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# ELECTRON-CYCLOTRON MASER AND SOLAR MICROWAVE MILLISECOND SPIKE EMISSION

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## I. Introduction

An intense solar microwave millisecond spike emission (SMMSE) event has been observed on May 16, 1981 by Zhao and Jin at Beijing Observatory ($\omega = 2\pi \times 2.84$ GHz)[1]. The peak flux density of the spikes is high to $5 \times 10^5$ s.f.u., and the corresponding brightness temperature (BT) reaches $\sim 10^{15}$ K.

In order to explain the observed properties of SMMSE, we propose in this paper that a beam of electrons with energy of tens kev injected from the acceleration region downwards into an emerging magnetic arch forms so-called hollow beam distribution and causes electron-cyclotron maser (ECM) instability. The growth rate of second harmonic X-mode is calculated and its change with time is deduced. It is shown that the saturation time of ECM is $t_s \approx 0.42$ ms and only at last short stage ($\Delta t < 0.2\ t_s$) the growth rate decreases to zero rather rapidly. So a SMMSE with very high BT ($T_b > 10^{15}$ K) will be produced if the ratio of number density of nonthermal electrons to that of background electrons, $n_s/n_e$, is larger than $4 \times 10^{-5}$.

## II. Model and Excitation of X-mode

If the induced electric field is larger than the Dreicer field, it will be able to accelerate electrons along the field to energy of tens kev in one millisecond. The injected electrons with a large pitch angle gyrate around the magnetic field B and form so- called hollow beam distribution (Fig. 1) as following equation[2]:

$$F(v_\perp, v_{\parallel}) = (\pi^{3/2}\, d_\perp^2\, d_{\parallel}\, v_{0\perp}^2\, v_{0\parallel})^{-1}$$

$$\exp\left\{-\left(\frac{v_\perp - v_{0\perp}}{d_\perp\, v_{0\perp}}\right)^2 - \left(\frac{v_{\parallel} - V_0}{d_{\parallel}\, v_{0\parallel}}\right)^2\right\}$$

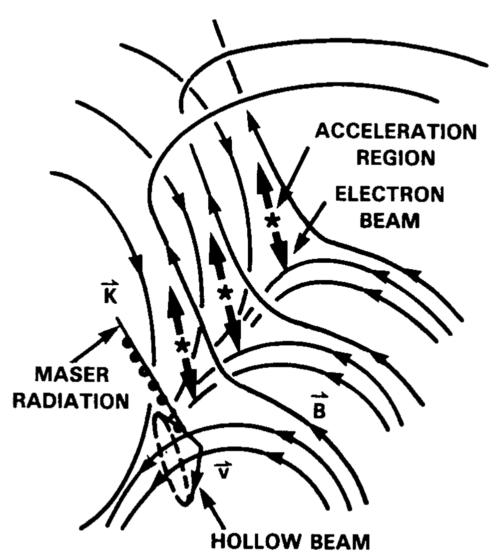


Figure 1. Model of hollow beam driven ECM for SMMSE.

It drives ECM and we have the growth rate at the m-th harmonic[2]

$$\omega_i^{(m)} = \frac{\pi^2 c^3}{2^{2m-1}[(m-1)!]^2} \cdot \frac{n_s}{n_e} \cdot \frac{\omega_p^2}{\omega^2} \left\{ \left[2-n^2 - \frac{2\omega_p^2}{\omega(\omega+\Omega_e)}\right]\left(1 - \frac{\omega_p^2}{\omega^2} - n^2\sin^2\alpha\right) - n^2\cos^2\alpha \left(1 - \frac{\omega_p^2}{\omega^2}\right)\right\} \frac{\Omega_e}{G\ n\ \sin\alpha} \cdot \int_{v_-}^{v_+} dv_{11} \cdot b_m^{2m-1}$$
$$\times \left(\frac{m\Omega_e}{\omega} \cdot \frac{\partial}{\partial \cdot v_m} + \frac{v_m}{c} n \cos\alpha \frac{\partial}{\partial\ v_{11}}\right) F\,(v_m,\ v_{11}) \qquad (2)$$

where

$$G = 2n^4\left[\frac{\omega^2\omega_p^2 \sin^2\alpha}{(\omega^2-\Omega_e^2)^2} + \frac{\omega_p^2}{\omega^2}\cos^2\alpha\right] + 4n^2\left[\frac{\omega_p^2(\omega_p^2-\Omega_e^2)}{(\omega^2-\Omega_e^2)^2} - 1 - \frac{\omega_p^2\Omega_e^2 \sin^2\alpha}{2\ (\omega^2-\Omega_e^2)^2}\right] + 2\left[\left(2 - \frac{\omega_p^2}{\omega^2}\right)\left(1 - \frac{\omega_p^2}{\omega^2-\Omega_e^2}\right)^2 - \frac{\omega_p^4\Omega_e^2}{\omega^2(\omega^2-\Omega_e^2)^2} + 2\left(1 - \frac{\omega_p^2}{\omega^2}\right)^2 \cdot \frac{\omega^2\Omega_e^2}{(\omega^2-\Omega_e^2)^2}\right]$$

$$\frac{v_m}{c} = \left\{1 - \left(\frac{\omega}{m\Omega_e}\right)^2 + 2\left(\frac{\omega}{m\Omega_e}\right)^2 \frac{v_{11}}{c} n \cos\alpha - \left[1 + \left(\frac{\omega}{m\Omega_e}\right)^2 n^2\cos^2\alpha\right) \frac{v_{11}^2}{c^2}\right\}^{1/2}$$

$$\frac{v_\pm}{c} = \frac{\left(\frac{\omega}{m\Omega_e}\right)^2 n \cos\alpha \pm \sqrt{1 - \left(\frac{\omega}{m\Omega_e}\right)^2 (1 - n^2\cos^2\alpha)}}{1 + \left(\frac{\omega}{m\Omega_e} n \cos\alpha\right)^2}$$

c is light velocity, n is refraction index satisfying the Appleton-Hartree dispersion relation, $\alpha$ is the angle between wave vector K and B, $\omega$ is wave frequency,

$$\Omega_e = \frac{eB}{m_e c},\ \omega_p^2 = \frac{4\pi n_e e^2}{m_e},\ K_\perp = K\sin\alpha,\ K_{11} = K\cos\alpha,\ b_m = \frac{K_\perp v_m}{\Omega_e}.$$

Assuming that the energy of nonthermal electrons E = 60 keV, pitch angle $\phi = 60°$, i.e., $v_{0\perp}/c = 0.386$, $v_{011}/c = 0.223$, and $d_\perp = 0.2$, $d_{11} = 0.4$, $\omega_p/\Omega_e = 0.4$, it is found that the second harmonic (m = 2) X-mode will grow fastest if $\omega = 2.02\ \Omega_e$, $\alpha = 65°$, and the growth rate is

$$\omega_i^{(2)} = 0.096 \frac{n_s}{n_e} \Omega_e$$

III. Saturation of ECM

The excitation of ECM leads to diffusion of energetic electrons in velocity space, which in turn weakens the instability. According to quasilinear theory we have[3]

$$\frac{\partial}{\partial t} F(v_\perp, v_{\parallel}) = D(t, v_\perp, v_{\parallel}) \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} v_\perp \frac{\partial}{\partial v_\perp} F(v_\perp, v_{\parallel}) \tag{3}$$

$$D(t, v_\perp, v_{\parallel}) = \frac{4\pi^2 e^2}{m_e^2} \int d\vec{K}\, \mathcal{E}_{\vec{K}}(t)\, \delta\left(\omega - \frac{m\Omega_e}{\gamma} - K_{\parallel} v_{\parallel}\right) \tag{4}$$

$$\frac{\partial}{\partial t} \mathcal{E}_{\vec{K}}(t) = 2\omega_i (\vec{K}, t) \mathcal{E}_{\vec{K}}(t) \tag{5}$$

where $\gamma = (1 - (v_\perp^2 + v_{\parallel}^2)/c^2)^{-1/2}$, $D(t, v_\perp, v_{\parallel})$ is the diffusion coefficient and $\mathcal{E}_{\vec{K}}$ the wave spectral density. Let

$$\tau = \int_0^t dt'\, D(t', v_\perp, v_{\parallel}) \tag{6}$$

we can rewrite Eq. (3) as

$$\frac{\partial F}{\partial \tau} = \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} v_\perp \frac{\partial F}{\partial v_\perp}$$

then obtain the distribution for $t > 0$ (i.e. $t > 0$).

$$F(\tau, v_\perp, v_{\parallel}) = \frac{1}{2\tau} \int_0^\infty dy\, F(0, y, v_{\parallel})\, e^{-\frac{v_\perp^2 + y^2}{4\tau}}\, I_0\left(\frac{v_\perp y}{2\tau}\right) y$$

where $F(0, v_\perp, v_{\parallel})$ is the initial distribution of Eq. (1), $I_0(x)$ is modified Bessel function of zero order with argument x.

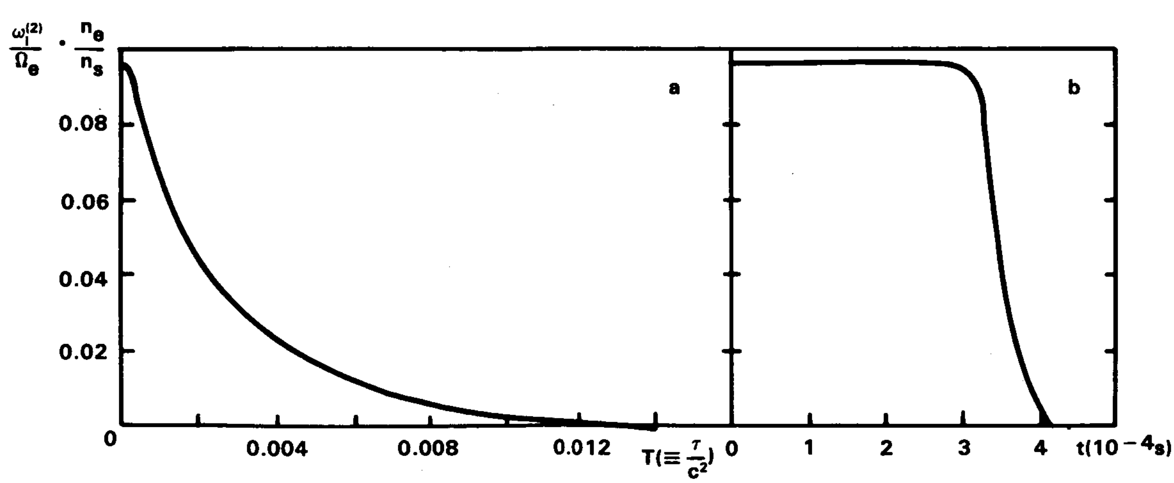


Figure 2. Plots of $\omega_i^{(2)}/\Omega_e \cdot n_e/n_s$: (a) vs $\tau$, (b) vs time t, all for X-mode with parameters: E = 60 keV, $\phi = 60°$, $\alpha = 65°$, $\omega_p = 0.4\Omega_e$, $d_\perp = 0.2$, $d_{\parallel} = 0.4$ and $\omega = 2.02\Omega_e$.

Substituting Eq. (7) into Eq. (2) and using the following relations

$$\left(\frac{\partial F}{\partial v_{\parallel}}\right)_t = \left(\frac{\partial F}{\partial v_{\parallel}}\right)_\tau + \left(\frac{\partial F}{\partial \tau}\right)_{v_{\parallel}} \left(\frac{\partial \tau}{\partial v_{\parallel}}\right)_t$$

$$\approx \left(\frac{\partial F}{\partial v_{\parallel}}\right)_\tau - \frac{\tau}{v_{\parallel}} \left(\frac{\partial F}{\partial \tau}\right)_{v_{\parallel}}$$

$$\left(\frac{\partial F}{\partial v_{\perp}}\right)_t \approx \left(\frac{\partial F}{\partial v_{\perp}}\right)_\tau$$

and Eqs. (4)-(6), we obtain the change of $\omega_i^{(2)}$ with $\tau$ (Fig. 2a) and then with time t (Fig. 2b), which gives the saturation time of ECM $t_S$ ($\tau = \tau_S$) corresponding to $\omega_i^{(2)} = 0$. We have $t_S \approx 0.42$ ms and find out that $\omega_i^{(2)}$ almost doesn't change until $t \approx 0.8\, t_S$ and only at last short stage ($\Delta t \lesssim 0.2\, t_S$) it decreases to zero rapidly. In addition we have obtained the following relations

$$t_s \approx \frac{1}{2\,\omega_{io}}\, \ell n\left(2\,\frac{W(t_s)}{W_o}\right) \tag{8}$$

$$W(t_s) \approx \frac{m_e^2}{4\pi^2 e^2}\, \tau_s\, \omega_{io}\, K_{\parallel}^{*}\, v_{0\parallel} \tag{9}$$

where $W(t_S)$ and $W_O$ are the saturation and initial energy density of the wave, respectively, $\omega_{iO}$ is the initial growth rate and $K_{\parallel}^{*} = \eta\omega/c \cos\alpha$.

The above calculation is rather complicated. For simplicity we give an evaluation for saturation value $\tau_S$.

Taking only the part of distribution (1) containing $v_\perp$ and introducing $\chi \equiv v_\perp/c$, $v \equiv v_{0\perp}/c$, $A = d_\perp v$, we have

$$F_o(\chi) = (\pi c^2 A^2)^{-1} \exp\left\{-\left(\frac{\chi - v}{A}\right)^2\right\}$$

$$\frac{\partial F_o}{\partial \chi} = -\frac{2(\chi - v)}{\pi c^2 A^4} \exp\left\{-\left(\frac{\chi - v}{A}\right)^2\right\}$$

If $\chi = \chi_O \equiv (1 - d_\perp/\sqrt{2})\, v$, then $\partial F_O/\partial\chi > 0$ (so $\omega_i > 0$) and has its maximum. Introducing $T = \tau/c^2$, $\chi = \chi_O$, $z = y/c$, we have for $\tau > 0$ ($t > 0$)

$$F(T,\chi) = \frac{1}{2\pi c^2 A^2 T}\int_o^\infty dz\, I_o\left(\frac{\chi z}{2T}\right) z \exp\left\{-\frac{\chi^2 + z^2}{4T} - \left(\frac{z - v}{A}\right)^2\right\}$$

$$\frac{\partial F}{\partial \chi} = \frac{1}{4\pi c^2 A^2 T^2}\int_o^\infty dz\left[z\, I_1\left(\frac{\chi z}{2T}\right) - \chi\, I_o\left(\frac{\chi z}{2T}\right)\right.\, z \exp\left\{-\frac{\chi^2 + z^2}{4T} - \left(\frac{z - v}{A}\right)^2\right\} \tag{10}$$

we can divide the integration in Eq. (10) into two parts:

$$Q_1 = \int_0^{z_0} dz\, z \left[ \frac{z^2\chi}{16T^2}(2T-\chi^2) + \frac{z^4\chi^3}{3\cdot 2^{10}T^4}(4T-3\chi^2) + \frac{z^6\chi^5}{9\cdot 2^{14}T^6}(T-\chi^2) - \chi \right]$$

$$\exp\left\{ -\frac{\chi^2+z^2}{4T} - \left(\frac{z-v}{A}\right)^2 \right\}$$

$$Q_2 \approx \sqrt{\frac{T}{\pi\chi}} \int_{z_0}^{\infty} dz\sqrt{z}\,(z-\chi)\exp\left\{ -\frac{\chi^2+z^2}{4T} - \left(\frac{z-v}{A}\right)^2 \right\}$$

where $z_0 \approx 10T/\chi$ and in $Q_1$ $(0 \le z \le z_0)$ the Bessel functions $I_0$, $I_1$ have been expressed approximately for small argument but in $Q_2$ $(z_0 \le z < \infty)$ for large argument. It is easy to see that in general we have $Q_1 < 0$ and $Q_2 > 0$.

If $z_0 << \chi$, i.e. $T << 0.1\,\chi^2$, $Q_2$ is dominant and $\partial F/\partial\chi > 0$. This is the case at the start of the growing wave.

If $z_0 >> \chi$, i.e. $T >> 0.1\,\chi^2$, $Q_1$ becomes dominant, so $\partial F/\partial\chi < 0$. This is the case after saturation.

If $z_0 \sim \chi$, i.e., $T \sim 0.1\,\chi^2$, we have $Q_2 > |Q_1|$ and $\partial F/\partial\chi > 0$, but $|Q_1|$ gets larger. This corresponds to the case that the growth rate decreases obviously.

Hence we can take $T_S = 0.1\,v^2$ or $\tau_S = 0.1\,v_{0\perp}^2$ as the saturation value when $Q_1$ can be comparable with $Q_2$. For the chosen parameter, $v_{0\perp}/c = 0.386$, we have $T_S \approx 0.015$ which is in agreement with numerical calculation ($T_S \approx 0.013$).

## IV. Gyroresonance Absorption and BT of Radiation

The gyroresonance absorption (GA) is a serious problem for maser radiation. The optical depth of the mth layer ($m \ge 2$) for $\chi$-mode is[4]

$$\tau_m = \left(\frac{\omega_p}{\omega}\right)^2 \frac{m^{2m}\sqrt{2}}{2^{m-1}m!} \cdot \frac{\omega L_B}{c} \left(\frac{T_e}{m_e c^2}\right)^{m-1}$$

with $L_B = B\left|\frac{dB}{dL}\right|^{-1}$. For $\omega = 2\pi \times 2.84$ GHz and $m = 3$ we have

$$\tau_3 \approx T^2 \left(\frac{\omega_p}{\omega}\right)^2 \left(\frac{T_e}{10^6}\right)^2 \frac{L_B}{10^8}$$

Because $L_B > 1000$ km in general, an intense maser radiation ($T_b > 10^{14}$K) can hardly be received unless $T_e < 2\times10^6$K and $\omega_p/\omega \lesssim 0.1$

It seems possible, however, that some magnetic arches with smaller dimension and higher magnitude may emerge from an active region where the magnetic configuration is more complicated, then it may be true that $L_B < 1000$ km (e.g. $L_B = 500$ km) at larger angle to $\vec{B}$ and the third GA will be weaker, so the radiation may escape from the corona.

The BT ($T_b$) and flux density (S) received are

$$T_b = \left(\frac{2\pi c}{\omega}\right)^3 \left(\frac{\Delta\omega}{\omega}\right)^{-1} \frac{W}{\Delta\Omega_r} e^{-\tau_3} \tag{12}$$

$$S = 2\,T_b \left(\frac{\omega}{2\pi c}\right)^2 \frac{A}{R^2} \tag{13}$$

where W is the energy density of the wave, $R = 1.5 \times 10^{13}$ cm is the distance between the Sun and the Earth, A is the surface area of the radiation source, which is about $10^{15}$ cm$^2$ for SMMSE, $\Delta\omega$ is the bandwidth of the radiation ($\Delta\omega/\omega \approx 0.01$ according to Droge[5] and our calculation), $\Delta\Omega_r$ is the radiation solid angle. It is easy to see from Fig. 3 that the propagation angle $\alpha$ will be from 60° to 70° if the pitch angle $\phi$ = 60°, hence $\Delta\Omega_r = 1$.

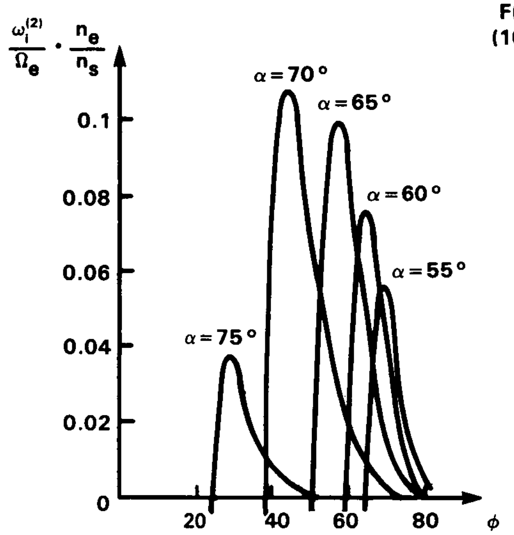


Figure 3. A plot of $\omega_i^{(2)}/\Omega_e \cdot n_e/n_s$ vs $\phi$ for different $\alpha$: 55°, 60°, 65°, 70° and 75°. Other parameters are the same as in Fig. 2.

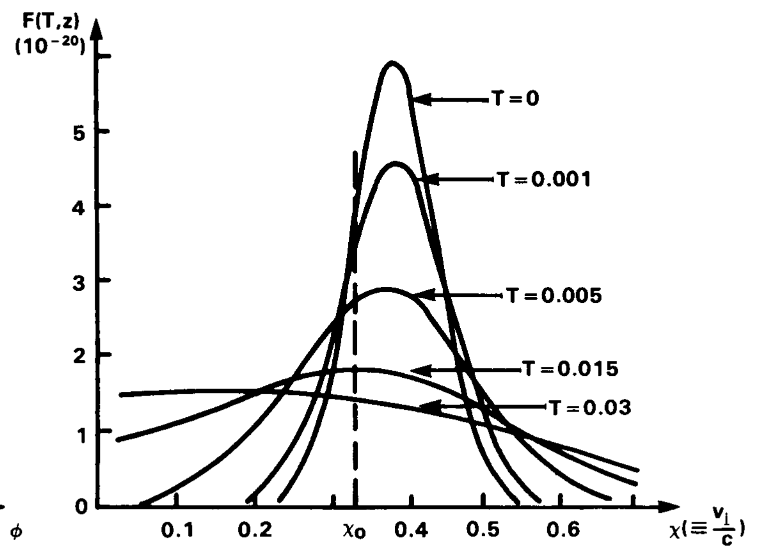


Figure 4. A plot of $v_\perp$ part of distribution of nonthermal electrons for different T with $\chi \equiv v_\perp/c$, $T \equiv \tau/c^2$, $v \equiv v_{0\perp}/c = 0.386$, and $\chi_0 = 0.86v$.

Assuming $n = 0.97$, $\alpha = 65°$ (for the fastest growing second harmonic of X-model) and $n_s/n_e = 4 \times 10^{-5}$, $\tau_s/c^2 = 0.013$, $W(t_s)/W_0 = 2 \times 10^{10}$, $e^{-\tau_3} = 0.1$, we have obtained from Eqs. (8)-(13)

$$t_s \approx 0.36 \text{ ms},\quad W(t_s) \approx 5.9 \times 10^{-5} \text{ ergs cm}^{-3}$$
$$T_b \approx 5 \times 10^{15} \text{ K},\quad S \approx 5.5 \times 10^5 \text{ s.f.u.}$$

These results are in agreement with observations.

## V. Summary and Concluding Remarks

It has been shown that ECM is an effective mechanism to produce electromagnetic radiation. Its BT may be higher than $10^{15}$ K and it can escape from the solar corona without any transformation. The simple procedure, high efficiency, short time scale, strong directivity of radiation, and high degree of circular polarization are its advantages over the plasma radiation mechanism to explain SMMSE. It is calculated in this paper that SMMSE with very high BT ($T_b \approx 5 \times 10^{15}$ K) would be produced by hollow beam driven ECM if E = 60 keV, $\phi = 60°$, $\alpha = 65°$, $n_s/n_e = 4 \times 10^{-5}$, $\omega_p/\Omega_e = 0.4$, $L_B = 500$ km, and $T_e = 2 \times 10^6$K.

It is shown that the range of pitch angle $\phi$ for exciting ECM is very wide (> 30°, see Fig. 3) and the energy of nonthermal electrons may be from ~10 keV to 100 keV or higher. Besides the hollow beam distribution, other velocity anisotropy as a loss-cone distribution can also excite ECM under appropriate conditions. Hence it seems that ECM is often produced above solar active regions. The main reasons why there appear rarely intense SMMSE events may be due to the powerful directivity of radiation and strong third GA.

It is easy to see from Eq. (7) and Fig. 4 that with the evolution of the energetic electron's distribution, the transverse velocity component of them is decreased but the parallel one isn't. As a result, the pitch angle is getting smaller, so the electrons, after losing most of their transverse energy, get through the magnetic mirror points and rush into the transition region to produce hard X-ray bursts, so with which the SMMSE correlate very well[6].

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