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IMPULSIVELY GENERATED FAST CORONAL PULSATIONS

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Abstract

Rapid oscillations in the corona will be discussed from a theoretical standpoint, developing some previous work on ducted, fast magnetoacoustic waves in an inhomogeneous medium. In the theory, impulsively (e.g. flare) generated mhd waves are ducted by regions of low Alfvén speed (high density) such as coronal loops. Wave propagation in such ducts is strongly dispersive and closely akin to the behaviour of Love waves in seismology, Pekeris waves in oceanography and guided waves in fibre optics. Such flare-generated magnetoacoustic waves possess distinctive temporal signatures consisting of periodic, quasi-periodic and decay phases. The quasi-periodic phase possesses the strongest amplitudes and the shortest time scales. Time scales are typically of the order of a second for inhomogeneities (coronal loop width) of 1000 km and Alfvén speeds of 1000 kms^{-1}, and pulse duration times are of tens of seconds.

Quasi-periodic signatures have been observed in radio wavelengths for over a decade and more recently by SMM. It is hoped that the theoretical ideas outlined above may be successfully related to these observations and thus aid the interpretation of oscillatory signatures recorded by SMM. Such signatures may also provide a diagnostic of coronal conditions.

New aspects of the ducted mhd waves, for example their behaviour in smoothly varying as opposed to tube-like inhomogeneities, are currently under investigation. The theory is not restricted to loops but applies equally to open field regions.

1. Introduction

Since the end of the 1960's many observations of coronal oscillations have been reported, the bulk of the evidence being provided by radio wave and hard X-ray data (Abrami, 1970, 1972; Rosenberg, 1970; McLean et al, 1971; Gotwols, 1972; Kai and Takayanagi, 1973; McLean and Sheridan, 1973; Kaufmann et al, 1977; Pick and Trottet, 1978; Tapping, 1978; Dennis et al, 1981; Trottet et al, 1981; Kane et al, 1983, Kiplinger et al, 1983 a,b; Takakura et al, 1983), though coronagraph observations have also been reported (Koutchmy et al, 1983; Pasachoff and Landman, 1984). Besides the long period (one minute and greater) radio pulsations reported by, for example, Trottet et al (1979) there are numerous observations of short period oscillations with periodicities of around one second. An interesting feature of these short period observations is the form of the observed signature, the temporal variation. Short bursts of oscillatory behaviour are seen to occur, the wave packet or wave train behaviour being described variously as periodic, quasi-periodic or as a sequence of spikes. Kiplinger (1984, private communication) has pointed out that of the various criteria that any successful model of these oscillations must meet, one is certainly that the process is not an enduring one. Theory must be able to explain the termination of the pulses. Kiplinger also stresses two other observed features of the hard X-ray (and also microwave and γ-ray) pulse train phemonena: the source appears to be more or less stationary and compact, and the oscillations are very irregular. A model should ascribe a time scale of the variations rather than a well-defined period i.e. one phase of the motion is quasi-periodic.

How are these rapid oscillations generated? It is suspected that the source is impulsive, perhaps a solar flare, as in the examples given by McLean and Sheridan (1973) and Kiplinger et al (1983 a,b). However, Tapping (1978) and Gaizauskas and Tapping (1980) have reported bursts of oscillations with which there appears to be no associated flare activity.

2. Model of a Coronal Structure

The open and closed 'loops' of the magnetically dominated corona vary in size and are often twisted, hot and dense. Structures may have cool cores and it is thought that brightness variations over them may imply density variations. Not all these observed properties can be incorporated into a model which is simple enough to exhibit the wave-like phenomena associated with the corona. Here the effects of gravity (not so important for a region of scale $\gtrsim 3 \times 10^3$km above the photosphere), curvature (more important for closed loops) and twist (important in knowing if an equilibrium exists) will be neglected. Since transverse dimensions are usually much smaller than longitudinal ones, the structure will be modelled by a straight cylinder and will be regarded as a density inhomogeneity. This may take the form of a uniform magnetic cylinder or alternatively the density may vary across the inhomogeneity but in either case we consider this to be an intrusion into an otherwise uniform magnetic environment.

It has been shown previously (Roberts, Edwin and Benz, 1984) that a uniform, high density coronal cylinder of radius a, its axis lying along the z-direction, acts as a wave guide for the free modes of oscillation ($e^{i(kz+\omega t)}$, ω, k real) of the structure. A high density ($v_{Ae} > v_A$, where v_A, v_{Ae} are the Alfvén speeds in the cylinder and its environment respectively) coronal loop can oscillate freely (that is without radiating energy to infinity) in the fast magnetoacoustic mode. Motions in the environment of the coronal loop arise simply in response to those within the loop; the loop acts as a wave duct trapping the fast mhd oscillations, which penetrate only a small distance into the environment.

The dispersion relation for these ducted, free modes of oscillation is (Meerson et al, 1978; Wentzel 1979; Wilson 1980; Spruit 1982; Edwin and Roberts 1983; Roberts et al, 1984)

$$\rho_o(k^2v_A^2 - \omega^2)m_e \frac{K_n'(m_e a)}{K_n(m_e a)} = \rho_e(k^2v_{Ae}^2 - \omega^2)n_o \frac{J_n'(n_o a)}{J_n(n_o a)}, \tag{1}$$

where

$$m_e^2 = \frac{(k^2v_{Ae}^2 - \omega^2)(\omega^2 - k^2c_e^2)}{(c_e^2 + v_{Ae}^2)(\omega^2 - k^2c_{Te}^2)}, \quad m_o^2 = \frac{(\omega^2 - k^2v_A^2)(\omega^2 - k^2c_o^2)}{(c_o^2 + v_A^2)(\omega^2 - k^2c_T^2)},$$

and ρ_o and ρ_e are the equilibrium densities inside and outside the cylinder, respectively. The sound and Alfvén speeds inside the cylinder are c_o and v_A, outside c_e and v_{Ae}. Also,

$$c_T^2 = \frac{c_o^2v_A^2}{(c_o^2 + v_A^2)}, \quad \text{and} \quad c_{Te}^2 = \frac{c_e^2v_{Ae}^2}{(c_e^2 + v_{Ae}^2)}.$$

K_n, J_n are Bessel functions (modified and first kind, respectively). $n = 0$ give the symmetric (sausage) modes (those in which the axis of the cylinder is undisturbed by the vibration) and $n = 1$ the kink or asymmetric modes.

Figure 1 illustrates the solutions of Equation (1) for a dense coronal loop: two sets of waves feature which, because the sound and Alfvén speeds differ by an order of magnitude, have widely separated frequencies. One class of oscillations has slow, acoustic time scales, for both kink and sausage modes, and the waves are only mildly dispersive. In a low β plasma the tube speed c_T is close to the sound speed c_o so the frequencies of both these slow modes are given (to a good approximation, provided ka is not too large) by

$$\omega \sim kc_T \approx kc_o. \tag{2}$$

Of the fast waves (with phase-speeds satisfying $v_A < \omega/k < v_{Ae}$) only the principal kink mode exists for all wave numbers k (i.e. does not have a cut-off). It has a typical frequency given by

$$\omega \sim kc_k, \tag{3}$$

where

$$c_k^2 = \frac{\rho_e v_{Ae}^2 + \rho_o v_A^2}{\rho_o + \rho_e}.$$

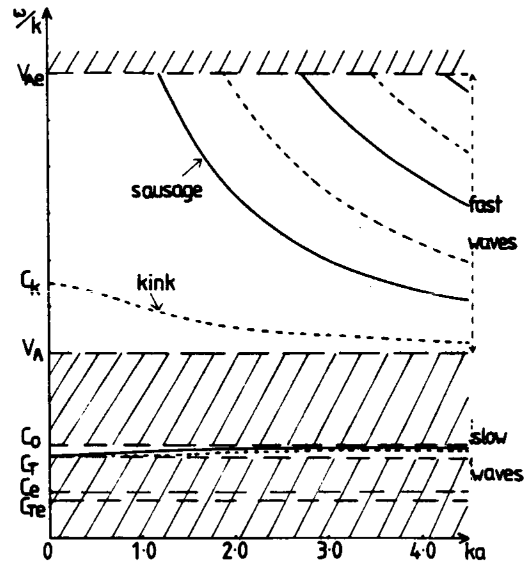

Figure 1 The phase speed ω/k (ω, k real) as a function of ka ($k > 0$) where a is the radius of a dense coronal structure, illustrated here for $v_{Ae} = 5c_o$, $c_e = 0.5c_o$ and $v_A = 2c_o$. The mildly dispersive, slow band of oscillations has infinitely many modes, on acoustic time scales, for both sausage (——) and kink (- - -) modes. Here only two are shown. Except for the principal kink mode, which exists for all wave numbers k, the fast waves have cut-offs. Hatching denotes regions where there are no free modes (After Edwin & Roberts, 1983).

In a coronal loop of length L, with its footpoints anchored in the dense chromosphere/photosphere both the slow and fast waves may occur as standing modes. Then Equation (2) and (3) would give rise to two time scales $\tau_s \sim 2L/c_o$ and $\tau_f \sim 2L/c_k$, corresponding to the slow and fast waves respectively. For typical coronal parameters, for example a sound speed of 200 km s^{-1}, an Alfven speed of $\sim$ 2000 km s^{-1} and a loop length of 2×10^4 – 10^5 km, τ_s and τ_f are estimated to be of order 200–1000s and 15–70s respectively.

3. Impulsively Generated Modes

A solar flare, considered as either a single or multiple impulsive source, will initiate propagating rather than standing waves. Alternatively, there may be other less energetic sources generating the waves, which may arise either in a closed coronal loop, if the motions have insufficient time to reflect from the far ends of the loop, or in open field regions.

To simplify the discussion and interpretation of these propagating waves it will be assumed that the plasma is cold. This is reasonable for the low β corona and involves no loss of essential physics as far as the fast modes are concerned. Mathematically it means that $c_o = c_e = 0$ in Figure 1 and that just the fast band of waves remains. For the fast sausage waves in particular the dispersion relation (1) with $c_e = c_o = 0$ reduces to

$$- n_o \frac{K_1(m_e a)}{K_o(m_e a)} = m_e \frac{J_1(n_o a)}{J_o(n_o a)} \tag{4}$$

where

$$n_o^2 = \frac{\omega^2 - k^2 v_A^2}{v_A^2} \quad \text{and} \quad m_e^2 = \frac{(k^2 v_{Ae}^2 - \omega^2)}{v_{Ae}^2}.$$

For a cylindrical structure of large radius, (4) reduces to

$$\tan(n_o a - \pi/4) = - n_o/m_e, \tag{5}$$

which, except for a phase shift of $\pi/4$, is identical to the dispersion relation given by Pekeris (1948) for the propagation of explosive sound in shallow water.

Thus Eqn (4) describes the highly dispersive, fast, symmetric (cold plasma) waves of Figure 1. If such waves are generated impulsively, then the resulting disturbance ϕ may be represented as a Fourier integral over all frequencies ω and wave numbers k. In general, a wave packet results. For example, if ω is related to k from (4) by $\omega = \pm W(k)$, then $W'(k)$ describes the group velocity of the waves and ϕ has the form (Pekeris, 1948; Whitham, 1974):

$$\phi \sim \begin{cases} A(z,t)e^{i\psi(z,t)}, & W''(k) \neq 0, \quad \text{(6a)} \\ \dfrac{1}{(t|W'''(k)|)^{1/3}} e^{i\psi(z,t)}, & W''(k) = 0, \quad \text{(6b)} \end{cases}$$

where

$$A(z,t) = \sqrt{\frac{2\pi}{t|W''(k)|}}\, e^{-(\pi i/4)\mathrm{sgn}W''(k)}, \text{ and } \psi(z,t) = zk - W(k)t.$$

Thus Eqn (6a) describes an oscillatory wave train with ψ giving the variation between local maxima and minima. However, since A, k and ω are not constant the wave train is not uniform. The distance and *time* between successive maxima, and the amplitude, are not constant i.e. there is a *quasi-periodic* behaviour. It is obvious from Eqns (6) that extrema, $W''(k) = 0$, of the group velocity are important since then expression (6b), the so called Airy phase, describes the disturbance rather than (6a).

The group velocity curve for the ω-k dispersion relation (Eqn (4)) for a *uniform* cylinder of density inhomogeneity is found to possess a minimum.

So too does the group velocity curve for a μ-power density intrusion into an otherwise uniform medium. If the density variation across the loop has the form

$$\rho_o(r) = \begin{cases} \rho_o(0)\left[1 - \left[\frac{r}{a}\right]^{\mu}\left[1 - \frac{\rho_e}{\rho_o}\right]\right], & 0 \leq r \leq a, \\ \rho_e, & r > a, \end{cases} \tag{7}$$

then the ω-k dispersion relation becomes (Edwin, 1984)

$$-\frac{v\,K_o(v)}{K_1(v)} - \frac{\left[1 - \frac{K_o(v)\,K_2(v)}{K_1^2(v)}\right] v^2}{\mu + 2} = \frac{u\,J_o(u)}{J_1(u)}, \tag{8}$$

where $u^2 = \frac{\mu}{\mu + 2} n_o^2 a^2$ and $v^2 = m_e^2 a^2$. Letting $\mu \to \infty$ in the relation (8) recovers the uniform density dispersion relation Eqn (4).

Typical group velocity plots for the lowest order sausage modes of the uniform ($\mu \to \infty$) and parabolic ($\mu = 2$) density variation cylinders are shown in Figure 2.

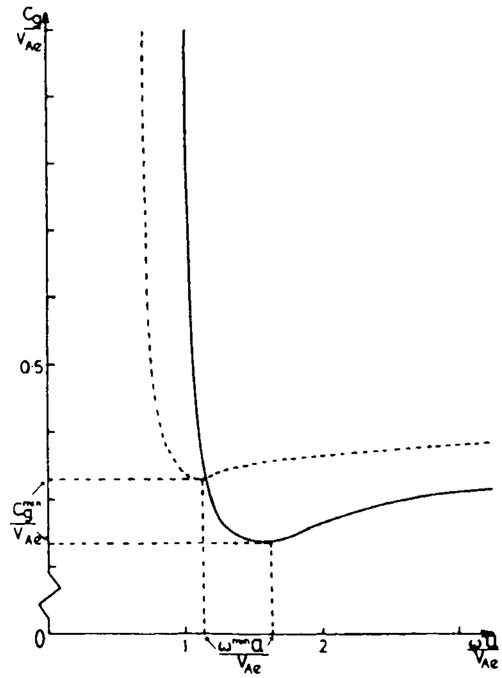

Figure 2 The dimensionless group velocity c_g/v_{Ae} as a function of dimensionless frequency $\omega a/v_{Ae}$ of the lowest order fast sausage mode of Figure 1, shown for uniform (---) and parabolic (——) density profiles across a coronal structure. Here $\rho_o(0)$, the density on the axis of the structure is 6 times that in the exterior of the loop, ρ_e. (After Edwin, 1984).

Since each curve has a minimum there is a point where $W''(k) = 0$. The disturbance resulting from an impulsively generated fast symmetric mode may thus be described as follows. At time $t = 0$ a pulse is generated at a point source $z = 0$, say, which is composed of all frequencies. The disturbance observed at a (large) distance $z = h$ from the source, on the basis of one of the group velocity curves in Figure 2, initially has a frequency given by

$$\omega_c = k_c v_{Ae}, \tag{9}$$

where the wave number k_c at cut-off is found from (4) or (8) with $\omega/k \simeq v_{Ae}$. Specifically,

$$k_c = \frac{f.j_s^{(0)}}{(\rho_o(0)/\rho_e - 1)^{\frac{1}{2}} a}, \quad s = 1,2,3,\ldots \tag{10}$$

where $f = 1$ and $\rho_o(0) = \rho_o$ for the uniform cylinder, and $f = (1 + 2/\mu)^{\frac{1}{2}}$ for the μ-power density variation; $j_s^{(0)}$ ($\simeq$ 2.40, 5.52, ...) are the zeros of the Bessel function J_o. This disturbance has taken a travel time of $t = h/v_{Ae}$ from the source and initiates the *periodic phase*. As time progresses the frequency and amplitude of this nearly sinusoidal wave (Eqn (6a)) grow slowly until a time h/v_A when high frequency information arrives from the source. So a new train of high frequency waves, due to the right-hand branch of the group velocity curve, is superimposed on those arriving from the left-hand branch. The result is a strong increase in amplitude and the oscillation becomes quasi-periodic (see (6a) where the amplitude varies inversely as the square root of the slope of the group velocity curve). During this *quasi-periodic phase* the frequencies of the waves in the right-hand and left-hand branches of the curve continue to decrease/increase respectively (see Figure 2) until at a time $t = h/c_g^{min}$, where c_g^{min} is the minimum group velocity, they coincide. The disturbance then consists of the single frequency ω^{min} (Figure 2) and, though the disturbance continues to oscillate with this constant frequency, the amplitude declines rapidly according to (6b) ($W(k) = \omega^{min}$) in the *Airy* or *decay phase*. The behaviour at a location $z = h$ from the impulsive source is well known (Pekeris, 1948; Ewing et al, 1957; Brekhovskikh, 1960; Tolstoy, 1973; Kennett, 1983) and the sequence of phases just described can be illustrated as in Figure 3.

In order to draw comparisons between observations and theory the particular frequencies and time scales associated with Figure 3 must be identified. The frequency representative of the periodic phase is given by (9) and (10), and setting $s = 1$ for the lowest order sausage mode. In terms of period this means that the *largest periodicity* of the disturbance is given by

$$\tau_c = \frac{2\pi a}{f j_1^{(0)} v_{Ae}} \left[\frac{\rho_o}{\rho_e} - 1\right]^{\frac{1}{2}}. \tag{11}$$

For a very dense inhomogeneity ($\rho_o >> \rho_e$), Eqn (11) reduces to

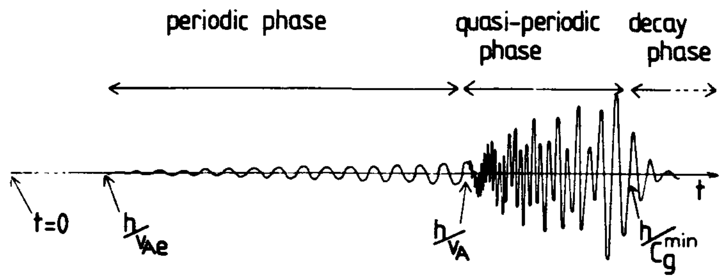

Figure 3 A sketch of the evolution of the lowest order fast sausage mode of Figure 1 in the low β extreme (c_o, c_e << v_A, v_{Ae}) showing the various phases in the disturbance as recorded at an observation level $z = h$ away from an impulsive source at $z = 0$.

$$\tau_c \simeq \frac{2.6a}{fv_A} . \qquad (12)$$

The other important frequency is that representative of the quasi-periodic phase, though this is a variable quantity because of the very quasi-periodicity. ω^{min}, which characterizes the frequency at the end of the quasi-periodic phase is larger than ω_c and so has an associated period τ^{min} which is *smaller* than τ_c. Thus rapid oscillations occur during the quasi-periodic phase.

4. Relation to observations

The mechanism by which fast magnetoacoustic waves ducted in a density enhancement of the corona may be manifest as radio wave observations was discussed in Roberts et al (1984). Here we merely summarise the properties of such mhd waves when they are impulsively generated, perhaps by a solar flare, and show that several of the desirable features of observed rapid oscillations can be explained by our model.

When the lowest order, fast, symmetric mode is generated impulsively it exhibits both periodic and quasi-periodic signatures as shown in Figure 3, the vast majority of the wave power appearing as a well-defined packet. The time signature thus resembles the pulsations recorded in the radio wave (and other) data (see e.g. Pick and Trottet, 1978; Tapping, 1978). The time scales associated with such a disturbance are indeed rapid and of the order of 1 second, as observations require. For example, taking a particle density of $\sim 10^9 cm^{-3}$, a magnetic field intensity of 40G and a loop radius of $\sim 10^3$km, Eqn (5) shows that the periodicity at cut-off for a uniform loop is 0.94s. Again it must be remembered that this is the largest time scale of the motion, the periodicity within and at the end of the quasi-periodic phase being much smaller. Also coronal loops having a gradual density variation with radius rather than forming a step-like density inhomogeneity, would according to the μ-power model have even larger values of ω^{min} and a correspondingly reduced time scale of oscillation in the quasi-periodic phase.

Apart from periodicities, there are time scales associated with the durations of the periodic and quasi-periodic phases. Observational determination of the onset of the periodic phase is expected to be difficult because the lower amplitude oscillations may be lost in background noise, but the quasi-periodic phase is theoretically distinctive (Figure 3) and so comparisons with estimates of its duration, τ_{dur}, given by

$$\tau_{dur} = h \left[\frac{1}{c_g^{min}} - \frac{1}{v_A}\right], \qquad (13)$$

and observed signatures of similar form should be possible. It was suggested also in Roberts <u>et al</u>, (1984) that the width of the inhomogeneity and the associated Alfvén speeds may be obtained by plotting a/h (in units of τ^{min}/τ_{onset}, where $\tau_{onset} = h/v_A$) against τ_{dur} (in units of τ_{onset}), so that the theory may be used to determine physical conditions in the corona using radio pulsation data.

The picture presented by Figure 3 depends largely on the comparative sizes of v_A, v_{Ae} and c_g^{min}. If the density of the inhomogeneity is not very different from that of its surroundings, so that v_A is close to v_{Ae} and the structuring is weak, then the quasi-periodic phase is of fairly short duration because $c_g^{min} \sim v_A$ (see Figure 2). Consequently, with the periodic phase lost in noise, the observed pulse train will start abruptly. Also it will decay more gradually because the Airy phase is correspondingly extended. (The curvature of the group velocity curve at the minimum is not so pronounced for $v_A \sim v_{Ae}$ so $|W''(k)|$ in (6b) is smaller.) This theoretical possibility may correspond to the event observed by McLean and Sheridan (1973). Density variations across the loop would produce a lower c_g^{min} and longer duration times (see Figure 2 and Eqn (13)).

5. <u>Concluding Remarks</u>

A mechanism has been offered to try to explain some of the quasi-periodic, rapid, oscillatory behaviour observed in the solar corona. It has been shown that the explanation does not depend on the coronal loop having uniform density. Variations in density across the loops would modify the duration of the quasi-periodic phase and the time scale of the periodic phase, but observations and results from radio wave, hard X-ray and other data could still be accounted for.

Most of the foregoing analysis (the group velocity curves, etc) has been carried out for both sausage and kink modes. (For the asymmetric kink mode in a wide cylinder ka >> 1, the phase-shifted Love's equation

$$\tan(n_o a - \pi/4) = m_e/n_o,$$

describing seismological waves ducted in the Earth's crust, is obtained instead of Eqn (5).) All modes would be generated by an impulsive source so a combination of the above features is to be expected. Moreover disturbances could be generated by a series of impulses at various times so that a quasi-periodic phase resulting from one disturbance may interact with the periodic phase of a subsequent impulse to give a much more complicated picture than that presented in Figure 3. Interactions between impulses and the various combinations of modes could therefore produce a far more distorted signature than that described here for a single mode and a single impulse.

Whilst it is recognised that the real situation in a coronal loop is unlikely to be that of a single, symmetric mode in a dissipationless, linear system arising from a single impulse, the theory nevertheless offers an idea which can be tested against further observations. The marriage of observation and theory offers the possibility of a fruitful diagnostic of *in situ* coronal conditions.

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