation of the radial mass transport rate which avoids such flaws. It has been pointed out by Greenberg (1986) that when ring particles with different semi-major axes collide inelastically, their center-of-mass velocity is often greater than the circular orbit velocity at the smaller semi-major axis or less than the circular orbit velocity at the larger semi-major axis. The net result of such collisions is often more closely spaced semi-major axes rather than radial spreading. This peculiar reversal of the velocity gradient is a consequence of the non-local character of eccentric orbits that are associated with the radii of their semi-major axes rather than the actual radial coordinate of the particle. Such non-local effects may not be adequately modeled by published kinetic theory because it is formulated in terms of the local density of particles in phase space. In particular, the assumption of a tri-axial Gaussian velocity distribution may preclude the local shear reversal described by Greenberg because even functions cannot model a velocity distribution that is systematically positive when the particle is at radii smaller than the semi-major axis and systematically negative when the particle is at radii larger than the semi-

major axis.

To avoid this difficulty, we have begun a direct calculation of the radial flux of semi-major axes due to particle collisions that does not require the determination of an effective shear viscosity from the off-diagonal component of the dispersion tensor. The calculation is motivated by analogous calculations of the diffusion of plasma guiding centers across magnetic fields (see for example Ichimaru 1973). The basic set-up of the calculation is similar to the one described by Hameen-Anttila (1978) where one must find the first and second moments of the collisional rate of change of the semi-major axes. By relating these quantities to the collisional rate of change of the velocity dispersion tensor, Hameen-Anttila obtained an effective shear viscosity identical to that obtained by Shu and Stewart (1985). At the present time we are seeking to remove a dubious approximation in Hameen-Anttila's calculation that is functionally equivalent to the assumption of a Gaussian velocity distribution that is made in the usual kinetic theory calculation. Preliminary results of our investigation suggest that the actual radial mass transport rate may differ substantially from the rate predicted from previously derived effective viscosities.

Variational Principle for Nonlinear Spiral Density Waves

Shu, Yuan, and Lissauer (1985a) and Borderies, Goldreich, and Tremaine (1985) derived a dispersion relation and a wave amplitude relation for nonlinear spiral density waves. We have found a Lagrangian functional that contains the same information as their averaged equations of motion (Stewart 1986). Both the nonlinear dispersion relation and the wave action transport equation can be easily obtained from the Lagrangian by the variational principles described in Whitham's (1974) book. Our formulation greatly simplifies the derivation of these relations and goes a long way toward clarifying the relationship of nonlinear spiral density waves to other nonlinear wave phenomena described in the general literature. We have added an isothermal pressure term to the Lagrangian to facilitate comparisons with earlier theories of linear spiral density waves in spiral galaxies.

In addition to the pedagogical value of this work, our Lagrangian formulation provides the means of systematic improvements of the published theory. An equivalent Hamiltonian field theory can be derived from the Lagrangian. In this formulation it becomes clear that Shu et al.'s (1985a) averaging procedure is equivalent to the standard mean field approximation that is widely used in solid state physics problems. Systematic improvements to the mean field theory that are commonly used in solid state physics may be adapted to the spiral density wave problem using our formalism. Since waves in planetary rings are dissipated by inelastic particle collisions, it is desirable to add dissipation to the Lagrangian formulation. We are currently investigating the possibility of adding dissipation by combining the Lagrangian formalism with a Fokker-Planck model for diffusion of the radial displacements and velocities about the mean motion in the wave. A Fokker-Planck approximation is feasible for this problem because the relative velocity between colliding particles is much less than the systematic velocity in nonlinear density waves. Since Fokker-Planck models are commonly employed to model wave dissipation in condensed matter physics, we are attempting to borrow the refined techniques that have been developed in that field. Our goal is to formulate a useful

alternative to the techniques used by Shu et al. (1985b) and by Borderies et al. (1986) to model collisional damping of nonlinear density waves in planetary rings.

III. ANTICIPATED CAPABILITIES AND GOALS FOR 7/1/87 - 12/31/87

Due to the unique capabilities of our proposed numerical simulation procedure currently under development, we expect to be able to perform meaningful tests of recent analytical formulations of particle disk dynamics. Preliminary results for an unperturbed disk have already been obtained: Computed values for the pressure tensor components are in good agreement with analytical formulae derived in Shu and Stewart (1985).

By simulating a wide range of particle number densities and perturber mass ratios, we hope to determine general criteria for moonlets to clear gaps or to produce observable disturbances in planetary rings. Previously derived criteria were based on very primitive models of the collisional damping process and limited by the assumption of complete damping of forced eccentricities between successive encounters with the perturber (eg. Greenberg 1983).

Given the detail of the observations of the wavy edges in Encke's division of Saturn's rings (Cuzzi and Scargle 1985), we hope to make special efforts to produce detailed models of moonlet-perturbed edges of planetary rings. The most recent modeling effort (Showalter et al. 1986) completely neglects the effects of particle-particle collisions that we are attempting to simulate.

An improved theory for radial mass transport in planetary rings is to be derived which avoids the assumptions of previous derivations. In particular, there is no necessity to identify the off-diagonal component of the velocity dispersion tensor with the effective viscosity.

The propagation of nonlinear spiral density waves in rings is to be described by a Lagrangian field theory. Higher order corrections to the approximate theory of Shu et al (1985a, b) and Borderies et al (1985, 1986) can be derived with this formulation. An alternative method of modeling collisional damping is to be formulated which is better able to handle large azimuthal wave numbers characteristic of moonlet perturbed systems.

We hope to place strong constraints on the degree of runaway growth during planetary accretion. This work may lead to the first simulation of the earliest stages of planetary accretion which avoids the isotropic "particle-in-a-box" approximation used by Greenberg et al. (1978) and others that have been criticized by Wetherill and Cox (1984, 1985).

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Evolution of Planetesimal Velocities¹

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Abstract

A self-consistent set of equations for the velocity evolution of a general planetesimal population is presented. The equations are given in a form convenient for calculations of the early stages of planetary accumulation, when it is necessary to model the planetesimal swarm by the methods of gas dynamics, rather than follow the orbital evolution of individual bodies. To illustrate the relative importance of the various terms of these equations, steady state velocities of a simple planetesimal population, consisting of two different sizes of bodies, are calculated. Dynamical friction is found to be an important mechanism for transferring kinetic energy from the larger planetesimals to the smaller ones, providing an energy source for the small planetesimals that is comparable to that provided by the viscous stirring process. When small planetesimals are relatively abundant, gas drag and inelastic collisions among the smaller bodies are of comparable importance for dissipating energy from the population.

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