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Precise Computer Controlled Positioning of Robot End Effectors Using Force Sensors

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Project Summary

A major problem in space applications of robotics and docking of spacecraft is the development of technology for automated precise positioning of mating components with smooth motion and soft contact. To achieve the above objective, a design method has been developed for optimally placing the closed-loop poles of a discretized robotic control system at exact prescribed loctions inside the unit circle of the complex z-plane. The design method combines the merits of the pole placement and the linear quadratic design approaches. The proposed design procedure is based on the assignment of one real eigenvalue or two complex conjugate (or real) eigenvalues at each design step. The method involves solutions of simple algebraic equations and thus is considered to be efficient for on-line or off-line computations. Also, in this project, two methods for the linearization of nonlinear model of a robotic manipulator have been presented.

Since automatic control of multi-degree freedom robotic manipulators involves high nonlinear equations of systems, we propose a pilot project involving the control of an one-dimensional system. This simple system can be readily implemented for testing the concepts and algorithms. The ideas developed in this project will provide proven principles for the development of the use of froce/torque sensors for robotic manipulators with more than one joint.

Based on the research results in the period of January 1987 to June 1987, five papers have been accepted for publication in the referred journals [16,17,18], and presentation at the 1987 Automatic Control Conference [19,20].

Current Work by the Investigators

1. Introduction

The dynamic characteristics of a linear system are influenced by the locations of its poles. Therefore, for a system to exhibit good response, both in the transient and steady states, it is necessary to place the closed-loop poles in desired positions. The design of discrete optimal control systems with prescribed eigenvalues has been studied by Solheim [1]. Solheim [1] stated that it is not, in general, possible to determine the resultant state weighting matrix Q for the discrete-time systems. To overcome the drawbacks, Amin [2] modified the recursive approach of Solheim [1] to guarantee the existence of the resultant Q. Both of the procedures involves solution of the discrete-time algebraic Riccati equation at each design step.

In this project, an optimal pole placement method is presented for the design of computer control systems for robotic manipulators that are modeled by highly coupled, nonlinear systems of equations. Two methods of linearization have been considered:

(1) One is based upon feedforward cancellation of the gravity terms and piecewise constant parameterization.

(2) The other is based upon the use of perturbation equations associated with a nominal trajectory.

This project is organized as follows:

Section 2 contains a brief review of the equations of motion for a robotic manipulator. Linearization techniques for a robotic manipulator are presented in Section 3. Finally, Section 4 presents discrete linear regulators with prescribed eigenvalues.

2 Equations of Motion

By applying either the Newton-Euler or the Lagrange's equations, the equations of motion for a robotic manipulator with n joints can be obtained and written in vector-matrix notation as

$$D(q)\ddot{q} + h(q,\dot{q}) + g(q) = \tau(t) \tag{1}$$

where

τ	is an $n \times 1$ vector of forces or torques applied to links,
$q, \dot{q}, \ddot{q}$	are $n \times 1$ vectors representing joint positions, velocities and accelerations,
$D(q)$	is an $n \times n$ generalized mass matrix,
$h(q,\dot{q})$	is an $n \times 1$ vector of Coriolis and centrifugal acceleration terms, and
$g(q)$	is an $n \times 1$ vector representing the effects of gravity.

The effects of viscous friction acting on the links can be taken into account by adding a term $V\dot{q}$ to Eq. (1) to get

$$D(q)\ddot{q} + V\dot{q} + h(q,\dot{q}) + g(q) = \tau(t) \tag{2}$$

where V is an $n \times n$ diagonal matrix containing the coefficients of friction for each joint.

We will now consider the effect of including the dynamics of the actuators. Assume that all joints are revolute and that each joint is driven by a DC servo motor through a gear chain. With the assumption that the armature-winding inductance is negligible and that the DC motors are operated in their linear range, i.e., the torque delivered by the motor is proportional to the armature-winding current by a constant K_t, the dynamic equations of the actuators can be written as

$$v = K_b\dot{\theta}_a + RK_t^{-1}\tau_a \tag{3a}$$

and

$$\tau_a - \tau' = J_a\ddot{\theta}_a + B_a\dot{\theta}_a \tag{3b}$$

where

v	is an $n \times 1$ vector of applied armature voltages,
τ_a	is an $n \times 1$ vector of torques delivered by the DC motors,
τ'	is an $n \times 1$ vector of torques applied by the links to the actuators,
$\theta_a, \dot{\theta}_a, \ddot{\theta}_a$	are $n \times 1$ vectors representing the angular positions, velocities and accelerations of the actuators shafts,
J_a, B_a	are $n \times n$ diagonal matrices representing the moment of inertia and the viscous friction coefficients of the actuators, and
R, K_b, K_t	are $n \times n$ diagonal matrices representing the armature-winding resistances, the back EMF constants and the torque constants of the DC motors.

Assuming that the gear backlash is negligible, we can write

$$\tau' = N_g^{-1}\tau \tag{4a}$$

$$\theta_a = N_g q \tag{4b}$$

$$\dot{\theta}_a = N_g \dot{q} \tag{4c}$$

$$\ddot{\theta}_a = N_g \ddot{q} \tag{4d}$$

where N_g is defined as an $n \times n$ constant diagonal matrix with each diagonal element specifying the corresponding joint gear ratio.

Using Eqs. (1)-(4), the dynamic equations for a robotic system including the dynamics of the actuators can be written as

$$D'(q)\ddot{q} + V'\dot{q} + h'(q,\dot{q}) + g'(q) = v(t) \tag{5a}$$

where

$$D'(q) = RK_t^{-1}[N_g J_a + N_g^{-1} D(q)] \tag{5b}$$

$$V' = N_g K_b + RK_t^{-1}[N_g B_a + N_g^{-1} V] \tag{5c}$$

$$h'(q,\dot{q}) = RK_t^{-1}N_g^{-1}h(q,\dot{q}) \tag{5d}$$

and

$$g'(q) = RK_t^{-1}N_g^{-1}g(q) \tag{5e}$$

3. Methods of Linearization

Two methods of linearization of a highly coupled nonlinear robotic control system are considered. The first method is based upon the feedforward cancellation of the gravity terms and the piecewise parameterization, while the second method is based upon the perturbation equations associated with a given nominal trajectory.

3.1 Cancellation of Gravity Terms

For simplicity, the dynamics of the actuators are neglected; however, the method is applicable even if the dynamic equation of the overall system including the actuator is used because of the structure of that equation. Note that the Coriolis and centrifugal term $h(q,\dot{q})$ is a quadratic vector form of $\dot{q}$ (see Raibert and Horn [3]). Hence this term can be expressed as

$$h(q,\dot{q}) = E(q,\dot{q})\dot{q} \tag{6}$$

where $E(q,\dot{q})$ is defined as an $n \times n$ matrix. The dynamical equation of the robotic system in Eq. (2) can be rewritten as

$$D(q)\ddot{q} + [V + E(q,\dot{q})]\dot{q} = \tau - g(q) \tag{7a}$$

or

$$\ddot{q} = -D^{-1}(q)[V + E(q,\dot{q})]\dot{q} + D^{-1}(q)[\tau - g(q)] \tag{7b}$$

Eq. (7) can be written in the state-space representation as

$$\dot{x}(t) = A(t)x(t) + b(t)u(t) \tag{8a}$$

where

$$A(t) = \begin{bmatrix} 0_n & I_n \\ 0_n & -D^{-1}(q)[V + E(q,\dot{q})] \end{bmatrix} \tag{8b}$$

$$B(t) = \begin{bmatrix} 0_n \\ D^{-1}(q) \end{bmatrix} \tag{8c}$$

and

$$x^T(t) = [q^T(t), \dot{q}^T(t)], \quad u(t) = \tau(t) - g[q(t)] \tag{8d}$$

The linearized dynamic equations of a robotic system can be computed at each sampling period where the nominal trajectory is known, i.e, the joint position $q(t)$ and the velocity $\dot{q}(t)$ are given. Thus, the control problem can be considered as a time-varying control problem.

Remark 1 For the implementation of a controller using this approach, a discrete-time model should be used instead of the continuous-time model. Also, assuming that $D(q)$ and $E(q,\dot{q})$ are piecewise constant, the system in Eq. (8) is a system of type 1 with the input $u(t) = \tau(t) - g(t)$. Thus if the computed system parameters are exactly the same as the true ones, the elimination of the steady state position error will be assured by the use of state-feedback control law, but not for the position error when the set point is a ramp input (i.e., the manipulator is programmed to move at constant velocity). In order to eliminate such errors, integral control may be applied.

3.2 Perturbation Equations

For simplicity, the dynamics of the actuators are omitted in the derivation of the perturbation equations; however, the results can be easily extended to include the actuators dynamics of the DC motors.

Suppose that the desired trajectory in the task-space (world space) of the hand (the gripper) of a manipulator is preplanned. The corresponding trajectory including $q^*(t), \dot{q}^*(t)$ and $\ddot{q}^*(t)$ can be precomputed as well as the nominal applied torque $\tau^*(t)$ (or voltage $v^*(t)$) required for motion along the specified trajectory.

The dynamical equations in Eq. (2) can be expressed as a sum of the nominal equation,

$$D(q^*)\ddot{q}^* + V\dot{q}^* + h(q^*, \dot{q}^*) + g(q^*) = \tau^*(t) \tag{9}$$

plus a perturbation equation,

$$\delta(D\ddot{q}) + V\delta\dot{q} + \delta h + \delta g = \delta\tau \tag{10}$$

The variations $\delta(D\ddot{q}), \delta h$, and δg can be expressed in terms of the following linear approximations,

$$\delta[D(q)\ddot{q}] = \hat{A}(t)\delta q + \hat{B}(t)\ddot{q} \tag{11a}$$

$$\delta h(q, \dot{q}) = \hat{C}(t)\delta q + \hat{E}(t)\delta\dot{q} \tag{11b}$$

$$\delta g(q) = \hat{F}(t)\delta q \tag{11c}$$

where

$$\hat{A}(t) = \frac{\partial(D\ddot{q})}{\partial q}|_{q^*,\dot{q}^*} \tag{12a}$$

$$\hat{B}(t) = \frac{\partial(D\ddot{q})}{\partial\ddot{q}}|_{q^*,\dot{q}^*} \tag{12b}$$

$$\hat{C}(t) = \frac{\partial h}{\partial q}|_{q^*} \tag{12c}$$

$$\hat{E}(t) = \frac{\partial h}{\partial\dot{q}}|_{q^*,\dot{q}^*} \tag{12d}$$

and

$$\hat{F}(t) = \frac{\partial g}{\partial q}|_{q^*} \tag{12e}$$

Note that the (i, j) element of the matrix $\hat{A}$ is found as

$$\hat{A}_{ij} = \frac{\partial\sum_{k=1}^{n} D_{ik}\ddot{q}_k}{\partial q_j}|_{q^*,\dot{q}^*} \tag{13a}$$

or

$$\hat{A}_{ij} = \sum_{k=1}^{n} [\frac{\partial D_{ik}}{\partial q_i} \ddot{q}_k]|_{q^*,\dot{q}^*} \tag{13b}$$

Therefore, the perturbation equations for the manipulator can be approximated as

$$D(t)\delta\ddot{q} + \bar{P}(t)\delta\dot{q} + \bar{Q}(t)\delta q = \delta\tau \tag{14}$$

where

$$\bar{P}(t) = V + \hat{E}(t) \tag{15a}$$

and

$$\bar{Q}(t) = \hat{A}(t) + \hat{C}(t) + \hat{F}(t) \tag{15b}$$

Thus, the equivalent state-space representation of the manipulator dynamics becomes

$$\delta\dot{x}(t) = A(t)\delta x(t) + B(t)\delta\tau(t) \tag{16}$$

where

$$x = [\delta q_1, \delta q_2, \cdots, \delta q_n, \delta\dot{q}_1, \delta\dot{q}_2, \cdots, \delta\dot{q}_n]^T \tag{17a}$$

$$A(t) = \begin{bmatrix} 0_n & I_n \\ -D^{-1}\bar{Q}(t) & -D^{-1}\bar{P}(t) \end{bmatrix} \tag{17b}$$

and

$$B(t) = \begin{bmatrix} 0_n \\ D^{-1}\bar{Q}(t) \end{bmatrix} \tag{17c}$$

Consider the linear state-feedback control law for the system in Eq. (16) as

$$\delta\tau(t) = -K(t)\delta x(t) \tag{18a}$$

The total input torque becomes

$$\tau(t) = \tau^*(t) + \delta\tau(t) \tag{18b}$$

Note that there are several design methods available in the literature for choosing the appropriate feedback gain K [4,5].

Remark 2 An important advantage of linear state-feedback control based on the linearized perturbation equations is that when the desired trajectory is preplanned, the feedback gain matrix K can be computed off-line and stored in a table. On the other hand, the parameterization approach does not require any prior knowledge of the path, but the system parameters must be either computed on-line or stored in a look-up table based upon segmentation of the workspace.

4 Discrete Linear Regulators with Prescribed Eigenvalues

This section deals with the design of linear discrete regulators with prescribed eigenvalues. The discrete optimal pole placement methods have been discussed by Solheim [6], Amin [2] and others [7]. However, these methods are based on the solution of the algebraic discrete Riccati equation. In this section, a design method for the synthesis of discrete optimal control systems with prescribed eigenvalues is presented. The proposed method is based on the solution of simple algebraic equations and thus is considered to be computationally efficient.

Consider the linear time-invariant controllable system described by

$$x(k+1) = Gx(k) + Hu(k); \quad x(0) \tag{19a}$$

$$y(k) = Cx(k) \tag{19b}$$

where $x(k)$ and $u(k)$ are the $n \times 1$ state and $m \times 1$ input vectors, respectively, and G, H and C are constant matrices of appropriate dimensions. Assume that the system matrix G is nonsingular.

The main objective is to find a feedback control law,

$$u(k) = -Fx(k) \tag{20}$$

which gives the closed-loop system a set of desired eigenvalues and at the same time minimizes the quadratic performance index,

$$J = \sum_{i=0}^{\infty} [x^T(i)Qx(i) + u^T(i)Ru(i)] \tag{21}$$

where Q and R are nonnegative and positive-definite symmetric matrices, respectively.

Applying the controller in Eq. (20) to the system in Eq. (19), the closed-loop system becomes

$$x(k+1) = (G - HF)x(k) \triangleq G_c x(k) \tag{22}$$

The optimal control law in Eq. (20) that minimizes the performance index in Eq. (21) is given by

$$u(k) = -(R + H^T PH)^{-1} H^T PG x(k) \triangleq -Fx(k) \tag{23a}$$

with

$$F = (R + H^T PH)^{-1} H^T PG \tag{23b}$$

or

$$F = R^{-1} H^T (P^{-1} + HR^{-1} H^T)^{-1} G \tag{23c}$$

where the P is positive definite matrix and is obtained by solving the discrete algebraic Riccati equation,

$$P = G^T PG - G^T PH(R + H^T PH)^{-1} H^T PG + Q \tag{23d}$$

The approach here is how to choose Q and R such that the closed-loop system in Eq. (22) has a set of prescribed eigenvalues. Similar to Solheim's method [6], a recursive procedure is developed. Also, the technique of modifying the input control weighting matrix R as in Amin's method [2] is considered in order to assure optimallity of the closed-loop system. Before presenting the new method, some preliminary results are needed.

Lemma 1 [2] Given a controllable system as in Eq. (19) and the performance index in Eq. (21) with the control weighting R. Assume that j feedbacks are obtained for j recursive optimal problems with

$$G_{i+1} = G_i - HF_i, \quad G_1 = G \tag{24}$$

If the control weighting matrix R_i satisfies the condition,

$$R_{i+1} = R_i + H^T P_i H, \quad R_1 = R \tag{25}$$

then the feedback control matrix,

$$F = \sum_{i=1}^{j} F_i \tag{26}$$

is the solution of the optimal control problem with weighting matrices,

$$\bar{Q} = \sum_{i=1}^{j} Q_i, \quad \bar{R} = R \tag{27}$$

Furthermore, the solution $\bar{P}$ of the equivalent discrete Riccati equation becomes

$$\bar{P} = \sum_{i=1}^{j} P_i \tag{28}$$

Lemma 2 Consider the controllable system

$$x(k+1) = Gx(k) + Hu(k) \tag{29a}$$

where H is a $2 \times m$ matrix and G is a 2×2 matrix defined by

$$G = \begin{bmatrix} g_1 & g_2 \\ g_3 & g_4 \end{bmatrix} \tag{29b}$$

Then, the closed-loop system obtained by solving the optimal control problem for a set of Q and R can be expressed as

$$G_c = G - HF = G - HR^{-1}H^T(P^{-1} + HR^{-1}H^T)^{-1}G \triangleq \bar{D}G \tag{30}$$

where $\bar{D}$ is defined as the lower triangular matrix,

$$\bar{D} = \begin{bmatrix} \bar{d}_1 & 0 \\ \bar{d}_2 & \bar{d}_3 \end{bmatrix} \tag{31}$$

Also, if $det(G_c)$ is chosen such that

$$0 \leq det(G_c)/det(G) \leq 1 \tag{32}$$

then, the elements of the matrix $\bar{D}$ are calculated from

$$det(G_c) = det(\bar{D})det(G) \tag{33a}$$

$$tr(G_c) = tr(\bar{D}G) \tag{33b}$$

or

$$\bar{d}_1\bar{d}_3 = det(G_c)/det(G) \tag{33c}$$

$$g_1\bar{d}_1 + g_2\bar{d}_2 + g_4\bar{d}_3 = tr(G_c) \tag{33d}$$

with $\bar{d}_1$ and $\bar{d}_3(\bar{d}_1 \neq \bar{d}_3)$ to be chosen as positive numbers less than one.

Lemma 3 Consider the system as in *Lemma* 2. Let a lower triangular matrix D be defined as

$$D = I_2 - \bar{D} \triangleq \begin{bmatrix} d_1 & 0 \\ d_2 & d_3 \end{bmatrix} \tag{34}$$

where $\bar{D}$ is obtained from *Lemma* 2. Then, there exists a lower triangular transformation T_s such that

$$D = T_s \Lambda T_s^{-1} \tag{35a}$$

where

$$\Lambda = diag[d_1, \quad d_3] \tag{35b}$$

and

$$T_s = \begin{bmatrix} 1 & 0 \\ \alpha & \beta \end{bmatrix} \tag{35c}$$

with $\beta \neq 0$ and

$$\alpha = \frac{d_2}{d_1 - d_3} \tag{35d}$$

Lemma 4 Consider the system as in *Lemmas* 2 and 3. For a given positive definite R, there exists a P matrix such as

$$P = T_s^{-T} \Lambda_p T_s^{-1} \tag{36a}$$

where

$$\Lambda_p = (\Lambda_2^{-1} - \Lambda_r)^{-1} \tag{36b}$$

with Λ_r chosen to be a diagonal matrix such that

$$HR^{-1}H^T = T_s \Lambda_r T_s^T \triangleq T_s \begin{bmatrix} \lambda_{r1} & 0 \\ 0 & \lambda_{r2} \end{bmatrix} T_s^T \tag{36c}$$

and

$$\Lambda_2 \Lambda_r = \Lambda \tag{36d}$$

Proof: From Eqs. (30) and (34), we can write

$$HR^{-1}H^T(P^{-1} + HR^{-1}H^T)^{-1} = D = T_s \Lambda T_s^{-1} \triangleq T_s \Lambda_r T_s^T T_s^{-T} \Lambda_2 T_s^{-1} \tag{37a}$$

Therefore, we can write

$$HR^{-1}H^T = T_s \Lambda_r T_s^T \tag{37b}$$

and

$$(P^{-1} + HR^{-1}H^T)^{-1} = T_s^{-T} \Lambda_2 T_s^{-1} \tag{37c}$$

The right hand side of Eq. (37b) can be rewritten as

$$T_s \Lambda_r T_s^T = \begin{bmatrix} \lambda_{r1} & \lambda_{r1}\alpha \\ \lambda_{r1}\alpha & \lambda_{r1}\alpha^2 + \lambda_{r2}\beta^2 \end{bmatrix} \tag{37d}$$

Let

$$HR^{-1}H^T \triangleq \begin{bmatrix} \bar{r}_1 & \bar{r}_2 \\ \bar{r}_2 & \bar{r}_3 \end{bmatrix} \tag{37e}$$

Solving for $\alpha, \beta, \lambda_{r1}$ and λ_{r2} from Eqs. (37d) and (37e), we get

$$\lambda_{r1} = \bar{r}_1, \quad \alpha = \frac{\bar{r}_2}{\bar{r}_1}, \quad \lambda_{r2}\beta^2 = \frac{det(HR^{-1}H^T)}{\bar{r}_1} \tag{37f}$$

Furthermore, using Eqs. (37a), (37b) and (37c), we get

$$P = T_s^{-T} \Lambda_p T_s^{-1} \tag{37c}$$

with Λ_p given by Eq. (36b) and T_s given by Eq. (35c).

Lemma 5 Consider the system as in *Lemmas* 2 and 3. Also, consider the similarity transformation matrix T_d given by

$$T_d = \begin{bmatrix} t_4 & -t_2 \\ -t_3 & t_1 \end{bmatrix} \tag{38a}$$

Let

$$HR^{-1}H^T = \begin{bmatrix} r_1 & r_2 \\ r_2 & r_3 \end{bmatrix}, \quad T_d^{-1}HR^{-1}H^T T_d^{-T} \triangleq \begin{bmatrix} \hat{r}_1 & \hat{r}_2 \\ \hat{r}_2 & \hat{r}_3 \end{bmatrix} \tag{38b}$$

Also, let

$$\hat{G} = T_d^{-1} G T_d \triangleq \begin{bmatrix} \hat{g}_1 & \hat{g}_2 \\ \hat{g}_3 & \hat{g}_4 \end{bmatrix} \tag{38c}$$

where

$$\hat{g}_1 = \frac{1}{det(T_d)}[g_1 t_1 t_4 + g_3 t_2 t_4 - t_3(g_2 t_1 + g_4 t_2)] \tag{38d}$$

$$\hat{g}_2 = \frac{1}{det(T_d)}(-g_1 t_1 t_2 - g_3 t_2^2 + g_2 t_1^2 + g_4 t_1 t_2) \tag{38e}$$

$$\hat{r}_1 = \frac{1}{det(T_d)^2}(r_1 t_1^2 + 2r_2 t_1 t_2 + r_3 t_2^2) \tag{38f}$$

and

$$\hat{r}_2 = \frac{1}{det(T_d)^2}[t_3(r_1 t_1 + r_2 t_2) + r_2 t_1 t_4 + r_3 t_2 t_4] \tag{38g}$$

Then, for Eq. (33*d*) to hold and T_s in Eq. (37) to exist, we choose $t_3 = 0$, $r_2 = 0$ and t_2 in Eq. (38*a*) to satisfy the following equation,

$$t_2^2 + b_1 t_2 + b_2 = 0. \tag{38h}$$

where

$$b_1 = \frac{t_1(r_1 g_3 + r_3 g_2)}{r_3(g_4 - v)} \tag{38i}$$

and

$$b_2 = \frac{t_1^2[r_1(g_1 - v)]}{r_3(g_4 - v)} \tag{38j}$$

with

$$v = \frac{(1 - d_3)tr(G) - tr(G_c)}{d_1 - d_3} \tag{38k}$$

satisfying the following equation,

$$min\{g_1, g_4\} \leq v \leq max\{g_1, g_4\} \tag{38l}$$

Proof: Equation (33d) can be rewritten as

$$g_1 = -\alpha g_2 + \frac{(1 - d_3)tr(G) - tr(G_c)}{d_1 - d_3} \triangleq -\alpha g_2 + v \tag{38m}$$

Using the transformation T_d, Eq. (38m) becomes

$$\hat{g}_1 = -\alpha \hat{g}_2 + v \tag{38n}$$

Substituting for $\hat{g}_1, \hat{g}_2, \alpha$ and v in Eq. (38n), Eq. (38n) can be rewritten as

$$t_2^2 + b_1 t_2 + b_2 = 0. \tag{38o}$$

with b_1 and b_2 as given by Eqs. (38i) and (38j), respectively. Note that t_2 in Eq. (38o) exists for any values of t_1 and t_4 provided that v in Eq. (38k) satisfies Eq. (38l).

<u>*Theorem* 1</u> Consider the system as in *Lemmas* 2, 3, 4 and 5. Let the input matrix H has rank equal to 2. Then, there exists a lower triangular matrix $D(= I_2 - \bar{D})$, given by *Lemma* 2, that places the closed-loop eigenvalues at prescribed values and causes the state weighting matrix,

$$\hat{Q} = T_d^{-T} Q T_d^{-1} \tag{39a}$$

with

$$Q = P - G^T PG + G^T D^T PG = P - G^T(I - D^T)PG \tag{39b}$$

to be positive semi-definite provided that Eq. (32) holds, and the optimal control law becomes

$$u(k) = -Fx(k) = -R^{-1}H^T(\hat{P}^{-1} + HR^{-1}H^T)^{-1}Gx(k) \tag{39c}$$

with

$$\hat{P} = T_d^{-T} P T_d^{-1} \tag{39a}$$

Proof: Using the transformation T_d, the right hand side of Eq. (37b) can be rewritten as

$$T_s \Lambda_r T_s^T = \begin{bmatrix} \lambda_{r1} & \lambda_{r1}\alpha \\ \lambda_{r1}\alpha & \lambda_{r1}\alpha^2 + \lambda_{r2}\beta^2 \end{bmatrix} \tag{40}$$

Let

$$T_d^{-1} H R^{-1} H^T T_d^{-T} \triangleq \bar{R} \triangleq \begin{bmatrix} \hat{r}_1 & \hat{r}_2 \\ \hat{r}_2 & \hat{r}_3 \end{bmatrix} \tag{41}$$

Solving for $\alpha, \beta, \lambda_{r1}$ and λ_{r2} from Eqs. (40) and (41), we get

$$\lambda_{r1} = \hat{r}_1, \quad \alpha = \frac{\hat{r}_2}{\hat{r}_1}, \quad \lambda_{r2}\beta^2 = \frac{det(\bar{R})}{\hat{r}_1} \tag{42}$$

Therefore, by using Eq. (42) and *Lemma* 4, P can be expressed as

$$P = \frac{v}{\lambda_{r2}\beta^2} \begin{bmatrix} \frac{det(\bar{R})u}{\hat{r}_1^2 v} + \alpha^2 & -\alpha \\ -\alpha & 1 \end{bmatrix} \tag{43}$$

Substituting for P from Eq. (43) and D from Eq. (35a) into Eq. (39b), we get

$$Q = \frac{v}{\lambda_{r2}\beta^2}\left\{ \begin{bmatrix} \frac{det(\bar{R})u}{\hat{r}_1^2 v} + \alpha^2 & -\alpha \\ -\alpha & 1 \end{bmatrix} - \frac{d_3}{v}\hat{G}^T \begin{bmatrix} \frac{det(\bar{R})d_1}{\hat{r}_1^2 d_3} + \alpha^2 & -\alpha \\ -\alpha & 1 \end{bmatrix} \hat{G} \right\} \tag{44a}$$

where

$$u = \frac{d_1}{1 - d_1}, \quad v = \frac{d_3}{1 - d_3}, \quad \hat{G} = T_d^{-1} G T_d \tag{44b}$$

It is seen that T_d in Eq. (44) can be chosen to satisfy Eq. (38h) and to make Q a positive semi-definite matrix.

From Eqs. (23) and (43), the optimal control law associated with R and $\hat{Q}$ in Eq. (39a) can be written as

$$u(k) = -Fx(k) = -R^{-1}H^T(\hat{P}^{-1} + HR^{-1}H^T)^{-1}Gx(k) \tag{44c}$$

<u>*Corollary* 1</u> Given a controllable system as in *Lemma* 2 with the input matrix H having a rank of one. If d_1 in D is chosen as zero and d_3 is chosen to satisfy Eq. (33c), an optimal closed-loop system with prescribed eigenvalues can be obtained provided that the closed-loop eigenavlues satisfy Eq. (32).

Proof: Since H has a rank of one, it can be transformed to the following structure,

$$H = \begin{bmatrix} 0_{1\times m} \\ h_{1\times m} \end{bmatrix} \tag{45a}$$

To assure that P is a positive definite matrix, Λ_2 in Eq. (36b) can be written as

$$\Lambda_2 = diag[w, \ \frac{d_3}{\lambda_{r2}}], \ \ for \ \ w > 0 \tag{45b}$$

Then, the corollary can be proved in a similar manner to *Theorem* 1.

<u>*Corollary* 2</u> Consider the controllable system in Eq. (19) where G and H are given as $A = \lambda_{1\times 1}$ and $H = h_{1\times m}$. Let $D = d(> 0)$ and $R > 0)$ and the desired closed-loop eigenvalue be $\bar{\lambda}$. The closed-loop system G_c has $\bar{\lambda}$ and is optimal with respect to R and

$$Q = q = p - \lambda\bar{\lambda}p > 0 \tag{46a}$$

provided that the closed-loop eigenvalue $\bar{\lambda}$ is chosen such that

$$0 \leq \lambda\bar{\lambda} < 1 \tag{46b}$$

Design Procedures

Consider the following controllable open-loop system described as

$$x(k+1) = Gx(k) + Hu(k) \tag{47}$$

Step 1 :

Find a transformation marix M_1 such that the given system matrix G can be converted to a block-diagonal form [8] as shown below:

$$M_1^{-1}GM_1 = block - diag[G_k, G_{k-1}, \cdots, G_1] \triangleq \bar{G} \tag{48a}$$

where each G_i is either a 2×2 or 1×1 block. Also, compute

$$M_1^{-1}H = [H_k^T, H_{k-1}^T, \cdots, H_1^T]^T \triangleq \bar{H} \tag{48b}$$

Step 2 :

Set $i = 1$, and initialize the feedback gain K and the matrices R_1, $\bar{P}$ and $\bar{Q}$ as shown below.

$$K = 0_{m\times n},\ R_1 = R,\ \bar{P} = 0_n,\ \bar{Q} = 0_n \tag{49}$$

Step 3 :

Assign the closed-loop poles to satisfy the requirements in *Theorem* 1 with $R_{i+1} = R_i + H^T P_i H (R_1 = R)$ and compute the feedback control F_i from Eq. (44c) and the state weighting matrix Q_i from Eq. (44a) using the pair (G_i, H_i).

Step 4 :

Compute

$$\hat{P} = M_1^{-T}[block - diag[0_{n-n_i}, P_i]]M_1^{-1} \tag{50a}$$

$$\bar{Q} = \bar{Q} + M_1^{-T}[block - diag[0_{n-n_i}, Q_i]]M_1^{-1} \tag{50b}$$

$$\bar{P} = \bar{P} + \hat{P} \tag{50c}$$

$$K = K + (R + H^T \hat{P} H)^{-1} H^T \hat{P} \bar{G} \tag{50d}$$

$$\bar{G} = \bar{G} - \bar{H}[0_{m\times(n-n_i)}, F_i] \triangleq \begin{bmatrix} \hat{G}_i & W_i \\ 0 & G_{ci} \end{bmatrix} \tag{50e}$$

<u>*Step* 5</u> :

Block-diagonalize the partially designed system matrix in Eq. (50b) and move the last block of $\bar{G}$, i.e., G_{ci} to the first block and accumulate the transformations in $M_1 = M_1 M_2$ to compute the new system matrix and input matrix as

$$\bar{G} = M_2^{-1}\bar{G}M_2 = \begin{bmatrix} G_{ci} & 0 \\ 0 & \hat{G}_i \end{bmatrix} \tag{51a}$$

and

$$\bar{H} = M_2^{-1}\bar{H} = [H_1^T, (H_2 - L_i H_1)^T]^T \tag{51b}$$

The transformation M_2 is of the form,

$$M_2 = \begin{bmatrix} L_i & I \\ I & 0 \end{bmatrix}, \quad M_2^{-1} = \begin{bmatrix} 0 & I \\ I & -L_i \end{bmatrix} \tag{51c}$$

The matrix L_i can be obtained by solving the following Lyapunov equation,

$$\hat{G}_i L_i - L_i G_{ci} + W_i = 0_{(n-n_i)\times n_i} \tag{51d}$$

<u>*Step* 6</u> :

Set $i = i + 1$. If $i > k$, stop, else go to step 3

Project Description

Since automatic control of multi-degree freedom robotic manipulators involves high order nonlinear equation of systems, we propose a pilot project involving the control one-dimensional system. This simple system can be readily implemented for testing the concepts and the algorithm.

The method to design a computer control system for the one-dimensional simplified mechanical model shown in Fig. 1 is presented in this section. The control law will be designed to position mass m_2 precisely adjacent to the barrier so that the reaction force from the wall is minimum. The mass m_1 represents the inertia of the manipulator while m_2 represents the mass of the end effector plus that of the object being positioned. The spring represents the compliance of the system. The system is driven by a linear motor that is equipped with a linear optical encoder for measurement of position. The force transmitted from the spring to mass m_1 will be measured using a force/torque sensor.

The equations of motion for the mechanical system shown in Fig. 2 can be written as

$$m_1\ddot{x}_1 + c\dot{x}_1 + k(x_1 - x_2) = u(t) \tag{52a}$$

$$m_2\ddot{x}_2 + c\dot{x}_2 - k(x_1 - x_2) = 0 \quad for \quad x_2 < d \tag{52b}$$

$$2m_2\dot{x}_2 = I \quad for \quad x_2 = d \tag{52c}$$

where c is the coefficient of friction and k is the spring constant.

Considering the motor dynamics, we get

$$v_a = k_b\dot{x}_1 + Ri \tag{53a}$$

where k_b is defined as the back EMF motor constant and v_a is the motor input voltage. Also,

$$u(t) = k_t i \tag{53b}$$

where k_t is the motor torque constant.

The idea developed in this project will provide proven principles for the development of the use of force/torque sensors for robotic manipulators with more than one joint.

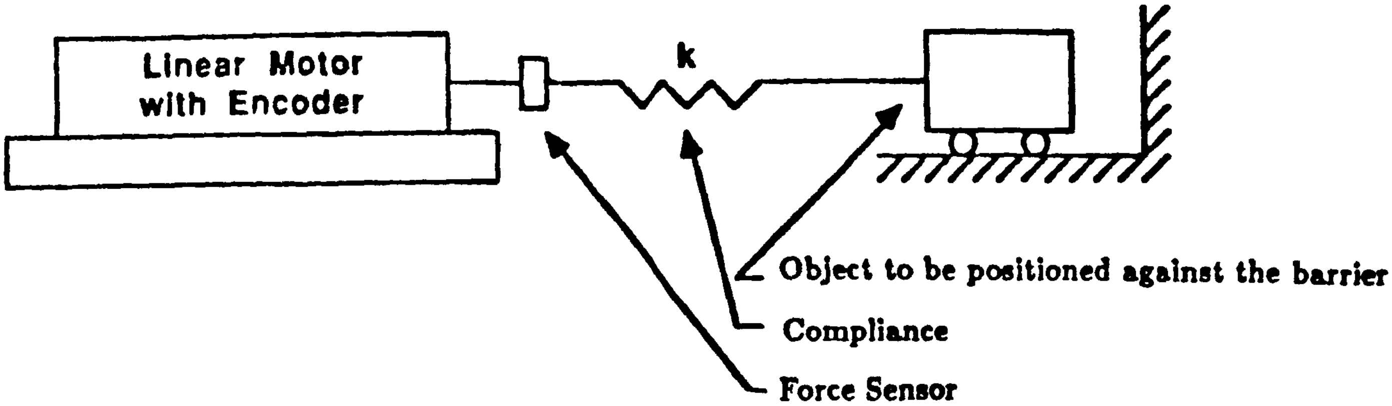

Figure 1 Mechanical System Representation

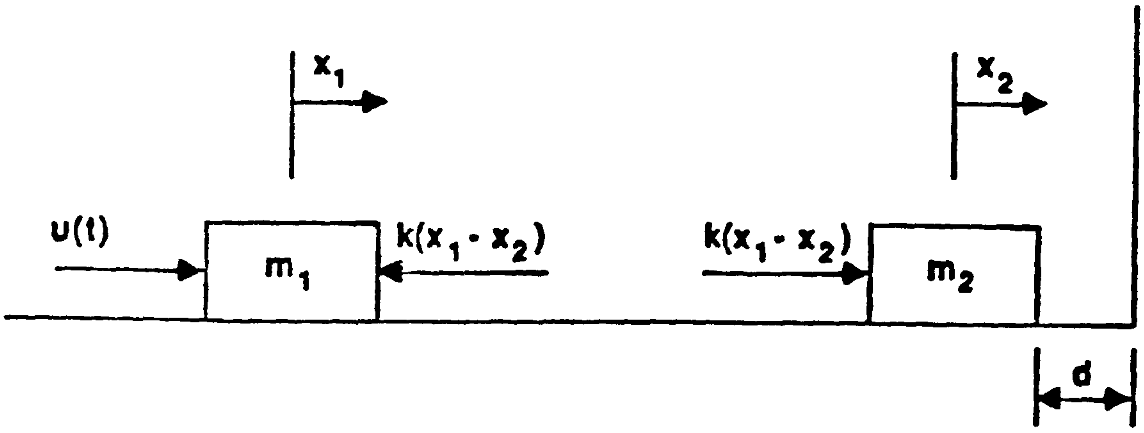

Figure 2 Block Diagram Representation of the Mechanical System

Discussion

The proposed problem represents the simplification to the one-dimensional case of precision positioning of an object. This is a pilot project to provide an investigation of the use of force sensor information in closed-loop controller design. The project will provide for the development of concepts that can be extended to general docking and assembly operations in space. In follow-on work the problem can be extended to the design of a control system for a multi-degree freedom manipulator using feedback from a force/torque sensor. This feedback will be determined by optimally placing the closed-loop poles of a discretized robotic control system at prescribed locations.

The advantage of incorporating force/torque sensor information in the closed-loop control design is that precise but soft positioning can be achieved in a smooth motion without generation of large forces resulting from mating of parts of docking operation.

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