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LANCZOS MODES FOR REDUCED-ORDER CONTROL OF FLEXIBLE STRUCTURES

by

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LANCZOS ALGORITHM - FE VERSION

$$\bar{r}_j = K^{-1}Mq_j$$

$$r_j = \bar{r}_j - \alpha_j q_j - \beta_j q_{j-1}$$

$$\alpha_j = q_j{}^T M \bar{r}_j$$

$$\beta_j = (r_{j-1}{}^T M r_{j-1})^{1/2}$$

$$q_{j+1} = (1/\beta_{j+1}) r_j$$

$$\beta_{j+1} = (r_j{}^T M r_j)^{1/2}$$

The algorithm for recursively forming Lanczos vector q_{j+1} from Lanczos vector q_j for the case of nonsingular K is shown. It involves an orthogonalization step and a normalization step.

STARTING VECTOR

$$F = b\varepsilon(t)$$

$$r_o = K^{-1}b$$

$$q_1 = (1/\beta_1) r_o$$

$$\beta_1 = (r_o{}^T M r_o)^{1/2}$$

Lanczos modes are most useful when the spatial distribution of the excitation is constant. This load distribution then determines the starting vector.

LANCZOS EQUATION FORMAT

$$M\ddot{u} + Ku = F$$

$$MK^{-1}M\ddot{u} + Mu = MK^{-1}F$$

$$u = Q_m z$$

$$Q_m = [q_1 q_2 \ldots q_m]$$

$$Q_m{}^T MK^{-1}MQ_m\ddot{z} + Q_m{}^T MQ_m z = Q_m{}^T MK^{-1}F$$

Lanczos vectors are used in a mode-superposition manner which is not exactly the same as the familiar Rayleigh-Ritz version of mode-superposition. Multiplication by the equation of MK^{-1} "smooths" the loading.

LANCZOS EQUATION FORMAT - CONT.

$$T_m\ddot{z} + z = g_m$$

$$g_m = Q_m{}^T MK^{-1}F$$

$$T_m = \begin{bmatrix} \alpha_1 & \beta_2 & & & & \\ \beta_2 & \alpha_2 & \beta_3 & & & \\ & \beta_3 & \cdot & \cdot & & \\ & & \cdot & \cdot & \cdot & \\ & & & \cdot & \cdot & \beta_m \\ & & & & \beta_m & \alpha_m \end{bmatrix}$$

The final Lanczos equations have a tridiagonal generalized mass matrix and have a unit matrix for the generalized stiffness matrix. The form of the generalized force vector g_m is very special.

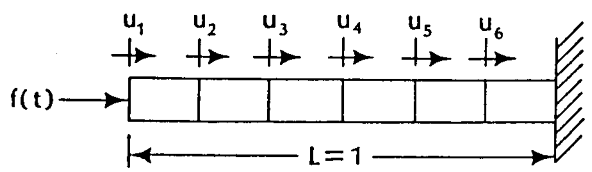
EXAMPLES

- Lanczos Modes and Equations
- Comparison of Normal Mode Models and Lanczos Mode Models

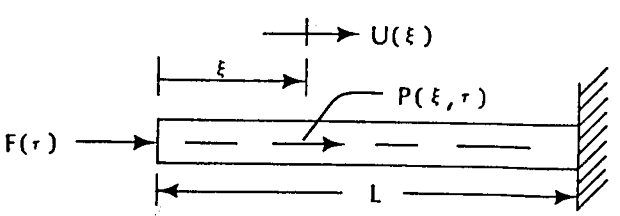
  + Poles and Zeros

  + Frequency Response Functions

  + Transient Response



FINITE ELEMENT MODEL



CONTINUOUS MODEL

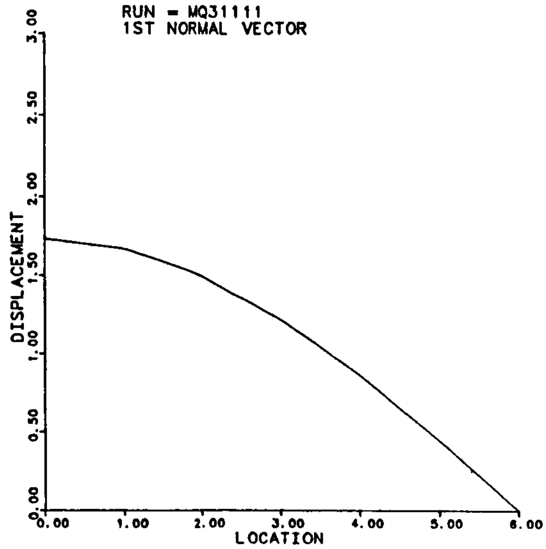
FE LANCZOS MODE MODEL

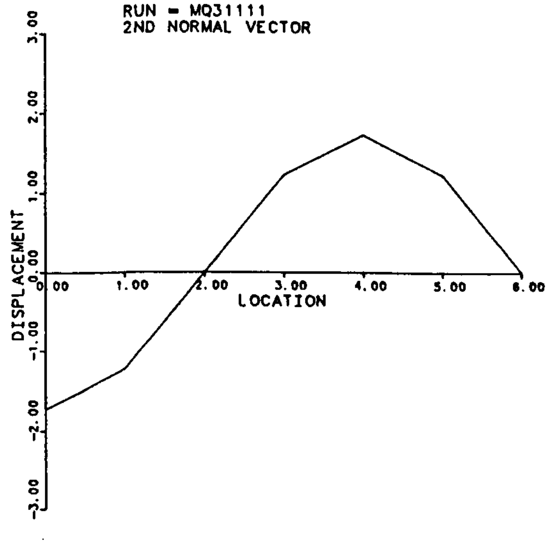
$$T_m \ddot{z} + z = g_m$$

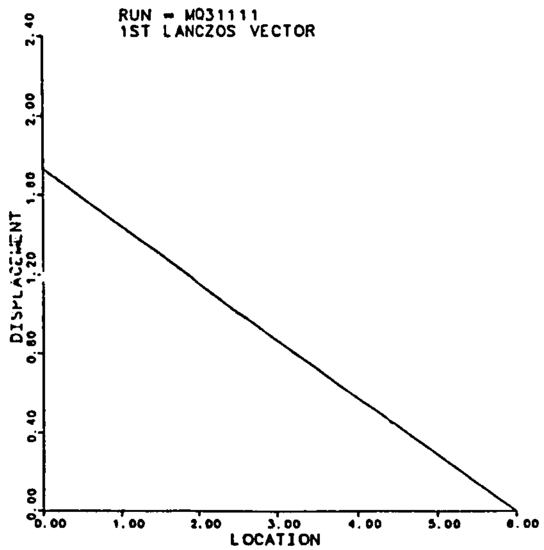
$$T_6 = \begin{bmatrix} 0.39770 & 0.04361 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.04361 & 0.04234 & 0.01238 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.01238 & 0.01564 & 0.00516 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.00516 & 0.00830 & 0.00221 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.00221 & 0.00505 & 0.00083 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.00083 & 0.00320 \end{bmatrix}$$

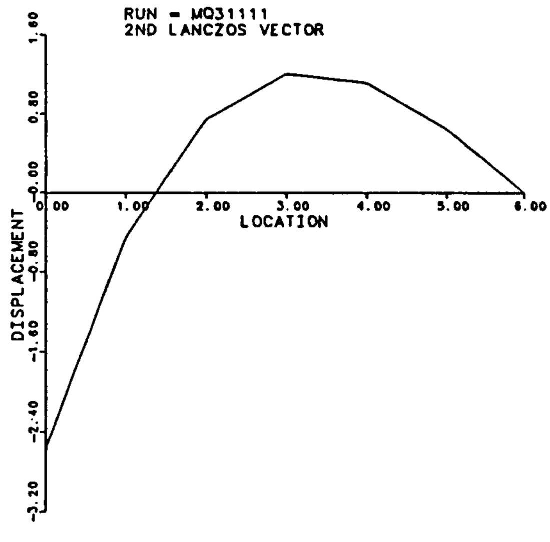
$$g_6 = \begin{Bmatrix} 0.57735 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{Bmatrix} f(t)$$

Note that there is tridiagonal inertia coupling of the Lanczos generalized coordinates, but note that the only Lanczos coordinate directly excited by the external force is the first coordinate.









Typical normal modes and Lanczos vectors are shown here for axial deformation of a clamped-free rod. The starting Lanczos vector is based on a single force applied at the "free" end. Although finite element normal modes do have some stain near the free end, "exact" normal modes would all be strain-free at the end where the excitation force is applied.

TRANSFER FUNCTIONS IN MODAL COORDINATES

$$\begin{bmatrix} s^2+2.48152 & 0 & 0 & 0 \\ 0 & s^2+23.36993 & 0 & 0 \\ 0 & 0 & s^2+70.87551 & 0 \\ 0 & 0 & 0 & s^2+156.16108 \end{bmatrix} \hat{\eta}(s)$$

$$= \begin{Bmatrix} 1.42231 \\ -1.48875 \\ 1.62980 \\ -1.85632 \end{Bmatrix} \hat{f}(s)$$

TRANSFER FUNCTIONS - NORMAL MODE MODELS

$$\hat{u}_1(s)/\hat{f}(s) = 10.34151[s^2+3.28270)^2][s^2+(6.82929)^2]$$

$$[s^2+(10.94259)^2]/\Delta_4$$

$$\Delta_4(s) = [s^2+(1.57528)^2][s^2+(4.83424)^2]$$

$$[s^2+(8.41876)^2][s^2+(12.49644)^2]$$

$$\hat{u}_1(s)\ /\hat{f}(s) = 4.23934[s^2+(3.52834)^2]/\Delta_2(s)$$

$$\Delta_2(s) = [s^2+(1.57528)^2][s^2+(4.83424)^2]$$

$$\hat{u}_1(s)\ /\hat{f}(s) = 2.02296\ /\ [s^2+(1.57528)^2]$$

TRANSFER FUNCTIONS - LANCZOS MODE MODELS

$$\hat{u}_1(s)/\hat{f}(s) = 18.29893[s^2+(3.17763)^2][s^2+(6.59014)^2]$$

$$[s^2+(10.97092)^2]/\Delta_4(s)$$

$$\Delta_4(s) = [s^2+(1.575285)^2][s^2+(4.83427)^2]$$

$$[s^2+(8.48668)^2][s^2+(15.21009)^2]$$

$$\hat{u}_1(s)\ /\hat{f}(s) = 7.14344[s^2+(3.06112)^2]/\Delta_2(s)$$

$$\Delta_2(s) = [s^2+(1.57529)^2][s^2+(5.19414)^2]$$

$$\hat{u}_1(s)\ /\hat{f}(s) = 2.51446\ /\ [s^2+(1.5857)^2]$$

POLES AND ZEROS OF REDUCED-ORDER MODELS

EXACT POLES (6DOF)	4DOF NORMAL	4DOF LANCZOS	2DOF NORMAL	2DOF LANCZOS
1.57528	1.57528	1.57529	1.57528	1.57529
4.83424	4.83424	4.83427	4.83424	5.19414
8.41876	8.41876	8.48668	-	-
12.49644	12.49644	15.21009	-	-

TABLE 2. POLES OF REDUCED-ORDER MODELS

EXACT POLES (6DOF)	4DOF NORMAL	4DOF LANCZOS	2DOF NORMAL	2DOF LANCZOS
3.17759	3.28270	3.17763	3.52834	3.06112
6.57266	6.82929	6.59014	-	-
10.39230	10.94259	10.97092	-	-
14.69693				
18.85315				

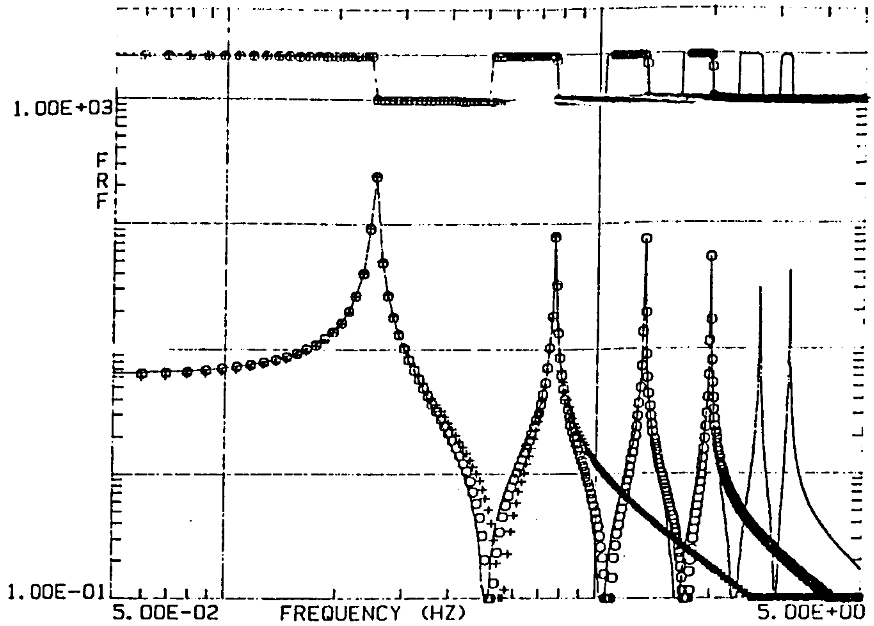
TABLE 3. ZEROS OF REDUCED-ORDER MODELS

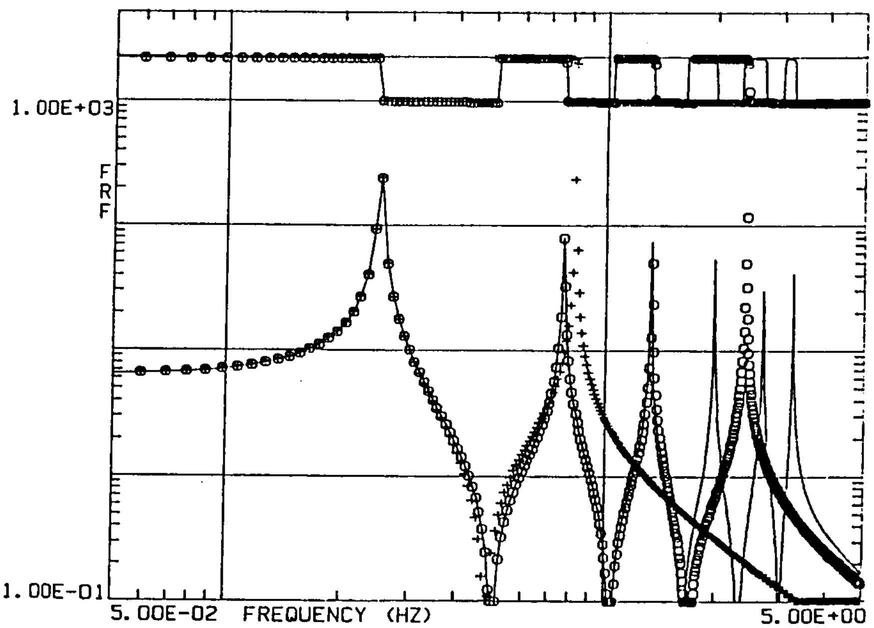
STATIC RESPONSE COMPARISON

	Normal Mode Model	Lanczos Mode Model	No. of Modes
$(\hat{u}_1/\hat{f})_{s=0}$	0.96961	0.99951	4
"	0.91005	0.99980	2
"	0.81521	1.00000	1
"	1.00000	1.00000	6(EXACT)

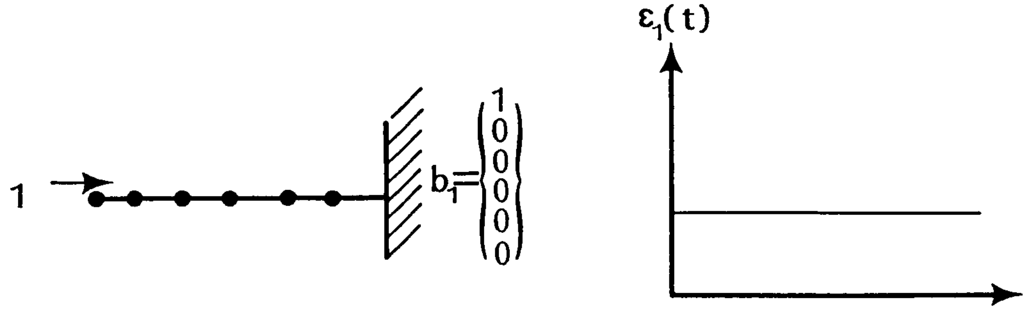
TABLE 1. TRANSFER FUNCTIONS EVALUATED AT s=0

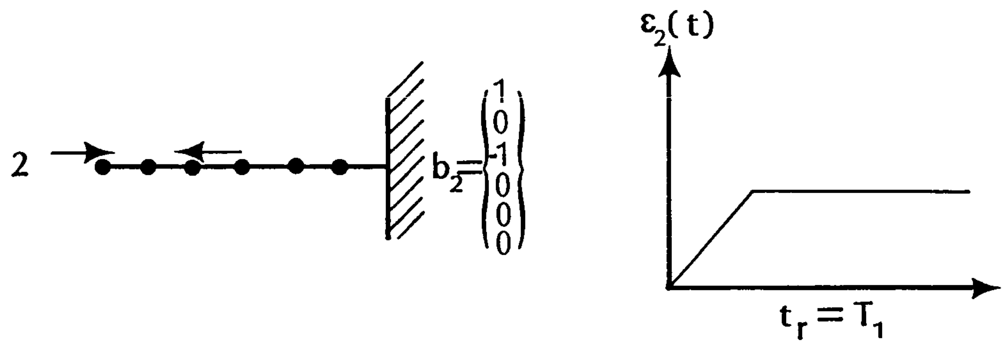
These two tables compare poles and zeros of normal mode models and Lanczos vector models and the static response of normal mode and Lanczos vector models.



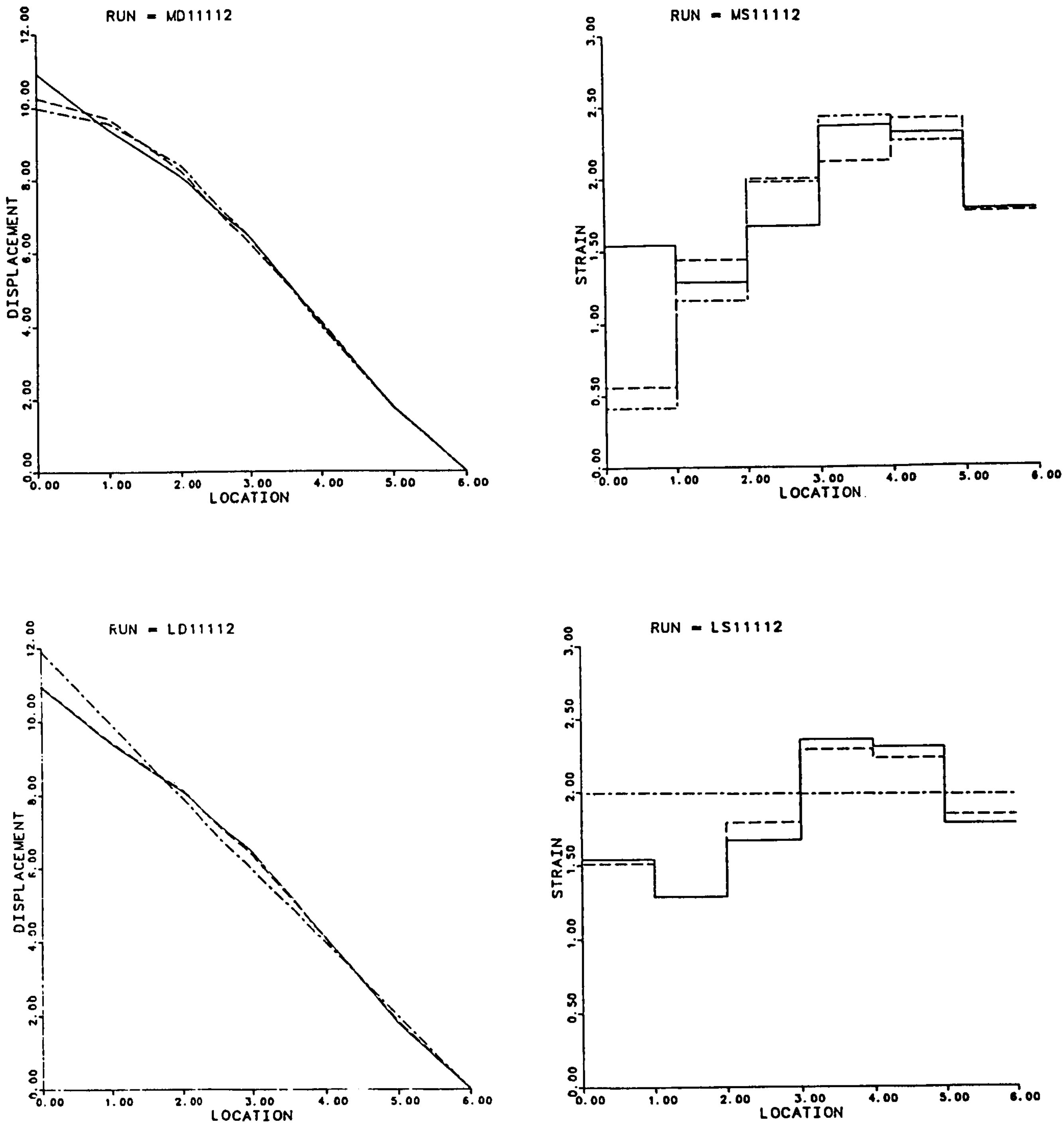


Direct frequency response functions (tip displacement/tip force) are shown for normal mode models and Lanczos vector models. The Lanczos models have improved low-frequency performance.

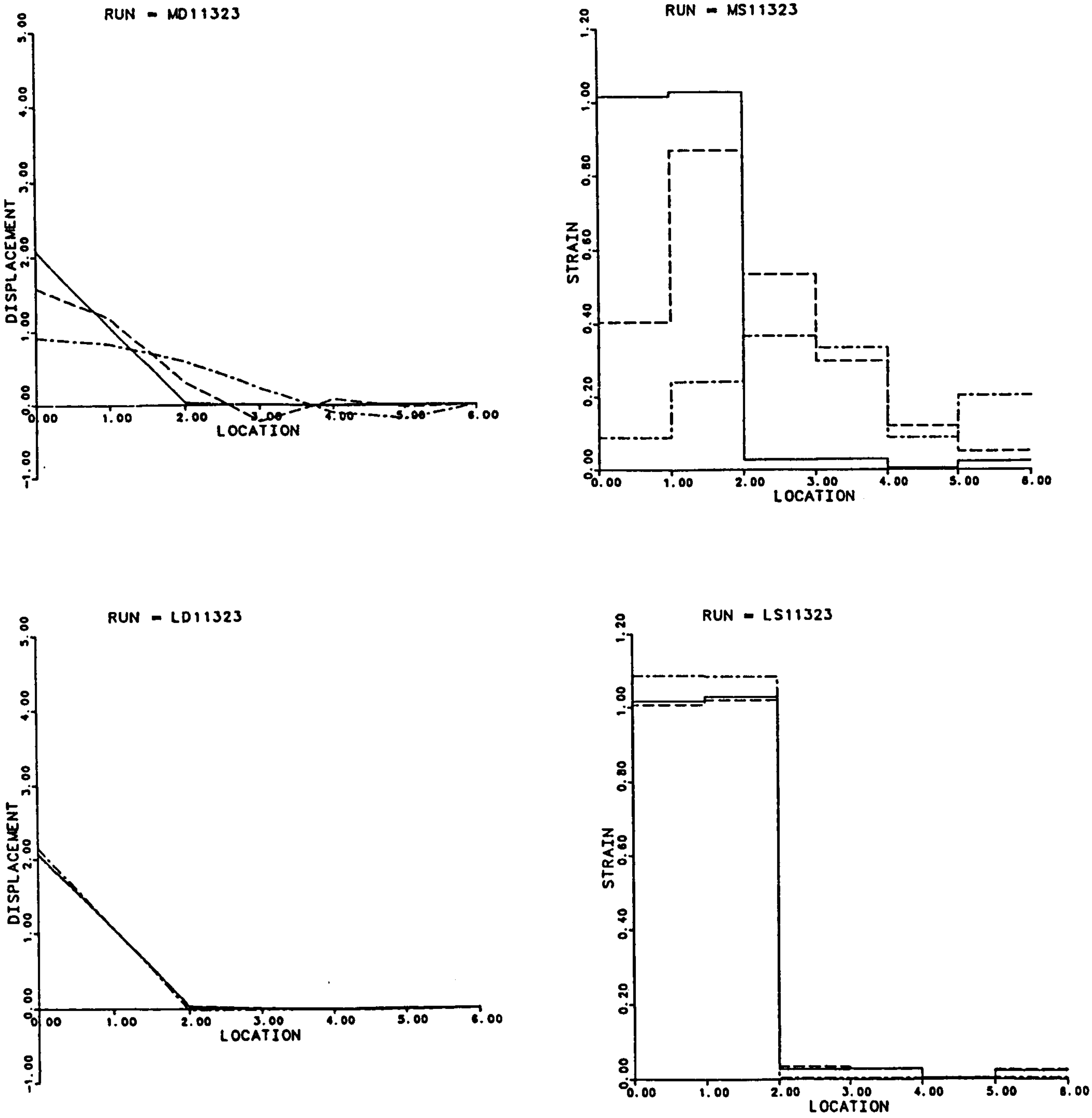




Control maneuvers, such as slewing, are transient response problems. The next examples compare transient response solutions of normal mode an Lanczos vector models when the structure is subjected to step and ramp excitation. Two force distributions are illustrated.



These are modal and Lanczos solutions for a step force applied at the tip. Both displacement and strain solutions are shown. Note that the Lanczos solutions converge a little better than the normal mode solutions do.



These are modal and Lanczos solutions for opposing forces applied at nodes 1 and 3 with a ramp time history. Note that the modal solutions are very poor, while the Lanczos solutions show excellent displacement and strain convergence.

CONCLUSIONS AND RECOMMENDATIONS

- Lanczos mode models represent low-frequency forced response better than do normal mode models.
- Lanczos mode models can be developed for both continuous and finite element structural representations.
- Lanczos mode models for systems with multiple inputs and/or rigid body modes should be developed.
- Numerical stability of the Lanczos algorithm should be assessed.
- Control system designs employing Lanczos mode models should be attempted.